DEA MODELS INVOLVING FUTURE PERFORMANCE

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Abstract: In many practical applications, past results are not sufficient for evaluating a DMU's performance in highly volatile operating environments, such as those with highly volatile crude oil prices and currency exchange rates. That is, in such environments, a DMU's whole performance may be seriously distorted if its future performance, which is sensitive to crude oil price volatility and/or currency fluctuations, is ignored in the evaluation process. Hence, this research aims at developing a new system of DEA models that incorporate a DMU's uncertain future performance, and thus can be applied to fully measure their efficiency.

Keyword: Data envelopment analysis, Performance evaluation, Forecast, Dynamic, Entropy.

1. INTRODUCTION

Companies in most, if not all, industries can expect to experience a sustained level of volatility over the next few years. For example, crude oil prices and currency exchange rates have been exhibiting high volatility recently due to both natural and human causes, and will continue to do so. It is evident that every company, regardless of industry, is inevitably affected in different degrees by crude oil prices and/or currency exchange rates. Of particular interest in this paper are the industries that are highly sensitive to macroeconomic indices such as crude oil prices and currency exchange rates. That is, the entirety of company performance in those industries tightly depends on future volatility of the macroeconomic indices. It follows that to thoroughly evaluate such companies' performance, the evaluator must assess not only their past and present records but also future potential. Obviously, it is very challenging to evaluate a company's performance that involves a past-present-future time span. Hence, this research aims to tackle the problem of how to fully evaluate company performance in highly volatile future environments.

DEA has been well recognized as a powerful evaluation tool, and has been applied to a wide variety of practical evaluation applications. It is a non-parametric linear programming technique that measures the relative efficiency of DMUs by capturing the interaction among a common set of multiple inputs and outputs. It is noted that conventional DEA models are designed for measuring the productive efficiency of DMUs based merely on historical data. However, such past results are not sufficient for evaluating a DMU's performance in highly volatile operating environments such as those with highly volatile crude oil prices and currency exchange rates. It is evident that, in such environments, if a DMU's future performance that is sensitive to crude oil price volatility and/or currency fluctuations is ignored in the evaluation process, then its whole performance may be seriously distorted. Hence, the performance-evaluation techniques that explicitly take future volatility into account are unavoidable and indispensable in practice.

However, to our knowledge, there are no DEA models proposed in the literature that take future performance volatility into account. We believe that Chang et al. (2015) is the only research work so far that simultaneously takes past, present and future performance indicators into account. Their proposed DEA models are, however, most suitable for conducting performance evaluations for DMUs in which future potential, e.g., R&D expenses, plays a vital role in their competitive success. That is, those DEA models are not designed for evaluating the DMUs' performance that is sensitive to macroeconomic indices such as crude oil prices and currency exchange rates. Therefore, this research seeks to develop a new system of DEA models that incorporate the DMUs' uncertain future performance, and thus can be applied to fully measure the efficiency of the DMUs in volatile environments.

2. GENERALIZED DYNAMIC EVALUATION STRUCTURES

Consider a past-present-future intertemporal evaluation structure that consists of (T+k) terms (1,2,…,T+k), where terms (1,…,T-1), term T and term (T+1,…,T+k), respectively, represent the past, present and future time structures. Figure 1 demonstrates such an evaluation structure. As shown in the figure, past and present terms (1,2,…,T) exhibit a typical dynamic structure; however, future terms (T+1,…,T+k) show a non-typical dynamic structure. Therefore, this past-present-future intertemporal evaluation structure is referred to as a *generalized dynamic* structure in this research. In addition, it is noted that this evaluation structure is an integration of three different single-term structures that correspond to term $t\left(t=1,...,T\right)$, term T+1, and term $l\left(l=T+2,...,T+k\right)$, respectively. Therefore, in what follows, we first introduce the three single-term evaluation structures. Then, based on these single-term structures, we construct the complete-term evaluation structure. However, to begin with, we need to define the carry-over activities between two consecutive terms. Here, we classify the carry-overs into two types to explicitly reflect their actual characteristics: discretionary (free) and non-discretionary (fixed) carry-overs. DMUs can freely handle free carry-overs such as current assets. By contrast, DMUs cannot control fixed carry-overs such as non-current assets. Note that in the generalized dynamic structure, there are carry-overs between pairs of terms $\left(t,t+1\right)$, $t=1,...,T$; however, there are no intermediate carry-overs between pairs of future terms $\left(t,t+1\right)$, $t=T+1,...,T+(k-1)$, due to the difficulty of forecasting the related values.

