GRIPS Discussion Paper 13-20

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October 2013

GRIPS
NATIONAL GRADUATE INSTITUTE FOR POLICY STUDIES

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ABSTRACT

The Department for Transport in the United Kingdom has been a pioneer in including indirect benefits in the cost–benefit analysis of a transport project. They identify three types of wider impacts, i.e., (1) agglomeration, (2) increased or decreased output in imperfectly competitive markets, and (3) labor market impacts, and provide detailed guidelines on how to estimate them. Extending a differentiated product model that provides the microfoundations of urban agglomeration economies to include all three types of the wider impacts, this paper examines whether the British methodology of estimating the wider benefits can be justified theoretically.

Keywords: wider benefits; cost–benefit analysis; agglomeration economies; new economic geography; price markup

JEL Classification: D43; R12; R13

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1. Introduction

Traditionally, cost–benefit analysis of a transport project has focused on direct benefits and costs such as reductions in travel time, operating costs for transport users, and increases in environmental costs, while ignoring indirect effects such as increases in regional production and appreciation of property prices. This practice can be justified in a first-best world with no price distortion, but, as shown in Harberger (1964), in the presence of price distortions, conventional cost–benefit analysis must be extended to include changes in deadweight losses.[1]

The Department for Transport in the United Kingdom has been a pioneer in including indirect benefits in cost–benefit analysis. The Standing Advisory Committee on Trunk Road Assessment (SACTRA) started studying this issue in 1996, which has resulted in guidelines on the wider impacts by Department for Transport (2012). They identify three types of wider impacts, i.e., (1) agglomeration, (2) increased or decreased output in imperfectly competitive markets, and (3) labor market impacts, and provide detailed guidelines on how to estimate them. The methodology has been applied to the Crossrail project in London (Crossrail Ltd, 2005; Colin Buchanan and Partners Limited, 2007) and the HS2 high-speed rail project (HS2 Ltd, 2011), among others.

Among the three types of wider benefits, the latter two have clear sources of price distortions, i.e., monopoly power and taxes, but distortions involved in the agglomeration benefits are not clear. The Department for Transport (2012) note that increased agglomeration leads to higher productivity through its impacts on business interaction, labor market interaction, and freight costs, but does not identify specific price distortions. Kanemoto (2013a, 2013b) use a model of differentiated products in deriving second-best benefit evaluation criteria for transportation improvements.[2] In a model of this type, agglomeration economies are derived from imperfect competition in the differentiated

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[1] See, for example, Venables and Gasiorek (1999), Department for Transport (2005), Graham (2005, 2006), Venables (2007), and Vickerman (2008). Arnott (2007) studies second-best congestion tolls in the presence of agglomeration externalities.

[2] These papers use the methodology developed in Behrens et al. (2010) to extend the Henry George Theorem to a second-best economy.

goods markets, and it is not clear whether the agglomeration benefits are additional to the second type of wider benefits. Extending the framework in these papers to include all three types of the wider benefits, this paper examines whether the methodology of estimating the wider benefits using the Department for Transportation guidelines has theoretical justifications. We also consider differentiated consumer goods in addition to differentiated intermediate goods, income taxes and the costs of public services to support residents and firms in a city, and heterogeneity of cities.[3] We also discuss how to estimate the parameters required to compute the wider benefits.

The organization of this article is as follows. Section 2 briefly summarizes the British approach to the estimation of wider economic benefits. Section 3 develops a theoretical model and Section 4 derives measures of indirect benefits in this model and discusses the practical implications of the theoretical results. Section 5 examines difficulties in empirical estimation of the parameter values associated with the wider benefits. Section 6 contains concluding remarks.

2. Estimation of wider economic benefits in the UK

This section summarizes the guidelines on how to appraise the three wider benefits in Department for Transport (2012) and then briefly explains the estimation of the wider economic benefits for the Crossrail project in London.

2.1. British guidelines on the estimation of wider benefits

The first type of wider benefit is called agglomeration benefits (or *WI*1 for short). The concept of effective density developed by Graham (2005, 2006) plays the key role in estimating them. The effective density is defined for each subarea in a city by a gravity-type equation, i.e., the weighted sum of the number of workers, with weights determined as a decreasing function of distance. The agglomeration benefits are calculated by applying an elasticity of productivity with respect to effective density. The elasticity depends on industry type and it is set at 0.021 for manufacturing, 0.034 for construction, 0.024 for consumer services, and 0.083 for producer services. The total increase in output caused by the change in effective density constitutes additional benefits

[3] Kanemoto (2013a) examines agglomeration benefits in a model of heterogeneous cities.

unlike in other types where only part of the increase in output represents the welfare change.

The second type of wider benefit is from an ‘output change in imperfectly competitive markets,’ which is the welfare impact that results from increased or decreased output in markets where the price diverges from the marginal cost, $WI3$. Although the impacts from increased competition, $WI2$, may exist in addition to $WI3$, the guidelines assume that they are negligible until there is further evidence to suggest otherwise. The welfare impacts of increased or decreased output in imperfectly competitive markets are calculated as a fixed proportion of total user benefits to business trips, where the proportion, called the up-rate factor, is set to be 0.1.

The third type, $WI4$, represents economic welfare impacts arising from labor market changes. The labor market changes caused by a transportation project are classified into two types: ‘labor supply change,’ which captures changes in labor participation of existing residents and ‘move to more/less productive jobs,’ which reflects changes in employment location. Because some of these impacts are already measured within the user benefits, only the ‘tax wedge’ element of the productivity increase is included in the welfare impacts, $WI4$. The impact of ‘labor supply change’ on GDP, $GP1$, is estimated using the elasticity of labor supply with respect to effective wages net of taxes and transport costs. The elasticity is estimated to be 0.1. To estimate the impact of the ‘move to more/less productive jobs’ on GDP, $GP3$, a Land Use Transport Interaction (LUTI) model must be used to forecast the employment and residential relocations caused by a transport project. A move from an area with a lower GDP per worker to an area with a higher one increases aggregate GDP.

The tax wedge applied to the impact on GDP from more/less people working, $GP1$, is 0.4 and that for the move to more/less productive jobs, $GP3$, is 0.3. The tax wedges reflect income tax, national insurance contributions, and corporation tax. The guidelines note that, in the central case, the impact of the move to more/less productive jobs, $GP3$, should be assumed to be zero, thus restricting its role to sensitivity analysis.

2.2. Economic appraisal of Crossrail

Crossrail Ltd (2005) estimates the wider economic benefits of the Crossrail project in London, following the earlier version of the guidelines (Department for Transport, 2005).

Table 1 summarizes the estimation results. From this table we can see that the estimates of the wider benefits are quite large, equal to almost half of the conventional user benefits.

The two biggest components of the wider benefits are the agglomeration benefits and the move to more productive jobs. The present value of the agglomeration benefits is £3,094m, where the increase in output from the increase in employment density arising from Crossrail is estimated to be around £100 per job per annum for central London jobs.

