

Technical efficiency based on cost gradient measure

Miki Tsutsui^a, Kaoru Tone^b, Yuichiro Yoshida^b

^a*Central Research Institute of Electric Power Industry, 2-11-2 Iwado Kita, Komae-shi, Tokyo 201-8511, Japan*

^b*National Graduate Institute for Policy Studies, 7-22-1 Roppongi, Minato-ku, Tokyo 106-8677, Japan*

Abstract

This study introduces a new scheme of data envelopment analysis (DEA) named cost gradient measure (CGM) to evaluate technical efficiency. In this model, we can obtain more cost conscious technical efficiency than those by other traditional DEA models such as CCR[7] and slacks-based measure (SBM) [19]. In addition, the CGM can avoid shortcomings of these traditional models, i.e. factor inefficiency scores can be measured for each input as opposed to CCR and SBM models. In this study, we show the generality of CGM that it includes CCR as a special case; and compare the CGM result with those of the other DEA models using illustrative data, and clarify favorite features of this model. In addition, we also apply these models to Japanese electric utilities and explain the characteristics of their results.

Keywords : cost gradient measure, DEA, technical efficiency, input price

1. Introduction

Data envelopment analysis (DEA) is one of representative methods to measure managerial efficiency of Decision making units (DMUs). It was originally introduced by Charnes, Cooper and Rhodes [7], and the basic model is called the CCR. Up to now, considerable number of researchers have developed DEA models, e.g. BCC[4] and SBM[19]. In addition, researchers also applied these models to empirical studies examining performance of various entities, e.g. business enterprises, hospitals, sports players and so forth [9, 2].

Since basic DEA models can measure technical efficiency using physical input and output data with no recourse to cost information, it was considered appropriate to apply

to non-profit organizations at first. However, recently, business entities also have become the object of DEA studies. They aim profit maximization in competitive markets, and thus monetary aspect is critically important for them. In this context, researchers should make effective use of cost data if available. This study proposes a new model named Cost Gradient Measure (CGM), which enables us to measure technical efficiency focusing on input factor costs. Thus we can obtain more cost conscious technical efficiency than those by other DEA models.

The remainder of the paper unfolds as follows. In Section 2, we review the representative DEA models, and then the CGM model is introduced in Section 3. In this section, we compare CGM with traditional radial and non-radial models such as CCR and SBM using illustrative data in order to clarify the favorable features of CGM. In addition, units-invariant issue in CGM is also explained. In Section 4, we propose several assumptions to conduct CGM without cost information. It is exhibited that CCR model can be derived from CGM under a certain assumption. In section 5, we apply CGM and the other models to Japanese electric utilities and demonstrate advantages of CGM using actual utility data. In the final section, we conclude this paper and mention future study subjects.

2. Review of the previous basic models

The most basic DEA model is called CCR, which was proposed by Charnes, Cooper and Rhodes [7]. It is formulated as

[CCR]

$$\begin{aligned}
 \theta^* &= \min_{\theta, \lambda} \theta \\
 &\text{subject to} \\
 \mathbf{x}_o &\geq \mathbf{X}\lambda \\
 \mathbf{y}_o &\leq \mathbf{Y}\lambda \\
 \lambda &\geq \mathbf{0},
 \end{aligned} \tag{1}$$

where \mathbf{X} and \mathbf{Y} are input and output matrices and \mathbf{x}_o and \mathbf{y}_o are input and output vectors for DMU_o , respectively. λ is the intensity vector and θ^* is efficiency score of CCR¹.

The basic idea of the input-oriented CCR model is depicted in Figure 1. In this figure, while DMUs A , B and C are on the efficient frontier and evaluated as efficient, DMU_o is located inside the frontier and evaluated as inefficient. In the CCR model, DMU_o will refer to the mid-point between A and B , where the radial line from the origin (0) to x_o intersects the frontier. This means DMU_o will be on the frontier if it can reduce its input (x_o) to the intersection (x_o^*) radially. Its reduction rate is calculated as $\frac{\overline{0x_o^*}}{\overline{0x_o}}$, which is the CCR efficiency score of DMU_o . It should be noted the factor-specific efficiency ($\frac{x_{io}^*}{x_{io}}$ for input i) is identical for all inputs in the CCR model (e.g. $\frac{x_{1o}^*}{x_{1o}} = \frac{x_{2o}^*}{x_{2o}}$.)

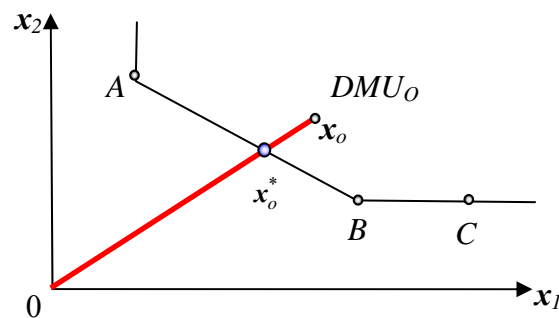


Figure 1: CCR

The CCR model features the proportional reduction of inputs, and thus, it might neglect slacks, e.g. $DMU C$ will be evaluated as efficient in Figure 1 because CCR

¹ For details, see [7,10].

neglects the input slack indicated as distance between B and C . Furthermore, the assumption of radial reduction might be too strong in some situations, e.g. it is unnatural that factor specific efficiency of capital input is equal to that of labor input.

To date, considerable numbers of extended DEA models have been introduced, and many of them are based on CCR. For instance, the BCC model [4] was variable returns-to-scale version of CCR, and the assurance region method [18] added restrictions to DEA multipliers. Furthermore, super-efficiency model proposed by Andersen and Petersen [1], Malmquist index decomposition developed by Färe et al. [14] and network DEA model proposed by Färe and Grosskopf [11,12] are developed based on the CCR.

In contrast to these radial models, the slacks-based measure (SBM) model proposed by Tone [19] is a representative of the non-radial models. The SBM does not assume proportional reduction of all inputs, and thus, for instance, the capital input can be reduced independently from labor input reduction. The SBM is formulated as follows:

[SBM]

$$\begin{aligned} \rho^* &= \min_{\lambda, s^-, s^+} 1 - \frac{1}{m} \sum_{i=1}^m \frac{s_i^-}{x_{io}} \\ &\text{subject to} \\ &\mathbf{x}_o = \mathbf{X}\lambda + \mathbf{s}^- \\ &\mathbf{y}_o = \mathbf{Y}\lambda - \mathbf{s}^+ \\ &\mathbf{s}^-, \mathbf{s}^+, \lambda \geq \mathbf{0} \end{aligned} \quad (2)$$

where \mathbf{s}^- and \mathbf{s}^+ are input and output slack vectors, and $\frac{s_i^-}{x_{io}}$ indicates factor specific inefficiency for input i ($i = 1, \dots, m$). Consequently, the efficiency score ρ^* means the average of factor specific inefficiencies.

In the SBM, the direction of input reduction will be decided in the mathematical

model as Figure 2 portrays, i.e. it is the direction in which the average of factor-specific efficiency for all inputs will be the minimum.

In the SBM model, we can obtain non-uniform factor efficiency scores differing from CCR. However, due to the nature of the linear programming solution, it tends to generate extreme solutions for slacks, e.g., zero and non-zero patterns of slacks.² This will be disadvantageous if we employ SBM to intertemporal analysis. For instance, the zero and non-zero patterns of slacks at time period t may significantly differ from those of time period $t+1$.

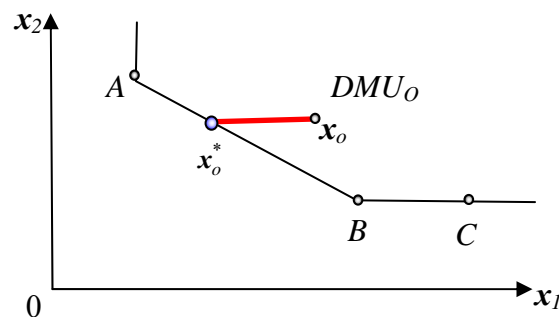


Figure 2: SBM

The extended SBM models also have been developed similarly to the CCR case, e.g. super SBM [20], network SBM [23], dynamic SBM [24] and so forth. See also Tone [21] for variations of SBM models.