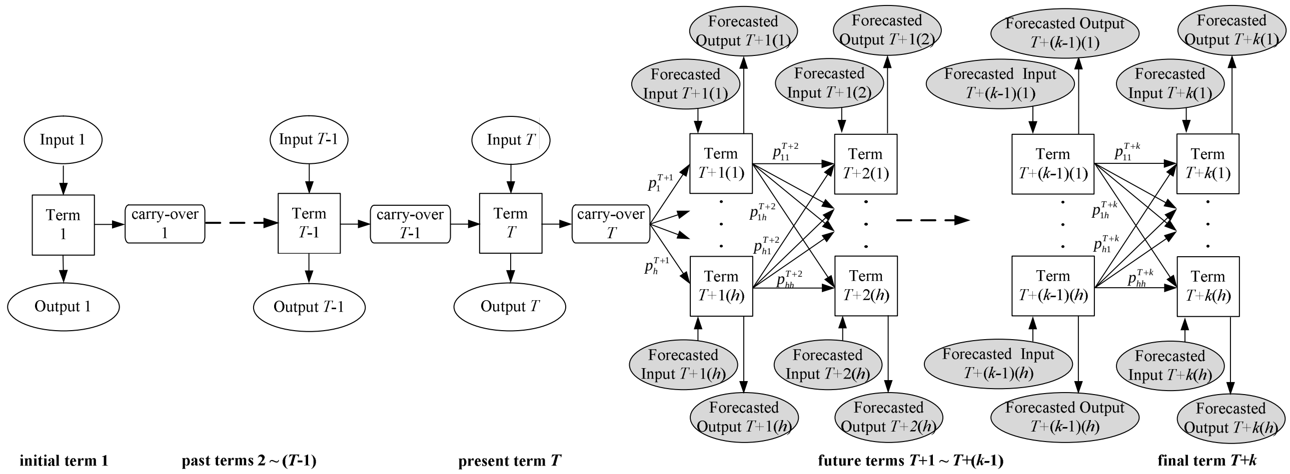

Figure 1: Generalized dynamic evaluation structure.

