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ABSTRACT: Extant studies take it for granted that there is a one-to-one mapping from a change in the equilibrium allocation to a change in welfare. We show that such a premise does not apply to fairly standard models of monopolistic competition. For any change in the equilibrium allocation, there exist an infinite number of possible welfare changes when the mass of varieties consumed differs between the two equilibria. Our results thus reveal a fundamental difficulty in measuring welfare changes when varieties are endogenous.

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1. Introduction

Extant studies presume that there is a one-to-one mapping from a change in the equilibrium allocation to a change in welfare. The aim of this paper is to show that the premise need not hold in various models of monopolistic competition. More precisely, we show that for any given change in the equilibrium allocation there are *infinitely many possible welfare changes* when the mass of varieties consumed differs between the two equilibria.

This one-to-many mapping arises for two reasons: the change in equilibrium, driven by an exogenous shock, is *invariant to some class of transformations* of the subutility function of each variety; and the associated welfare change *depends on this class of transformations* when consumption diversity changes.

These two results can be explained as follows. First, there exist transformations that do not affect the two conditions used for deriving the Marshallian demand function *at equilibrium*, namely that the budget constraint is satisfied with equality, and that the marginal rate of substitution equals the relative price. Therefore, the equilibrium allocation, and thus the change in the equilibrium allocation, are invariant to these transformations.

Second, such transformations, however, alter the Hicksian demand function by influencing the contribution of each variety to the utility constraint of the expenditure minimization problem. Thus, different transformations reflecting different values of an additional variety generate different expenditure functions without affecting the change in equilibrium. As a result, the associated welfare change, as measured by the expenditure functions, is not uniquely determined without knowing the exact contribution of each variety to utility.

Our finding has important implications for welfare assessments in models of monopolistic competition. Since there are infinitely many possible transformations of the subutility function that are consistent with any given change in the equilibrium allocation, we need to choose the "right" transformation to measure the welfare change. Yet, it does not affect the equilibrium and can thus hardly be identified. Even if we could identify the true transformation, it would not need to be identical to the one implicitly used in the literature. Thus, existing welfare evaluations based on a particular choice of transformation need to be qualified as they can under- or overestimate the true welfare change.

The rest of the paper is organized as follows. In Section 2, we set the stage by deriving our key results for the case of the subutility function with constant elasticity of substitution (CES). Since the seminal work by Dixit and Stiglitz (1977), this specification has been used in many fields of economics such as trade, growth, and geography (e.g., Krugman, 1980; Grossman and Helpman, 1993; Matsuyama, 1995; Melitz, 2003). More recently, the case with variable elasticity of substitution (VES) has attracted increasing attention (e.g., Zhelobodko, Kokovin, Parenti, and Thisse, 2012; Dhingra and Morrow, 2014). Therefore, in Section 3, we analyze the case of general subutility functions. We first derive a class of transformations that do not affect the equilibrium allocation in Section 3.1, and then show in Section 3.2 that those transformations however do affect welfare evaluation. In Section 4, we extend our key results and relate them to special cases that have been considered in the literature. Section 5 concludes.

2. CES subutility function

Consider an economy with a single consumption good, supplied as a continuum of differentiated varieties. Let $[0, \overline{N}]$ and $q_i \geq 0$ denote the potential range of varieties and the consumption of variety i. We define the utility function as

$$U = \int_0^{\overline{N}} \mathit{\Phi}\left(u(q_i)\right) \mathrm{d}i,$$

where $u(q_i) = q_i^{(\sigma-1)/\sigma}$ with $\sigma > 1$ is the CES subutility function and $\mathit{\Phi}$ is a monotonic transformation of u.[1] Throughout the paper, we assume that $\mathit{\Phi}(u(0)) = 0$, $\mathit{\Phi}$ is continuously differentiable, and $\mathit{\Phi} \circ u$ is strictly increasing and strictly concave. In this section we consider

$$\mathit{\Phi}(u(q_i)) = \begin{cases} a + bu(q_i) & \text{for } q_i \in (\varepsilon, \infty) \\ \int_0^{u(q_i)} \psi(t)\mathrm{d}t & \text{for } q_i \in [0, \varepsilon] \end{cases}, \tag{1}$$

[1] Most existing studies using the CES subutility function (e.g., Krugman, 1980; Melitz, 2003) assume that $\mathit{\Phi}(u(q_i)) = u(q_i)$, and that the utility function is given by $[\int_0^{\overline{N}} q_i^{(\sigma-1)/\sigma} \mathrm{d}i]^{\sigma/(\sigma-1)}$. This is a monotonic transformation G of our utility function U, with $G(U) = U^{\sigma/(\sigma-1)}$, which yields the same welfare change than ours for any given shock. We thus disregard such a transformation in this section. We return to this point in Section 3.

where $a \geq 0$, $b > 0$, and $\varepsilon > 0$. When $a = 0$, $b = 1$, and ε is small enough, $\Phi(u(q_i))$ boils down "almost everywhere" to the CES function that is widely used in the literature. Note that, even when $a \neq 0$ and $b \neq 1$, $\Phi(u(q_i))$ displays CES for all $q_i \in (\varepsilon, \infty)$. Note also that if $a = 0$, $b = 1$, and $\psi(t) = 1$ for all t, then $\Phi(u(q_i)) = u(q_i)$ for all $q_i \in [0, \infty)$, which corresponds to the untransformed case.

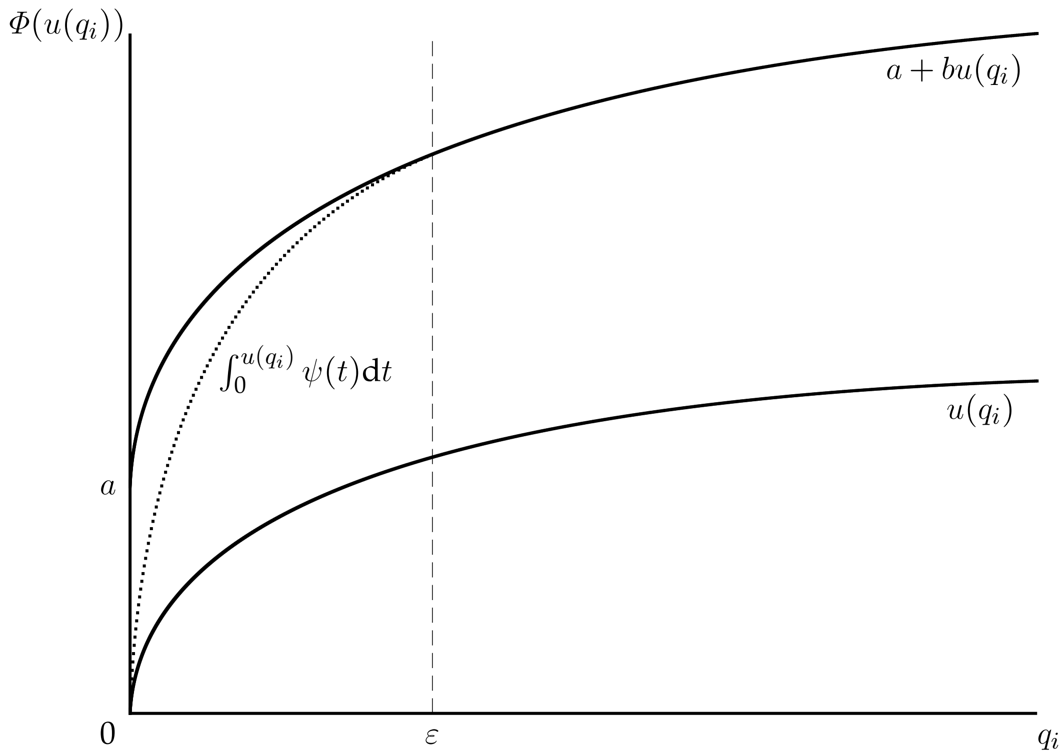
For Φ in (1) to meet the assumptions mentioned above, we assume that ψ satisfies the following three conditions: (i) $\psi > 0$ and $\psi' \leq 0$; (ii) $\int_0^{u(\varepsilon)} \psi(t)\mathrm{d}t = a + bu(\varepsilon)$; and (iii) $\psi(u(\varepsilon))u'(\varepsilon) = bu'(\varepsilon)$. Condition (i) ensures that $\Phi(u(q_i))$ is strictly increasing and strictly concave not only for $q_i \in (\varepsilon, \infty)$ but also for $q_i \in [0, \varepsilon]$. Condition (ii) implies that the left and right limits coincide at $q_i = \varepsilon$, so that $\Phi(u(q_i))$ is continuous for all $q_i \in [0, \infty)$. Condition (iii) finally implies that the left and right derivatives of $\Phi(u(q_i))$ coincide at $q_i = \varepsilon$, so that $\Phi(u(q_i))$ is differentiable for all $q_i \in [0, \infty)$. We show in Appendix A.1 how to construct a function ψ satisfying conditions (i) to (iii) for any given $\varepsilon > 0$.