The wider benefits from the move to more productive jobs totals £3,232m. This is based on a scenario with 5,000 additional central London jobs by 2016 and 33,000 by 2026. The productivity differential between central London and outer London and the rest of the UK is estimated to be about £10,000–£12,000 per person per annum. The discounted present value of the total increase in GDP from the relocation of employment is then £10,772 million over 60 years. Applying the tax wedge of 30% yields the wider benefits from the move to more productive jobs.

The estimate of the benefits arising as a result of imperfect competition equals 10% of the benefits of work trips, which gives a present value of £486m.

Table 1. Benefits and costs of Crossrail

Benefits and costs	Value (£m PV)
Total costs	13,902
Less net rail revenues	–6,149
Plus indirect tax reductions	1,207
Net cost to Government	**8,960**
Conventional user benefits	**16,093**
Agglomeration benefits	3,094
Imperfect competition	486
Move to more productive jobs	3,232
Labor force participation	349
Total wider economic benefits	**7,161**

Source: Crossrail Ltd (2005)

The GDP increase arising from the increased labor force participation is valued as 21% of the time savings accruing to commuters, which equals £872m. Applying the tax wedge of 40% yields benefits of £349m.

Colin Buchanan and Partners Limited (2007) provided revised estimates of the wider benefits based on more optimistic scenarios and parameter values. Major examples of the changes are: higher employment growth scenarios, removal of the 30% cap on productivity gains from employment relocation, and a higher agglomeration elasticity of 0.125 compared with the earlier one of 0.07. These changes increase the wider economic benefits to £22.4bn (mid scenario).

3. The model

3.1. Basic structure of the model

This section presents a simple model of monocentric cities that can produce all three types of wider benefits considered by the British guidelines. In this model, agglomeration economies result from product differentiation in consumption and intermediate goods, reflecting the benefits of a larger variety of goods and services for consumers such as specialty shops and restaurants and a wider array of business-to-business services provided by financial companies, law and accounting offices, and consulting firms among others.[4]

The basic structure of the model is as follows. The economy consists of two monocentric cities, where all workers commute to the central business district (CBD). Both cities produce a homogeneous tradable good and have differentiated consumption and intermediate goods, where the intermediate goods are used in the production of the tradable good. The tradable good can be transported between cities at no cost, whereas the differentiated goods are not transportable outside the city. All production takes place in the CBD. The size of the CBD is fixed and, for simplicity, the spatial configuration inside the CBD is ignored, except for the transportation costs of differentiated goods that are assumed to be uniform within the CBD. A transportation project reduces these transportation costs in addition to the commuting costs of workers.

The cities may be heterogeneous with different technological and other conditions. Workers and consumers are, however, homogeneous. They are mobile and free to choose a city in which to live and work. The total population (or the number of households) in

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[4] Duranton and Puga (2004) distinguish three types of microfoundations of urban agglomeration: sharing, matching, and learning mechanisms. Our framework is an example of the sharing model in this classification.

the economy is $\overline{P}$, divided into the two cities with populations P^j, $j = 1,2$: $\overline{P} = P^1 + P^2$. We assume that absentee landlords own land in the cities.

3.2. Allocation within a city

This subsection characterizes the market equilibrium within a city. Although production functions are generally different across cities, we omit superscript *j* in this subsection. To avoid notational complexity, our model has only one factor of production, i.e., labor, but an extension to a multifactor model—such as capital and land—is straightforward.

3.2.1. Consumer choice

The utility function of a worker is $u = U\left(x_0, \{x_i\}_{i \in M_X}, x_N\right)$, where x_i is the consumption of differentiated good *i*, M_X is the set of available differentiated goods, x_0 is the homogeneous tradable good, and x_N is the consumption of leisure. The labor supply by a worker is $y_N = \bar{x}_N - x_N$, where $\bar{x}_N$ is the total time available for work and leisure. The tradable good is taken as a numeraire.

We assume a simple monocentric city, where all urban workers commute to the CBD, and the lot sizes of all houses are fixed and equal. We ignore the structural part of a house and assume that the alternative cost of urban land is zero. As noted above, the size of the CBD is fixed. In this simple framework, the total commuting cost in a monocentric city can be expressed as a function of the population of a city *P* and a transportation cost parameter *k*: $TC(P,k)$. If a worker is added to a city, the total commuting cost increases by the commuting cost at the edge because the city must expand to accommodate the new worker. The commuting cost for a resident living at the edge of the city, denoted by $T(P,k)$, must then satisfy:

$$T(P,k) = \frac{\partial TC(P,k)}{\partial P}. \tag{1}$$

We concentrate on a symmetric equilibrium where quantities consumed are equal for all differentiated goods with positive consumption: $x_i = x$ for $i = 1, \cdots, m_X$, where m_X is the number of (or, more precisely, the mass of the set of) differentiated consumption goods that are actually consumed. The utility function can be written as:

$$u = u(x_0, x, m_X, x_N), \tag{2}$$

and the budget constraint for a resident living at the edge of the city who pays zero rent is

$$(1-t)wy_N = x_0 + m_X p_X x + T(P,k), \tag{3}$$

where w is the before-tax wage rate, t is the income tax rate, p_X is the price of a differentiated consumption good, and labor supply y_N satisfies $y_N = \bar{x}_N - x_N$. The price of the homogeneous tradable good is 1 because it is taken as a numeraire. The rent schedule is determined such that the utility levels are equal everywhere in a city. The first-order condition for utility maximization is:

$$\frac{\partial u / \partial x}{\partial u / \partial x_0} = m_X p_X \,; \; \frac{\partial u / \partial x_N}{\partial u / \partial x_0} = (1-t)w \,; \; \frac{\partial u / \partial m_X}{\partial u / \partial x_0} \geq p_X x \,. \tag{4}$$

The price in the first equation is multiplied by m_X because x represents the common consumption level of all m_X varieties. The second equality shows that the marginal rate of substitution between leisure and the numeraire equals the after-tax wage rate. The optimality condition for the variety, m_X, is in an inequality form because even if the consumer wants more variety, it may be constrained by the supply side conditions.

3.2.2. Production of differentiated consumption goods

Production of differentiated consumption good *i*, $i \in M_X$, by a firm is denoted by Y_i. The number of varieties is m_X, which is exogenous in the short run and endogenous in the long run with free entry of producers. Only one firm produces a particular variety. Production requires only labor input, $N_i = c_X Y_i + a_X$, where the fixed cost part is a_X and the marginal cost is c_X. The profit of a firm is then $\Pi_i = (p_i - wc_X)Y_i - wa_X$. The transportation costs of differentiated goods are included in the production cost, and a transportation project reduces the marginal cost and/or the fixed cost.

Each producer is small and maximizes his/her profit, taking all the variables other than his/her own price as fixed. We concentrate on a symmetric equilibrium where the prices and quantities of all consumer goods are equal and denoted by p_X and x, respectively. The first-order condition for profit maximization implies that the price markup satisfies:

$$\mu_X = \frac{p_X - wc_X}{wc_X} = \frac{1}{\eta_X - 1} \geq 0, \tag{5}$$

where η_X denotes the price elasticity of perceived demand, which is equal for all varieties.

The total labor requirement for differentiated good production is:

$$N_X = m_X(c_X Y_X + a_X), \tag{6}$$

and the market equilibrium in the differentiated good market requires $Y_X = Px$. Note that N_X denotes the sum of labor inputs for all differentiated consumer-good producers, $N_X = \sum N_i$, while Y_X denotes the production of an individual producer.