First, the evaluation structure with respect to term $t\left(t=1,\ldots,T\right)$ is associated with input set *t*, output set *t*, incoming carry-over *t*, and outgoing carry-over *t*; it is however noted that the incoming carry-over 1 from initial term 0 is usually unknown and is thus omitted (see Tone and Tsutsui, 2010). Second, the non-typical dynamic evaluation structure with respect to future term $T+1$ is comprised of *h* sub-terms denoted as $T+1(l), l=1,\ldots,h$. That is, it is assumed that there are *h* possible states associated with future term $T+1$; for example, there could have *h* possible crude oil prices or US dollar currency exchange rates in term $T+1$. Each sub-term $T+1(l)\left(l=1,\ldots,h\right)$ is associated with a transition probability (weight) from present term *T* to sub-term $T+1(l)$ denoted as p_l^{T+1}, such that $\sum_{l=1}^{h} p_l^{T+1}=1$. How to determine $p_l^{T+1}, l=1,\ldots,h$ is detailed in the next section. In addition, each sub-term $T+1(l)\left(l=1,\ldots,h\right)$ is associated with input set $T+1(l)$, output set $T+1(l)$, and incoming carry-over $T+1(l)$ with weight p_l^{T+1}. Third, the structure associated with future terms $T+2, T+3,\ldots,T+k$ is slightly different from that which is associated with future term $T+1$. More precisely, the only difference between the two structures is that there are no incoming carry-over activities with respect to future terms $T+2, T+3,\ldots,T+k$ because of the difficulty of forecasting their corresponding values. However, two consecutive terms between future terms $T+2, T+3,\ldots,T+k$ are still connected with occurrence conditional probability. That is, there is a transition probability (weight) from sub-term $T+g(z)\left(z=1,\ldots,h\right)$ of future term $T+g\left(g=1,\ldots,k-1\right)$ to sub-term $T+\left(g+1\right)(l)\left(l=1,\ldots,h\right)$ of future term $T+\left(g+1\right)$ that is denoted as $p_{zl}^{T+(g+1)}$. How to determine these transition probabilities is also detailed in the next section. Furthermore, each sub-term $T+g(l)\left(l=1,\ldots,h\right)$ of future term $T+g\left(g=2,\ldots,k\right)$ is associated with input set $T+g(l)$ and output set $T+g(l)$ with weight $\sum_{z=1}^{h} p_{zl}^{T+g}$. It is noted that the assumption here that there are also *h* possible states associated with future terms $T+2, T+3,\ldots,T+k$ is just for presentation convenience, but a requirement.

Lastly, Figure 1 demonstrates the complete generalized dynamic evaluation structure, displaying time spanning past-present-future periods that are constructed based on the three single-term evaluation structures described above.

3. FUTURE PERFORMANCE FORECASTS

Notice that the forecasted inputs (e.g., production costs) and outputs (e.g., selling profits) depicted in Figure 1 are actually functions of variables (e.g., crude oil prices and currency exchange rates) that are sensitive to highly volatile operating environments. It is quite possible, and common, that different DMUs have different degrees of sensitivity to the variables. Therefore, in such circumstances, to completely evaluate the DMUs, the evaluator must take future performance volatility into account, which is exactly the major point of this research. In addition, each of these variables, e.g., currency exchange rates, may be measured in several different currencies. For example, a DMU may procure resources (input costs) from and sell products (output revenues) to different countries so that it faces different currencies and thus varying currency exchange rates. Theoretically, a variable that involves *n* different currencies should be treated as *n* different variables. However, in doing so, the numbers of inputs and outputs, and thus the size of the generalized dynamic evaluation structure shown in Figure 1, will exponentially and dramatically increase. It follows that the differentiation power of the corresponding generalized dynamic DEA models will significantly decrease. Hence, in this instance, we use a single currency to measure the variables by converting other currencies into that single currency. For example, consider crude oil prices or currency exchange rates based on US dollars by converting other foreign currencies into US dollars.

There exist a variety of forecasting methods to predict the values of the above variables (Montgomery et al., 1990). However, none of them can be considered to be superior to the others in every respect (see e.g., Armstrong, 2001; Ouenniche et al., 2014). Nonetheless, there are some well-accepted principles, such as short-term forecasts that are generally more accurate than medium- and long-term ones; aggregate forecasts that are generally more precise than single ones; and simple methods that are preferable to complex methods because they are easier to understand and explain. It is noted that the development and the choice of forecasting techniques are not the focus of this research. This study utilizes the moving average method (see e.g., Montgomery et al., 1990) to estimate the future performance forecasts because the moving average method is one of the most well-known and established forecasting methods in practice (Sanders and Manrodt, 1994; Armstrong 2001). Furthermore, this research directly applies the data from public domain resources, which generally do not provide detailed information. Under such circumstances, entropy in information theory offers a feasible way for measuring the uncertainty of the probability distributions of random variables (future inputs and outputs in this research) (see, e.g., Kapur, 1989). Kapur (1989, p. 11) states that, “We should take all given information into account and we should scrupulously avoid taking into account any information that is not given to us.” This leads to the renowned maximum-entropy principle that, “aims to give us as uniform or as broad a distribution as possible, subject to the constraints being satisfied (Kapur, 1989, p. 11).” Moreover, based on data availability, future inputs and outputs are treated as discrete random variables that take a finite number of values.