The Weighted SBM model [25] is one extension of SBM models. This applies weighted average of factor specific efficiency instead of arithmetic average in the objective function in (2) as:

$$\mu^* = \min_{\lambda, s^-, s^+} 1 - \sum_{i=1}^m W_i \frac{s_i^-}{x_{io}} \quad (3)$$

² See Avkiran et al. [3].

where W_i is the weight for input i and satisfies $\sum_{i=1}^m W_i = 1$. Tsutsui and Goto [25] employed cost shares as W_i in the WSBM model, because weighted factor specific inefficiency (s_i^- / x_{io}) can be redefined as the ratio of slack value and the total cost as follows:

$$\mu^* = \min_{\lambda, s^-, s^+} 1 - \sum_{i=1}^m CS_{io} \frac{s_i^-}{x_{io}} = \min_{\lambda, s^-, s^+} 1 - \sum_{i=1}^m \frac{w_{io} s_i^-}{C_o} \quad (3')$$

where w_{io} is the factor price of input i for DMU_{*o*}, C_o indicates its total cost, and thus, the cost share of input i is $CS_{io} = w_{io} x_{io} / C_o$.

The representative non-radial DEA models other than SBM can be enumerated as Russell measure [17], the additive models [8], the multi-directional efficiency analysis (MEA) model [5] and so forth.

In this paper, we propose the cost gradient measure (CGM) model which can measure cost conscious technical efficiency and overcome the shortcomings of traditional radial and non-radial models.

3. Cost Gradient Measure (CGM)

In this section, we propose a new scheme to measure technical efficiency named cost gradient measure (CGM).

3.1. Introduction of CGM

In this paper, we assume an input-oriented model, in which possible reduction of inputs under given outputs will be measured as inefficiency. As we mentioned in the previous section, the direction of inputs reduction of the CCR model is radial from the observed input to the origin. The direction of inputs reduction in SBM is non-radial and determined by a linear programming solution. Contrary to them, the direction of inputs

reduction in the CGM model is based on the values of input factors, i.e. factor prices.

Figure 3 visually describes the CGM. The direction of inputs reduction is along with that of input price $w_o = (w_{1o}, w_{2o})$. Consequently, it is the normal to the cost plane C_o and is the most cost intensive and steepest descent direction, i.e. DMU_o can reduce the total cost at a maximum rate.

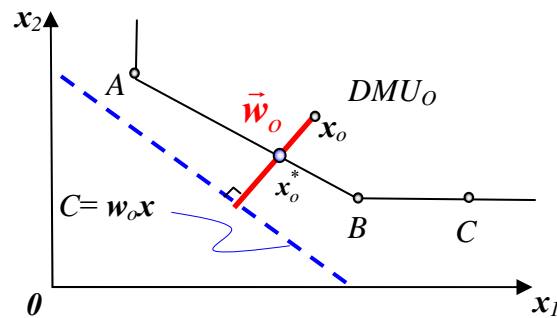


Figure 3: Cost gradient measure

From the managers' point of view, it only takes their private information on their input prices to identify the cost gradient. On one hand, input-price information is intrinsic and thus readily available to each DMU. On the other, during a given production period, managers may not know yet about the shape of the production frontier in that specific period, and thus they may not be able to immediately jump to the allocation that is both technically and allocatively efficient within the same production period. Instead, they will follow the steepest descent on the total cost surface which saves the total cost at the maximum rate. This provides the basis for the use of CGM.

3.2. CGM formulation

3.2.1. Notation

In this paper, we deal with the DMU set $D = \{j; j=1, \dots, n\}$ and the input and output

sets $IN=\{i; i=1,\dots,m\}$ and $OUT=\{h; h=1,\dots,r\}$, respectively. The amount of input i and output h for DMU_o ($o \in D$) are denoted as x_{io} and y_{ho} . Its input cost is c_{io} and the total cost are measured as $C_o = \sum_{i \in IN} c_{io}$. The factor price of input i is denoted w_{io} . In the actual business world, we may not obtain the factor price data. In such case, we can alternatively use the average unit factor cost measured by $w_{io} = c_{io} / x_{io}$.

3.2.2. Formulation

We assume the production possibility set P as follows:

[Production Possibility Set: P]

$$P = \{(x,y) | x \geq \mathbf{X}\lambda, y \leq \mathbf{Y}\lambda, \lambda \geq 0\}. \quad (4)$$

The cost gradient measure (CGM) can be formulated³ as

[CGM]

$$\begin{aligned} \tau^* &= \max_{\tau, \lambda, s^{\bar{=}}, s^-, s^+} \tau \\ &\text{subject to} \\ x_o &= \mathbf{X}\lambda + s^{\bar{=}} + s^- \\ y_o &= \mathbf{Y}\lambda - s^+ \\ s^{\bar{=}} &= \tau \mathbf{w}_o \\ s^{\bar{=}}, s^-, s^+, \lambda &\geq \mathbf{0} \end{aligned} \quad (5)$$

where \mathbf{w}_o denotes input factor price vector of x_o . ($s^{\bar{=}} + s^-$) and s^+ are input and output slack vectors. In this formula, input slack is separated into two parts; proportional slack $s^{\bar{=}}$ to \mathbf{w}_o and remaining slack s^- .

The motivation behind this formulation is that possible reduction of input factors is

³ This formulation can be regarded as the directional distance function model [16, 13] defining the directional vector as \mathbf{w}_o .

proportional to its input value (w_o). In this case, the gradient direction of input reduction is normal to the cost isoquant plane which is expressed as

[Cost Isoquant]

$$w_o x_o = w_{1o} x_{1o} + \dots + w_{mo} x_{mo} = C_o \quad (6)$$

Let an optimal solution of (5) be $(\tau^*, \lambda^*, s^{=*}, s^{-*}, s^{+*})$. Here, s^{-*} may be not unique. Hence we solve the second stage LP to obtain s^{-**} after fixing $\tau = \tau^*$ in Equation (5) with the objective function as

$$\max_{\lambda, s^-, s^+} \sum_{i=1}^m \frac{s_i^-}{x_{io}} \quad (7)$$

Then we can obtain the technically efficient input as

[Technically Efficient Input]

$$x_o^{TE*} = \mathbf{X}\lambda^* = x_o - \tau^* w_o - s^{-**} \quad (8)$$

We define the technical efficiency index (TE) for DMU_o as weighted sum of technical efficient input divided by actual input as

[Technical Efficiency Index: TE]

$$TE = \sum_{i=1}^m CS_{io} \cdot \frac{x_{io}^{TE*}}{x_{io}} = \sum_{i=1}^m \frac{w_{io} x_{io}^{TE*}}{C_o} = \frac{C_o^{TE*}}{C_o} \leq 1 \quad (9)$$

where CS_{io} utilized as a weight is the cost share of input i . This definition is the same as that of WSBM.

In addition, the factor technical efficiency index (TE_i) for input i can be measured as a portion of TE as

[Factor Technical Efficiency Index: TE_i]

$$TE_i = \frac{w_{io} x_{io}^{TE*}}{C_o} . \quad (10)$$

As we mentioned, CGM provides technical efficiency score based on the input reduction that maximizes the total cost savings. In any production period, company managers may not know the shape of the production frontier and cannot move to the allocatively efficient input mix directly. Therefore, moving to the most cost-reduction-intensive direction would be a choice as a midway to the optimal allocation. That is the underlying idea of the technical efficiency based on the CGM.

In addition, the CGM result exhibits favorable features. Compared with the result of CCR, CGM can provide non-uniform factor technical efficiency scores defined in (10), which enables us to conduct further analysis to clarify causes of inefficiency. In addition, CGM can avoid sharp contrast of slacks compared with SBM and it can provide more practical factor efficiency scores. Consequently, it may safely be said that CGM can overcome shortcomings of traditional models.