As an increase in variety benefits all consumers, the shadow price of variety for differentiated consumer goods is $\pi_{m_X} \equiv N(\partial u / \partial m_X)/(\partial u / \partial x_0)$. The markup for variety can then be defined as:

$$\mu_{m_X} = \frac{\pi_{m_X} - p_X Y_X}{p_X Y_X} \geq 0, \tag{7}$$

where the inequality follows from the utility maximization condition (4).

We consider two cases: the short-run case where the variety, m_X, is fixed and the long-run case where it is determined by free entry of firms. In the latter case, the zero profit condition of free entry is:

$$p_X Y_X = w(c_X Y_X + a_X). \tag{8}$$

Combining this equation with the first-order condition for profit maximization yields:

$$Y_X = \frac{a_X}{c_X}\left(\eta_X - 1\right). \qquad (9)$$

Thus, the price elasticity of perceived demand determines the production level of each variety. Substituting this into the labor requirement condition, we obtain the total labor force in the differentiated consumption goods industry as the price elasticity multiplied by the fixed cost and the variety:

$$N_X = a_X m_X \eta_X . \qquad (10)$$

3.2.3. Production of the tradable good

The production of the tradable good requires differentiated intermediate inputs only,[5] and the production function is $y_0 = f\left(\{y_i\}_{i \in M_Y}\right)$, where y_0 and y_i denote the homogeneous final good and differentiated intermediate input *i*, respectively, and M_Y is the set of available intermediate goods. We assume that the production function is symmetric in intermediate inputs and that it is well behaved, so profit maximization yields a unique interior solution. The final good is homogeneous, and its transportation cost is zero. The mass of the set of intermediate goods that are actually used for production (i.e., $y_i > 0$) is denoted by m_Y and is simply called the variety.

We again concentrate on a symmetric equilibrium where all y_i's are equal. The production function of the final good can then be written as a function of the quantity of an input, *y*, and variety, m_Y: $y_0 = \hat{f}(y, m_Y)$. The zero profit condition from free entry, $\hat{f}\left(y, m_Y\right) = m_Y p_Y y$, combined with the first-order condition for profit maximization ensures that producers choose the production scale at which constant returns to scale prevail: $\partial \hat{f}(y, m_Y) / \partial y = \hat{f}(y, m_Y) / m_Y y$. This condition determines the scale of production y as a function of variety, m_Y: $y = y(m_Y)$.

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[5] Our framework of differentiated intermediate goods generalizes the model of Abdel-Rahman and Fujita (1990) from its constant-elasticity-of-substitution (CES) production function to a general functional form.

The choice of variety, m_Y, is constrained by the entry decisions of intermediate-good producers. Even if adding another variety increases profit, it may not be available in the market. The first-order condition is therefore in an inequality form: $\partial\hat{f} / \partial m_Y \geq p_Y y$. In fact, the inequality is strict in most cases.

Symmetry makes the aggregate production function of a city particularly simple. Denote the total production of an intermediate good in a city by Y_Y. The number of producers then equals Y_Y / y, and multiplying the production function of a final-good producer by this yields the aggregate production function:

$$Y_0 = F(Y_Y, m_Y) \equiv Y_Y \frac{\hat{f}(y(m_Y), m_Y)}{y(m_Y)}, \tag{11}$$

where Y_0 denotes the total production of the final good in a city. Thus, the aggregate production function is linear with respect to the total production of an intermediate good in a city, and agglomeration economies arise through the increase in the variety. The first-order conditions for profit maximization can be rewritten as:

$$\frac{F(Y_Y, m_Y)}{Y_Y} = \frac{\partial F(Y_Y, m_Y)}{\partial Y_Y} = p_Y m_Y \tag{12}$$

$$\pi_{m_Y} \equiv \frac{\partial F(Y_Y, m_Y)}{\partial m_Y} \geq p_Y Y_Y, \tag{13}$$

where the price of the differentiated intermediate good can be written as a function of variety, $p_Y(m) = \hat{f}(y(m_Y), m_Y) / m_Y y(m_Y)$, and π_{m_Y} can be interpreted as the shadow price of variety m_Y. Note that with the variety m_Y fixed, the price of an intermediate good is determined by the conditions on the final-good side only and does not depend on those on the intermediate-goods side.

3.2.4. Production of differentiated intermediate goods

Next, let us turn to the producers of the intermediate goods. The production technology parallels that for the consumer goods although the parameter values (i.e., marginal and fixed costs) can be different. The labor input required for producing Y_i of variety *i* is $N_i = c_Y Y_i + a_Y$, where the fixed cost a_Y and the marginal cost c_Y are assumed to be constant (measured in terms of labor units). Given the perceived demand function, an intermediate-good producer maximizes the profit $p_i Y_i - w(c_Y Y_i + a_Y)$. The first-order condition for profit maximization and the free entry condition in the endogenous variety case yield the same conditions as in the differentiated consumer good case. The only thing we have to do is simply to change the subscripts from *X* to *Y* in equations (5) to (10).

3.3. Initial market equilibrium

Free and costless migration of workers equalizes the utility levels in the two cities: $u\left(x_0^1, x^1, m_X^1, x_N^1\right) = u\left(x_0^2, x^2, m_X^2, x_N^2\right)$. The total labor supply in city *j* ($j = 1, 2$) is $N^j = P^j y_N^j$, which is divided between the consumption and intermediate goods sectors: $N^j = N_X^j + N_Y^j$. Furthermore, the market equilibrium for a consumer good requires $P^j x^j = Y_X^j$. The equilibrium condition for the tradable good is:

$$\sum_{j=1}^{2}\left[Y_0^j - (x_0^j + g_P^j)P^j - g_N^j N^j - TC^j(P^j, k^j)\right] = 0, \tag{14}$$

where g_P^j is the cost of public services that an additional resident imposes on the government in area *j* and g_N^j is the cost of public services associated with labor participation, e.g., the cost of child care for working couples.

4. The benefits of transportation investment in a city

We now examine the general equilibrium impacts of a marginal increase in transportation capacity in city 1, denoted by k^1. The capacity increase has impacts on the commuting costs of workers and the transport costs of differentiated goods. A marginal increase in capacity reduces the total commuting cost by $\partial TC(P^1, k^1)/\partial k^1$. It also improves intra-CBD transportation, which reduces the transport costs of differentiated consumption goods as well as the costs of delivering business-to-business services (e.g., those provided by law and accounting offices and consulting firms) by reducing the time costs of intra-city business trips. The impacts of this sort are represented by reductions in the marginal costs of differentiated goods, $dc_Y^1 / dk^1 < 0$ and $dc_X^1 / dk^1 < 0$, assuming that the marginal costs include the delivery costs. The transportation improvements also have impacts on the fixed costs of production, $da_Y^1 / dk^1 < 0$ and $da_X^1 / dk^1 < 0$, which represent reductions in setup costs for producers, for example, by reducing the construction costs of office buildings and the costs of business start-up services. The marginal direct benefit of the capacity investment is then the sum of these three types of impacts:

$$DB^1 = MB_T^1 + MB_{c_Y}^1 + MB_{c_X}^1 + MB_{a_Y}^1 + MB_{a_X}^1 , \tag{15}$$

where $MB_T^1 = -\partial TC / \partial k^1$ is the marginal reduction in commuting costs, and $MB_{c_Z}^1 = -w^1 m_Z^1 Y_Z^1 (dc_Z^1 / dk^1)$ and $MB_{a_Z}^1 = -w^1 m_Z^1 (da_Z^1 / dk^1)$, $Z = Y, X$, are the marginal reductions in marginal and fixed costs of differentiated goods, respectively. Our particular focus is on the indirect benefits that are additional to this marginal direct benefit.

