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Second-Best Cost-Benefit Analysis in Monopolistic Competition Models of Urban Agglomeration

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Abstract

Many sources of urban agglomeration involve departures from the first-best world. By modeling the microstructure of agglomeration economies, we derive second-best benefit evaluation formulae for urban transportation improvements. Previous work has investigated the same problem, but without explicitly modeling the sources of agglomeration economies. Accordingly, our analysis examines whether earlier results remain valid when monopolistic competition with differentiated products provides the microfoundation of the agglomeration economies. By explicitly introducing the rural sector and multiple cities, we also show that the agglomeration benefits depend on where the new workers are from.

Keywords: cost-benefit analysis; agglomeration economies; monopolistic competition; new economic geography; second-best economies

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1. Introduction

Many of the sources of urban agglomeration, such as gains from variety, better matching, and knowledge creation and diffusion, involve departures from the first-best world.[1] The benefit evaluation of a transportation project must then take into account agglomeration benefits along with any direct user benefits. A number of economists have studied this issue, and policymakers in some countries, such as the United Kingdom, have been attempting to include these considerations in their project assessments.[2]

Based on past empirical work, urban agglomeration economies are substantial. For instance, a review by Rosenthal and Strange (2004, p. 2133) summarizes the empirical findings as follows: "In sum, doubling city size seems to increase productivity by an amount that ranges from roughly 3−8%." Agglomeration economies on the consumer side are also substantial, as argued by Glaeser et al. (2001), with estimates by Tabuchi and Yoshida (2000) suggesting economies in the order of 7−12 percent. Certainly, the benefit estimates could exceed 10 percent after combining production and consumption agglomeration economies.

By modeling the microstructure of agglomeration economies, this paper derives second-best benefit evaluation formulae for urban transportation improvements. Venables (2007) investigated the same problem but without explicitly modeling the sources of agglomeration economies. Accordingly, our analysis examines whether the results in this prior work remain valid when monopolistic competition with differentiated products provides the microfoundation of agglomeration economies. By explicitly introducing the rural sector and multiple cities, we also show that the agglomeration benefits depend on where the new workers are from.

Extending the Henry George Theorem to a second-best setting with distorted prices, Behrens et al. (2010) showed that the optimality condition for the number of cities (or equivalently, the optimal size of a city) must be modified to include Harberger's excess

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[1] See Duranton and Puga (2004) for a review of the theoretical analysis of various sources of urban agglomeration, Fujita and Thisse (2002) for the New Economic Geography approach, and Kanemoto (1990) for the analysis of a nonmonocentric city model.

[2] See, for example, Venables and Gasiorek (1999), Department of Transport (2005), (2008), Graham (2005, 2006), and Vickerman (2007).

burden, that is, the weighted sum of induced changes in consumption, with the weights being the price distortions. New Economic Geography (NEG)-type models of monopolistic competition contain distortions of two forms: a price distortion for each variety of the differentiated good, and a distortion associated with the number of available varieties consumed. Although the former is well known, the latter has largely escaped the attention of the existing literature. Importantly, because these two types of distortions work in opposite directions, the net effect is uncertain. In this article, we examine whether we can obtain similar results with transportation investment projects. Moreover, in yet another departure from Venables (2007), we explicitly introduce the rural sector and multiple cities. We show that the results hinge on whether the new workers are from the rural sector or other cities.

The remainder of the paper is as follows. In Section 2, we present a model of urban agglomeration economies based on monopolistic competition in differentiated intermediate products. Section 3 derives second-best benefit measures of transportation investment. In Section 4, we extend the analysis to a model of differentiated consumer goods. Section 5 concludes.

2. The model

Our model adds three elements to Venables (2007): the microstructure of agglomeration, multiple cities, and an explicit rural sector.[3] We examine agglomeration economies on both the production and consumption sides, using monopolistic competition models with product differentiation in the intermediate or consumer goods. The differentiated goods are not transportable to outside a city. The economy contains n cities and a rural area, where all cities are monocentric, i.e., all workers commute to the central business district (CBD). All cities have the same topographical and technological conditions. Workers/consumers are mobile and free to choose where, between the cities and the rural area, to live and work.

Our first model assumes differentiated intermediate inputs, where the production of an urban final good requires differentiated intermediate inputs. We later replace the intermediate inputs with differentiated consumer goods to examine the generality of our

[3] We ignore income tax distortions because Venables' analysis is applicable to our model without modification.

results. We assume the final good is homogeneous. The final good can be transported costlessly between cities and the rural area, but, as stated above, intermediate goods can be used only within a city. Final good producers are competitive within a city, taking both output and input prices as fixed. We assume free entry for final good producers. For simplicity, we assume the rural area produces the same final product, albeit with a different technology. While the final good is consumed directly by consumers, it is also used in the production of transportation services.

The intermediate good producers are monopolistically competitive. We assume free entry for intermediate good production as well as for final good production. Following Venables (2007), we use a monocentric city model with commuting transportation and assume absentee landlords own land in both urban and rural areas.

Production of the urban final good

The production of the urban final good requires differentiated intermediate inputs only, and the production function is $y_0 = F(\{y_i\}_{i\in M})$, where y_0 and y_i respectively denote the homogeneous final good and differentiated intermediate input *i*, and M is the set of available intermediate goods. Unlike in typical NEG models, we do not assume specific functional forms. We only assume the production function is symmetric in the y_i's, and that it is well behaved, so profit maximization yields a unique interior solution. The mass of the set of intermediate goods that are actually used for production (i.e., $y_i > 0$) is denoted by *m* and called the variety. An example of production functions satisfying these conditions is an additively separable function,

$$y_0 = \left(\int_0^m f(y_i)di\right)^{1+\rho}, \tag{1}$$

which includes the constant elasticity of substitution (CES) form commonly used in NEG models:

$$y_0 = \left(\int_0^m \alpha(y_i)^{\frac{\sigma-1}{\sigma}} di\right)^{1+\rho}. \tag{2}$$

The final good, y_0, is homogeneous and its transportation cost is zero.

The final good industry is competitive within a city and we assume free entry. The profit of a producer is $\pi = y_0 - \int_0^m p_i y_i di$, where p_i is the price of intermediate good i and we normalize the price of the final good as one (1). A producer takes the prices of intermediate goods, as well as that of the final good, as fixed.