Figure 1 depicts an example of Φ for some given ε. As one can see, the idea is to take an affine transformation of the CES subutility function $u(q_i) = q_i^{(\sigma-1)/\sigma}$ for $q_i \in (\varepsilon, \infty)$ and to complement it with $\int_0^{u(q_i)} \psi(t)\mathrm{d}t$ that is strictly increasing and strictly concave for $q_i \in [0, \varepsilon]$ (the dotted part below ε). This "almost affine" transformation changes the contribution of each variety to utility, yet as shown below leaves the marginal rate of substitution and the budget constraint unchanged when $q_i > \varepsilon$ for all varieties produced in the economy. We will show that there exists an $\varepsilon > 0$ such that the equilibrium consumption satisfies $q_i > \varepsilon$ for all varieties produced in the economy.

We assume that consumers supply one unit of labor inelastically. Let $w > 0$ and $p_i > 0$ denote the wage rate and the price of variety i. Each consumer solves the following utility maximization problem

$$\begin{aligned} \max_{\{q_i,\, i \in [0,\overline{N}]\}} \quad & U = \int_0^{\overline{N}} \Phi(u(q_i))\mathrm{d}i \\ \text{s.t.} \quad & \int_0^{\overline{N}} p_i q_i \mathrm{d}i = w, \\ & q_i \geq 0 \quad \forall i \in [0, N^p], \\ & q_i = 0 \quad \forall i \in (N^p, \overline{N}], \end{aligned}$$

Figure 1: An illustration of the $\Phi(u(q_i))$ function.



where $N^p > 0$ is the mass of varieties produced in the economy. Since our results hold irrespective of the determinants of N^p, we abstract from those determinants in what follows.[2] The constraint $q_i = 0$ for all $i \in (N^p, \overline{N}]$ implies that varieties that are not supplied cannot be consumed. Letting λ be the marginal utility of income, the first-order condition for variety $i \in [0, N^p]$ is given by

$$\Phi'(u(q_i))u'(q_i) \leq \lambda p_i. \tag{2}$$

We can choose $\varepsilon > 0$ in (1) such that $q_i > \varepsilon$ for all $i \in [0, N^p]$, which implies that (2) must hold with equality for all $i \in [0, N^p]$. This can be seen as follows. First, from condition (iii) above, we know that $\Phi'(u(\varepsilon))u'(\varepsilon) = \psi(u(\varepsilon))u'(\varepsilon) = bu'(\varepsilon) = b[(\sigma-1)/\sigma]\varepsilon^{-1/\sigma}$, which is strictly decreasing in ε

[2] In most models of monopolistic competition, the determinants include population size and fixed costs for production or entry. Thus, changes in population size or fixed costs affect the mass N^p of varieties supplied.

and which can be made arbitrarily large for a sufficiently small ε. Second, let $p^{\max} \equiv \max_{i\in[0,N^P]}\{p_i\}$ denote the maximum price. Then, since $p^{\max} < \infty$, we can choose $\varepsilon > 0$ in (1) such that the first-order condition (2) is violated at ε:

$$\Phi'(u(\varepsilon))u'(\varepsilon) > \lambda p^{\max}. \tag{3}$$

Last, by concavity of $\Phi \circ u$, the solution to $\Phi'(u(q_i))u'(q_i) = (\Phi \circ u)'(q_i) = \lambda p_i$ must be such that $q_i > \varepsilon$ for all $i \in [0, N^p]$.

When $q_i > \varepsilon$ holds for all $i \in [0, N^p]$, λ is independent of ε and implicitly defined as

$$\int_0^{N^p} p_i \cdot ((\Phi \circ u)')^{-1}(\lambda p_i)\mathrm{d}i = w, \tag{4}$$

where the mass N^p of varieties produced is the same as the mass N of varieties consumed. We thus denote N^p by N. Since $\Phi(u(q_i))$ displays CES for $q_i \in (\varepsilon, \infty)$, equation (4) yields a closed form solution $\lambda = [b(\sigma - 1)/\sigma][w/P(N)^{1-\sigma}]^{-1/\sigma}$, where $P(N) = [\int_0^N p_i^{1-\sigma}\mathrm{d}i]^{1/(1-\sigma)}$ is the CES price aggregate.

Furthermore, we can explicitly derive the threshold value $\overline{\varepsilon}$ below which we obtain $q_i > \varepsilon$ for all $i \in [0, N]$. Indeed, plugging the solution for λ into (3), we have $\varepsilon < (p^{\max})^{-\sigma}P(N)^{\sigma-1}w$. Thus, $\overline{\varepsilon} = (p^{\max})^{-\sigma}P(N)^{\sigma-1}w$, where the right-hand side is exogenous to consumers because $N = N^p$. Since the maximum price $p^{\max}$, the price aggregate $P(N)$, and the wage w are strictly positive and finite, $\overline{\varepsilon}$ must be strictly positive. Hence, there exists an $\varepsilon \in (0, \overline{\varepsilon})$ such that $q_i > \varepsilon$ for all $i \in [0, N]$. Note that in the symmetric case, in which all prices are the same, the threshold reduces to $\overline{\varepsilon} = w/(Np)$.

In the remainder of this section, we thus assume that $\varepsilon < \overline{\varepsilon}$ and focus on the case with $q_i > \varepsilon$ and $\Phi(u(q_i)) = a + bq_i^{(\sigma-1)/\sigma}$. Then, the first-order condition for variety $i \in [0, N]$ is given by $b[(\sigma - 1)/\sigma]q_i^{-1/\sigma} = \lambda p_i$, so that

$$\left(\frac{q_i}{q_j}\right)^{-\frac{1}{\sigma}} = \frac{p_i}{p_j}$$

for varieties $i, j \in [0, N]$. The marginal rate of substitution between any two varieties is thus unaffected by the transformation Φ. This, together with the budget constraint, which is also unaffected by Φ, implies that the Marshallian demand function for $q_i > \varepsilon$ is independent of a and b: $q_i = p_i^{-\sigma}P(N)^{\sigma-1}w$. Let

$q^{\min} \equiv \min_{i\in[0,N]}\{q_i\}$ denote the minimum consumption. Then, the condition $\varepsilon < \overline{\varepsilon}$ can be restated as $\varepsilon < \overline{\varepsilon} \equiv q^{\min} = (p^{\max})^{-\sigma}P(N)^{\sigma-1}w$. In the symmetric case, in which all prices are the same, the inequality becomes $\varepsilon < \overline{\varepsilon} = q = w/(Np)$.

To sum up, when the subutility function is of the CES form, there always exists an $\varepsilon \in (0, \overline{\varepsilon})$ such that $q_i = p_i^{-\sigma}P(N)^{\sigma-1}w > \varepsilon$ for all $i \in [0, N]$ with $N = N^p$. Hence, the values of a and b do not affect the Marshallian demand function for $q_i > \varepsilon$.

Since a and b are parameters of the utility function, they do not appear in the supply side of the economy. Hence, given the Marshallian demand function, the first-order condition for profit maximization, as well as the free entry condition and the resource constraint, is unaffected by a and b. Consequently, the equilibrium allocation, and thus the change in the equilibrium allocation driven by some shocks that affect the mass of varieties supplied, are independent of a and b.

The important point to note, however, is that the welfare change due to such shocks depends on a and b. To see this, we consider the expenditure minimization problem subject to the target utility level $\overline{U}$ as follows:

$$\begin{aligned}
\min_{\{q_i,\ i\in[0,\overline{N}]\}} \quad & \int_0^{\overline{N}} p_i q_i \mathrm{d}i \\
\text{s.t.} \quad & \int_0^{\overline{N}} \Phi(u(q_i))\mathrm{d}i = \overline{U}, \\
& q_i \geq 0 \quad \forall i \in [0, N^p], \\
& q_i = 0 \quad \forall i \in (N^p, \overline{N}],
\end{aligned}$$

where $N^p > 0$ is the mass of varieties produced in the economy and the constraint $q_i = 0$ for all $i \in (N^p, \overline{N}]$ implies that varieties that are not supplied cannot be consumed.