This paper concentrates on the analysis of marginal changes because integrating them yields discrete changes. If a jump occurs along the equilibrium path, integration is not possible at the jump point, but we can deal with this by separately evaluating a change in welfare at that point.

We use the Allais surplus (Allais, 1943, 1977) to measure the benefits of a transport project because it is simple and consistent at the same time. We may use different consumer surplus concepts such as Marshallian consumer surplus and Hicksian compensating and equivalent variations, however, the Marshallian measure has the well-known difficulty of path dependence, and, as pointed out by Kanemoto and Mera (1985), compensating and equivalent variations yield complicated formulas in a general equilibrium setting. The Allais surplus is the amount of the numeraire good that can be extracted from the economy with the utility level fixed. In our model, the Allais surplus is given by:

$$S = \sum_{j=1}^{2} S^j \ ; \ S^j = Y_0^j - (x_0^j + g_P^j) P^j - g_N^j N^j - TC^j (P^j, k^j) , \tag{16}$$

where S^j is the surplus in city *j*.

We evaluate the change in the Allais surplus caused by a marginal increase in transport capacity k^1 in city 1. Differentiating the Allais surplus with respect to the capacity along the equilibrium path and using the relationship in (1) between the total commuting cost and the commuting cost for a resident at the edge of a city, we obtain:

$$\frac{dS}{dk^1} = MB_T^1 + \sum_{j=1}^{2}\left(\frac{dY_0^j}{dk^1} - P^j\frac{dx_0^j}{dk^1} - (x_0^j + g_P^j + T^j)\frac{dP^j}{dk^1} - g_N^j\frac{dN^j}{dk^1}\right). \tag{17}$$

Applying the first-order conditions for utility and profit maximization and market equilibrium conditions to this equation yields the following proposition. The proof is relegated to Appendix A.

Proposition 1 (Extension of Harberger's measure of welfare change): *The change in the Allais social surplus caused by a marginal change in transport capacity in city 1 satisfies*

$$\frac{dS}{dk^1} = DB^1 + \sum_{j=1}^{2}\left(AB_Y^j + AB_X^j + GB^j\right),$$

where DB^1 *is the direct benefit;* AB_X^j *and* AB_Y^j *are imperfect competition benefits in city j for the differentiated consumer and intermediate goods sectors, respectively; and* GB^j *represents the government benefit consisting of the tax benefits and public service costs in city j:*

$$AB_X^j = w^j\left(\mu_X^j m_X^j c_X^j \frac{dY_X^j}{dk^1} + \mu_{m_X}^j \frac{N_X^j}{m_X^j}\frac{dm_X^j}{dk^1}\right),$$

$$AB_Y^j = w^j\left(\mu_Y^j m_Y^j c_Y^j \frac{dY_Y^j}{dk^1} + \mu_{m_Y}^j \frac{N_Y^j}{m_Y^j}\frac{dm_Y^j}{dk^1}\right),$$

$$GB^j = (t^j w^j - g_N^j)P^j\frac{y_N^j}{dk^1} + \left((t^j w^j - g_N^j)y_N^j - g_P^j\right)\frac{dP^j}{dk^1}.$$

The indirect benefits contain imperfect competition benefits from monopolistic pricing, tax benefits from tax distortion, and public service costs associated with increased labor supply. The imperfect competition benefits apply Harberger's measure of welfare change to monopolistic price distortions in the differentiated goods sector. The Harberger measure is the weighted sum of induced changes in production, with the weights being the corresponding price distortions. When the variety is endogenous, we have to extend the Harberger measure to include the variety distortions in addition to price distortions. Proposition 1 expresses this extended Harberger measure using the price and variety markups.

Another component of Harberger's excess burden comes from income tax distortion: the indirect benefits include increases in the tax revenue caused by changes in labor supply and employment location. An increase in the tax revenue might be accompanied by a rise in the costs of public services. For example, the immigration of workers may require additional government services, and an increase in labor supply of existing households might increase the need for public services such as child care. The net benefit is positive only when the supply of public services involves significant increasing returns to scale.

The first term in GB^j corresponds to wider benefits from more/less people working ($GP1$) in the British guidelines. The guidelines appear to ignore two issues regarding the wider benefits of this type. First, increases in public service costs to support new workers must be deducted from the tax benefits. On the contrary, the guidelines assume that an increase in labor supply leads to reductions in benefits, which is one of the reasons why the tax wedge is set at 40%, as opposed to 30% for $GP3$ from the migration of workers. Careful analysis of public service costs is necessary to check whether this 10% increase in the tax wedge can be justified. Second, as in the agglomeration benefits, negative impacts on other cities must be included in the benefit estimation.

The second term in GB^j stands for the benefits from the move to more/less productive jobs impact, $GP3$. As in the case of $GP1$, required increases in public service costs and negative impacts on other areas must be included. Department for Transport (2012) explicitly takes care of the second point by requiring the use of productivity differentials from national average GDP per worker.

4.1. The fixed variety case

We now examine the imperfect competition benefits in more detail. In the short-run case where the varieties are fixed, the variety markups disappear and the agglomeration benefits are determined by price markups only. The price distortion of a differentiated good induces distortion in the wage rate in addition to the income tax distortion. This distortion makes the social value of labor higher than the market wage rate, whereas the income tax distortion causes the market wage rate to exceed the shadow price of labor for a worker. In the fixed variety case, the wage markup induced by the price distortion equals the price markup. This can be seen as follows.

The total labor force requirement (6) for differentiated consumption goods and its counterpart for intermediate goods yield:

$$Y_X = \frac{N_X - m_X a_X}{m_X c_X} \equiv \hat{Y}_X(N_X, c_X, a_X; m_X)\,;\; Y_Y = \frac{N_Y - m_Y a_Y}{m_Y c_Y} \equiv \hat{Y}_Y(N_Y, c_Y, a_Y; m_X)\,. \qquad (18)$$

In this case, there is a one-to-one relationship between the output and labor input, given the marginal and fixed costs parameters. Because optimization by consumers and producers ensures that the price of a differentiated good equals its social marginal value, the social marginal value of labor is p_X / c_X for a consumer good and p_Y / c_Y for an intermediate good. The wage markups can then be defined as:

$$\mu_{N_X} = \frac{1}{w}(\frac{p_X}{c_X} - w) = \mu_X\,;\; \mu_{N_Y} = \frac{1}{w}(\frac{p_Y}{c_Y} - w) = \mu_Y\,. \qquad (19)$$

Thus, the wage markups equal the price markups.