In addition, CGM is general in that it can derive the CCR model as a special case by replacing the 3rd constraint in (5) by $s^- = \tau x_o$. This generality of CGM will be further discussed in Section 4.

The dual model of (5) can be expressed as

[CGM-dual]

$$\begin{aligned} & \min_{v,u} v x_o - u y_o \\ & \text{subject to} \\ & v w_o = 1 \\ & v X - u Y \geq 0 \\ & v, u \geq 0, \end{aligned} \quad (11)$$

where \mathbf{v} and \mathbf{u} are unknown multiplier vectors. By redefining the objective function as

$$\max_{\mathbf{v}, \mathbf{u}} \mathbf{u}\mathbf{y}_o - \mathbf{v}\mathbf{x}_o, \quad (12)$$

the dual form of CGM can be regarded as a profit maximization model with virtual output price \mathbf{u} and input cost \mathbf{v} .

3.3. Numerical Example

Numerical example will clarify the favorable features of the CGM model compared with the traditional models. We generate illustrative data of two inputs and one output for six DMUs as shown in Table 1.

Table 1: Illustrative data

DMU	hput1			hput2			C (total cost)	Output Y
	x1	c1	w 1 (price) =c1/x1	x2	c2	w 2 (price) =c2/x2		
A	4	12	3	12	60	5	72	100
B	10	60	6	4	40	10	100	100
C	4	40	10	5	10	2	50	50
D	9	18	2	18	90	5	108	150
E	14	70	5	5	10	2	80	100
F	9	63	7	8	16	2	79	100

Figure 4 depicts the technical efficiency scores of CCR, SBM, WSBM and CGM.

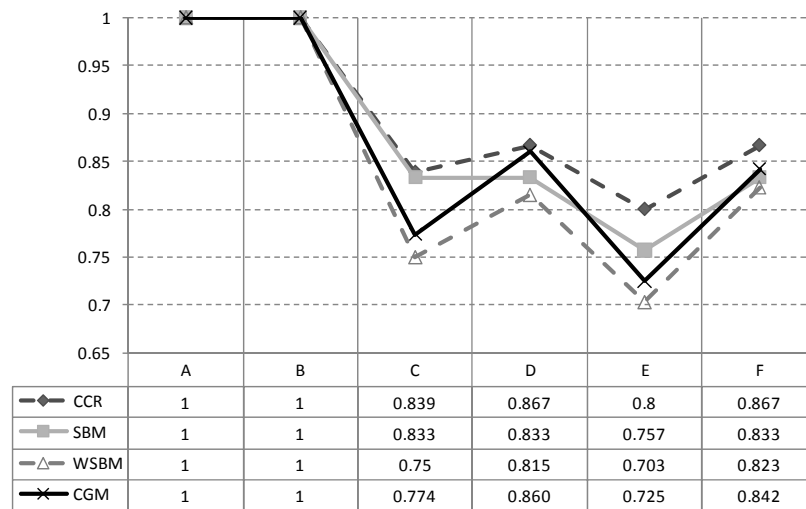


Figure 4: Comparison of technical efficiency scores

While DMUs *A* and *B* are on the frontier and evaluated as efficient in the all models, the scores of the remaining DMUs are varying among the models. To clarify the characteristics of the models, it will be better to focus on factor inefficiencies, which are measured as slack divided by observed input amount $(\frac{s_i^{-*}}{x_{io}})^4$. Figure 5 describes factor inefficiencies of inputs 1 and 2 (x_1 and x_2). In the case of CCR, it is obtained by $1-\theta$.

⁴ In the case of CGM, the numerator is $s_i^{=*}+s_i^{-*}$.

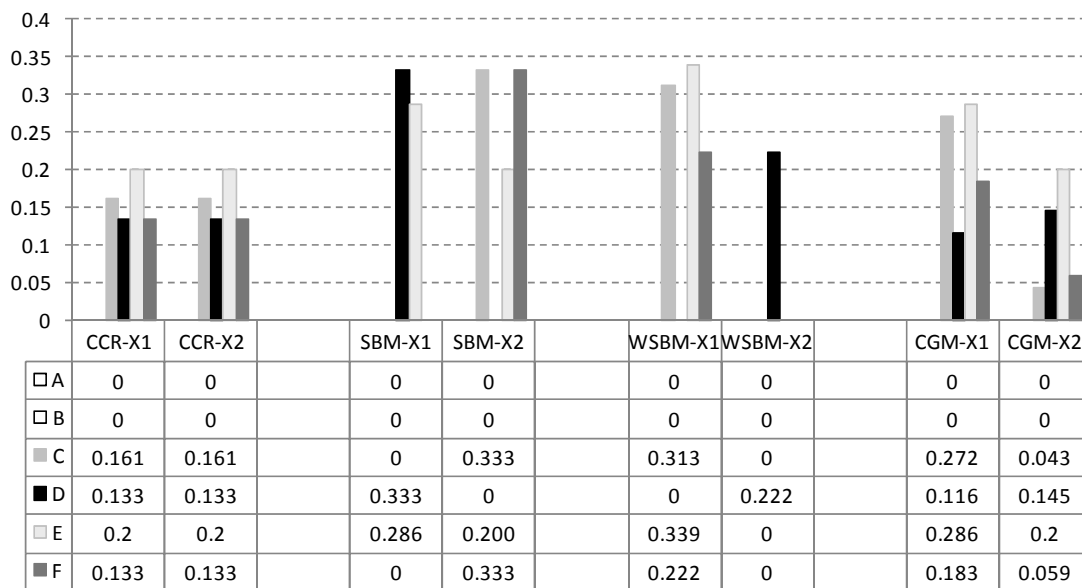


Figure 5: Comparison of factor inefficiency scores

For DMUs *A* and *B*, inefficiency scores are zero, since they are efficient as explained in Figure 4. In the CCR model, factor inefficiencies are homogeneous between x_1 and x_2 , which is caused by the radial model. In the SBM model, DMUs *C*, *D* and *F* are scored zero in x_1 or x_2 . This is a typical example of extreme solution (zero or positive) of the SBM model. The WSBM model also generates sharp contrast solutions for all models. However, inefficiencies are measured in the different side (another input factor) from those of SBM. Contrary to these models, CGM provides non-uniform inefficiency scores for x_1 and x_2 , and avoids zero or positive solutions. From the view point of application to the real world data, it must be unpractical that we assume the factor inefficiencies (e.g. capital and labor inefficiencies) are all identical. In addition, it is also unrealistic if factor inefficiencies are measured only one side of inputs. We deem CGM can provide more reasonable efficiency scores.

The important feature of CGM, besides the technical advantage mentioned above, is that these CGM inefficiency scores are measured based on the direction which enables maximum cost saving in a short run. This should be the most significant difference from

the traditional models.

3.4. Units-invariant issue

The efficiency scores provided by CGM formulated in (5) are dependent on the units in which the inputs are measured in. If we emphasize the idea to reduce inputs along with the cost gradient, failing to units-invariant property is inevitable. However, it is problematic to have different results depending on the units, e.g. thousand or million. Faced by this difficulty, we propose two countermeasures as technical remedy to proceed further.

3.4.1. Data standardization

If the original data are standardized by a certain statistics of the dataset, we can always obtain the same standardized data even if the original value is multiplied by k .

For example, when we standardized input data x_i by data average ($x'_i = \frac{\sum_{j=1}^n x_{ij}}{n}$), we can

obtain \bar{x}_{ij} even in multiplied case as

$$\bar{x}_{ij} = \frac{x_{ij}}{x'_i} = \frac{kx_{ij}}{kx'_i}. \quad (13)$$

In other words, the standardization process produces “unitless value”. The CGM model with the unitless \bar{x}_{ij} enables us to obtain the efficiency scores independent from units.

However, in this case, x'_i and \bar{x}_{ij} in (13) will be changed when we inserted/eliminated a certain DMU into/from dataset. This results in changes of efficiency scores for all other DMUs in the dataset. It may be unreasonable to have different scores depending on the DMU set.