As before we assume that $\varepsilon \in (0, \overline{\varepsilon})$, so that $\Phi(u(q_i)) = a + bq_i^{(\sigma-1)/\sigma}$ for all $i \in [0, N^p]$ and $N = N^p$ hold. We then obtain the compensated demand function as follows:

$$q\left(p_i, P(N), \overline{U}, A(N), b\right) = p_i^{-\sigma}P(N)^{\sigma}\left[\frac{\overline{U} - A(N)}{b}\right]^{\frac{\sigma}{\sigma-1}},$$

where $A(N) = Na$. The expenditure function is then given by

$$e\left(P(N), \overline{U}, A(N), b\right) = P(N)\left[\frac{\overline{U} - A(N)}{b}\right]^{\frac{\sigma}{\sigma-1}}. \tag{5}$$

With $a = 0$ and $b = 1$, expression (5) reduces to $e(P(N), \overline{U}) = P(N)\overline{U}^{\sigma/(\sigma-1)}$, which is independent of $A(N)$ and b. Holding $\overline{U}$ constant, we take the rate of change of (5) to define the welfare change between two equilibria as follows:[3]

$$-\mathrm{d}\ln e = -\mathrm{d}\ln P(N) + \frac{\sigma}{\sigma-1}\frac{Na}{b\int_0^N q_i^{(\sigma-1)/\sigma}\mathrm{d}i}\mathrm{d}\ln N, \tag{6}$$

which measures by how much consumers can reduce expenditure in response to some shocks affecting the mass of varieties supplied.[4] Using a more general model, we show in Section 3 that (6) is the same as either equivalent or compensating variation for small shocks (marginal changes).

Expression (6) illustrates our main result: *the mapping from a change in the equilibrium mass of varieties* $\mathrm{d}\ln N$ *to a change in welfare* $-\mathrm{d}\ln e$ *is one-to-many, depending on the values of* a *and* b*.* To see this one-to-many mapping more clearly, consider a special case in which all firms have common marginal cost mw, where m is the constant marginal labor requirement and the wage rate w is taken as the numeraire. Given the CES markup pricing $p_i = [\sigma/(\sigma-1)]mw$, we then have the rate of change in the price aggregate as follows:

$$\mathrm{d}\ln P(N) = \frac{1}{1-\sigma}\mathrm{d}\ln N, \tag{7}$$

[3]When $a = 0$, expression (6) can be related to the change in the expenditure function used by, e.g., Feenstra (1994) and Redding and Weinstein (2016) to measure the impacts of new varieties on welfare.

[4]When the utility function is transformed as $G(U) = U^{\sigma/(\sigma-1)}$, the expenditure function becomes

$$e(P(N), \overline{U}, A(N), b) = P(N)\left[\frac{\overline{U}^{\frac{\sigma-1}{\sigma}} - A(N)}{b}\right]^{\frac{\sigma}{\sigma-1}}.$$

The welfare change is unaffected by G because $\overline{U}$ is held constant and does not enter (6).

because N is the only endogenous variable in $P(N)$.[5] Imposing symmetry on q_i and plugging (7) into (6), the welfare change becomes

$$-\,\mathrm{d}\ln e = \frac{1}{\sigma-1}\left(1+\frac{\sigma a}{bq^{\frac{\sigma-1}{\sigma}}}\right)\mathrm{d}\ln N. \tag{8}$$

The second term in the parenthesis captures the differential welfare effect of having non-zero a terms for the varieties: *for any change in the equilibrium mass of varieties* $\mathrm{d}\ln N$*, there exist infinitely many possible welfare changes* $-\mathrm{d}\ln e$*, depending on the values of* a *and* b*.* The larger the value of a, the greater the welfare impact of a change in the mass of varieties. Note that when $a \neq 0$, the value of b also matters for the welfare change.

The existing literature takes it for granted that there is a one-to-one mapping from $\mathrm{d}\ln N$ to $-\mathrm{d}\ln e$ by implicitly assuming that $a = 0$ holds (e.g., Krugman, 1980; Grossman and Helpman, 1993; Melitz, 2003). Indeed, when $a = 0$, the relationship between the change in the equilibrium allocation and the change in welfare reduces to $-\mathrm{d}\ln e = -\mathrm{d}\ln P(N) = [1/(\sigma-1)]\mathrm{d}\ln N$. Thus, as before, consumers spend less to achieve the target utility level $\overline{U}$ when the mass of varieties increases due to some exogenous shock, i.e., $\mathrm{d}\ln N > 0$, $\mathrm{d}\ln P(N) < 0$, and $-\mathrm{d}\ln e > 0$. However, how much less they spend is uniquely determined by the value of the elasticity of substitution σ because $-\mathrm{d}\ln e$ does not depend on the values of a and b. This conventional welfare measure underestimates the welfare change if the true transformation of the subutility function involves a positive a term. In Section 4 we will also illustrate a case where this widely used measure can overestimate the welfare change.

Our results reveal a fundamental difficulty in measuring welfare changes: the information on the equilibrium allocations before and after a shock does not allow us to evaluate welfare changes when varieties are endogenous. The

[5] When marginal cost m differs across firms (e.g., Melitz, 2003), equation (7) is given by

$$\mathrm{d}\ln P(N) = \frac{1}{1-\sigma}\left[\frac{1}{N}\int_0^N \left(\frac{m_i}{m_N}\right)^{1-\sigma}\mathrm{d}i\right]^{-1}\mathrm{d}\ln N,$$

which depends not only on the change in the mass of varieties but also on the relative efficiency of the marginal firm. Since the latter is exogenous, our main result holds for the case with heterogeneous firms.

reasons are that observing the two equilibrium quantities $q_i = 0$ and $q_i \in (\varepsilon, \infty)$ is not sufficient to capture the exact contribution of each variety to utility, and that the Marshallian demand function can depend on the values of a and b only when $q_i \in (0, \varepsilon]$. Thus, in principle, if we could observe the Marshallian demand function for all $q_i \in [0, \infty)$ for all new varieties $i \in (N, N + \mathrm{d}N]$, we could use Roy's identity to obtain the indirect utility function, which in turn could be solved for the expenditure function associated with the observed demand function (Hausman, 1981). However, in practice, this is very demanding since it requires observations $q_i \in (0, \varepsilon]$ that do not realize in equilibrium.[6]

3. General subutility functions

The example in the previous section was built for the specific case of the CES subutility function. We now show that our results extend to general subutility functions u. As in Section 2, we consider an economy with a single consumption good, supplied as a continuum of differentiated varieties. Preferences are additively separable across varieties. We denote by $[0, \overline{N}]$ the potential range of varieties in the economy, and by $[0, N^p]$, with $N^p < \overline{N}$, the range of varieties produced.

Let $p_i > 0$ and $q_i \geq 0$ denote the price and the consumption of variety $i \in [0, \overline{N}]$. Let u be a strictly increasing and strictly concave, twice continuously differentiable subutility function that satisfies $u(0) = 0$. Each consumer solves the following utility maximization problem:

$$
\begin{aligned}
\max_{\{q_i,\ i \in [0,\overline{N}]\}} \quad & U = G\left(\int_0^{\overline{N}} \Phi\left(u(q_i)\right) \mathrm{d}i\right) \\
\text{s.t.} \quad & \int_0^{\overline{N}} p_i q_i \mathrm{d}i = w, \\
& q_i \geq 0,\ \forall i \in [0, N^p], \\
& q_i = 0,\ \forall i \in (N^p, \overline{N}],
\end{aligned}
$$

[6]What is worse, the function ψ, which characterizes the Marshallian demand function for $q_i \in (0, \varepsilon]$, is not uniquely determined. We illustrate one example in Appendix A. Yet, there are many other possible specifications for ψ that affect $q_i \in (0, \varepsilon]$ while keeping the Marshallian demand function evaluated at equilibrium points, $q_i = 0$ and $q_i \in (\varepsilon, \infty)$, unchanged.

where G is a positive monotonic transformation of the utility function and the constraint $q_i = 0$ for all $i \in (N^p, \overline{N}]$ implies that varieties that are not supplied cannot be consumed. As in Section 2, we assume that $\Phi(u(0)) = 0$, that Φ is continuously differentiable, and that $\Phi \circ u$ is strictly increasing and strictly concave.

In this section we consider a general case

$$\Phi(u(q_i)) = \begin{cases} F\left(u(q_i)\right) & \text{for } q_i \in (\varepsilon, \infty) \\ \int_0^{u(q_i)} \psi(t)\mathrm{d}t & \text{for } q_i \in [0, \varepsilon] \end{cases}, \tag{9}$$

where, unlike equation (1), u need not be of the CES form and F need not be an affine transformation of u. Note that if $F(u(q_i)) = u(q_i)$ and $\psi(t) = 1$ for all t, then $\Phi(u(q_i)) = u(q_i)$, which corresponds to the untransformed case.

For Φ in (9) to meet the assumptions mentioned above, we assume that ψ satisfies the following three conditions: (i) $\psi > 0$ and $\psi' \leq 0$; (ii) $\int_0^{u(\varepsilon)} \psi(t)\mathrm{d}t = F(u(\varepsilon))$; and (iii) $\psi(u(\varepsilon))u'(\varepsilon) = F'(u(\varepsilon))u'(\varepsilon)$, where we impose $F'(u(\varepsilon)) > 0$. Condition (i) ensures that $\Phi(u(q_i))$ is strictly increasing and strictly concave for all $q_i \geq 0$. Conditions (ii) and (iii) are required for continuity and differentiability of $\Phi(u(q_i))$ for all $q_i \geq 0$. We show in Appendix A.2 how to construct a function ψ satisfying conditions (i) to (iii) for any given $\varepsilon > 0$.