The above analysis suggests that this research utilize the maximum entropy approach to determine p_l^{T+1}, the transition probability from present term T to sub-term $T+1(l)$ of future term $T+1$, and $p_{zl}^{T+(g+1)}$, the transition probability from sub-term $T+g(z)$ of future term $T+g$ to sub-term $T+(g+1)(l)$ of future term $T+(g+1)$, that are described in the preceding section.

4. GENERALIZED DYNAMIC DEA MODELS

This research proposes a new system of DEA models with embedded the generalized dynamic structure that is described in Section 2. However, the dynamic DEA models with typical dynamic structure such as those proposed in Tone and Tsutsui (2010) can be used as building blocks to develop the generalized dynamic DEA models that incorporate DMUs' uncertain future performance.

To construct the generalized dynamic DEA models, it is assumed that there are *n* DMUs (j = 1, ...,n) over (T+k) terms (t = 1,...,T+k). In each term t (t = 1,...,T), DMUs have common m inputs (i = 1,...,m) and s outputs (i = 1,...,s). On the other hand, in each term t (t = T+1,...,T+k), DMUs have common r inputs (i = 1,...,r), and/or d outputs (i = 1,...,d). That is, it is important to note that depending on the considered problems, the future terms T+1,...,T+k may not simultaneously associate both inputs and outputs. Furthermore, let x_{ijt} (i = 1,...,m) and y_{ijt} (i=1,..., s) represent, respectively, the input and output of DMU j in term t (t = 1,...,T), and $u_{ijtl}(i=1,...,r)$ and $v_{ijtl}(i=1,...,d)$ represent, respectively, the input and output of DMU j in sub-term $t(l)$ of future term t(t = T+1,...,T+k). Note and recall that both input u_{ijtl} and output v_{ijtl} are functions of variables, such as crude oil prices and currency exchange rates that are measured by a common currency, e.g., the US dollar.

In addition, recall that it is assumed that each future term t(t = T+1,...,T+k) is comprised of h sub-terms (possible states) $t(l)$,l = 1,…,h. Moreover, denote the free and fixed carry-overs (links), respectively, as z_{ijt}^{free} (i = 1,…,$nfree$; j = 1,…,n; t = 1,...,T) and z_{ijt}^{fix} (i = 1,…,$nfix$; j = 1,…,n; t = 1,…,T), where *nfree* and *nfix* are the number of free and fixed links, respectively. Recall that there are no carry-over activities with respect to future terms due to the high degree of forecast difficulty.

4.1. Production Possibility Sets

Based on the notation defined above, the production possibility set $\left\{\left(x_{it}, y_{it}, u_{itl}, v_{itl}, z_{it}^{free}, z_{it}^{fix}\right)\right\}$ with respect to the generalized dynamic DEA models is defined as follows:

$$x_{it} \geq \sum_{j=1}^{n} x_{ijt}\lambda_j^t \quad (i=1,...,m;\, t=1,...,T)$$
$$y_{it} \leq \sum_{j=1}^{n} y_{ijt}\lambda_j^t \quad (i=1,...,s;\, t=1,...,T)$$
$$u_{itl} \geq \sum_{j=1}^{n} u_{ijtl}\delta_{jl}^t \quad (i=1,...,r;\, t=T+1,...,T+k;\, l=1,...,l)$$
$$v_{itl} \geq \sum_{j=1}^{n} v_{ijtl}\delta_{jl}^t \quad (i=1,...,d;\, t=T+1,...,T+k;\, l=1,...,l)$$
$$z_{it}^{free} \quad \text{unrestricted} \quad (i=1,...,nfree;\, t=1,...,T)$$
$$z_{it}^{fix} = \sum_{j=1}^{n} z_{ijt}^{fix}\lambda_j^t \quad (i=1,...,nfix;\, t=1,...,T)$$
$$\sum_{j=1}^{n} \lambda_j^t = 1 \quad (t=1,...,T)$$
$$\sum_{j=1}^{n} \delta_{jl}^t = 1 \quad (t=T+1,...,T+k;\, l=1,...,h)$$
$$\lambda_j^t \geq 0 \quad (j=1,...,n;\, t=1,...,T)$$
$$\delta_{jl}^t \geq 0 \quad (j=1,...,n;\, l=1,...,h;\, t=T+1,...,T+k)$$