Substituting (18) into the aggregate production function for the homogeneous final good (11) yields:

$$Y_0 = \hat{F}(N_Y, c_Y, a_Y; m_Y) \equiv F(\hat{Y}(N_Y, c_Y, a_Y; m_Y), m_Y)\,, \qquad (20)$$

where $\partial \hat{F} / \partial N_Y = p_Y / c_Y$ from (12) and (18). In our model, where all production takes place in the CBD and the size of the CBD is fixed, the effective density can be defined simply by the number of workers. The elasticity of productivity with respect to effective density is then:

$$\rho_Y = \frac{\partial(\hat{F} / N_Y)}{\partial N_Y} \frac{N_Y}{\hat{F} / N_Y} = \frac{N_Y}{\hat{F}} \frac{\partial \hat{F}}{\partial N_Y} - 1 = \frac{N_Y}{\hat{F}} \frac{p_Y}{c_Y} - 1, \tag{21}$$

where we call this the elasticity of productivity with respect to employment density. This elasticity equals the wage markup (and hence the price markup) only if the wage rate equals the average product of labor: $w = \hat{F} / N_Y$. In such a case, the profit of a producer is zero: $p_Y Y_Y - w N_Y / m_Y = 0$. If the profit is positive, then the elasticity is smaller than the markups: $\rho_Y < \mu_{N_Y} = \mu_Y$.

The following proposition obtains the imperfect competition benefits in the fixed variety case. The proof is in Appendix B.

Proposition 2 (Fixed variety): *In the short run where the numbers of differentiated goods are fixed, the imperfect competition benefits in area j satisfy*

$$AB_X^j = \mu_X^j (MB_{c_X}^j + MB_{a_X}^j + w^j \frac{dN_X^j}{dk^1}),$$

$$AB_Y^j = \mu_Y^j (MB_{c_Y}^j + MB_{a_Y}^j + w^j \frac{dN_Y^j}{dk^1}).$$

The wage markups equal the price markups in the fixed variety case, $\mu_{N_Y}^j = \mu_Y^j$ *and* $\mu_{N_X}^j = \mu_X^j$. *The wage markup for the intermediate good is larger (smaller) than the elasticity of productivity with respect to employment density if the profit of a producer is positive (negative):*

$$\mu_{N_Y}^j \gtrless \rho_Y^j = \frac{\partial \hat{F}}{\partial N_Y^j} \frac{N_Y^j}{\hat{F}} - 1, \; if \;\; p_Y Y_Y - w \frac{N_Y}{m_Y} \gtrless 0.$$

This proposition shows that the imperfect competition benefits are composed of three elements. The first element represents reductions in the transport costs of differentiated goods multiplied by corresponding price markups. This corresponds to 'output change in imperfectly competitive markets,' *WI*3, in Department for Transport (2012), because, although the terminology is general, the guidelines limit its application to user benefits to business trips. The guidelines recommend estimating this by multiplying the total user benefits of business trips by the up-rate factor of 0.1. If the up-rate factor equals the price markup, this coincides with our result. The second element captures the additional benefits associated with reductions in the fixed costs.

The third element can be interpreted as agglomeration benefits, *WI*1, which are given by the increases in labor supply multiplied by wage rates and price markups. The wage markups equal the price markups in the fixed variety case. For differentiated intermediate goods, the elasticity of productivity with respect to employment density is related to the wage markup, but they are equal only if the profit of a differentiated good producer is zero. The method of using the effective density to estimate agglomeration benefits therefore yields a biased estimate.

In the fixed variety case, the agglomeration benefits (*WI*1) as well as 'output change in imperfectly competitive markets' (*WI*3) are derived from output changes in imperfectly competitive markets. *WI*1 reflects the increase in output caused by an increase in labor supply, whereas *WI*3 captures the increase in output with labor supply fixed caused by a reduction in transportation costs.

The proposition shows that, in the fixed variety case, the price markups can be used to estimate agglomeration benefits. Harris (1999) reports estimates of price–cost margins for manufacturing industries in the UK. Many other studies estimate price markups (or price–cost margins) using various techniques. These include pioneering studies by Appelbaum (1982), Bresnahan (1982), and Hall (1986) and a recent more sophisticated contribution by de Loecker and Warzynski (2012).

When estimating agglomeration benefits, reductions in agglomeration benefits in city 2 have to be included in the benefit estimation. Crossrail Ltd (2005) estimates the agglomeration impacts only for central London, ignoring adverse effects on other areas. This might have caused serious overestimation of agglomeration benefits.

4.2. The endogenous variety case

If the variety is endogenous and determined by the free entry of producers, the imperfect competition benefits are much more complicated. We first obtain wage markups for workers in the intermediate goods industry because they are simpler than those in the consumer goods industry.

Because the tradable good is taken as a numeraire and its price is fixed at one, demand for an intermediate good is particularly simple. As shown in Kanemoto (2013a), it becomes a function of the variety only in a symmetric equilibrium, and the price elasticity can be written as a function of the variety: $\eta_Y = \eta_Y(m_Y)$. Then, we can invert $N_Y = a_Y m_Y \eta(m_Y)$ to write the variety as a function of the total labor force in the intermediate good industry of a city: $m_Y = \tilde{m}_Y(N_Y, a_Y)$. The production of a variety also becomes a function of the labor force N_Y:

$$Y_Y = \tilde{Y}_Y(N_Y, c_Y, a_Y) \equiv (a_Y / c_Y)\big(\eta(\tilde{m}_Y(N_Y, a_Y)) - 1\big).$$

Substituting these functions into the aggregate production function (11), we obtain a reduced-form aggregate production function:

$$Y_0 = F(\tilde{Y}(N_Y, c_Y, a_Y), \tilde{m}(N_Y, a_Y)) \equiv \tilde{F}(N_Y, c_Y, a_Y). \tag{22}$$

The wage markup can then be defined as:

$$\mu_{N_Y} = \frac{1}{w}\left(\frac{\partial \tilde{F}}{\partial N_Y} - w\right). \tag{23}$$

In the same way as in the fixed variety case, the elasticity of productivity with respect to employment density is:

$$\rho_Y = \frac{\partial(\tilde{F}/N_Y)}{\partial N_Y}\frac{N_Y}{\tilde{F}/N_Y} = \frac{\partial \tilde{F}}{\partial N_Y}\frac{N_Y}{\tilde{F}} - 1. \tag{24}$$

Because the profit of a differentiated good producer is zero, this equals the wage markup:

$$\rho_Y = \mu_{N_Y}. \tag{25}$$

The demand structure is more complicated for differentiated consumption goods than for intermediate goods because they are consumed only within a city. In particular, their price elasticity depends on variables other than the variety, e.g., the total population of a city. Because of this, the above procedure to obtain the wage markup cannot be applied to consumption goods. In a special case where the price elasticity is constant, we can apply the same argument as in the intermediate good case. It can be seen that the price elasticity of demand is constant if the utility function is of the CES form.

The following proposition rewrites the results in Proposition 1, using the wage markup. The proof is in Appendix C.