Let λ denote the marginal utility of income. The first-order condition for variety $i \in [0, N^p]$ is given by

$$\Phi'(u(q_i))u'(q_i) \leq \widetilde{\lambda} p_i, \tag{10}$$

where $\widetilde{\lambda} \equiv \lambda / G'(\cdot)$. As in Section 2, we can show under what conditions there exists $\varepsilon > 0$ such that $q_i > \varepsilon$ for all $i \in [0, N^p]$, so that (10) holds with equality for all $i \in [0, N^p]$. This can be seen as follows. First, from condition (iii) above, we know that $\Phi'(u(\varepsilon))u'(\varepsilon) = \psi(u(\varepsilon))u'(\varepsilon) = F'(u(\varepsilon))u'(\varepsilon)$, which is strictly decreasing in ε because by assumption $\Phi \circ u$ is a strictly concave function. Second, let $p^{\max} \equiv \max_{i \in [0, N^P]}\{p_i\}$. Contrary to the CES case in Section 2, the marginal utility at $q_i = \varepsilon$ may be bounded even when ε gets arbitrarily small, i.e., $\lim_{\varepsilon \to 0^+} F'(u(\varepsilon))u'(\varepsilon) < \infty$ may hold. A sufficient condition for $q_i > \varepsilon$ to hold for all $i \in [0, N^p]$ is given by

$$F'(u(\varepsilon))u'(\varepsilon) > \widetilde{\lambda} p^{\max} \tag{11}$$

because, by concavity of $F \circ u$, the solution to $F'(u(q_i))u'(q_i) = \widetilde{\lambda} p_i$ must be such that $q_i > \varepsilon$ for all $i \in [0, N^p]$.

Assume that condition (11) is satisfied. Then, equation (10) must hold with equality for all $i \in [0, N^p]$, which can be restated as $F'(u(q_i))u'(q_i) = \widetilde{\lambda} p_i$ for all $i \in [0, N^p]$. In that case, $\widetilde{\lambda}$ is independent of ε and implicitly defined as

$$\int_0^{N^p} p_i \cdot ((F \circ u)')^{-1}(\widetilde{\lambda} p_i)\mathrm{d}i = w, \tag{12}$$

where the mass N^p of varieties produced is the same as the mass N of varieties consumed. Note that, by concavity of $F \circ u$, the solution for $\widetilde{\lambda}(\mathbf{p}, N, w)$ in (12) exists and is uniquely determined, where $\mathbf{p}$ is a vector of statistics that characterizes the distribution of prices $\{p_i\}_{i \in [0,N]}$.[7]

Furthermore, we can pin down the threshold value $\overline{\varepsilon}$ below which condition (11) holds. Indeed, plugging $\widetilde{\lambda}(\mathbf{p}, N, w)$ into (11), and noting that $F'(u(\varepsilon))u'(\varepsilon)$ is decreasing in ε, we know that $\varepsilon < \overline{\varepsilon}(p^{\max}, \mathbf{p}, N, w)$, where $(p^{\max}, \mathbf{p}, N, w)$ are exogenous to consumers because $N = N^p$. In Section 3.1 we restate the sufficient condition (11) to ensure $q_i > \varepsilon$ for all $i \in [0, N^p]$ by using Marshallian demand functions, and derive a closed form solution for $\overline{\varepsilon}$ in the symmetric case. We also illustrate a closed form solution for $\overline{\varepsilon}$ in an asymmetric case in Appendix B.

In the remainder of this section, we assume that $\varepsilon < \overline{\varepsilon}$ holds, which allows us to focus on the case with $q_i > \varepsilon$ and thus with $\Phi(u(q_i)) = F(u(q_i))$.

3.1 *Identifying a class of monotonic transformations that do not affect the equilibrium allocation*

We now identify a class of monotonic transformations F that do not affect the Marshallian demand functions and, therefore, the equilibrium allocation $\{q_i, \forall i \in [0, \overline{N}]\}$. Since we consider the case in which $\varepsilon < \overline{\varepsilon}$, we have $q_i > \varepsilon$ for all $i \in [0, N]$, so that the first-order conditions for utility maximization with respect to q_i and q_j imply that

$$\frac{F'\left(u(q_i)\right)u'(q_i)}{F'\left(u(q_j)\right)u'(q_j)} = \frac{p_i}{p_j} \tag{13}$$

[7] In the CES case of Section 2, $\mathbf{p}$ is unidimensional and subsumed by the price aggregate $P(N)$.

for $i, j \in [0, N]$. Expression (13) shows that the monotonic transformation G of the *utility* function does not affect the marginal rate of substitution between varieties i and j. Combining (13) and the budget constraint, the Marshallian demand function for each variety is invariant to G, reflecting the well-known property that the utility is ordinal. However, as can be seen from (13), the same does not generally apply to the monotonic transformation F of the *subutility* function.

We analyze under what conditions F does not affect the Marshallian demand functions and thus the equilibrium allocation is invariant to the transformation. To this end, we first consider the benchmark case without the transformation, i.e., $F(u(q_i)) = u(q_i)$ for all $i \in [0, \overline{N}]$. We then have $F' \equiv 1$, so that expression (13) becomes:

$$\frac{u'(q_i)}{u'(q_j)} = \frac{p_i}{p_j} \tag{14}$$

for $i, j \in [0, N]$. Hence, the monotonic transformation F of the subutility function does not affect the marginal rate of substitution, i.e., expressions (13) and (14) are equivalent, if and only if

$$F'\left(u(q_i)\right) \equiv F'\left(u(q_j)\right), \quad \forall q_i, q_j > \varepsilon. \tag{15}$$

Condition (15) holds if and only if F is of the following form:

$$F\left(u(q_i)\right) = a + b\, u(q_i), \quad \forall q_i > \varepsilon. \tag{16}$$

To prove this result, assume that (16) holds. Then, it is easy to see that (15) holds. Conversely, assume that (15) holds. Then, for any given $q_i > \varepsilon$, $F'\left(u(q_i)\right) = F'\left(u(q_j)\right)$ must hold regardless of $q_j > \varepsilon$, so that $F'\left(u(q_i)\right) = \mathcal{C}$, where $\mathcal{C}$ is a constant. This differential equation implies a general solution of the form in (16).[8] In what follows, we assume that $a \geq 0$, and that $b > 0$.

It can be verified that when (16) holds, the monotonic transformation F of the subutility function does not affect the equilibrium allocation. Indeed, in that case, both (13) and (14) lead to

$$q_j = (u')^{-1}\left(u'(q_i)\frac{p_j}{p_i}\right) \tag{17}$$

[8] In order to encompass the existing literature, we consider in Section 4 the case in which b can be a function of the mass N of varieties consumed.

for $i, j \in [0, N]$. Plugging this expression into the budget constraint and noting that $q_i = 0$ for all $i \in (N, \overline{N}]$ yield

$$\int_0^N p_j \cdot (u')^{-1} \left(u'(q_i) \frac{p_j}{p_i} \right) \mathrm{d}j = w \tag{18}$$

for $i \in [0, N]$, which implicitly defines the Marshallian demand function

$$q_i = q(p_i, \mathbf{p}, N, w), \tag{19}$$

where $\mathbf{p}$ is a vector of statistics that characterizes the distribution of prices $\{p_i\}_{i \in [0,N]}$. As can be seen from (18), the demand function in (19) does not depend on the transformation F of the subutility function. Since that transformation neither shows up in the cost function nor in the resource constraint, it will not affect the equilibrium allocation $\{q_i, \forall i \in [0, \overline{N}]\}$. Hence, we abstract from the supply side of the economy.

Using the Marshallian demand function (19), the sufficient condition $\varepsilon < \overline{\varepsilon}(p^{\max}, \mathbf{p}, N, w)$ for $q_i > \varepsilon$ to hold for all $i \in [0, N]$ can be rewritten as $\varepsilon < q(p^{\max}, \mathbf{p}, N, w)$. This can be seen as follows. First, when $\varepsilon < \overline{\varepsilon}(p^{\max}, \mathbf{p}, N, w)$ holds, we obtain $q_i = (F \circ u)'(\widetilde{\lambda} p_i)$ for all $i \in [0, N]$ from the first-order condition. Since $(F \circ u)'$ is strictly decreasing by strict concavity of $F \circ u$ and since a change in any p_i does not affect $\widetilde{\lambda}$ due to the continuum assumption, q_i must decrease in p_i. Let $q^{\min} \equiv \min_{i \in [0,N]}\{q_i\}$ denote the minimum quantity. Then, $q^{\min} = (F \circ u)'(\widetilde{\lambda} p^{\max}) = q(p^{\max}, \mathbf{p}, N, w)$. Hence, $q_i > \varepsilon$ for all $i \in [0, N]$, which is equivalent to $q^{\min} > \varepsilon$, holds when $q(p^{\max}, \mathbf{p}, N, w) > \varepsilon$. Alternatively, the sufficient condition for such an $\varepsilon > 0$ to exist can be restated as $q^{\min} > 0$.