In the above production possibility set, $\lambda^t \in R^n\,(t=1,...,T)$ and $\delta_l^t \in R^n$ $(l=1,...,h;$ $t=T+1,...,T+k)$ are the intensity vectors, and the third and fourth to last constraints correspond to the variable returns-to-scale assumption (if the constraints are omitted, then the production possibility set is associated with the assumption of constant returns to scale). Furthermore, it is noted that x_{ijt} and y_{ijt}, and u_{ijtl} and v_{ijtl} on the right-hand side of the above constraints are, respectively, observed and forecasted positive data, while x_{it}, y_{it}, u_{itl}, and v_{itl} on the left-hand side of the constraints are all variables. Moreover, notice that the constraints in the production possibility set are defined separately for each term. Hence, to ensure the continuity of link flows (carry-overs) between two consecutive terms of the past (1,…,T-1), present (T) and the first future (T+1) terms, we need to include the following conditions:

$$\sum_{j=1}^{n} z_{ijt}^{free}\lambda_j^t = \sum_{j=1}^{n} z_{ijt}^{free}\lambda_j^{t+1} \quad (\forall i=1,...,nfree; t=1,...,T-1);$$

$$\sum_{j=1}^{n} z_{ijt}^{fix}\lambda_j^t = \sum_{j=1}^{n} z_{ijt}^{fix}\lambda_j^{t+1} \quad (\forall i=1,...,nfix; t=1,...,T-1);$$

$$\sum_{j=1}^{n} z_{ijT}^{free}\lambda_j^T = \sum_{l=1}^{h} p_l^{T+1}\left(\sum_{j=1}^{n} z_{ijT}^{free}\delta_{jl}^{T+1}\right) \quad (\forall i=1,...,nfree);$$

$$\sum_{j=1}^{n} z_{ijT}^{fix}\lambda_j^T = \sum_{l=1}^{h} p_l^{T+1}\left(\sum_{j=1}^{n} z_{ijT}^{fix}\delta_{jl}^{T+1}\right) \quad (\forall i=1,...,nfix).$$

4.2. DEA Models Involving Future Performance

Based on the production possibility set that is constructed in the preceding subsection, this research develops the DEA models that incorporate uncertain future performance. It is emphasized that all the proposed models are non-radial slacks-based measure (SBM) models (Tone, 2001). That is, these models consider the excesses associated with inputs and/or the shortfalls associated with outputs as the main targets of the evaluation. In addition, due to that, depending on the considered problems, the future terms T+1,...,T+k may not simultaneously associate both inputs and outputs.

The procedures for constructing input-oriented, output-oriented and non-oriented models are similar. Therefore, we here simply introduce the input-oriented model. For modeling convenience, we first denote DMU$_o$(o = 1,…,n) as follows:

$$x_{iot} = \sum_{j=1}^{n} x_{ijt}\lambda_j^t + s_{iot}^- \quad (i=1,...,m; t=1,...,T) \tag{1}$$

$$y_{iot} = \sum_{j=1}^{n} y_{ijt}\lambda_j^t - s_{iot}^+ \quad (i=1,...,s; t=1,...,T) \tag{2}$$

$$z_{iot}^{free} = \sum_{j=1}^{n} z_{ijt}^{free}\lambda_j^t + s_{iot}^{free} \quad (i=1,...,nfree; t=1,...,T) \tag{3}$$