We focus on a symmetric equilibrium where the prices (and hence quantities) of differentiated goods are all equal. We can then write the production function of the final good as a function of the equal input level y and variety m:

$$y_0 = F(\{y_i\}_{i \in M})\big|_{y_i = y} \equiv \phi(y,m),$$

where in the additively separable case $\phi(y,m) = (mf(y))^{1+\rho}$. Because the first-order condition for profit maximization is $\partial F / \partial y_i = p_i$ for input i, this function satisfies:

$$\frac{\partial \phi}{\partial y} = m\frac{\partial F}{\partial y_i} = mp. \tag{3}$$

Variety is determined by the entry decisions of suppliers, but, in order for them to be used by final good producers, adding another variety must be profitable, so that:

$$\frac{\partial \phi}{\partial m} = \frac{\partial F}{\partial m} \geq py. \tag{4}$$

We will find that the inequality is strict in most cases. The free-entry/zero-profit condition is:

$$\phi(y,m) = mpy. \tag{5}$$

Combining (3) and (5) yields a familiar condition that the marginal product equals the average product in a free-entry equilibrium:

$$\frac{\partial \phi(y,m)}{\partial y} = \frac{\phi(y,m)}{y}. \tag{6}$$

This condition determines the input level y as a function of variety m :

$$y = y(m). \tag{7}$$

The demand function for input i can be written generally as a function of input prices: $y_i = d(\{p_i\}_{i \in M})$. Denote the number of final good producers by k. Then, the market demand for input i is:

$$Y_i = D(\{p_i\}_{i \in M}, k) \equiv kd(\{p_i\}_{i \in M}), \ \ i \in M.$$

Production of differentiated intermediate goods

Next, let us turn to the producers of the intermediate good. An intermediate good producer has monopoly power because of product differentiation. Under the standard monopolistic competition assumption, however, a producer is small enough to ignore impacts on other producers and the number of final good producers. The perceived demand function is then $Y_i = D(p_i; \{p_{i'}\}_{i' \neq i}, k)$, with variables other than the producer's own price fixed.[4] The perceived price elasticity of demand is:

$$\zeta_i = -\frac{\partial D_i}{\partial p_i}\frac{p_i}{Y_i} = -\frac{\partial (kd_i)}{\partial p_i}\frac{p_i}{ky_i} = -\frac{\partial d_i}{\partial p_i}\frac{p_i}{y_i} = \zeta\left(p_i; \{p_{i'}\}_{i' \neq i}\right), \tag{8}$$

where the elasticity does not depend on the number of final good producers *k* because it is taken as fixed by a producer.

Production of an intermediate good requires only labor as an input. The labor input required for producing Y_i of variety i is $N_i = cY_i + a$, where the fixed cost and the marginal cost are fixed at *a* and *c* (measured in terms of labor units), respectively. Given the perceived demand function, an intermediate good producer maximizes the profit $\pi_i = p_i Y_i - w(cY_i + a)$, where *w* is the wage rate.

In a symmetric equilibrium with $p_i = p$ and $Y_i = Y$ for any $i \in M$, the price elasticity of demand (8) becomes a function of price *p* and variety *m*: $\zeta = \zeta(p, m)$. The first-order condition for profit maximization can then be written as:

$$\frac{p - wc}{p} = \frac{1}{\zeta(p, m)}, \tag{9}$$

i.e., the profit margin is the inverse of the price elasticity. The free-entry condition is:

$$pY - w(cY + a) = 0. \tag{10}$$

Because the number of final good producers equals Y / y, the aggregate production function in a city can be written as a function of the quantity of an intermediate good produced, *Y*, and the variety, *m*:

[4] This formulation assumes the Bertrand-type behavior in which a producer takes the prices of other producers as fixed. We may use the Cournot assumption that quantities supplied by other producers are taken as fixed. The same qualitative results are obtained in the Cournot case, although the values of price elasticities are in general different.

$$Y_0 = \widetilde{F}(Y,m) \equiv \frac{Y}{y(m)}\phi(y(m),m), \tag{11}$$

where from (3), (4), and (6) the aggregate production function satisfies:

$$\frac{\partial \widetilde{F}(Y,m)}{\partial Y} = mp, \tag{12}$$

$$\frac{\partial \widetilde{F}(Y,m)}{\partial m} = \frac{Y}{y}\frac{\partial \phi}{\partial m} \geq pY. \tag{13}$$

From the first-order condition (9) and the zero-profit condition (10), we obtain:

$$Y = \frac{a}{c}(\zeta(p,m)-1). \tag{14}$$

Now, denote the total labor force in a city by $N = \int_0^m N_i di$. Then, because all workers in a city work in the differentiated intermediate good industry, the labor requirement for differentiated good production yields:

$$N = m(cY + a). \tag{15}$$

Using the three equations, (12), (14), and (15), we can solve for three variables, p, m, and Y, as functions of N:

$$p = \widetilde{p}(N),\ Y = \widetilde{Y}(N), \text{ and } m = \widetilde{m}(N). \tag{16}$$

From (15), $\widetilde{m}(N)$ and $\widetilde{Y}(N)$ satisfy:

$$\widetilde{m}'(N) = \frac{1 - mc\widetilde{Y}'(N)}{c\widetilde{Y}(N) + a}. \tag{17}$$

Commuting costs and urban land

An urban worker consumes housing of quality $\bar{h}$, which is assumed to be exogenously fixed. We ignore $\bar{h}$ as it is fixed, and for simplicity assume housing only requires land as an input.

As in Venables (2007), we assume that urban workers do not receive a share of the land rent revenue. The budget constraint for an urban worker is:

$$w = x_0 + t(z) + r(z) \text{ for all } z \in [0, \hat{z}],$$

where $t(z)$ is the commuting cost for a worker living at distance z from the CBD, $r(z)$ is housing rent, and $\hat{z}$ is the city edge. We assume that the rent is zero at the periphery of the city. Note that commuting requires only the final good as an input. In equilibrium, the housing rent differentials completely offset the commuting cost differentials so that the consumption of the final good equals $x_0 = w - t(\hat{z})$ for all workers in a city.