Proposition 3 (Endogenous variety): *In the long run where the numbers of differentiated goods are determined by free entry, the imperfect competition benefits for differentiated intermediate goods in area j satisfy*

$$AB_Y^j = \mu_Y^j MB_{c_Y} + (\mu_{N_Y}^j \eta_Y^j - 1) MB_{a_Y} + \mu_{N_Y}^j w^j \frac{dN_Y^j}{dk^1}.$$

Furthermore, the markup for intermediate goods equals the degree of increasing returns to scale of the aggregate production function:

$$\mu_{N_Y}^j = \frac{\partial \widetilde{F}}{\partial N_Y^j} \frac{N_Y^j}{\widetilde{F}} - 1.$$

The same results hold for differentiated consumer goods if the price elasticity of demand is constant.

If the varieties are endogenous, the agglomeration benefits include the variety markups in addition to the price markups. This proposition shows that the wage markup captures both of them in the intermediate goods case. Furthermore, it equals the elasticity of productivity with respect to employment density for the intermediate goods. Unfortunately, no counterpart exists for consumer goods because the utility levels are not observable.

The first term in the imperfect competition benefits corresponds to the 'output change in imperfectly competitive markets,' *WI*3. When the variety changes, the impacts from increased competition, *WI*2, may arise. As seen above, the variety, $m_Y = \widetilde{m}_Y(N_Y, a_Y)$, does not depend on the marginal cost, but a decrease in the fixed cost increases the variety and reduces the output level of each variety. As shown in Kanemoto (2013b) and replicated here, the latter effect cancels out the direct benefit completely. The magnitude of the former effect depends on the wage markup and the price elasticity of demand. The assumption of Department for Transport (2012) that $WI2$ is negligible can be theoretically justified for changes in marginal costs.

Kanemoto (2013a, 2013b) show that the agglomeration benefits may be negative if increasing variety is anticompetitive. In such a case, increased variety is accompanied by a fall in the production level of each variety, which magnifies the price distortion. If this effect is larger than that of the increased variety, the net benefit becomes negative. The fact that virtually all the empirical studies found agglomeration economies to be positive would mean that this case is unlikely to occur in reality.

For differentiated consumer goods, agglomeration benefits include a complicated term that depends on the difference between the price and variety markups, whereas differentiated intermediate goods do not have this term in our model with symmetric production functions. Intermediate goods have another convenient feature that the agglomeration benefits can be estimated from the degree of increasing returns to scale of the aggregate production function even in the endogenous variety case. For consumer goods, this is not possible and some other method must be used to estimate agglomeration benefits. This issue will be examined in the next section.

5. Empirics of parameter estimation

Next, we turn to the issue of how to estimate the key parameters of the wider economic benefits. If entry of new firms can be ignored, it suffices to estimate the price markups at least when the source of agglomeration economies is product differentiation. Harris (1999) and Davies (1999) in the SACTRA report discuss plausible values for price–cost margins. Davies (1999) concludes that a typical price–cost margin is somewhat less than 0.2, which yields a price markup of somewhat less than 0.25. Recent estimates by De Loecker and Warzynski (2012) are in the range of 17–28%. Although many empirical studies on price markups (or, equivalently, price–cost margins) exist, it is difficult to assess the reliability of the estimates because the marginal cost is not observable directly.

More empirical research is necessary to be confident about the estimates of additional benefits.

The endogenous variety case is much more challenging because variety markups are difficult to estimate. The difficulty arises from the fact that the shadow price of variety is unobservable. In the case of intermediate goods, however, the aggregate production functions of final goods yield the estimates of agglomeration economies as shown in the preceding section.

There are a good number of empirical studies on urban agglomeration, with excellent review articles such as Rosenthal and Strange (2004), Melo et al. (2009), and Puga (2010). Most of the empirical studies rely on cross-sectional differences in productivity or wages, i.e., they are high in large/dense metropolitan areas. It is not clear that a rise in employment density caused by an improvement in transportation will have the same quantitative impact on productivity as the cross-sectional difference between areas with different density levels. The strongest evidence on this issue has been provided by Combes et al. (2008, 2010), which examine the effects of quality differences among workers. In particular, Combes et al. (2010) shows that accounting for the endogeneity of the quality and quantity of labor makes the estimates of the density elasticity smaller, yielding an elasticity of around 2%.

In the case of differentiated consumer goods, there is no aggregate production function that can be used to estimate the wage distortion. This causes a serious problem for the estimation of long-run agglomeration benefits. One possible solution is to use variation in housing or land prices. Because the utility level is equalized between cities, the benefits of increased variety are reflected in higher land prices. We now explore the possibility of using this mechanism to estimate agglomeration benefits on the consumption side.

As the utility level must be equal across cities, we have:

$$0 = E(1, p^j, m^j, w^j; u) + T(P^j, k^j),$$

where

$$E(1, p_X^j, m_X^j, w^j; u) = \min\left\{(1-t^j)w^j(x_N^j - \bar{x}_N^j) + x_0^j + m_X^j p_X^j x^j : u(x_0^j, x^j, x_N^j) \geq u\right\}$$

is the expenditure function. For a small change from a symmetric equilibrium, we have:

$$E_p dp_X^j + E_m dm_X^j + E_w dw^j + T_P dP^j + T_k dk^j = 0 .$$

This equation can be rewritten as:

$$m_X^j x^j dp_X^j + (p_X^j x^j - \frac{\partial u / \partial m_X^j}{\partial u / \partial x_0^j}) dm_X^j - (1-t^j) y_N^j dw^j + T_P dP^j + T_k dk^j = 0 .$$

Applying $N^j x^j = Y_X^j$ to this equation yields:

$$\mu_{m_X}^j dm_X^j = \frac{1}{p_X^j Y_X^j} \left\{ m_X^j Y_X^j dp_X^j - (1-t^j) y_N^j N^j dw^j + N^j T_P dP^j + N^j T_k dk^j \right\}.$$

We can estimate the variety markup $\mu_{m_X}^j$ from this relationship, and, combining this with the price markup, we obtain the wage markup. Unfortunately, this approach has a serious weakness: if there are city-specific amenities other than agglomeration economies, separating them from the benefits of increasing variety is difficult.

6. Concluding remarks

Kanemoto (2013a, 2013b) examine the indirect (or 'wider') benefits of a transportation improvement in a framework with explicit microfoundations of urban agglomeration. This paper extends the model to include the three elements of wider economic benefits identified in the guidelines of the Department for Transport (2012). Although the basic structure of these guidelines has a theoretical foundation, we found several points that might lead to biased estimates.

First, these guidelines ignore the negative impacts on other cities when estimating agglomeration benefits and benefits from labor supply increases. An increase in agglomeration for one city is accompanied by decreases for other cities. Ignoring the negative impacts on other cities causes overestimation of the benefits.

Second, the tax benefits of increased labor participation and relocation to high productivity areas are at least partially offset by increases in public service costs. The guidelines assume the opposite for an increase in labor participation: a 10% reduction in government costs in addition to the tax wedge of 30%. A careful examination of public service costs is necessary for reliable benefit estimates of labor market impacts.

Third, when varieties are endogenous, agglomeration benefits involve variety markups in addition to price markups. Agglomeration benefits can become negative if increasing variety is anticompetitive.