$$z_{iot}^{fix} = \sum_{j=1}^{n} z_{ijt}^{fix}\lambda_j^t \quad (i=1,...,nfix; t=1,...,T) \tag{4}$$

$$\sum_{j=1}^{n} z_{ijt}^{free}\lambda_j^t = \sum_{j=1}^{n} z_{ijt}^{free}\lambda_j^{t+1} \quad (\forall i=1,...,nfree; t=1,...,T-1) \tag{5}$$

$$\sum_{j=1}^{n} z_{ijt}^{fix}\lambda_j^t = \sum_{j=1}^{n} z_{ijt}^{fix}\lambda_j^{t+1} \quad (\forall i=1,...,nfix; t=1,...,T-1) \tag{6}$$

$$\sum_{j=1}^{n} \lambda_j^t = 1 \quad (t=1,...,T) \tag{7}$$

$$\lambda_j^t \geq 0 \quad (\forall j,t) \tag{8}$$

$$s_{iot}^- \geq 0 \quad (\forall i,t) \tag{9}$$

$$s_{iot}^+ \geq 0 \quad (\forall i,t) \tag{10}$$

$$s_{iot}^{free}: \text{unrestricted in sign} \quad (\forall i,t) \tag{11}$$

$$u_{iotl} = \sum_{j=1}^{n} u_{ijtl}\delta_{jl}^t + e_{iotl}^- \quad (i=1,...,r; t=T+1,...,T+k; l=1,...,h) \tag{12}$$

$$v_{iotl} = \sum_{j=1}^{n} v_{ijtl}\delta_{jl}^t - e_{iotl}^+ \quad (i=1,...,d; t=T+1,...,T+k; l=1,...,h) \tag{13}$$

$$\sum_{j=1}^{n} z_{ijT}^{free}\lambda_j^T = \sum_{l=1}^{h} p_l^{T+1}\left(\sum_{j=1}^{n} z_{ijT}^{free}\delta_{jl}^{T+1}\right) \quad (\forall i=1,...,nfree) \tag{14}$$

$$\sum_{j=1}^{n} z_{ijT}^{fix}\lambda_j^T = \sum_{l=1}^{h} p_l^{T+1}\left(\sum_{j=1}^{n} z_{ijT}^{fix}\delta_{jl}^{T+1}\right) \quad (\forall i=1,...,nfix) \tag{15}$$

$$\sum_{j=1}^{n} \delta_{jl}^t = 1 \quad (t=T+1,...,T+k; l=1,...,h) \tag{16}$$

$$\delta_{jl}^t \geq 0 \quad (\forall j,l,t) \tag{17}$$

$$e_{iotl}^- \geq 0 \quad (\forall i,l,t) \tag{18}$$

$$e_{iotl}^+ \geq 0 \quad (\forall i,l,t) \tag{19}$$

The input-oriented generalized dynamic DEA model corresponding to DMU$_o$(o = 1,…,n) can be expressed as follows:

$$\theta_o^* = \min \frac{1}{\sum_{t=1}^{T+k}\alpha^t}\left[\begin{array}{l}\sum_{t=1}^{T}\alpha^t\left[1-\frac{1}{m}\left(\sum_{i=1}^{m}\frac{\rho_i^- s_{iot}^-}{x_{iot}}\right)\right]\\ +\sum_{t=T+1}^{T+k}\alpha^t\sum_{l=1}^{h}w_l^{-t}\left[1-\frac{1}{r}\left(\sum_{i=1}^{r}\frac{\mu_i^- e_{iotl}^-}{u_{iotl}}\right)\right]\end{array}\right] \quad (20)$$

subject to (1)-(19).