As a statistic data for standardization, we can also use maximum value of the dataset ($x_i'' = \max_j(x_{ij})$) to obtain the unit-less value in the same manner as (13). When a certain DMU is inserted into or removed from the dataset, x_i'' will be changed only if the input of this DMU is equal to the max value.

Generally, in the traditional models such as CCR and SBM, inefficient DMUs do not influence shape of the efficiency frontier, i.e. the efficiency scores of all other DMUs will not change when inefficient DMUs, which are not on the efficiency frontier, are inserted/eliminated from the dataset. In contrast, it may feel strange that the efficiency scores change in response to manipulation of the inefficient DMUs in the above cases standardized by average or maximum values.

To overcome these problems, we propose a scheme to standardize dataset in the practical manner. Contrary to the above, it is not so strange that the efficiency scores change when efficient DMUs on the frontier are manipulated, because the shape of the frontier will be changed. Therefore, our recommendation is using the data of the most factor-productive DMUs, which are able to be on the efficiency frontier. In the case of data in Table 1, the most-factor productive DMUs are *A* and *B*, as shown in Table 2. We use average input and output of *A* and *B* for standardization.

Table 2: Data standardization

DMU	Input		Output	Factor Productivity	
	x1	x2	Y	Y/x1	Y/x2
A	4	12	100	25	8.33
B	10	4	100	10	25
C	4	5	50	12.5	10
D	9	18	150	16.67	8.33
E	14	5	100	7.14	20
F	9	8	100	11.11	12.5
Average of A and B	7	8	100		

Most factor-productive number
 Most factor-productive DMUs and their average

We define FP as the most factor-productive DMU set for all factors as

$$FP = \left\{ j \mid \frac{y_{hj}}{x_{ij}} = \max_k \left(\frac{y_{hk}}{x_{ik}} \right), (\forall i, h) \right\}. \quad (14)$$

Input and output data for standardization are respectively obtained after making average among DMUs belonging to FP as

$$\bar{x}_i^{FP} = \frac{\sum_{j \in FP} x_{ij}}{|FP|} \quad (\forall i) \quad \text{and} \quad \bar{y}_h^{FP} = \frac{\sum_{j \in FP} y_{hj}}{|FP|} \quad (\forall h). \quad (15)$$

The CGM with standardized data is formulated as follows:

[CGM-FP]

$$\begin{aligned} \tau^* &= \max_{\lambda, \tau, s^-, s^+} \tau \\ \text{subject to} \\ \bar{\mathbf{x}}_o &= \bar{\mathbf{X}}\lambda + \tau \bar{\mathbf{w}}_o + \mathbf{s}^- \\ \bar{\mathbf{y}}_o &= \bar{\mathbf{Y}}\lambda - \mathbf{s}^+ \\ \mathbf{s}^-, \mathbf{s}^+, \lambda &\geq \mathbf{0} \end{aligned} \quad (16)$$

where $\bar{\mathbf{X}} = (\bar{x}_1, \dots, \bar{x}_n)$, $\bar{\mathbf{x}}_o = (\bar{x}_{1o}, \dots, \bar{x}_{mo})$, $\bar{x}_{io} = \frac{x_{io}}{\bar{x}_i^{FP}}$ ($\forall i$), $\bar{\mathbf{Y}} = (\bar{y}_1, \dots, \bar{y}_n)$,

$\bar{\mathbf{y}}_o = (\bar{y}_{1o}, \dots, \bar{y}_{ro})$, $\bar{y}_{ho} = \frac{y_{ho}}{\bar{y}_h^{FP}}$ ($\forall h$), $\bar{\mathbf{w}}_o = (\bar{w}_{1o}, \dots, \bar{w}_{mo})$ and $\bar{w}_{io} = w_{io} \bar{x}_i^{FP}$ ($\forall i$).

In addition, the technical efficient input is redefined as

[Technically Efficient Input]

$$\mathbf{x}_o^{TE*} = (\bar{\mathbf{x}}_o - \tau^* \bar{\mathbf{w}}_o - \mathbf{s}^{-**}) \bar{\mathbf{x}}^{FP} \quad (17)$$

The technical efficiency score can be measured by (9). This CGM-FP model generates technical efficiency scores independently of both units and manipulation of inefficient DMUs.

3.4.2. Units-invariant CGM model

We formulate the idea of CGM faithfully in the sense that we reduce inputs along the cost gradient direction. As we pointed out in the previous section, this model lost units-invariant property. However, if we relax the constraint, unit-invariant CGM model can be obtained as follows.

$$\begin{aligned}
 & \text{[CGM-UI]} \\
 & \tau^* = \max_{\lambda, \tau, s^-, s^+} \tau \\
 & \text{subject to} \\
 & \mathbf{x}_o = \mathbf{X}\lambda + \tau\tilde{\mathbf{w}}_o + \mathbf{s}^- \\
 & \mathbf{y}_o = \mathbf{Y}\lambda - \mathbf{s}^+ \\
 & \mathbf{s}^-, \mathbf{s}^+, \lambda \geq \mathbf{0}
 \end{aligned} \tag{18}$$

where \mathbf{w}_o in (5) is changed to

$$\tilde{\mathbf{w}}_o = (w_{1o}x_{1o}^2, w_{2o}x_{2o}^2, \dots, w_{mo}x_{mo}^2) = (c_{1o}x_{1o}, c_{2o}x_{2o}, \dots, c_{mo}x_{mo}). \tag{19}$$

$\tilde{\mathbf{w}}_o$ implies weighted input by cost or cost share⁵. The CGM-UI model is units-invariant, and we are not bothered by units. See Appendix A for a proof.

3.4.3. Comparison of the results

Figure 6 compares the efficiency scores among the CGM models, i.e. native CGM, CGM-FP, CGM-UI and models standardized by the average (CGM-AVE) and the max value (CGM-MAX). For comparison, we append the result of CCR.

⁵ If it is multiplied by inverse of total cost ($1/C_o$), we can consider it is weighted input by cost share.

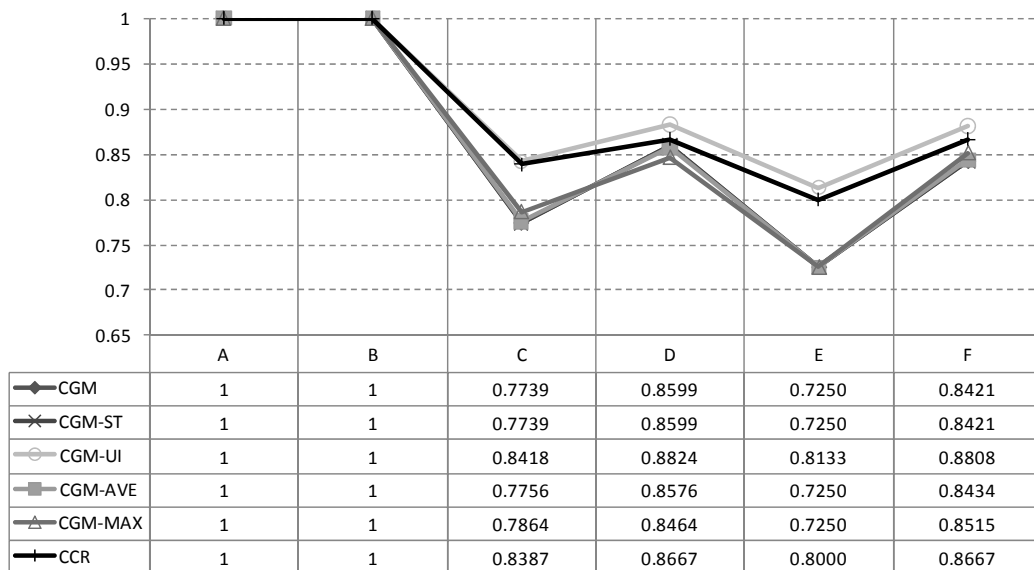


Figure 6 : Comparison of technical efficiency scores of Units-invariant models

In this artificial data, the results of the CGM family are quite similar to that of the native CGM⁶ formulated in (5) except CGM-UI that is relatively close to that of CCR.