The equilibrium condition for the housing market is $N = \int_0^{\hat{z}} n(z)dz$, where $n(z)$ is the density of workers that can be accommodated at distance z from the center. The density $n(z)$ is exogenously determined by topography and land use regulation. Solving this equation for the boundary of the city $\hat{z}$, we obtain $\hat{z} = \hat{z}(N)$. Transport costs for a worker at the periphery of the city can then be written as a function of the population N and a parameter t indicating the unit cost of transportation:

$$t(\hat{z}) = T(N,t). \tag{18}$$

Venables (2007) assumed $n(z) = (1+\theta)z^{\theta}$ and $t(z) = tz^{\lambda}$. In this example, $\hat{z} = N^{1/(1+\theta)}$ and $T(N,t) = tN^{\gamma-1}$, where $\gamma = (1+\theta+\lambda)/(1+\theta)$.

Denote the aggregate transportation costs in a city (measured in terms of the final product) by:

$$TC(N,t) = \int_0^{\hat{z}} t(z)n(z)dz. \tag{19}$$

Then,

$$\frac{\partial TC(N,t)}{\partial N} = T(N,t). \tag{20}$$

That is, if a worker is added to a city, this person must be located at the edge of the city and the total transportation cost increases by the commuting cost of a worker at the edge, (18). We consider a transportation improvement project that marginally reduces the cost parameter t. Its direct benefit, denoted by MB_t, is a *decrease* in the total transportation cost caused by a marginal *reduction* in t, or equivalently, an *increase* in the cost by a marginal *increase* in t:

$$MB_t = \frac{\partial TC(N,t)}{\partial t}. \tag{21}$$

In the example of Venables (2007), these are $TC(N,t) = tN^{\gamma}/\gamma$ and $MB_t = N^{\gamma}/\gamma$.

Equilibrium conditions

The total population in the economy is $\overline{N}$, which is divided into n cities with population N^j, $j = 1,\cdots,n$, and the rural area with population N^A:

$$\overline{N} = \sum_j N^j + N^A. \tag{22}$$

The number of cities n is assumed to be fixed. As noted before, we assume that the rural product is the same as the urban final product. The production function of the rural sector is $G(N^A)$, where the wage rate equals the marginal product: $w^A = G'(N^A)$. The consumption of a rural worker is then $x_0^A = w^A$. Free migration equalizes the consumption levels in all cities and the rural area, i.e., $x_0^A = x_0^j, j = 1,\cdots,n$. This implies net income equalization:

$$w^j - T(N^j, t^j) = w^A,\ \ j = 1,\cdots,n. \tag{23}$$

3. Benefits of transportation investment

We now examine the general equilibrium impacts of small transportation improvements in cities. Our goal is to estimate the benefits of a marginal reduction in transportation costs in city 1, taking into account the effects on urban agglomeration.

Price distortions

Before examining the general-equilibrium impacts of a transportation project, we define price distortions. First, the marginal social benefit of an intermediate good can be measured by an increase in the final good production caused by a marginal increase in an intermediate input: $MB_Y = \partial F / \partial y_i$. From the first-order condition of profit maximization, this equals the price of an intermediate input: $MB_Y = p$. Because the marginal social cost is its production cost $MC_Y = wc$, the price distortion of an intermediate good is:

$$\tau_Y = MB_Y - MC_Y = p - wc. \tag{24}$$

The marginal social benefit of increasing the variety of differentiated goods is the resulting increase in the production of the final good $MB_m \equiv \partial\widetilde{F}(Y,m)/\partial m$, and the marginal cost is the cost of producing the additional variety $MC_m \equiv w(cY + a)$. The price distortion of variety is then:

$$\tau_m = MB_m - MC_m = \frac{\partial \widetilde{F}(Y,m)}{\partial m} - w(cY + a)\,. \tag{25}$$

Substituting $m = \widetilde{m}(N)$ and $Y = \widetilde{Y}(N)$ obtained in (16) into the aggregate production function (11), we obtain a reduced-form aggregate production function linking aggregate production to the total labor force in a city:

$$Y_0 = \widetilde{F}(\widetilde{Y}(N), \widetilde{m}(N)) \equiv \widetilde{Y}_0(N)\,.$$

The marginal social benefit of a worker in the differentiated good industry is then:

$$MB_N = \widetilde{Y}_0{}'(N) = \frac{\partial \widetilde{F}}{\partial Y}\widetilde{Y}'(N) + \frac{\partial \widetilde{F}}{\partial m}\widetilde{m}'(N) = mMB_Y\widetilde{Y}'(N) + MB_m\widetilde{m}'(N)\,.$$

The marginal social cost of a worker equals the wage rate w, $MC_N = w$, and the wage distortion is the difference between these two:

$$\tau_N = MB_m\widetilde{m}'(N) + mMB_Y\widetilde{Y}'(N) - w\,.$$

Using the definitions of price and variety distortions in (24) and (25) and noting the relationship between $\widetilde{m}'(N)$ and $\widetilde{Y}'(N)$ in (17), we can rewrite this equation as:

$$\tau_N = \tau_m\widetilde{m}'(N) + m\tau_Y\widetilde{Y}'(N)\,. \tag{26}$$

Thus, the wage distortion captures both the price and variety distortions of differentiated intermediate goods.

Harberger formula

Now, we turn to the impacts of a transportation project. We first derive a general formula that can be interpreted as an extension of the Harberger triangles to urban agglomeration. Given that there is only one consumption good in our model, we can define the social surplus as the total amount of the good available for consumption by urban and rural workers and absentee landlords:

$$S = \sum_j \left(Y_0^j - TC^j\right) + G(N^A)\,.$$

Substituting the aggregate production function (11), the total cost function (19), and the population constraint (22) into this yields:

$$S = \sum_j \left(\widetilde{F}(Y^j, m^j) - TC(N^j, t^j)\right) + G(\overline{N} - \sum_j N^j)\,.$$

Our task is to evaluate a change in the social surplus caused by a marginal change in transportation costs t^j. Totally differentiating the social surplus equation, we obtain:

$$dS = \sum_j \left(\frac{\partial \widetilde{F}}{\partial Y^j} dY^j + \frac{\partial \widetilde{F}}{\partial m^j} dm^j - \frac{\partial TC}{\partial N^j} dN^j - \frac{\partial TC}{\partial t^j} dt^j \right) - G' \sum_j dN^j .$$