Fourth, the effective density approach to estimating the agglomeration benefits can be justified for differentiated intermediate goods if the zero-profit condition is satisfied. In the fixed variety case, this leads to underestimation if the profits of differentiated intermediate goods producers are positive. Furthermore, the approach cannot be applied to differentiated consumption goods.

Fifth, in the endogenous variety case, a decrease in fixed costs affects market competitiveness because it increases the variety and reduces the production of each variety. A change in the marginal cost does not have this effect, which provides a theoretical justification for the guidelines' assumption that the impacts from increased competition are negligible.

Many problems also exist in the estimation of key parameters such as price and variety markups and the density elasticity of productivity. Especially challenging is the estimation of the variety markups for consumer goods. One possible solution is to use variation in housing or land prices. This approach has a serious weakness because it is difficult to separate the effects of differences in variety from those in other amenities such as climate, landscape, and culture.

Another serious difficulty is estimating the causal relationship between density/size and productivity. Combes et al. (2008, 2010) provide evidence of overestimation when the quality of labor is correlated with urban agglomeration.

There are many unresolved issues left for future research. Particularly important are the following two. First, this paper analyzed only one type of agglomeration economy, i.e., those arising from sharing the variety of products. We do not expect that other types of agglomeration economies yield drastically different results, but this is to be confirmed in future research. Second, we still do not know much about agglomeration economies on the consumption side. We need theoretical and empirical research on the consumption-side agglomeration benefits.

Acknowledgments

An earlier version of this paper was presented at the Annual Conference of the International Transportation Economics Association 2013. I thank the participants there for useful comments. Financial support from Kakenhi (23330076) is gratefully acknowledged.

Appendix A. Proof of Proposition 1

Using the budget constraint (3), we can rewrite the change in the surplus as:

$$dS^j = MB_T^j dk^j + dY_0^j - P^j dx_0^j - \left((1-t^j)w^j y_N^j - p_X^j m_X^j x^j + g^j\right)dP^j . \tag{26}$$

Because the utility level is kept constant for the Allais surplus, we have:

$$\frac{\partial u}{\partial x_0^j} dx_0^j + \frac{\partial u}{\partial x^j} dx^j + \frac{\partial u}{\partial m_X^j} dm_X^j + \frac{\partial u}{\partial x_N^j} dx_N^j = du = 0 .$$

Applying to this equation the first-order conditions for consumer optimization (4) and the definition of the shadow price of variety yields:

$$P^j dx_0^j = -m_X^j p_X^j P^j dx^j - \pi_X^j dm_X^j - (1-t^j) w^j P^j dx_N^j .$$

From $N^j = P^j y_N^j$ and $x_N^j = \bar{x}_N^j - y_N^j$, we have $P^j dx_N^j = y_N^j dP^j - dN^j$. Substituting this and $P^j dx^j = dY_X^j - x^j dP^j$ into the above equation yields:

$$P^j dx_0^j = (1-t^j) w^j (dN^j - y_N^j dP^j) - p_X^j m_X^j (dY_X^j - x^j dP^j) - \pi_{m_X}^j dm_X^j .$$

Furthermore, from profit maximization of a final good producer, we have:

$$dY_0^j = \frac{\partial F(Y_Y^j, m_Y^j)}{\partial Y_Y^j} dY_Y^j + \frac{\partial F(Y_Y^j, m_Y^j)}{\partial m_Y^j} dm_Y^j = p_Y^j m_Y^j dY_Y^j + \pi_{m_Y}^j dm_Y^j .$$

Substituting these equations into (26) yields:

$$\begin{aligned} dS^j = MB_T^j &+ \left(p_Y^j m_Y^j dY_Y^j + \pi_{m_Y}^j dm_Y^j - w^j dN_Y^j\right) + \left(p_X^j m_X^j dY_X^j + \pi_{m_X}^j dm_X^j - w^j dN_X^j\right) \\ &+ t^j w^j dN^j - g^j dP^j \end{aligned} . \tag{27}$$

Now, from the labor requirement for the differentiated consumer good (6) and the corresponding condition for intermediate goods, we have:

$$dN_Z^j = \frac{N_Z^j}{m_Z^j} dm_Z^j + m_Z^j c_Z^j dY_Z^j + m_Z^j Y_Z^j dc_Z^j + m_Z^j da_Z^j \text{, for } Z = X, Y .$$

Substituting this into (27) and using definitions of the variety markups and the direct benefits yields Proposition 1.

Appendix B. Proof of Proposition 2

In the fixed variety case, we do not have the variety change terms in the imperfect competition benefits and they are simply:

$$AB_Y^j = \mu_Y^j w^j m_Y^j c_Y^j \frac{dY_Y^j}{dk^1};\; AB_X^j = \mu_X^j w^j m_X^j c_X^j \frac{dY_X^j}{dk^1}.$$

From $dN_Y^j = m_Y^j c_Y^j dY_Y^j + m_Y^j Y_Y^j dc_Y^j + m_Y^j da_Y^j$ and the corresponding equation for the consumer good, we can rewrite these equations to obtain Proposition 2.

Appendix C. Proof of Proposition 3

We first examine the imperfect competition benefits for differentiated intermediate goods. From the definitions of $\tilde{m}_Y(N_Y, a_Y)$ and $\tilde{Y}_Y(N_Y, c_Y, a_Y)$, we obtain:

$$\frac{dm_Y}{dk^1} = \frac{\partial \tilde{m}_Y}{\partial N_Y}\frac{dN_Y}{dk^1} + \frac{\partial \tilde{m}_Y}{\partial a_Y}\frac{da_Y}{dk^1}$$

$$\frac{dY_Y}{dk^1} = \frac{\partial \tilde{Y}_Y}{\partial N_Y}\frac{dN_Y}{dk^1} - \frac{Y_Y}{c_Y}\frac{dc_Y}{dk^1} + \left(\frac{Y_Y}{a_Y} + \frac{a_Y}{c_Y}\eta_Y' \frac{\partial \tilde{m}_Y}{\partial a_Y}\right)\frac{da_Y}{dk^1}.$$

Substituting these equations into the agglomeration benefits and applying the definition of the wage markup yields:

$$AB_Y = w\mu_{N_Y}\frac{dN_Y}{dk^1} - w\mu_Y m_Y Y_Y \frac{dc_Y}{dk^1} + wm_Y\frac{da_Y}{dk^1} + w\left(\mu_Y m_Y a_Y \eta_Y' + \mu_{m_Y}\frac{N_Y}{m_Y}\right)\frac{\partial \tilde{m}_Y}{\partial a_Y}\frac{da_Y}{dk^1},$$

where we have used the relationship, $\mu_Y = a_Y / c_Y Y_Y$, to obtain the third term on the right-hand side. From the definitions of direct benefits, we can rewrite this as:

$$AB_Y = \mu_Y MB_{c_Y} - MB_{a_Y} + \left(w\mu_Y m_Y a_Y \eta_Y' + w\mu_{m_Y}\frac{N_Y}{m_Y}\right)\frac{\partial \tilde{m}_Y}{\partial a_Y}\frac{da_Y}{dk^1} + w\mu_{N_Y}\frac{dN_Y}{dk^1}.$$