where α^t is the *term weight* corresponding to term $t(t=1,...,T+k)$ that is specified by the evaluator, w_l^{-t}, defined in Section 3, is the evaluator-specified *future sub-term input weight* corresponding to sub-term $t(l)(l=1,...,h)$ of future term $t(t=T+1,...,T+k)$, and ρ_i^-, μ_i^- are the evaluator-specified *past-present input weight* and *future input weight* that correspond, respectively, to past-present input $i(i=1,...,m)$ and future input $i(i=1,...,r)$. In addition, the weights are set to satisfy the following conditions:

$$\sum_{l=1}^{h}w_l^{-t}=1(t=T+1,...,T+k), \quad \sum_{i=1}^{m}\rho_i^-=m, \text{ and}$$

$$\sum_{i=1}^{r}\mu_i^-=r.$$

It is evident that the objective function involves *T*+*hk* efficiency-related scores measured by the relative slacks of inputs, where *T* scores are related to the *T* past-present terms, and *hk* scores are related to the *k* future terms, with each consisting of *h* sub-terms. That is, the objective function is defined as the weighted average of *T*+*hk* efficiency-related scores measured by the relative slacks of inputs. Note that each score is unit-invariant with a value less than or equal to 1 (the latter is realized when all the corresponding slacks are zero). It follows that the objective function value is less than or equal to 1. Recall that *future sub-term input weight* $w_l^{-t}(l=1,...,h;t=T+1,...,T+k)$ in objective function (20) is derived from $\overline{p}_l^{T+1}(l=1,...,h)$ and $\overline{p}_{zl}^{t}(z,l=1,...,h;t=T+2,...,T+k)$.

Let the optimal solution to the above model be, $\lambda_j^{t*},\delta_{jl}^{t*},s_{iot}^{-*},s_{iot}^{+*},s_{iot}^{free*},e_{iotl}^{-*},e_{iotl}^{+*} \quad (\forall i,j,t,l)$. It is important to note that, since s_{iot}^{free} is unrestricted in sign (i.e., if $s_{iot}^{free}>0$, then the current value z_{iot}^{free} is excessive and if $s_{iot}^{free}<0$, then z_{iot}^{free} is deficient), slacks in the free links are not considered in the objective function of the input-oriented past-present DEA model. However, as shown in Tone and Tsutsui (2010), the slacks can be taken into account in either of the following two ways: (1) the *ex post* way; and (2) the binary mixed integer fractional programming approach. We refer the reader to Tone and Tsutsui (2010) for the latter approach and consider only the former method. That is, let $s_{iot}^{free*-}=\max\{0,s_{iot}^{free*}\}$ and $s_{iot}^{free*+}=-\min\{0,s_{iot}^{free*}\}$.

Then, we can define the input-oriented overall efficiency θ_o^* as

$$\theta_o^* = \frac{1}{\sum_{t=1}^{T+k}\alpha^t}\left[\begin{array}{l}\sum_{t=1}^{T}\alpha^t\left[1-\frac{1}{m+nfree}\left(\sum_{i=1}^{m}\frac{\rho_i^- s_{iot}^{-*}}{x_{iot}}+\sum_{i=1}^{nfree}\frac{s_{iot}^{free*-}}{z_{iot}^{free}}\right)\right]\\ +\sum_{t=T+1}^{T+k}\alpha^t\sum_{l=1}^{h}w_l^{-t}\left[1-\frac{1}{r}\left(\sum_{i=1}^{r}\frac{\mu_i^- e_{iotl}^{-*}}{u_{iotl}}\right)\right]\end{array}\right]$$

Besides, in such a generalized dynamic evaluation structure, θ_o^* is actually the weighted average of *T*+*hk* efficiency scores that are represented by $\theta_{ot}^*, t=1,...,T$ and $\theta_{otl}^*, t=T+1,...,T+k, l=1,...,h$. That is,

$$\theta_{ot}^* = 1-\frac{1}{m+nfree}\left(\sum_{i=1}^{m}\frac{\rho_i^- s_{iot}^{-*}}{x_{iot}}+\sum_{i=1}^{nfree}\frac{s_{iot}^{free*-}}{z_{iot}^{free}}\right),(t=1,...,T)$$