Applying equilibrium conditions obtained in the preceding section to this equation yields the Harberger formula. First, using $G' = w^A$, the equal-income condition (23), and the definitions of marginal social benefits, we obtain:

$$dS = \sum_j \left(m^j MB_Y^j dY^j + MB_m^j dm^j - MB_t^j dt^j \right) - \sum_j w^j dN^j . \qquad (27)$$

Next, the total differentiation of the labor force requirement in the differentiated urban sector (15) yields:

$$dN^j = (cY^j + a) dm^j + m^j c dY^j .$$

Substituting this into the equation above and using the definitions of price distortions, (24) and (25), we can further rewrite (27) as:

$$dS = -\sum_j MB_t^j dt^j + \sum_j \left(m^j \tau_Y^j dY^j + \tau_m^j dm^j \right), \qquad (28)$$

where $MB_t^j = -\partial TC / \partial t^j$ is the marginal direct benefit of a reduction in t^j defined in (21). This is an extension of Harberger's measure of welfare change (Harberger, 1964), i.e., a change in surplus can be decomposed into the direct benefit and the changes in the excess burden, where the excess burden is given by the weighted sum of induced changes, with weights being the price distortions. As noted by Behrens et al. (2010), the Harberger formula must be extended to include the variety distortion when the variety is endogenous.

Using the price distortion of labor (18), we can simplify (28) as:

$$dS = -\sum_j MB_t^j dt^j + \sum_j \tau_N^j dN^j . \qquad (29)$$

Thus, the excess burden can be measured by the wage distortion only. This result shows that the agglomeration externality measure in Venables (2007) is valid if it is obtained from a reduced-form aggregate production function with differentiated intermediate inputs.

Benefits of transportation investment in a city

Next, we consider a change in transportation costs in city 1, starting from a symmetric equilibrium where all cities are identical replicas of each other. Its direct benefit is the change in the total transportation cost in city 1: $MB_t^1 = \partial TC(N^1, t^1)/\partial t^1$. From the Harberger formula (28), the change in the social surplus is:

$$-\frac{dS}{dt^1} = MB_t^1 - \left(m\tau_Y \frac{dY^1}{dt^1} + \tau_m \frac{dm^1}{dt^1} \right) - (n-1)\left(m\tau_Y \frac{dY}{dt^1} + \tau_m \frac{dm}{dt^1} \right), \qquad (30)$$

where superscript 1 denotes city 1 and variables without a superscript refer to other cities, and we have used the fact that all the variables are equal, including the price distortions and the variety at the initial symmetric equilibrium: $\tau_Y^1 = \tau_Y$, $\tau_m^1 = \tau_m$, and $m^1 = m$. Note that we attach a minus sign to dS/dt^1 to indicate the impact of a marginal *decrease* in transportation costs (i.e., $-dt^1$).

If we use the wage distortion, we obtain:

$$-\frac{dS}{dt^1} = MB_t^1 + \tau_N \left(-\frac{dN^1}{dt^1} - (n-1)\frac{dN}{dt^1} \right) = MB_t^1 + \tau_N \frac{dN^A}{dt^1} \qquad (31)$$

from (29). Thus, if a transportation improvement in a city increases the total urban population (or decreases the rural population, $-dN^A/dt^1 < 0$), then there will be positive additional benefits. A transportation improvement in city 1 tends to increase its population. This creates agglomeration benefits in addition to the direct user benefits because the social value of an additional worker exceeds the wage rate. However, this process also reduces the size of other cities, and the adverse effects on other cities (partially) offset the benefits in city 1. If the population of the rural area (or equivalently, the total population of the urban areas) is fixed, then these effects cancel each other out and there will be no extra benefits besides the direct benefit: $-dS/dt^1 = MB_t^1$.

Now, we show that the extra benefits are always nonnegative if the stability condition for population migration is satisfied. Equilibrium within a city determines the wage rate as a function of its population, $w^j = \tilde{w}(N^j)$. The equilibrium condition for population movement (23) can then be rewritten as:

$$\tilde{w}(N^1) - T(N^1, t^1) = \tilde{w}(N) - T(N, t) = G'(N^A).$$

The effect of a marginal change in t^1 is then:

$$\left(\tilde{w}' - T_N\right)\frac{dN^1}{dt^1} - T_t = \left(\tilde{w}' - T_N\right)\frac{dN}{dt^1} = G''(N^A)\left(-\frac{dN^1}{dt^1} - (n-1)\frac{dN}{dt^1}\right).$$

From the first equality, we obtain:

$$\frac{dN^1}{dt^1} = \frac{dN}{dt^1} + \frac{T_t}{\tilde{w}' - T_N}.$$

Substituting this into the second equality yields:

$$\frac{dN}{dt^1} = -\frac{G''(N^A)T_t}{\left(\tilde{w}' - T_N + nG''(N^A)\right)\left(\tilde{w}' - T_N\right)}.$$

Combining these two relationships with the population constraint (22), we obtain:

$$\frac{dN^A}{dt^1} = -\frac{dN^1}{dt^1} - (n-1)\frac{dN}{dt^1} = -\frac{T_t}{\tilde{w}' - T_N + nG''(N^A)}.$$

Now, one of the necessary conditions for stability is that if a random perturbation increases the population in all cities equally and decreases that in the rural area accordingly, the utility in cities becomes lower than that in the rural area, inducing counteractive population movement from cities to the rural area:

$$\frac{d}{dN}(\tilde{w}(N) - T(t,N) - G') = \tilde{w}' - T_N + nG''(N^A) \leq 0.$$

This implies:

$$\frac{dN^A}{dt^1} = -\frac{T_t}{\tilde{w}' - T_N + nG''(N^A)} \geq 0,$$

and we obtain the result that the additional benefits are always nonnegative:

$$-\frac{dS}{dt^1} = MB_t^1 + \tau_N\frac{dN^A}{dt^1} \geq MB_t^1.$$

The additively separable case

If the production function of the final good is additively separable as in (1), demand for an intermediate good by a final good producer, y, and its total supply by an intermediate good producer, Y, do not depend on variety m. This can be seen as follows.