Now, it is easy to see that $\tilde{m}_Y(N_Y, a_Y)$ and $\tilde{Y}_Y(N_Y, c_Y, a_Y)$ satisfy:

$$\frac{\partial \tilde{Y}_Y}{\partial N_Y} = \frac{a_Y}{c_Y} \eta'_Y \frac{\partial \tilde{m}_Y}{\partial N_Y}$$

$$\frac{\partial \tilde{m}_Y}{\partial N_Y} = -\frac{1}{m_Y \eta_Y} \frac{\partial \tilde{m}_Y}{\partial a_Y}.$$

Using these relationships, we can rewrite the wage markup as:

$$\mu_{N_Y} = -\left(\mu_Y m_Y a_Y \eta'_Y + \mu_{m_Y} \frac{N_Y}{m_Y} \right) \frac{1}{m_Y \eta_Y} \frac{\partial \tilde{m}_Y}{\partial a_Y}.$$

Substituting this into the imperfect competition benefits above yields:

$$AB_Y = \mu_Y MB_{c_Y} + (\mu_{N_Y} \eta_Y - 1) MB_{a_Y} + \mu_{N_Y} w \frac{dN_Y}{dk^1}.$$

In the constant elasticity case for consumer goods, we can follow the same procedure as in the differentiated good case to obtain:

$$AB_X = \mu_X MB_{c_X} + (\mu_{N_X} \eta_X - 1) MB_{a_X} + \mu_{N_X} w \frac{dN_X}{dk^1}.$$

References

Abdel-Rahman, H., Fujita, M., 1990. Product variety, Marshallian externalities, and city sizes. Journal of Regional Science 30, 165–183.

Allais, M., 1943. A la Recherche d’une Discipline´ Economique, vol. I. Paris: Imprimerie Nationale.

Allais, M., 1977. Theories of general economic equilibrium and maximum efficiency. In: Schwödiauer, G. (ed.) Equilibrium and Disequilibrium in Economic Theory. D. Reidel Publishing Company, Dordrecht, pp. 129–201.

Appelbaum, E., 1982. The estimation of the degree of oligopoly power. Journal of Econometrics 19, 287–299.

Arnott, R., 2007. Congestion tolling with agglomeration externalities. Journal of Urban Economics 62, 187–203.

Behrens, K., Kanemoto, Y., Murata, Y., 2010. The Henry George Theorem in a second-best world. Faculty of Economics, University of Tokyo, CIRJE Discussion Paper No. CIRJE-F-773.

Bresnahan, T.F., 1982. The oligopoly solution concept is identified. Economics Letters 10, 87–92.

Crossrail Ltd., 2005, Economic appraisal of Crossrail 2005, 1–10. (http://www.crossrail.co.uk/assets/download/742, access date: 28 May 2013).
Colin Buchanan and Partners Limited, 2007. The economic benefits of Crossrail, (http://www.crossrail.co.uk/assets/download/743, access date: 29 29 January 2013).
Combes, P.-P., Duranton, G., Gobillon, L., 2008. Spatial wage disparities: Sorting matters! Journal of Urban Economics 63, 723–742.
Combes, P.-P., Duranton, G., Gobillon, L., Roux, S., 2010. Estimating agglomeration effects with history, geology, and worker fixed effects, in: Glaeser, E.L. (Ed.), Agglomeration Economics. Chicago University Press, Chicago, IL, pp. 15–65.
Davies, S., 1999. Review of the evidence on the incidence of imperfect competition in the UK, in: Report to Standing Advisory Committee on Trunk Road Assessment. DETR, London, pp. 91–104.
De Loecker, J., Warzynski, F., 2012. Markups and firm-level export status. American Economic Review 102, 2437–2471.
Department for Transport, 2005. Transport, wider economic benefits, and impacts on GDP, Discussion Paper (http://webarchive.nationalarchives.gov.uk/+/http://www.dft.gov.uk/pgr/economics /rdg/webia/webmethodology/sportwidereconomicbenefi3137.pdf, access date: 22 May 2011).
Department for Transport, 2012. The wider impacts sub-objective, TAG Unit 3.5.14, (http://www.dft.gov.uk/webtag/documents/expert/pdf/U3_5_14-wider-impacts120723.pdf, access date: 13 May 2013).
Duranton, G., Puga, D., 2004. Micro-foundations of urban agglomeration economies, in: Henderson, J.V., Thisse, J.-F. (Eds.), Handbook of Regional and Urban Economics, Volume 4. Elsevier, Amsterdam, San Diego and Oxford, pp. 2063–2117.
Graham, D., 2005. Wider economic benefits of transport improvements: Link between agglomeration and productivity. Stage 1 Report, Department for Transport, London.
Graham, D., 2006. Wider economic benefits of transport improvements: Link between agglomeration and productivity. Stage 2 Report, Department for Transport, London.
Hall, R.E., 1986. Market structure and macroeconomic fluctuations. Brookings Papers on Economic Activity, 285–322.
Harberger, A.C., 1964. The measurement of waste. American Economic Review 54, 58–76.
Harris, R.I.D., 1999. Incidence of imperfect competition in UK sectors and regions, in: Report to Standing Advisory Committee on Trunk Road Assessment. DETR, London, pp. 77–90.
HS2 Ltd., 2011. Valuing the benefits of HS2 (London – West Midlands), 1–18 (http://www.hs2.org.uk/sites/default/files/inserts/Valuing%20the%20Benefits%20of%20HS2%20%28London-West%20Midlands%29.pdf, access date: 28 May 2013).
Kanemoto, Y., 2013a. Second-best cost-benefit analysis in monopolistic competition models of urban agglomeration. Journal of Urban Economics 76, 83–92.
Kanemoto, Y., 2013b. Evaluating benefits of transportation in models of new economic geography. Economics of Transportation 2, 53–63.
Kanemoto, Y., Mera, K., 1985. General equilibrium analysis of the benefits of large transportation improvements. Regional Science and Urban Economics 15, 343–363.
Melo, P.C., Graham, D.J., Noland, R.B., 2009. A meta-analysis of estimates of urban agglomeration economies. Regional Science and Urban Economics 39, 332–342.

Puga, D., 2010. The magnitude and causes of agglomeration economies. Journal of Regional Science 50, 203–219.

Rosenthal, S.S., Strange, W.C., 2004. Evidence on the nature and sources of agglomeration economies, in: Henderson, J.V., Thisse, J.-F. (Eds.), Handbook of Regional and Urban Economics. Volume 4. Cities and Geography. Elsevier, Amsterdam; San Diego and Oxford, pp. 2119–2171.

Venables, A., Gasiorek, M., 1999. The welfare implications of transport improvements in the presence of market failure. Part 1, Report to Standing Advisory Committee on Trunk Road Assessment. DETR, London.

Venables, A.J., 2007. Evaluating urban transport improvements: cost–benefit analysis in the presence of agglomeration and income taxation. Journal of Transport Economics and Policy 41, 173–188.

Vickerman, R., 2008. Transit investment and economic development. Research in Transportation Economics 23, 107–115.