;

$$\theta_{otl}^* = 1-\frac{1}{r}\left(\sum_{i=1}^{r}\frac{\mu_i^- e_{iotl}^{-*}}{u_{iotl}}\right),(t=T+1,...,T+k,l=1,...,h).$$

Therefore, the input-oriented overall efficiency, i.e., θ_o^*, can be defined as follows:

$$\theta_o^* = \frac{1}{\sum_{t=1}^{T+k} \alpha^t} \left(\sum_{t=1}^{T} \alpha^t \theta_{ot}^* + \sum_{t=T+1}^{T+k} \alpha^t \sum_{l=1}^{h} w_l^{-t} \theta_{otl}^* \right).$$

Definition 1. (input-oriented term efficient) If $\theta_{ot}^* \left(t = 1,...,T \right) = 1$ and $\theta_{otl}^* (t = T+1,...,T+k,$ $l = 1,...,h) = 1$, then DMU$_o$ is referred to as *input-oriented term efficient* with respect to past-present term $t(t=1,...,T)$ and sub-term $t(l)(l=1,...,h)$ of future term $t(t=T+1,...,T+k)$, respectively.

Definition 2. (input-oriented overall efficient) If $\theta_o^* = 1$, then DMU$_o$ is referred to as *input-oriented overall efficient*.

Theorem 1. DMU$_o$ is *input-oriented overall efficient*, if and only if all *T* + *hk* terms are *input-oriented term efficient*, i.e., $\theta_{ot}^* = 1, t = 1,...,T$ and $\theta_{otl}^* = 1, t = T+1,...,T+k, l = 1,...,h$.

5. EMPIRICAL STUDY

The proposed generalized dynamic DEA models are new to the DEA literature. Therefore, to analyze and evaluate this new system of DEA models, we conduct an empirical study based on the real data concerning high-tech IC design companies in Taiwan. It is well known that the IC design industry is extremely competitive. An IC design company usually procures raw materials from a few different countries, seeking to lower its operational costs. And, at the same time, seeks to sell its products to as many countries as possible to increase profits. Hence, an IC design company's operations performance is very sensitive to today's highly volatile international currency exchange rates. To conduct this empirical study, we extract the empirical data, comprising 40 IC design companies, from the *Taiwan Economic Journal* (TEJ) database, utilizing only the latest periods, year 2010 to year 2013. In addition, this research applies the moving average method to predict year 2014 forecasts based on the TEJ data from years 2010 to 2013. The results show that 12 out of the 40 DMUs are input-oriented *overall efficient*. Note that a DMU is input-oriented *overall efficient* if and only if the DMU's whole terms are input-oriented *term efficient*. Besides, the empirical results also show that if future performance indicators are omitted when conducting a performance evaluation, then the DMUs' performance may either be overestimated or underestimated.

6. CONCLUSIONS

This study proposes a new system of generalized dynamic DEA models that simultaneously and explicitly take DMUs' past, present and future actions into account to evaluate the DMUs' overall performance. To date, there are very limited DEA studies in the literature that consider a DMU's future performance. Actually, to the best of our knowledge, this study is the first to attempt developing DEA models for evaluating a DMU's future performance in highly volatile operating environments, with, for example, highly volatile crude oil prices and/or currency exchange rates. In addition, it is worth mentioning that this study applies the maximum entropy approach to deal with uncertain future circumstances. We believe that entropy theory can play an import role in developing the past-present-future intertemporal DEA models.

Unfortunately, due to data availability, we cannot estimate the cost of sales (input) and net revenue (output) from forecasted currency exchange rates. Recall that the forecasted inputs and outputs should be the functions of foreign exchange rates. Therefore, we have no choice but to apply the moving average method to directly forecast future inputs and outputs from historical data. That is, the forecasts cannot fully reflect the highly volatile operating environments. We believe that detailed data, if available, can further reveal the value of the proposed new past-present-future intertemporal DEA models.

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