First, in a symmetric equilibrium we have $\phi(y,m) = (mf(y))^{1+\rho}$, and (6) becomes:

$$(1+\rho)f'(y) = f(y)/y. \qquad (32)$$

This equation determines y.

Second, the first-order condition for profit maximization of a final good producer is $p_i = (1+\rho)(y_0)^{\frac{\rho}{1+\rho}} f'(y_i)$. Because the output of the final good y_0 is taken as fixed by an intermediate good producer, the price elasticity of perceived demand (8) becomes:

$$\zeta_i = -\frac{(\partial d / \partial p_i) p_i}{y_i} = -\frac{f'(y_i)}{y_i f''(y_i)} = \frac{1}{R_R(y_i)},$$

where $R_R(y) \equiv -yf''/f'$ is equivalent to the measure of relative risk aversion in expected utility theory. Because *y* is determined by (32), the price elasticity is fixed. Equation (14) then determines the production level of an intermediate good as $Y = (\zeta - 1)a/c$. An important implication of this is that transportation improvements do not affect the production level Y, and any change in intermediate good production occurs only through variety *m*.

Because the total production of an intermediate good, *Y*, and the amount of an intermediate good used by a final good producer, *y*, are both fixed, the price elasticity of demand is constant and equals the inverse of the measure of relative risk aversion, R_R. The price distortion then satisfies: $\tau_Y = pR_R \geq 0$. Although the price distortion exists in the additively separable case, it does not cause any excess burden because the output level *Y* does not change. The variety distortion is $\tau_m = \rho w(cY + a) \geq 0$, which is the cost of producing a variety multiplied by the returns to scale parameter ρ. The wage distortion is proportional to the price distortion of variety and satisfies: $\tau_N = \tau_m /(cY + a) = \rho w$. Note that the measure of relative risk aversion R_R is the key parameter for the price distortion, whereas the returns to scale parameter ρ determines the variety and wage distortions.

If we restrict the functional form to CES as in (2), condition (32) holds only when $\rho = 1/(\sigma - 1)$. In this case the scale of a final good producer is indeterminate, but the aggregate production exists and satisfies $Y_0 = \alpha^{\sigma/\sigma-1} m^{\sigma/\sigma-1} Y$ in a symmetric case. The price elasticity is constant at $\zeta = \sigma$, and the production level can be solved explicitly as $Y = (\sigma - 1)a/c$. Because in the CES case the returns to scale parameter ρ and the elasticity of substitution parameter σ are perfectly linked, one parameter (either σ or ρ) determines all the distortions: $\tau_Y = wc/(\sigma - 1)$, $\tau_m = wa(\sigma/(\sigma - 1))$, and

$\tau_N = w/(\sigma - 1)$.

4. Differentiated consumer goods

This section examines whether or not the results obtained in the differentiated intermediate goods model carry over to differentiated consumer goods. Another extension is to introduce a homogeneous good sector in cities so that urban workers have a choice between the two industries for their job opportunities.

With differentiated consumer goods, how to evaluate the utility change caused by a transportation improvement becomes an issue. Because we assume free and costless migration of workers, the utility levels are equal wherever they locate. We want to evaluate a change in this common utility level in pecuniary units. As is well known, we may use different consumer surplus concepts, such as Marshallian consumer surplus and compensating and equivalent variations. The Marshallian measure has a well-known difficulty of path dependence. As pointed out by Kanemoto and Mera (1985), compensating and equivalent variations yield complicated formulae in a general-equilibrium setting. Here, we use the Allais surplus because it provides a simple measure while being consistent (unlike the Marshallian measure). The Allais surplus is defined as the amount of the numéraire good that can be extracted from the economy with the utility levels being fixed at the initial levels.

Firm *i* in the differentiated consumer good industry in city *j* hires N_{Ci}^j workers, where the number of firms in the industry is m^j. The homogeneous good industry in city *j* employs N_0^j workers. The total number of workers is $N^j = N_C^j + N_0^j$ in city *j*, where $N_C^j = \sum_i N_{Ci}^j$ is the total number of workers in the differentiated consumer good sector. The homogeneous good is produced also in the rural area with technology different from that in urban areas. The number of workers in the rural area is N^A. The population constraint is then $\overline{N} = \sum_{j=1}^{n} N^j + N^A$, where *n* is the number of cities. An urban worker earns wage rate w^j. All workers in a city work at the CBD. Henceforth we omit superscript *j* when this does not cause confusion.

The homogeneous good is either consumed directly or used in intracity transportation. The formulation of the transportation sector is the same as before, and the rural area produces only the homogeneous good. The homogeneous good can be transported costlessly between cities and the rural area but the differentiated goods cannot be transported outside a city.

The utility function of a worker is $U(\mathbf{x}) = U(x_0, \{x_i\}_{i \in M})$, where x_0 is the homogeneous good, x_i is the consumption of differentiated good i, and M is the set of available differentiated goods. The utility function is assumed to be symmetric in the x_i's in M. As in the preceding sections, we assume that the lot size of a house is fixed and we ignore the structural part of housing. The homogeneous good is taken as the numéraire. A rural worker cannot consume the differentiated goods and hence $x_i = 0$ for any $i \in M$.

The budget constraint for an urban worker is $w = \int_0^m p_i x_i di + x_0 + T(N,t)$, where w is the wage rate for an urban worker, p_i is the price of differentiated good i, and $T(N,t)$ is the transportation cost for a worker living at the edge of the city as in preceding sections. The budget constraint for a rural worker is $w^A = x_0^A$, where w^A is the wage rate in the rural area.

In a symmetric equilibrium where quantities consumed are equal for all differentiated goods, we can write the utility function as:

$$\widetilde{U}(x_0, x, m) \equiv U(x_0, \{x\}_{i \in M}),$$

where m is the number of varieties of differentiated goods actually consumed by a household. Then, the first-order conditions for expenditure minimization yield:

$$\frac{\partial \widetilde{U}(x_0, x, m) / \partial x}{\partial \widetilde{U}(x_0, x, m) / \partial x_0} = mp,$$

$$\frac{\partial \widetilde{U}(x_0, x, m) / \partial m}{\partial \widetilde{U}(x_0, x, m) / \partial x_0} \geq px,$$

and the compensated demand function as $x = \widetilde{x}(p,m)$, where we suppress the fixed utility level.