To avoid the units-invariant issue, we can select the model CGM-FP or CGM-UI depending on the main concern of analysts themselves, i.e. the idea of CGM or units-invariant property.

4. CGM without cost data

In the CGM model, technical efficiency scores are measured along the direction of input values, and thus, the cost information is required. If factor cost c_i is available, we can obtain the average factor price (unit cost), total input cost and cost share as follows:

⁶ In other cases, the results may differ from the native one, e.g. in the case that projection on the frontier is located on the different facet from the point projected in the native model.

$$\begin{aligned}
\text{[Average factor price]: } w_i &= c_i / x_i \\
\text{[Total input cost]: } C &= \sum_{i \in IN} c_i \\
\text{[Cost share]: } CS_i &= c_i / C = w_i x_i / C
\end{aligned} \tag{20}$$

However, if factor cost c_i is not available, w_i , C and CS_i is also not available. Even in such case, we can apply the CGM model under several assumptions.

4.1. Assumption 1: uniform factor cost

We assume that factor cost c_i is uniform for all factors, and thus:

$$\begin{aligned}
\text{[Factor cost]: } c_1 &= \dots = c_m = c = C/m. \text{ (} c \text{ and } C \text{ are unknown)} \\
\text{[Average factor price]: } w_i &= c / x_i \\
\text{[Total input cost]: } C &= mc \\
\text{[Cost share]: } CS_i &= 1/m
\end{aligned} \tag{21}$$

In this case, the cost gradient can be assumed as $[\mathbf{1}/x_o] = (1/x_{1o}, \dots, 1/x_{mo})$ for DMU_o , and consequently, the assumed cost plain is $C = [\mathbf{1}/x_o]x$. Figure 7 graphically explains this case.

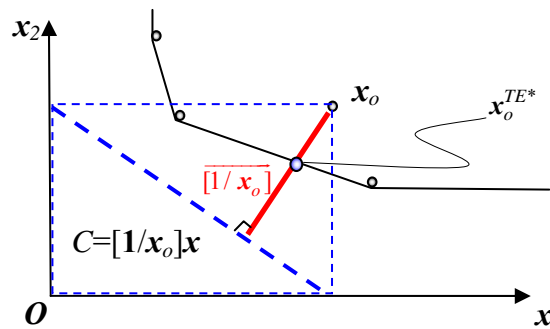


Figure 7: Assumption of proportionality between factor prices and inputs

To solve the CGM model without cost, the 3rd constraint in (5) is replaced by

$$\bar{s} = \tau [1/x_o]. \quad (22)$$

In addition, the technical efficiency index is measured as

$$TE = \sum_{i=1}^m CS_{io} \cdot \frac{x_{io}^{TE*}}{x_{io}} = \frac{1}{m} \sum_{i=1}^m \frac{x_{io}^{TE*}}{x_{io}}, \quad (23)$$

which is interestingly identical to that of SBM⁷.

In this assumption, we can obtain the technical efficiency scores without c and C .

4.2. Assumption 2: uniform factor price

We assume that the average factor price w_i is uniform for all factors, and thus:

[Average factor price]: $w_1 = \dots = w_m = w$ (w is unknown)

[Factor cost]: $c_i = wx_i$

[Total input cost]: $C = \sum_{i \in IN} c_i$ (24)

[Cost share]: $CS_i = \frac{x_i}{\sum_{i \in IN} x_i}$

In this case, the cost gradient can be assumed as $\mathbf{1}$ for DMU _{o} , and consequently, the assumed cost plain is $C = \mathbf{e}x$, i.e. a line of -45 degrees as shown in Figure 8.

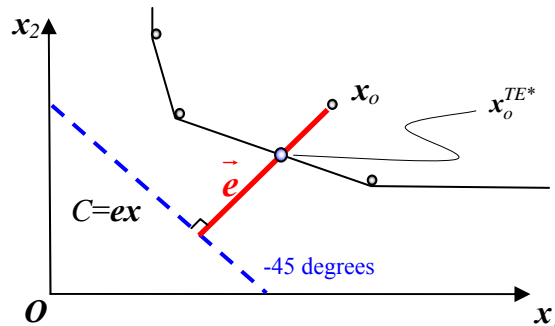


Figure 8: Assumption of uniform factor price

To solve the CGM model without cost, the 3rd constraint in (5) is replaced by

⁷ Precisely, (23) includes the second input slack \bar{s} in (5), which is not generated in the SBM.

$$\bar{s} = \tau e \quad (25)$$

Along with the definition of the technical efficiency index (9), TE can be calculated as

$$TE = \sum_{i=1}^m CS_{io} \cdot \frac{x_{io}^{TE^*}}{x_{io}} = \sum_{i=1}^m \frac{x_{io}}{\sum_i x_{io}} \cdot \frac{x_{io}^{TE^*}}{x_{io}} = \frac{1}{\sum_i x_{io}} \sum_{i=1}^m x_{io}^{TE^*}, \quad (26)$$

which is considered as a sort of additive form. On the other hand, if the uniform weight is applied to TE definition (9) instead of the cost share, we can obtain TE index same as (23).

4.3. Assumption 3: factor prices are proportional to inputs

We assume that factor price w_i is proportional to input x_i , and thus:

[Average factor price]: $w_i = \alpha x_i$ (α is unknown)

[Factor cost] : $c_i = \alpha x_i^2$

[Total input cost]: $C = \alpha \sum_{i \in IN} x_i^2$ (27)

[Cost share]: $CS_i = \frac{x_i^2}{\sum_{i \in IN} x_i^2}$

In this case, the cost gradient is x_o for DMU_o , and consequently, the assumed cost plain is $C = x_o x$. This case can be depicted as Figure 9.

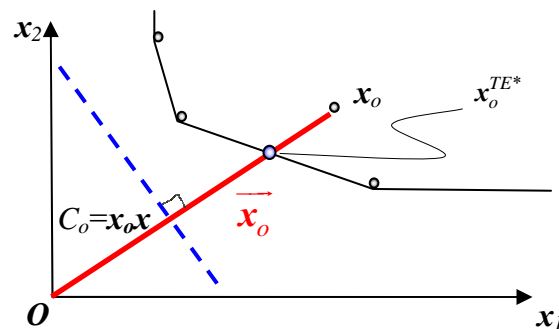


Figure 9: Assumption of uniform factor cost

Interestingly, we can find that this is the CCR model. To solve the CGM model in this case, the 3rd constraint in (5) is replaced by

$$\bar{s} = \tau \mathbf{x}_o. \quad (28)$$

The technical efficiency index defined in (9) will be

$$TE = \frac{\sum_{i=1}^m CS_{io} \cdot \frac{x_{io}^{TE*}}{x_{io}}}{\sum_{i=1}^m \frac{x_{io}^2}{\sum_i x_{io}^2} \cdot \frac{x_{io}^{TE*}}{x_{io}}} = \frac{1}{\sum_i x_{io}^2} \sum_{i=1}^m x_{io} x_{io}^{TE*}. \quad (29)$$

However, if we employ uniform weight and ignore the second slack \bar{s} in (5), the CCR efficiency score θ can be obtained as

$$TE = \frac{1}{m} \sum_{i=1}^m \frac{x_{io}^{TE*}}{x_{io}} = \frac{1}{m} \sum_{i=1}^m \frac{x_{io} - s_{io}^-}{x_{io}} = \frac{1}{m} \sum_{i=1}^m (1 - \tau) = \theta. \quad (30)$$

In summary, CCR can be derived from CGM under this assumption, i.e. factor price is proportional to input, and also we can obtain the CCR technical efficiency score under the uniform weight.

5. Application to electric utilities in Japan

In section 3, we explained CGM models with artificial data. In this section, we will apply this model to actual data of electric utilities in Japan.