Given $(p^{\max}, \mathbf{p}, N, w)$, the threshold $\overline{\varepsilon}$ or $q^{\min} = q(p^{\max}, \mathbf{p}, N, w)$ can be implicitly defined by using the budget constraint. Indeed, from the first-order conditions for q_i and $q^{\min} = \overline{\varepsilon}$, we can restate the sufficient condition (11) for $q_i > \varepsilon$ to hold for all varieties produced as follows:

$$\exists \varepsilon \in (0, \overline{\varepsilon}), \text{ where } \overline{\varepsilon} \text{ is given by } \int_0^N p_j \cdot (u')^{-1} \left(u'(\overline{\varepsilon}) \frac{p_j}{p^{\max}} \right) \mathrm{d}j = w. \tag{20}$$

In the symmetric case, where $p_i = p$ for all $i \in [0, N]$, we can derive a closed form solution $\overline{\varepsilon} = w/(Np) > 0$ from (20). Thus, there exists an ε such that $\varepsilon \in (0, \overline{\varepsilon})$. In asymmetric cases, we need to adjust for price dispersion.

In Appendix B, we illustrate a closed form solution for $\overline{\varepsilon} > 0$ in such an asymmetric case.[9]

We have hence shown the following result.

Proposition 1 (Equilibrium invariance) *Assume that* $u(0) = 0$, $u' > 0$, *and* $u'' < 0$, *that* F *satisfies (16), and that condition* (20) *holds. Then, the positive monotonic transformation* Φ *of the subutility function affects neither the Marshallian demand functions nor the equilibrium allocation.*

Proof See above. □

Proposition 1 shows that there exists a class of monotonic transformations Φ of the subutility function u that do not affect the equilibrium allocation and thus the change in the equilibrium allocation. The reason for this result is that the transformations satisfying (16) do not affect the two conditions for deriving the Marshallian demand function at equilibrium, namely that the budget constraint is satisfied with equality, and that the marginal rate of substitution equals the relative price. Our result implies that the values of a and b in (16) can hardly be identified from (the change in) the equilibrium allocation. However, we show in Section 3.2 that without the information on a and b welfare changes cannot be measured when varieties are endogenous.

3.2 *Measuring welfare changes using the class of monotonic transformations that do not affect the equilibrium allocation*

We have so far identified the class of monotonic transformations that do not affect the equilibrium allocation. We now show, however, that such transformations do affect welfare changes: for any change in the mass N^p of varieties supplied – and thus, given condition (20), for any change in the mass N of varieties consumed – there exist infinitely many possible welfare changes measured by the expenditure function.

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[9] As shown in Behrens and Murata (2007), deriving a closed form solution from (20) requires a separability property of $(u')^{-1}$. It is satisfied in the special cases of CES and constant absolute risk aversion (CARA).

To see this, we start with the following expenditure minimization problem subject to the target utility level $\overline{U}$:

$$\begin{aligned} \min_{\{q_i,\ i\in[0,\overline{N}]\}} \quad & \int_0^{\overline{N}} p_i q_i \mathrm{d}i \\ \text{s.t.} \quad & G\left(\int_0^{\overline{N}} \Phi(u(q_i))\mathrm{d}i\right) = \overline{U}, \\ & q_i \geq 0 \quad \forall i \in [0, N^p], \\ & q_i = 0 \quad \forall i \in (N^p, \overline{N}], \end{aligned}$$

where $N^p > 0$ is the mass of varieties produced in the economy and the constraint $q_i = 0$ for all $i \in (N^p, \overline{N}]$ implies that varieties that are not supplied cannot be consumed.

As before we focus on the case in which there exists an $\varepsilon \in (0, \overline{\varepsilon})$, so that $\Phi(u(q_i)) = F(u(q_i))$ for all $i \in [0, N^p]$ and $N = N^p$ hold. Then, the first-order conditions for expenditure minimization with respect to q_i and q_j for $i, j \in [0, N]$ imply (13). Focusing on the class of monotonic transformations in (16), we then obtain (17) as before. Noting that $q_i = 0$ for all $i \in (N, \overline{N}]$, the utility constraint becomes

$$\int_0^N [a + bu(q_j)]\,\mathrm{d}j = G^{-1}(\overline{U}), \tag{21}$$

which, together with (17), can be rewritten as

$$A(N) + b\int_0^N u\left((u')^{-1}\left(u'(q_i)\frac{p_j}{p_i}\right)\right)\mathrm{d}j = G^{-1}(\overline{U}), \tag{22}$$

where $A(N) = Na$. The compensated demand function is thus given by

$$q_i = q\left(p_i, \mathbf{p}, N, \overline{U}, A(N), b\right), \tag{23}$$

where $\mathbf{p}$ is a vector of statistics that characterizes the distribution of prices $\{p_i\}_{i\in[0,N]}$. Plugging (23) into the objective function, we obtain the expenditure function as follows:

$$e\left(\mathbf{p}, N, \overline{U}, A(N), b\right) = \int_0^N p_i q\left(p_i, \mathbf{p}, N, \overline{U}, A(N), b\right)\mathrm{d}i. \tag{24}$$

In the standard case without the monotonic transformation F of the subutility function, i.e., $a = 0$ and $b = 1$, we have

$$e\left(\mathbf{p}, N, \overline{U}\right) = \int_0^N p_i q\left(p_i, \mathbf{p}, N, \overline{U}\right) \mathrm{d}i. \tag{25}$$

We measure welfare changes using the expenditure function (24), which includes (25) as a special case. Holding the target utility level $\overline{U}$ constant, we analyze by how much consumers can reduce their expenditure after a shock that affects $\{\mathbf{p}, N\}$ and, therefore, $A(N)$. We know from Proposition 1 under which conditions the equilibrium allocation is invariant to a monotonic transformation Φ. When these conditions are met before and after the shock, *the change in the equilibrium allocation* is also unaffected by the transformation Φ.[10] However, the values of a and b can affect welfare changes as shown by the following proposition.

Proposition 2 (Welfare variance) *Suppose that the assumptions in Proposition 1 hold. Then, the welfare change is given by*

$$\begin{aligned} -\mathrm{d}\ln e = & -\int_0^N \frac{p_i q_i}{e} \mathrm{d}\ln p_i \mathrm{d}i \\ & - \left[\frac{N p_N q_N}{e} - \frac{N u(q_N)}{\int_0^N u'(q_i) q_i \mathrm{d}i} - \frac{N a}{b \int_0^N u'(q_i) q_i \mathrm{d}i} \right] \mathrm{d}\ln N. \end{aligned} \tag{26}$$

Thus, for any given change in the equilibrium mass of varieties $\mathrm{d}\ln N$*, there exist infinitely many possible welfare changes* $-\mathrm{d}\ln e$*, depending on the values of* a *and* b*.*

Proof See Appendix C. □

Several comments are in order. First, Proposition 2 implies that the welfare change, driven by a change in N, is sensitive to the values of a and b. Except for the special case with $a = 0$, any shock that affects the mass of varieties generates an additional welfare change as can be seen from (26). Since Proposition 1 shows that the values of a and b do not affect the equilibrium allocation (and thus do not affect the change in N), the commonly made assumption, $a = 0$

[10] If the conditions of Proposition 1 hold in the initial equilibrium, they will hold by continuity for small shocks that affect this equilibrium.

and $b = 1$, may not be innocuous when it comes to measuring welfare changes. Indeed, for any given change in N, there exist infinitely many possible welfare changes depending on the values of a and b.

Second, since a can take any value between zero and infinity (and b can take any positive value) without affecting the equilibrium, we need to identify the values of a and b in order to uniquely pin down the welfare change. However, as argued after Proposition 1, we can hardly identify those values from the equilibrium allocation because the transformation Φ does not affect the equilibrium allocation. Even if we could identify it, a need not be zero and b need not be one. Therefore, existing results on welfare changes, driven by a change in N, need to be qualified as they rely on a particular normalization.

Third, as shown in Appendix D, the welfare change $-\mathrm{d}\ln e$ may be viewed as either equivalent or compensating variation for small shocks (marginal changes). To establish this relationship, we follow the standard practice that $w = 1$ by choice of numeraire, both before and after the shock. Thus, our results are not driven by the way we measure welfare changes.