Production of differentiated consumer good i, $i \in M$, is denoted by Y_i. Consumer goods are differentiated and there is only one firm producing a particular variety in a community. Production technology is the same as before and the labor requirement for differentiated good production is $N_{Ci} = cY_i + a$. The profit of a firm is $\pi_i = (p_i - wc)Y_i - wa$.

Each producer is small and maximizes his/her profit, taking all the variables other than his/her own price as fixed. The perceived demand function is then:

$$Y_i = D(p_i;\{p_{i'}\}_{i'\neq i}, N) = Nx(p_i, \{p_{i'}\}_{i'\neq i}),$$

with $\{p_{i'}\}_{i'\neq i}$ and N taken as fixed. The perceived price elasticity of demand is:

$$\zeta_i = -\frac{\partial D_i}{\partial p_i}\frac{p_i}{Y_i} = -\frac{\partial x_i}{\partial p_i}\frac{p_i}{x_i} = \zeta\left(p_i;\{p_{i'}\}_{i'\neq i}\right),$$

where the second equality is obtained because a producer takes the population of the city, N, as fixed. In the symmetric equilibrium that we focus on, the price elasticity of demand can be written as $\zeta = \zeta(p, m)$. The first-order condition for profit maximization is then the same as that obtained in the intermediate input case, (9). The free-entry/zero-profit condition is also the same as (10).

The labor requirement for differentiated good production is:

$$N_C = m(cY + a), \tag{33}$$

and the market equilibrium in the differentiated good market requires:

$$Y = N\tilde{x}(p, m). \tag{34}$$

The production function of the homogeneous good is $Y_0 = G^U(N_0)$ in cities. The wage rate in a city has to be equalized between the differentiated and homogeneous good sectors so that:

$$w = G^{U}{}'(N - N_C). \tag{35}$$

So far we have obtained five equations, (9), (10), (33), (34), and (35), involving six endogenous variables in a city, m, Y, N, p, w, and N_C. These equations are sufficient to solve for the first five variables as a function of N_C : $m = \tilde{m}(N_C)$, $Y = \tilde{Y}(N_C)$, $N = \tilde{N}(N_C)$, $p = \tilde{p}(N_C)$, and $w = \tilde{w}(N_C)$.

The production function of the homogeneous good in the rural area is $Y_0 = G^A(N_0)$. The wage rate in the rural area is then $w^A = G^{A}{}'(N_0)$.

Price distortions

In the differentiated consumer good case, the marginal social benefit of a differentiated good is its marginal utility evaluated in monetary units, which equals its price from utility maximization: $MB_Y = (\partial U / \partial x_i)/(\partial U / \partial x_0) = p$. The marginal social cost is the same as before, $MC_Y = wc$, and the price distortion is $\tau_Y = p - wc$. The marginal social benefit of variety can be obtained as follows. Adding a variety increases the utility of a city resident by $\partial \widetilde{U}(x_0, x, m) / \partial m$. Because all residents benefit from the introduction of a new variety, we have to sum this over all residents in a city. Converting this into pecuniary terms yields the social benefit: $MB_m = N(\partial \widetilde{U} / \partial m)/(\partial \widetilde{U} / \partial x_0)$. The social cost is the same as before, $MC_m = w(cY + a)$, and the variety distortion is:

$$\tau_m = MB_m - MC_m = N \frac{\partial \widetilde{U} / \partial m}{\partial \widetilde{U} / \partial x_0} - w(cY + a). \tag{36}$$

The marginal social benefit of labor in the differentiated good sector arises from an increase in variety as well as an increase in output. An additional worker increases the production of each differentiated good by $\widetilde{Y}'(N_C)$ and variety by $\widetilde{m}'(N_C)$. The sum of the benefits from these two routes is $MB_N = MB_m \widetilde{m}'(N_C) + mMB_Y \widetilde{Y}'(N_C)$. The social cost of labor is the value of the marginal product in the homogeneous good sector, which equals the urban wage rate: $MC_N = w$. The wage distortion is then:

$$\tau_N = MB_m \widetilde{m}'(N_C) + mMB_Y \widetilde{Y}'(N_C) - w. \tag{37}$$

This formula is the same as that obtained in the intermediate differentiated good case except for the number of workers N_C. Because workers in the homogeneous good industry do not involve agglomeration economies, we have to exclude them from the source of price distortions. Following the same procedure as before, we can rewrite the wage distortion as:

$$\tau_N = \tau_m \widetilde{m}'(N_C) + m\tau_Y \widetilde{Y}'(N_C). \tag{38}$$

Harberger formula

Now, we are ready to examine the welfare impacts of transportation improvements. Because the utility levels are equal in equilibrium and the Allais surplus assumes that they are fixed at the initial levels, we have:

$$\widetilde{U}(x_0, x, m) = \widetilde{U}(x_0^A, 0, 0) = \overline{U} ,$$

with a fixed $\overline{U}$. Total differentiation then yields:

$$\frac{\partial \widetilde{U}}{\partial x_0} dx_0 + \frac{\partial \widetilde{U}}{\partial x} dx + \frac{\partial \widetilde{U}}{\partial m} dm = \frac{\partial \widetilde{U}}{\partial x_0} dx_0^A = 0 .$$

Dividing the utility change by the marginal utility of the numéraire, and noting the definitions of the marginal social benefits of a differentiated good and variety, we can rewrite this as:

$$dx_0^j + m^j MB_Y^j dx^j + \frac{MB_m^j}{N^j} dm^j = dx_0^A = 0 . \qquad (39)$$

By definition, the Allais surplus S satisfies:

$$S = \sum_j G^U(N_0^j) + G^A(N^A) - \sum_j N^j x_0^j - N^A x_0^A - \sum_j TC(N^j, t^j) .$$

Starting from an equilibrium where all cities have identical allocations, we consider changes in transportation costs, t . Totally differentiating the Allais surplus and substituting the wage rates for the marginal productivities of labor, we obtain:

$$\begin{aligned} & dS + \sum_j N^j dx_0^j + N^A dx_0^A \\ & = \sum_j \left(w^j dN_0^j - (x_0^j + T(N^j, t^j)) dN^j - MB_t(N^j) dt^j \right) + \left(w^A - x_0^A \right) dN^A , \end{aligned}$$

where we also used the result that the derivatives of the total transportation cost $TC(N, t)$ satisfy (20) and (21). Substituting (39) and the budget constraints for urban and rural workers[5] into this equation, and noting $dY = d(Nx) = xdN + Ndx$, we can further rewrite this as:

$$dS = \sum_j \left(- MB_t^j dt^j + m^j MB_Y^j dY^j + MB_m^j dm^j - w^j dN_C^j \right).$$