5.1. Data

There exist ten vertically integrated electric utilities in Japan and each utility exclusively had served electricity to customers in their own territory. In 2000, the industry was partially liberalized and a few new entrants became to be able to serve electricity to only

large customers. In this study we utilize data before liberalization, i.e. 1992 to 1999, when no substantial structural change occurred⁸. Furthermore, we exclude Okinawa electric power company from the analysis, because it is very small and served customers in isolated island. Consequently, we utilize data of nine companies as DMUs for eight years. In this study, 72 observations for nine DMUs from 1992 to 1999 are pooled consisting one efficiency frontier, since we assumed no structural change in this period.

We suppose that the activity of electric utilities is providing electric power to customers utilizing capital, labor and material inputs.⁹ As a capital input, we made a division index [6] for representative facilities of generation (total capacity of power plants), transmission (line length), transformation (total maximum output of substation), distribution (total transformer capacity) and general administration (capital stock). Labor input is number of employees including estimated number of outsourcing staff¹⁰. As a material input, we employed consumed fuel in the power generation division. The consumption of fossil and nuclear¹¹ fuel was converted into quantity of heat (calories). Electric power sale is divided into two outputs: sales to industrial and domestic customers.

All data are obtained from [15] and the major statistics of three inputs and two outputs are listed in Table 3.

⁸ As a first step of liberalization, the generation market was partially opened in 1995 and incumbent utilities started to call for bids for new additional fossil power plants. However, the effect of this institutional change was very limited.

⁹ Vertically integrated power utility operates generation, transmission, distribution and sales divisions inside one company. Focusing on these various functions, we would be better off employing network DEA model taking into account the interaction among divisions [11, 12 and 23]. It is possible to apply CGM to the network models, however, the inside structure of the vertical integration is ignored in this study to simplify the model.

¹⁰ The number of outsourcing staff was estimated from the cost of outsourcing divided by average unit cost of regular employees. Unit cost of outsourcing might be lower than regular employees, and thus, the number of outsourcing staff might be underestimated.

¹¹ It is difficult to measure the heat quantity of nuclear fuel. In this study, we estimated it using number of molecule and its kinetic energy.

Table 3: Major statistics for nine utilities

		1992	1993	1994	1995	1996	1997	1998	1999
X1 Capital Input (Divisia Index)	Average	1.362	1.421	1.473	1.515	1.574	1.607	1.649	1.672
	Max	3.638	3.742	3.868	3.910	4.044	4.195	4.233	4.319
	Min	0.339	0.372	0.404	0.409	0.421	0.429	0.465	0.468
X2 Labor Input (#)	Average	24,703	25,541	25,797	26,074	25,357	25,182	24,859	24,592
	Max	62,842	64,905	66,753	65,792	64,806	64,481	64,228	63,862
	Min	7,437	7,503	7,333	7,824	7,713	7,830	7,919	7,903
X3 Fuel Input (10 ⁹ kcal)	Average	155,524	154,386	172,135	172,465	175,546	176,995	179,835	184,381
	Max	534,117	528,255	571,883	579,486	584,476	592,846	591,354	611,472
	Min	24,716	26,206	32,504	34,638	35,379	35,130	38,769	43,090
Y1 Sales to Industrial (GWh)	Average	54,490	54,392	57,905	58,752	60,303	61,702	61,557	62,746
	Max	163,952	164,284	175,369	177,843	180,895	186,467	186,063	190,252
	Min	13,979	14,062	14,593	14,917	15,527	15,872	16,325	16,574
Y2 Sales to Domestic (GWh)	Average	21,132	21,734	23,705	24,706	25,097	25,556	26,483	27,295
	Max	66,100	67,381	73,486	76,508	76,531	78,910	80,984	83,974
	Min	4,854	5,027	5,431	5,685	5,866	5,968	6,121	6,437

5.2. Comparison of results

The results of CGM (CGM-FP), CCR, SBM and WSBM models are listed in Table 4. Figures 10 and 11 focus on the results of two utilities, DMU *E* and *H*, as a typical example.

Table 4: Results of CGM, CCR, SBM and WSBM models

	1992	1993	1994	1995	1996	1997	1998	1999
CGM								
A	0.816	0.811	0.847	0.864	0.910	0.924	0.973	1
B	0.886	0.880	0.884	0.891	0.912	0.888	0.907	0.934
C	0.787	0.796	0.809	0.822	0.836	0.843	0.893	0.912
D	0.892	0.910	0.914	0.951	0.952	0.994	0.913	0.945
E	0.930	0.876	0.910	0.891	0.958	0.990	0.987	1
F	0.955	0.950	0.981	1	0.984	0.988	0.992	1
G	1	1	0.987	0.987	0.946	0.967	0.940	0.956
H	1	0.946	0.950	0.912	0.927	0.935	0.882	0.874
I	0.896	0.892	0.918	0.942	1	0.976	0.986	1
CCR								
A	0.843	0.845	0.874	0.893	0.924	0.942	0.979	1
B	0.909	0.885	0.903	0.907	0.916	0.896	0.930	0.950
C	0.880	0.893	0.854	0.866	0.873	0.882	0.915	0.924
D	0.916	0.931	0.948	0.949	0.957	0.995	0.927	0.957
E	0.944	0.900	0.928	0.912	0.964	0.991	0.994	1
F	0.978	0.954	0.989	1	0.993	0.997	0.992	1
G	1	1	0.985	0.997	0.998	0.972	0.950	0.961
H	1	0.979	0.982	0.940	0.970	0.977	0.925	0.923
I	0.919	0.910	0.938	0.979	1	0.986	0.985	1
SBM								
A	0.816	0.817	0.846	0.863	0.886	0.902	0.947	1
B	0.866	0.865	0.870	0.878	0.894	0.879	0.908	0.927
C	0.771	0.784	0.767	0.772	0.782	0.801	0.846	0.866
D	0.885	0.902	0.903	0.942	0.949	0.986	0.908	0.943
E	0.901	0.822	0.877	0.848	0.938	0.985	0.986	1
F	0.928	0.920	0.962	1	0.967	0.983	0.989	1
G	1	1	0.975	0.986	0.962	0.964	0.935	0.939
H	1	0.957	0.945	0.925	0.942	0.948	0.889	0.892
I	0.864	0.865	0.890	0.934	1	0.968	0.977	1
WSBM								
A	0.800	0.787	0.822	0.832	0.855	0.865	0.914	1
B	0.858	0.842	0.856	0.861	0.881	0.850	0.880	0.900
C	0.781	0.784	0.780	0.796	0.810	0.813	0.860	0.880
D	0.887	0.894	0.903	0.933	0.940	0.978	0.890	0.926
E	0.886	0.807	0.859	0.832	0.903	0.977	0.978	1
F	0.942	0.937	0.975	1	0.978	0.987	0.990	1
G	1	1	0.975	0.981	0.943	0.957	0.920	0.922
H	1	0.938	0.926	0.909	0.923	0.923	0.850	0.866
I	0.868	0.873	0.895	0.937	1	0.968	0.978	1