Fourth, expression (26) applies to both CES and general subutilities. Indeed, when p_i does not depend on N, and when firms are symmetric so that $Np_Nq_N = e$, (26) boils down to (8). Both the CES and general cases require $q_i > \varepsilon$ for all $i \in [0, N]$. In both cases, a sufficient condition for this to hold is given by $\bar{\varepsilon} = q^{\min} = q(p^{\max}, \mathbf{p}, N, w) > \varepsilon$. In the CES case $q(p^{\max}, \mathbf{p}, N, w) = (p^{\max})^{-\sigma}P(N)^{\sigma-1}w$, and therefore $q^{\min} > 0$ regardless of the values of $(p^{\max}, \mathbf{p}, N, w)$. Hence, there exists an $\varepsilon > 0$ such that $q^{\min} > \varepsilon$. In the case of general subutilities, however, $q^{\min}$ need not be strictly positive. Thus, we need to focus on the set of $(p^{\max}, \mathbf{p}, N, w)$ to ensure $q^{\min} > 0$. When all prices are the same, this condition is satisfied since $q^{\min} = w/(Np) > 0$ holds. When prices are different, we need to adjust for price dispersion (see Appendix B).

Fifth, the extant literature using CES subutility regards variety expansion in the consumption good as being equivalent to variety expansion in the intermediate good or to a rise in total factor productivity due to specialization (e.g., Grossman and Helpman, 1993, Chapter 3). However, our results suggest that this reinterpretation relies on the implicit normalization, $a = 0$ and $b = 1$.

Thus, the equivalence does not hold in general. In particular, in the case of the intermediate good, the values of a and b affect the total output in equilibrium, whereas in the case of the consumption good, the values of a and b do not affect the equilibrium allocation while altering welfare evaluation.

Last, it is worth emphasizing that the change in consumption diversity, $\mathrm{d}\ln N \neq 0$, over the variety space plays a crucial role in Proposition 2 as can be seen from (26). Thus, our results do not apply to models with a fixed number of varieties (e.g., Eaton and Kortum, 2002) or models in which new varieties are defined over a characteristics space of some fixed dimension (e.g., Lancaster, 1966).

4. Extension

We have so far assumed that the monotonic transformation Φ does not depend on N. To encompass the existing literature (e.g., Benassy, 1996; Blanchard and Giavazzi, 2003), we now assume that Φ depends not only on $u(q_i)$ but also on N. In that case, condition (20) need not be sufficient to ensure $q_i > \varepsilon$ for all $i \in [0, N]$. The reasons are that condition (20) is based on (11), and that the latter abstracts from the impact of a new variety on the existing varieties. To take such an impact into account we first impose the "love of variety" condition for all varieties to be consumed, and then derive the condition for $q_i > \varepsilon$ to hold for all $i \in [0, N^p]$.

Assume that the transformation of the subutility function in given by $\Phi_i \equiv \Phi(u(q_i), N)$, where

$$\Phi(u(q_i), N) = \begin{cases} F\left(u(q_i), N\right) & \text{for } q_i \in (\varepsilon, \infty) \\ \int_0^{u(q_i)} \psi(t, N)\mathrm{d}t & \text{for } q_i \in [0, \varepsilon] \end{cases} .$$

Letting

$$\eta_{\Phi_i,u} \equiv \frac{u}{\Phi_i}\frac{\partial \Phi_i}{\partial u}, \quad \eta_{\Phi_i,N} \equiv \frac{N}{\Phi_i}\frac{\partial \Phi_i}{\partial N}, \quad \text{and} \quad \eta_{u,q_i} \equiv \frac{q_i}{u}\frac{\partial u}{\partial q_i}, \tag{27}$$

we show in Appendix E that the "love of variety" condition is given by

$$\int_0^N \frac{\Phi_i}{N}\left(\frac{N p_N q_N}{w}\eta_{\Phi_i,u}\eta_{u,q_i} - \eta_{\Phi_i,N}\right)\mathrm{d}i \leq \Phi_N. \tag{28}$$

Intuitively, condition (28) implies that the benefit of the marginal variety $i = N$ is no less than the cost of purchasing it. Note that $\eta_{\Phi_i,N}$ captures the impact of a new variety on the existing varieties.

In Sections 2 and 3, we assume that $\eta_{\Phi_i,N} = 0$. In that case, we show in Appendix E that the "love of variety" condition reduces to

$$(\Phi \circ u)'(q_N) \leq \frac{\Phi_N}{q_N}, \tag{29}$$

i.e., the marginal transformed utility is not greater than the average transformed utility. Note that condition (29) holds since $\Phi \circ u$ is assumed to be concave. This result shows that if Φ is independent of N, the "love of variety" condition always holds provided that $\Phi \circ u$ is a concave function. Hence, condition (20) is sufficient for our results in Sections 2 and 3.

When Φ depends on N, things are more complicated. The reason is that adding a variety at the margin may reduce the value of all inframarginal varieties. Thus, consumers need not consume all available varieties, in which case condition (20) is violated. Recovering (20) requires the "love of variety" condition (28) for all available varieties to be consumed, i.e., $N = N^p$.

Let λ be the marginal utility of income. The first-order condition for variety $i \in [0, N^p]$ is given by

$$\frac{\partial \Phi(u(q_i), N)}{\partial u} u'(q_i) \leq \widetilde{\lambda} p_i, \tag{30}$$

where $\widetilde{\lambda} \equiv \lambda / G'(\cdot)$. Assume that the "love of variety" condition (28) holds. Since (28) ensures positive consumption for all $i \in [0, N^p]$, we have $N = N^p$, so that (30) must hold with equality for all $i \in [0, N^p]$. Since N^p is exogenous to consumers, we can focus in the same way as in Sections 2 and 3 on the case in which there exists an $\varepsilon \in (0, \bar{\varepsilon})$ such that $q_i > \varepsilon$ for all $i \in [0, N^p]$.

Assume $\varepsilon \in (0, \bar{\varepsilon})$ and consider an affine transformation of the CES subutility function, i.e., $\Phi(u(q_i), N) = F(u(q_i), N) = a + b(N) q_i^{(\sigma-1)/\sigma}$, where b depends on the mass N of varieties consumed. In that case, (28) becomes

$$\frac{a}{b(N) q^{\frac{\sigma-1}{\sigma}}} + \frac{N b'(N)}{b(N)} \geq -\frac{1}{\sigma}, \tag{31}$$

where we impose symmetry across varieties (see Appendix E).[11] When condition (31) holds, the welfare change is given by (see Appendix C)

$$-\,\mathrm{d}\ln e = -\mathrm{d}\ln P(N) + \frac{\sigma}{\sigma-1}\left[\frac{a}{b(N)q^{\frac{\sigma-1}{\sigma}}} + \frac{b'(N)N}{b(N)}\right]\mathrm{d}\ln N. \tag{32}$$

The existing literature, based mostly on the CES subutility function, has sometimes assumed different functional forms for $b(N)$ for particular purposes. We have shown in Section 2 that the CES case, with $a = 0$ and $b(N) = b$, yields the welfare change

$$-\,\mathrm{d}\ln e = -\mathrm{d}\ln P(N) = \frac{1}{\sigma-1}\mathrm{d}\ln N, \tag{33}$$

where we have used (7). This can also be seen by setting $a = 0$ and $b'(N) = 0$ in (32). In this case, the welfare change depends solely on the value of σ.

As is well known, the parameter σ plays two roles in the CES model: the elasticity of substitution; and the "love of variety" effect, i.e., holding aggregate consumption Nq fixed, an increase in N raises the utility level. To disentangle the latter from the former, Benassy (1996) considers that $G(x) = x^{\sigma/(\sigma-1)}$, $\psi(t,N) = b(N)$, $a = 0$, and $b(N) = N^{(\nu+\frac{1}{1-\sigma})(\frac{\sigma-1}{\sigma})}$ with $\nu \in [0,\infty)$, so that

$$U = \left[\int_0^N N^{(\nu+\frac{1}{1-\sigma})(\frac{\sigma-1}{\sigma})} q_i^{\frac{\sigma-1}{\sigma}}\mathrm{d}i\right]^{\frac{\sigma}{\sigma-1}}. \tag{34}$$

Thus, when varieties are symmetric, we have $U = N^{\nu}(Nq)$ with the "love of variety" effect being N^{ν}. Since $a = 0$ and $Nb'(N)/N = (\nu + \frac{1}{1-\sigma})(\frac{\sigma-1}{\sigma})$, the "love of variety" condition (31) reduces to $\nu \geq 0$. In that case, our welfare measure (32) becomes

$$-\,\mathrm{d}\ln e = -\mathrm{d}\ln P(N) + \left(\nu + \frac{1}{1-\sigma}\right)\mathrm{d}\ln N = \nu\,\mathrm{d}\ln N, \tag{35}$$