Now, totally differentiating (33) and using the definitions of marginal social costs, we obtain:

$$dN_C^j = \frac{1}{w^j} \left(MC_m^j dm^j + m MC_Y^j dY^j \right).$$

Substituting this into the above equation yields the Harberger formula:

[5] Note that the budget constraints are satisfied at the initial equilibrium although in general they are not after transportation improvements.

$$dS = -\sum_j MB_t^j dt^j + \sum_j \left(\tau_m^j dm^j + m^j \tau_Y^j dY^j\right), \tag{40}$$

which is the same as that in the differentiated intermediate good model. Furthermore, using $m = \tilde{m}(N_C)$, $Y = \tilde{Y}(N_C)$, and wage distortion (38), we can rewrite this as:

$$dS = -\sum_j MB_t^j dt^j + \sum_j \tau_N^j dN_C^j .$$

Because urban workers in the homogeneous good industry do not cause agglomeration economies, the wage distortion applies to those in the differentiated good industry only.

Now, we consider a change in transportation costs in city 1, starting from a symmetric equilibrium. Because the Harberger formula (40) is the same as before, (30) holds also in the differentiated consumer good case. Condition (31) has to be modified as:

$$-\frac{dS}{dt^1} = MB_t^1 + \tau_N \left(-\frac{dN_C^1}{dt^1} - (n-1)\frac{dN_C}{dt^1} \right).$$

The last term is zero if the transportation project does not change the total labor force in the differentiated good industry. As noted earlier, only workers in the differentiated good industry contribute to agglomeration benefits. An important implication for real-world applications is that because the utility level cannot be measured directly, it is difficult to estimate the wage distortion, unlike in the differentiated intermediate input case where the reduced-form production function yields the estimate of the wage distortion.

The additively separable case

In the symmetric additively separable case, we can write the utility function as $U = U(x_0, M)$ with $M = \int u(x(i))di$. In a symmetric equilibrium the utility level of a worker is $U = U(x_0, mu(x))$. The price elasticity of demand for a differentiated good is:

$$\zeta = -\frac{\partial x_i}{\partial p_i}\frac{p_i}{x_i} = -\frac{u'(x_i)}{u''(x_i)x_i} = \frac{1}{R_R(x_i)} = \hat{\zeta}(x_i),$$

which depends only on x_i. In a symmetric equilibrium, we have:

$$\zeta = \hat{\zeta}(x) = \hat{\zeta}(Y/N).$$

Although the price elasticity does not depend on variety *m*, it does depend on the population size of a city in addition to the output level *Y* because it is determined by per capita consumption. Hence, transportation improvements change the output level Y, and the excess burden from the price distortion does not vanish, unlike in the intermediate input model.

The price distortion of the intermediate good is proportional to the measure of relative risk aversion, as in the intermediate input case: $\tau_Y = pR_R(x)$. The difference is that the relative risk aversion is not fixed in this case because the per capita consumption is endogenously determined by $x = \widetilde{Y}(N_C)/\widetilde{N}(N_C)$.

The variety distortion is proportional to the difference between the average and marginal utilities $AU = u(x)/x$ and $MU = u'(x)$:

$$\tau_m = Np\frac{x}{u'}\left(AU - MU\right).$$

Thus, the variety distortion depends crucially on the absolute level of utility. As noted in Behrens, et al. (2010), this is an important difference between the expected utility theory and models with endogenous product diversity. In expected utility theory where utility is unique up to an affine transformation, absolute utility levels do not matter. In monopolistic competition models, the value of a new variety is the difference between the utility level with equilibrium consumption and that with zero consumption, which is not affine invariant.

Because Y depends on N_C, the price distortion of labor depends on the price distortion of the intermediate good as well as the variety distortion, unlike in the intermediate input case:

$$\tau_N = \tau_m \frac{1}{cY + a} + m\left(\tau_Y - \tau_m \frac{c}{cY + a}\right)\widetilde{Y}'(N_C).$$

In the CES case with:

$$U = x_0^{1-\rho}\left(\int_0^m (x_i)^{\frac{\sigma-1}{\sigma}} di\right)^{\rho}, \ \sigma > 1,$$

the price elasticity is constant at $\zeta = \sigma$. It is straightforward to see that the production of a differentiated good Y is fixed and the same as that in the intermediate input case: $Y = (\sigma - 1)a/c$. Because of this, the CES assumption is sufficient to gurantee that the price distortion of the intermediate good has no impact on the excess burden. As noted above, however, additive separability is not sufficient unlike in the differentiated intermediate good case. The price, variety, and wage distortions are the same as those in the intermediate input case: $\tau_Y = wc/(\sigma - 1)$, $\tau_m = wa(\sigma/(\sigma - 1))$, and $\tau_N = w/(\sigma - 1)$.

5. Concluding remarks

This paper obtained cost–benefit measures for the case where monopolistic competition with differentiated products provides the microfoundation of agglomeration economies. We first examined a model with differentiated intermediate goods. The major results in this model are as follows. First, the Harberger formula for excess burden represents the extra benefits of transportation investment additional to the direct benefit if we include variety distortion in addition to price distortion. This measure of excess burden can also be expressed by using a wage distortion that captures both variety and price distortions. The agglomeration externality measure in Venables (2007) obtained from a reduced-form aggregate production function is equivalent to this measure.

Second, an improvement in urban transportation in one city increases the population in that city but reduces the populations in other cities. If the population of the rural area (or equivalently, the total population of the urban areas) is fixed, then the changes in the excess burden cancel each other out and only the direct benefit remains. Further, if migration between the rural area and cities is possible, then a transportation improvement increases the total urban population and there will be positive additional benefits.