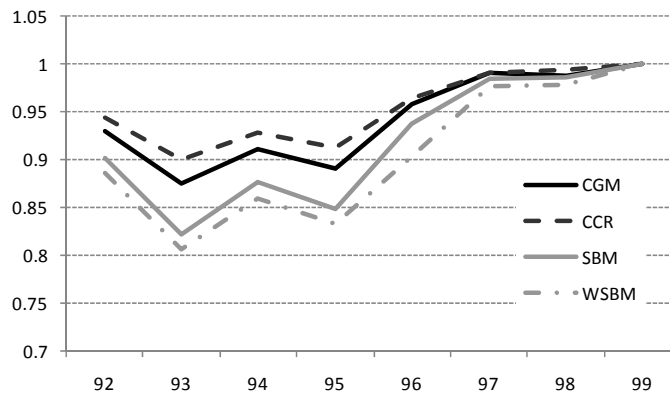


Figure 10: Technical efficiency of DMU E

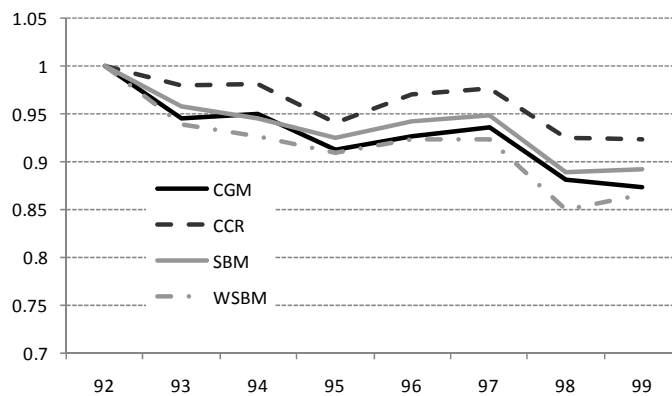
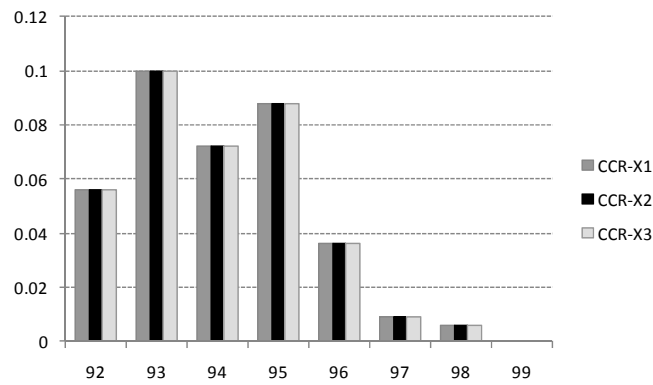
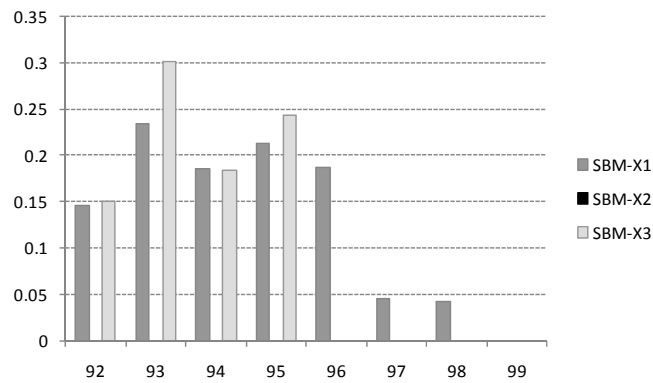
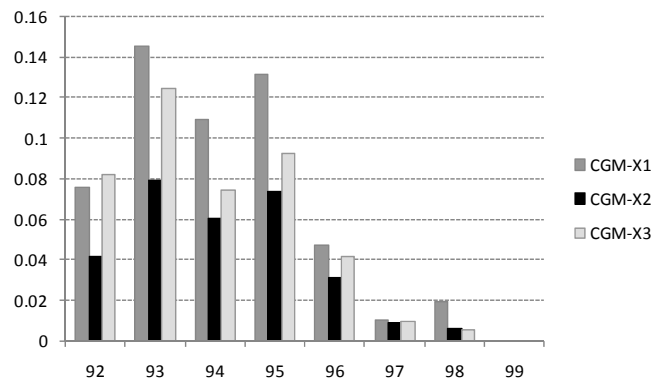


Figure 11: Technical efficiency of DMU H

It is known that the results of CCR are not less than SBM and WSBM, while no explicit relationship exists between WSBM and SBM [25]. In addition, there are no explicit relationship between CGM and the other models. Generally speaking, the results of CGM tend to be located between CCR and SBM/WSBM. In Figures 10 and 11, the results of CGM are almost all between CCR and SBM/WSBM¹². However, their characteristics are slightly different at every DMU. The results of DMU *E* are relatively near to CCR, while those of DMU *H* are close to SBM/WSBM. This is caused by the difference of cost gradient of DMUs.

In order to clarify the advantageous of CGM, Figures 12-14 depict the results of factor inefficiencies of DMU *E* calculated as slack divided by observed data in the same manner as Figure 5. For the results of CCR in Figure 12, it is obtained by $1-\theta$, and thus, inefficiencies of all input factors are completely same. The results of SBM show the various factor inefficiencies, i.e. capital inefficiency (x_1) is appeared during the period except in 1999, while fuel inefficiencies (x_3) are during 92 to 95 in Figure 13. On the other hand, no labor inefficiency is observed during the entire study period in the SBM. The CGM results shown in Figure 14 are scored between CCR and SBM, i.e. factor inefficiencies are impartially measured for all factors unlike SBM, and their scores are diverse in contrast to CCR.

¹² In this case, we have similar results between SBM and WSBM in contrast to the numerical example with artificial data in Figure 5.

Figure 12: Factor inefficiency of DMU *E* (CCR case)Figure 13: Factor inefficiency of DMU *E* (SBM case)Figure 14: Factor inefficiency of DMU *E* (CGM case)

We focus on the labor inefficiency in order to verify the practicality of the model, i.e. whether labor inefficiency is occurred in DMU *E* or not. Figures 15 and 16 portray labor productivity indices (y_1/x_2 and y_2/x_2) for DMUs *E* and *F*. The DMU *F* is one who

performs best of all in labor productivities. It can be pointed out that labor productivities of DMU *E* do not outperform those of DMU *F* during the study period, even if their scores are very close in Figure 15. And thus, this suggests that labor inefficiency might be occurred in DMU *E* rather than DMU *F*. However, as Figure 17 indicated, no labor inefficiency is measured for DMU *E* during the whole period in SBM results in spite of occurrence of inefficiency for DMU *F* in several periods. This fact might bring us discomfort¹³. Contrary to the SBM results, CGM labor inefficiencies in DMU *F* are always below those of DMU *E* except the final period, in which both of them are evaluated as efficient. Consequently, it can be said that CGM results is more reasonable in this case.

¹³ It should be noted that the factor inefficiency measured in DEA model is a relative index based on multiple inputs and outputs evaluation, while the partial factor productivity is an absolute index based on specific input and output. Therefore, it is no wonder that the relative ranking of factor inefficiency does not match to that of partial productivity. However, it would be more practical if they are completely matched.

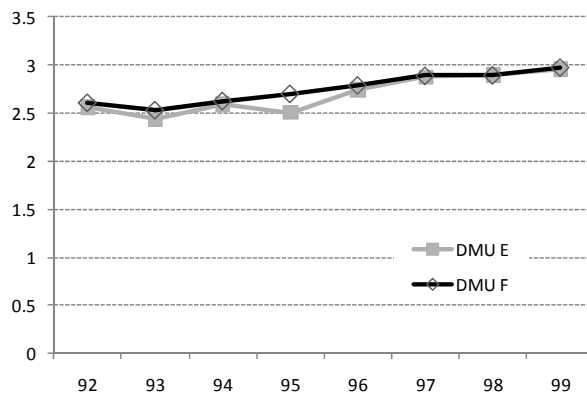


Figure 15: Comparison of Labor productivity index (y_1/x_2 case)

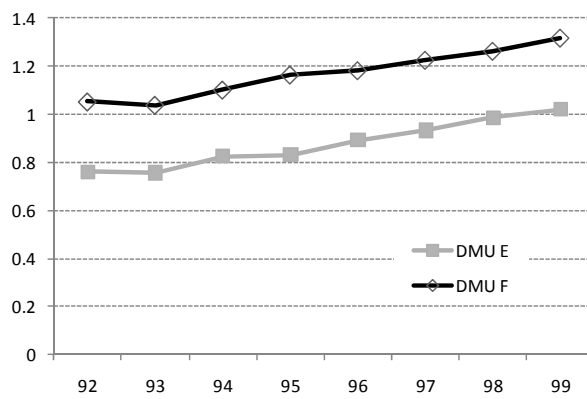


Figure 16: Comparison of Labor productivity index (y_2/x_2 case)

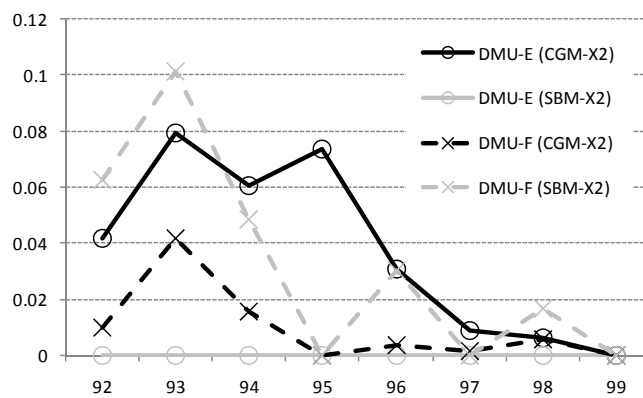


Figure 17: Comparison of Labor inefficiency index

As we already pointed out, the SBM may generate extreme results such as labor

inefficiency of DMU E in Figure 13, and it is likely to lead us unreasonable interpretation of results. It would be advantage of CGM to avoid such extremeness.