[11] Dixit and Stiglitz (1975, Section 4.4) derive a similar condition when $a = 0$ and when firms are homogeneous. With $a > 0$ and with firm heterogeneity, the expression becomes

$$\frac{a}{b(N)q_N^{\frac{\sigma-1}{\sigma}}} + \frac{Nb'(N)}{b(N)}\left[\frac{1}{N}\int_0^N\left(\frac{m_i}{m_N}\right)^{1-\sigma}\mathrm{d}i\right] \geq -\frac{1}{\sigma},$$

where N is the mass of varieties supplied in the economy and where m_i and m_N are marginal labor requirements for producing varieties i and N, respectively. The foregoing expression differs from the standard "love of variety" condition without firm heterogeneity, i.e., $u'(q) \leq u(q)/q$, that depends solely on preferences.

where we have used (7). This result shows that, depending on the value of ν, the absolute value of the welfare change can take any value between zero and infinity for a given change in the equilibrium mass of varieties $\mathrm{d}\ln N$. Since a given shock generates the same change in the equilibrium allocation for any value of $\nu \geq 0$, the benchmark case (33) overestimates the welfare change as compared to (35) if $\nu < 1/(\sigma-1)$ and underestimates it if $\nu > 1/(\sigma-1)$ when the true utility function is given by (34). It is worth pointing out that this result can be generalized to an arbitrary function $b(N)$: $b'(N) < 0$ implies that (33) overestimates the welfare change and vice versa. One can even completely neutralize the welfare effect of "love of variety" without affecting the equilibrium allocation. To see this, assume that $a = 0$ and $b(N) = N^{-1/\sigma}$ (see, e.g., Blanchard and Giavazzi, 2003). This specification is a special case of Benassy (1996), namely $\nu = 0$. In this case, our welfare measure (32), becomes

$$-\mathrm{d}\ln e = -\mathrm{d}\ln P(N) + \frac{1}{1-\sigma}\mathrm{d}\ln N = 0,$$

where we have again used (7). In this specification, welfare is unaffected by a shock that changes the mass of varieties as the positive effect via the price aggregate $P(N)$ is offset by the negative effect of the $b(N)$ term.

These three simple cases show that there is a class of models that yield the same change in the equilibrium allocation but predict substantially different welfare changes. For any given change in N, the absolute value of the welfare change can range from zero to infinity, depending on the assumption on $b(N)$. Thus, welfare evaluations based on specific normalizations in the existing literature can over- or underestimate the true welfare change when varieties are endogenous.

5. Concluding remarks

It is well known that preferences are invariant to *any* positive monotonic transformation of a *utility* function. In this paper, we have shown that the Marshallian demand function evaluated at equilibrium can remain invariant to infinitely many transformations of a *subutility* function. While such transformations do not affect the equilibrium allocation, they generate quite different welfare evaluations when consumption diversity varies. Thus, unless we know the true transformation, it is impossible to pin down the magnitude of the welfare change. Since the transformation does not affect the equilibrium allocation, it can be hardly identified. Even if we could identify it, it need not coincide with implicit normalizations made in the literature. Accordingly, existing results on welfare changes with endogenous consumption diversity need to be qualified.

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Appendix

The appendix is structured as follows. In Appendix A, we illustrate how to construct a ψ function that satisfies the properties spelled out in the main text. Appendix B provides a closed-form solution for the threshold $\bar{\varepsilon}$ in a VES case. In Appendix C, we derive the general expression for the welfare change between two equilibria. Appendix D establishes the equivalence between our measure of welfare changes and equivalent and compensating variations. Last, Appendix E deals with the "love of variety" condition.

Appendix A. Illustration of ψ

A.1. When $\Phi(u(q_i)) = a + bu(q_i)$ holds for $q_i > \varepsilon$. Assume that ψ in (1) satisfies the following three conditions: (i) $\psi > 0$ and $\psi' \leq 0$; (ii) $\int_0^{u(\varepsilon)} \psi(t)\mathrm{d}t = a + bu(\varepsilon)$; and (iii) $\psi(u(\varepsilon))u'(\varepsilon) = bu'(\varepsilon)$, where $u(\varepsilon) = \varepsilon^{(\sigma-1)/\sigma}$. As an illustration, assume that $\psi(t) = \alpha - \beta t$. We can then find $\alpha > 0$ and $\beta \geq 0$ such that ψ satisfies conditions (i) to (iii) for any $\varepsilon > 0$. To see this, let us start with condition (ii), which implies that $\alpha u(\varepsilon) - (\beta/2)[u(\varepsilon)]^2 = a + bu(\varepsilon)$. Condition (iii) implies that $\alpha - \beta u(\varepsilon) = b$. These two conditions give a system of linear equations in α and β, which yields a unique solution

$$\alpha = \frac{2a}{u(\varepsilon)} + b > 0 \quad \text{and} \quad \beta = \frac{2a}{[u(\varepsilon)]^2} \geq 0.$$

Finally, since $\psi' = -\beta \leq 0$ and $\psi(u(\varepsilon)) = \alpha - \beta u(\varepsilon) = b > 0$, $\psi(u(q_i))$ is strictly positive for all $q_i \in [0, \varepsilon]$. Hence, condition (i) holds, which completes the proof.

A.2. When $\Phi(u(q_i)) = F(u(q_i))$ holds for $q_i > \varepsilon$. Assume that ψ in (9) satisfies the following three conditions: (i) $\psi > 0$ and $\psi' \leq 0$; (ii) $\int_0^{u(\varepsilon)} \psi(t)\mathrm{d}t = F(u(\varepsilon))$; and (iii) $\psi(u(\varepsilon))u'(\varepsilon) = F'(u(\varepsilon))u'(\varepsilon)$. As an illustration, assume again that $\psi(t) = \alpha - \beta t$. We can then find $\alpha > 0$ and $\beta \geq 0$ such that ψ satisfies conditions (i) to (iii) for any $\varepsilon > 0$. To see this, let us start with condition (ii), which implies that $\alpha u(\varepsilon) - (\beta/2)[u(\varepsilon)]^2 = F(u(\varepsilon))$. Condition (iii) implies that $\alpha - \beta u(\varepsilon) = F'(u(\varepsilon))$, where $F' \equiv \partial F/\partial u$. These two conditions give a system of linear equations in α and β, which yields a unique solution

$$\alpha = \frac{F(u(\varepsilon))}{u(\varepsilon)} \left[2 - \frac{F'(u(\varepsilon))u(\varepsilon)}{F(u(\varepsilon))}\right] > 0$$
$$\beta = \frac{2F(u(\varepsilon))}{[u(\varepsilon)]^2} \left[1 - \frac{F'(u(\varepsilon))u(\varepsilon)}{F(u(\varepsilon))}\right] \geq 0,$$

where the inequalities come from $F'(u(\varepsilon))u(\varepsilon)/F(u(\varepsilon)) \leq 1$ since F is nonconvex by assumption. Finally, since $\psi' = -\beta \leq 0$ and $\psi(u(\varepsilon)) = \alpha - \beta u(\varepsilon) = F'(u(\varepsilon)) > 0$, $\psi(u(q_i))$ is strictly positive for all $q_i \in [0, \varepsilon]$. Hence, condition (i) holds, which completes the proof.

Appendix B. Illustration of a closed form solution for $\overline{\varepsilon} > 0$ in an asymmetric case

To illustrate a closed form solution for $\overline{\varepsilon} > 0$ in an asymmetric case, consider the CARA subutility function $u(q_i) = 1 - \mathrm{e}^{-\gamma q_i}$ as in Behrens and Murata (2007). In that case, we can derive $\widetilde{\lambda}$ from (12) as follows:

$$\widetilde{\lambda}(\mathbf{p}, N^p, w) = \exp\left[\ln(\gamma b) - \frac{\gamma w}{N^p \overline{p}} - \frac{1}{N^p \overline{p}} \int_0^{N^p} p_i \ln p_i \mathrm{d}i\right],$$

where $\overline{p} = (1/N^p) \int_0^{N^p} p_i \mathrm{d}i$ is the average price. Hence, for $q_i > \varepsilon$ to hold for all $i \in [0, N^p]$ in this asymmetric case, we need to choose ε such that

$$\varepsilon \in (0, \overline{\varepsilon}), \quad \text{where} \quad \overline{\varepsilon} \equiv \frac{1}{N^p \overline{p}} \left[w + \frac{1}{\gamma} \int_0^{N^p} p_i \ln\left(\frac{p_i}{p^{\max}}\right) \mathrm{d}i\right]. \tag{36}$$

Thus, $\overline{\varepsilon}$ can be expressed only in terms of the variables that are exogenous to consumers. In the special case where $p_i = \overline{p} = p^{\max}$ for all $i \in [0, N^p]$, $\overline{\varepsilon}$ is given by $w/(N^p \overline{p}) = \overline{q}$, where $\overline{q}$ is the quantity in the symmetric case. Note that the last equality comes from the budget constraint. Condition (36) tells us that in the case of symmetric prices, we can choose any $\varepsilon \in (0, \overline{q})$. When prices are different, we need to adjust for price dispersion as in the second term of (36). Clearly, for any given wage w, there is an upper bound on price dispersion for $\overline{\varepsilon} > 0$ to exist. Conversely, for any price distribution (with $p_i > 0$ for all $i \in [0, N^p]$), there exists a wage rate such that $\overline{\varepsilon} > 0$ exists.