We next examined the case where agglomeration economies originate from differentiated consumer goods. Most of the results in the earlier model carry over to this case but there are some differences. First, because some of the urban workers work in the homogeneous good industry, which does not produce agglomeration economies, the wage distortion is applied only to workers in the differentiated good industry. Second, because no data exist on utility levels, it is difficult to estimate the wage distortion. One way of overcoming this difficulty is to use the approach taken by Tabuchi and Yoshida (2000) and Asahi, Hikino, and Kanemoto (2008), which relies on the fact that housing prices reflect, among other things, agglomeration economies on the consumption side. Third, in the additively separable case, the output level of a differentiated good is fixed in the intermediate input model, but it depends on the city size in the differentiated consumer good model. Because of this, the price distortion does not cause any excess burden in the former model, but this does not hold in the latter model.

There are two practical implications of our findings. First, at least in a model of differentiated intermediate products, one can use a reduced-form aggregate production function, as in Venables (2007), to estimate the 'wider' benefits of transportation improvements. Second, whether or not substantial agglomeration benefits exist depends on where the new workers are from. If they are from another city with similar agglomeration economies, there will be little additional benefit. Conversely, if they are from rural areas with no agglomeration economies, or from small cities with only small agglomeration economies, the additional benefits may be substantial.[6]

Graham (2005, 2006, 2007a, 2007b) and Department of Transport (2005, 2008) employ a framework unlike that of Venables (2007) in modeling urban agglomeration. These particular studies use the concept of 'effective density' to measure relative proximity to urban activities, as defined for each location using a gravity model-type equation; for example, the weighted sum of the number of workers, with weights determined as a decreasing function of distance. However, even in a model of this type, we need to consider the adverse effects on areas that lose workers. We defer to future work the analysis of a second-best benefit measure based on the microfoundations of effective density.

If transportation improvements cause a merger of two cities, agglomeration might be increased without reducing agglomerations in other cities. In order to analyze a merger in our model, transportation improvements have to open up the possibility of transporting differentiated goods to another city. Using simulation models of this type, Venables and Gasiorek (1999) showed that the additional benefits are substantial amounting to around 30% to 40% of the direct benefits. Another direction for future work is to apply the technique developed in this paper to examine the generality of their results.

[6] Agglomeration economies tend to be larger in larger metropolitan areas. See Kanemoto et al. (2005) for an example of such a finding.

References

Asahi, C., S. Hikino, and Y. Kanemoto (2008): 'Consumption Side Agglomeration Economies in Japanese Cities', CIRJE Discussion Paper, No. CIRJE-F-561,.

Behrens, K., Y. Kanemoto, and Y. Murata (2010): 'The Henry George Theorem in a Second-Best World', CIRJE Discussion Paper, No. CIRJE-F-773, Faculty of Economics, University of Tokyo, and CEPR Discussion Paper, No. 8120.

Department for Transport (2005): 'Transport, Wider Economic Benefits, and Impacts on GDP', Discussion Paper (http://webarchive.nationalarchives.gov.uk/+/http://www.dft.gov.uk/pgr/economics/rdg/webia/webmethodology/sportwidereconomicbenefi3137.pdf, access date: 22/5/2011).

Department for Transport (2008): 'The Additionality of Wider Economic Benefits in Transport Appraisal', (http://www.dft.gov.uk/pgr/economics/rdg/webia/webtheory/additionalityweb, access date: 22/5/2011).

Duranton, G. and D. Puga (2004): 'Microfoundations of Urban Agglomeration Economies', In V. Henderson and J. Thisse (eds.), *Handbook of Regional and Urban Economics*, Vol. 4, North Holland, Amsterdam, 2063–2117.

Fujita, M. and J.F. Thisse (2002): *Economics of Agglomeration*, Cambridge University Press, Cambridge.

Glaeser, E. L., J. Kolko, and A. Saiz (2001): 'Consumer City', *Journal of Economic Geography*, 1, 27–50.

Graham, D. (2005): Wider Economic Benefits of Transport Improvements: Link between Agglomeration and Productivity. Stage 1 Report, Department for Transport, London.

Graham, D. (2006): Wider Economic Benefits of Transport Improvements: Link Between Agglomeration and Productivity, Stage 2 Report, Department for Transport, London.

Graham, D. (2007a): 'Agglomeration, Productivity and Transport Investment', *Journal of Transport Economics and Policy*, 41, 1–27.

Graham, D. (2007b): 'Variable Returns to Agglomeration and the Effect of Road Traffic

Congestion', *Journal of Urban Economics*, 62, 103–120.

Harberger, A.C. (1964) 'The measurement of waste', *American Economic Review*, 54, 58–76.

Kanemoto, Y. (1990): 'Optimal Cities with Indivisibility in Production and Interactions between Firms', *Journal of Urban Economics*, 27, 46–59.

Kanemoto, Y. and Mera, K. (1985): 'General Equilibrium Analysis of the Benefits of Large Transportation Improvements', *Regional Science and Urban Economics*, 15, 343-363.

Kanemoto, Y., T. Kitagawa, H. Saito, and E. Shioji (2005): 'Estimating Urban Agglomeration Economies for Japanese Metropolitan Areas: Is Tokyo Too Large? ', in A. Okabe (ed.), *GIS-Based Studies in the Humanities and Social Sciences*, Taylor & Francis, Boca Raton, 229–241.

Rosenthal, S. and W. Strange (2004): 'Evidence on the Nature and Sources of Agglomeration Economies', in J.V. Henderson and J. Thisse (eds.), *Handbook of Regional and Urban Economics*, Vol. 4, North Holland, Amsterdam, 2119–2171.

Tabuchi, T. and A. Yoshida (2000): 'Separating Urban Agglomeration Economies in Consumption and Production', *Journal of Urban Economics*, 48, 70–84.

Venables, A.J. (2007): 'Evaluating Urban Transport Improvements: Cost–Benefit Analysis in the Presence of Agglomeration and Income Taxation', *Journal of Transport Economics and Policy*, 41, 173–188.

Venables, A. and M. Gasiorek (1999): *The Welfare Implications of Transport Improvements in the Presence of Market Failure Part 1*, Report to Standing Advisory Committee on Trunk Road Assessment, DETR, London.

Vickerman, R. (2007): 'Recent Evolution of Research into the Wider Economic Benefits of Transport Infrastructure Investments', OECD/ITF Joint Transport Research Centre Discussion Paper, No. 2007-9.