6. Conclusions

This study introduced a new scheme named Cost Gradient Measure (CGM) to measure DEA technical efficiency using cost information of DMUs. This model provides technical efficiency index aiming maximum cost saving in the short run. In addition, CGM has several practical features in contrast to the traditional radial and non-radial models, e.g. we can conduct further analysis using factor inefficiency indices. In addition, this can be employed on a certain assumption even if cost information of DMUs cannot be obtained.

The CGM is a basic model to measure technical efficiency, and thus it can be applied to the extended models, e.g. intertemporal analysis such as Malmquist index [14], decomposition of overall efficiencies [22], network and dynamic models [11,12,23 and 24] and so forth. These are our future research subjects. Furthermore, this model can be extended to the non-oriented model by employing both input and output prices. This is also an important future subject.

Reference

- [1] Andersen P and Petersen NC (1993) "A procedure for ranking efficient units in data envelopment analysis" *Management Science* 39, 1261-1264.
- [2] Avkiran NK and Parker BR (2009) "Pushing the DEA research envelope" *Socio-Economic Planning Sciences*, doi:10.1016/j.seps.2009.06.001.
- [3] Avkiran NK, Tone K and Tsutsui M (2007) "Bridging radial and non-radial measures of efficiency in DEA" *Annals of Operations Research* 164(1), 127-138.

- [4] Banker RD, Charnes A and Cooper WW (1984) “Some Models for Estimating Technical and Scale Inefficiencies in Data Envelopment Analysis” *Management Science*, 30, 1078-92.
- [5] Bogetoft P and Hougaard JL (1999) “Efficiency evaluations based on potential (non-proportional) improvements” *Journal of Productivity Analysis* 12, 233-247.
- [6] Caves DW, Christensen LR, Tretheway MW. U.S. Trunk Air Carriers, 1972-1977 : A Multilateral Comparison of Total Factor Productivity. In: Cowing TG, Stevenson RE, editors. *Productivity Measurement in Regulated Industries*. New York: Academic Press. 1981:47-76.
- [7] Charnes A, Cooper WW and Rhodes E (1978) “Measuring the Efficiency of Decision Making Units” *European Journal of Operational Research*, 2, 429-444.
- [8] Charnes A, Cooper WW, Golany B and Seiford L (1985) “Foundations of data envelopment analysis for Pareto-Koopmans efficient empirical production functions”, *Journal of Econometrics* 30(1-2), 91-107.
- [9] Cook WD and Seiford LM (2008) “Data envelopment analysis (DEA) – Thirty years on” *European Journal of Operational Research* 192(1),1-17.
- [10] Cooper WW, Seiford LM and Tone K (2007) *Data envelopment analysis: A comprehensive text with models, applications, references and DEA-Solver software*, Springer.
- [11] Färe R and Grosskopf S (1996) *Intertemporal Production Frontiers; With Dynamic DEA*, Kluwer Academic Publishers.
- [12] Färe R and Grosskopf S (2000) “Network DEA” *Socio-Economic Planning Sciences*, 34, 35-49.
- [13] Färe R and Grosskopf S (2010) “Directional distance functions and slacks-based measures of efficiency” *European Journal of Operational Research* 200(1), 320-322.
- [14] Färe R, Grosskopf S, Lindgren B and Roos P (1989) “Productivity Developments in Swedish Hospitals: A Malmquist Output Index Approach” *In: Data Envelopment Analysis: Theory, Methodology and Applications*, edited by Charnes A, Cooper WW, Lewin AY and Seiford LM (1994), Chap.13, Kluwer Academic Publishers.
- [15] FEPC “Handbook of Electric Power Industry” Federation of Electric Power Companies, 1990-2001 (each year). (in Japanese).

- [16] Luenberger DG (1992) “Benefit Functions and duality”, *Journal of Mathematical Economics* 21, 461–481.
- [17] Russell RR (1985) “Measures of Technical Efficiency” *Journal of Economic Theory* 35,109-126.
- [18] Thompson RG, Singleton FD, Thrall RM and Smith BA (1986) “Comparative site evaluation for locating a high-energy physics lab in Texas”, *interfaces* 16, 35-49.
- [19] Tone K (2001) “A slacks-based measure of efficiency in data envelopment analysis” *European Journal of Operational Research*, 130, 498-509.
- [20] Tone K (2002) “A slacks-based measure of super-efficiency in data envelopment analysis” *European Journal of Operational Research* 143, 32-41.
- [21] Tone K (2009) “Variations on the theme of slacks-based measure of efficiency in DEA”, *European Journal of Operational Research*, doi:10.1016/j.ejor.2009.01.027.
- [22] Tone K and Tsutsui M (2007) “Decomposition of cost efficiency and its application to Japanese-US electric utility comparisons” *Socio-Economic Planning Sciences*, 41(2).
- [23] Tone K and Tsutsui M (2009a) “Network DEA: A slacks-based measure approach”, *European Journal of Operational Research*, 197, 243-252.
- [24] Tone K and Tsutsui M (2009b) “Dynamic DEA: A slacks-based measure approach” *OMEGA*, (forthcoming).
- [25] Tsutsui M and Goto M (2009) “A multi-division efficiency evaluation of U.S. electric power companies using a weighted slacks-based measure”, *Socio-Economic Planning Sciences* 43, 201-208.

Appendix A: Proof of units-invariance of CGM-UI model

Suppose that we change the units of measurement of \mathbf{X} by $\hat{\mathbf{X}} = \mathbf{D}\mathbf{X}$ where $\mathbf{D} = [d_{ii}] \in \mathbf{R}^{m \times m}$ is a diagonal matrix whose element $d_{ii} (> 0)$ ($i = 1, \dots, m$) represents the unit change factor for input i . Thus the unit price w_{ij} of input i changes to

$$\hat{w}_{ij} = w_{ij} / d_{ii} \quad (i = 1, \dots, m; j = 1, \dots, n).$$

Between the two measurement units, we have the equality

$$\hat{w}_{ij}\hat{x}_{ij} = w_{ij}x_{ij} \quad (i = 1, \dots, m; j = 1, \dots, n).$$

Thus, between the new and old directions we have the relationship:

$$\hat{\tilde{\mathbf{w}}}_o = (\hat{w}_{1o}\hat{x}_{1o}^2, \dots, \hat{w}_{mo}\hat{x}_{mo}^2)^T = (w_{1o}x_{1o}\hat{x}_{1o}, \dots, w_{mo}x_{mo}\hat{x}_{mo})^T = \mathbf{D}(w_{1o}x_{1o}^2, \dots, w_{mo}x_{mo}^2)^T = \mathbf{D}\tilde{\mathbf{w}}_o$$

Hence the first constraint in (18) becomes, in the new units,

$$\hat{\mathbf{x}}_o \geq \hat{\mathbf{X}}\boldsymbol{\lambda} + \hat{\tau}\hat{\tilde{\mathbf{w}}}_o = \hat{\mathbf{X}}\boldsymbol{\lambda} + \hat{\tau}\mathbf{D}\tilde{\mathbf{w}}_o$$

Multiplying \mathbf{D}^{-1} from the left, we have the same constraint with (18). Thus we have the same solution and score. Q.E.D.