Appendix C. Derivation of the welfare change

C.1. Constant b. Totally differentiating the expenditure function (24), we can define the welfare change as

$$-\mathrm{d}\ln e = -\frac{1}{e}\left(\int_0^N p_i q_i \mathrm{d}\ln p_i \mathrm{d}i + N p_N q_N \mathrm{d}\ln N + \int_0^N p_i q_i \mathrm{d}\ln q_i \mathrm{d}i\right), \tag{37}$$

where $\mathrm{d}\ln q_i$ involves all partial derivatives of the arguments of the compensated demand function (23). Consider the transformation identified in (16) and the utility constraint given by (21). Using $A(N) = Na$, the latter can be rewritten as

$$\int_0^N u(q_i)\mathrm{d}i = \frac{G^{-1}(\overline{U}) - A(N)}{b}.$$

Taking the rate of change of both sides, we have

$$\frac{u(q_N)N\mathrm{d}\ln N}{\int_0^N u(q_i)\mathrm{d}i} + \frac{\int_0^N u'(q_i)q_i\mathrm{d}\ln q_i\mathrm{d}i}{\int_0^N u(q_i)\mathrm{d}i} = -\frac{A(N)\mathrm{d}\ln A(N)}{b\int_0^N u(q_i)\mathrm{d}i}. \tag{38}$$

The first-order conditions for the expenditure minimization imply $u'(q_i)p_j = u'(q_j)p_i$. Multiplying both sides by q_j and integrating the resulting expression, we have

$$u'(q_i) = \frac{p_i}{e}\int_0^N u'(q_j)q_j\mathrm{d}j.$$

Plugging this expression into (38) yields

$$-\frac{\int_0^N p_i q_i\mathrm{d}\ln q_i\mathrm{d}i}{e} = \frac{\frac{A(N)}{b}\mathrm{d}\ln A(N) + u(q_N)N\mathrm{d}\ln N}{\int_0^N u'(q_j)q_j\mathrm{d}j}.$$

Finally, plugging this expression into (37), we obtain

$$\begin{aligned}-\mathrm{d}\ln e = &-\int_0^N \frac{p_i q_i}{e}\mathrm{d}\ln p_i\mathrm{d}i - \left[\frac{Np_N q_N}{e} - \frac{Nu(q_N)}{\int_0^N u'(q_i)q_i\mathrm{d}i}\right]\mathrm{d}\ln N \\ &+ \frac{A(N)}{b\int_0^N u'(q_i)q_i\mathrm{d}i}\mathrm{d}\ln A(N),\end{aligned}$$

which can then be rewritten as (26).

C.2. Variable b. When b depends on N, we can replace b with $b(N)$ in the compensated demand function (23), in the transformation identified in (16), and in the utility constraint given by (21). Using $A(N) = Na$, the latter can be rewritten as

$$\int_0^N u(q_i)\mathrm{d}i = \frac{G^{-1}(\overline{U}) - A(N)}{b(N)}.$$

Taking the rate of change of both sides, we have

$$\frac{u(q_N)N\mathrm{d}\ln N}{\int_0^N u(q_i)\mathrm{d}i}+\frac{\int_0^N u'(q_i)q_i\mathrm{d}\ln q_i\mathrm{d}i}{\int_0^N u(q_i)\mathrm{d}i}=-\frac{A(N)\mathrm{d}\ln A(N)}{b(N)\int_0^N u(q_i)\mathrm{d}i}-\mathrm{d}\ln b(N). \quad (39)$$

The first-order conditions for the expenditure minimization imply $u'(q_i)p_j = u'(q_j)p_i$. Multiplying both sides by q_j and integrating the resulting expression, we have

$$u'(q_i)=\frac{p_i}{e}\int_0^N u'(q_j)q_j\mathrm{d}j.$$

Plugging this expression into (39) yields

$$-\frac{\int_0^N p_iq_i\mathrm{d}\ln q_i\mathrm{d}i}{e}=\frac{\frac{A(N)}{b(N)}\mathrm{d}\ln A(N)+\left[\int_0^N u(q_i)\mathrm{d}i\right]\mathrm{d}\ln b(N)+u(q_N)N\mathrm{d}\ln N}{\int_0^N u'(q_j)q_j\mathrm{d}j}.$$

Finally, plugging this expression into (37), we obtain

$$\begin{aligned}-\mathrm{d}\ln e=&-\int_0^N\frac{p_iq_i}{e}\mathrm{d}\ln p_i\mathrm{d}i-\left[\frac{Np_Nq_N}{e}-\frac{Nu(q_N)}{\int_0^N u'(q_i)q_i\mathrm{d}i}\right]\mathrm{d}\ln N\\&+\frac{A(N)}{b(N)\int_0^N u'(q_i)q_i\mathrm{d}i}\mathrm{d}\ln A(N)+\frac{\int_0^N u(q_i)\mathrm{d}i}{\int_0^N u'(q_i)q_i\mathrm{d}i}\mathrm{d}\ln b(N).\end{aligned}$$

In the transformed CES case with symmetric varieties, we obtain

$$-\mathrm{d}\ln e=\frac{1}{\sigma-1}\left[1+\frac{a\sigma}{b(N)q^{\frac{\sigma-1}{\sigma}}}+\sigma\frac{b'(N)N}{b(N)}\right]\mathrm{d}\ln N.$$

Appendix D. The relationship among $-\mathrm{d}\ln e$, compensating variation, and equivalent variation

We show that the welfare change $-\mathrm{d}\ln e$ may be viewed as the marginal change in compensating variation (CV) or equivalent variation (EV). Thus, using $-\mathrm{d}\ln e$ to measure welfare changes is not driving our main results.

Let $t \geq 0$ denote continuous time and consider the change in CV and EV from time 0 to time t. Since $w = 1$ by choice of numeraire as in the existing literature, we have $e(\mathbf{p}^0, N^0, U^0, A(N^0), b(N^0)) = e(\mathbf{p}^t, N^t, U^t, A(N^t), b(N^t)) = w$, so that CV can be expressed as

$$\begin{aligned} \text{CV} &= e(\mathbf{p}^t, N^t, U^t, A(N^t), b(N^t)) - e(\mathbf{p}^t, N^t, U^0, A(N^t), b(N^t)) \\ &= e(\mathbf{p}^0, N^0, U^0, A(N^0), b(N^0)) - e(\mathbf{p}^t, N^t, U^0, A(N^t), b(N^t)) \\ &= \int_0^{N^0} p_i^0 q_i(p_i^0, \mathbf{p}^0, N^0, U^0, A(N^0), b(N^0)) \mathrm{d}i \\ &\quad - \int_0^{N^t} p_i^t q_i(p_i^t, \mathbf{p}^t, N^t, U^0, A(N^t), b(N^t)) \mathrm{d}i. \end{aligned}$$

Note that U is evaluated at time 0 in both terms of the last equation. To obtain a marginal change in CV, we differentiate this expression with respect to t and evaluate it at $t = 0$ as follows:

$$\begin{aligned} \left.\frac{\mathrm{dCV}}{\mathrm{d}t}\right|_{t=0} &= -\left.\frac{\mathrm{d}N^t}{\mathrm{d}t} p_{N^t}^t q_{N^t}(p_{N^t}^t, \mathbf{p}^t, N^t, U^0, A(N^t), b(N^t))\right|_{t=0} \\ &\quad - \left.\int_0^{N^t} \frac{\mathrm{d}p_i^t}{\mathrm{d}t} q_i(p_i^t, \mathbf{p}^t, N^t, U^0, A(N^t), b(N^t)) \mathrm{d}i\right|_{t=0} \\ &\quad - \left.\int_0^{N^t} p_i^t \frac{\mathrm{d}q_i(p_i^t, \mathbf{p}^t, N^t, U^0, A(N^t), b(N^t))}{\mathrm{d}t} \mathrm{d}i\right|_{t=0}. \end{aligned}$$

Since all variables are evaluated at $t = 0$, we can suppress the time index by letting $N^t = N^0 = N$ as follows:

$$\frac{\mathrm{dCV}}{\mathrm{d}t} = -\frac{\mathrm{d}N}{\mathrm{d}t} p_N q_N - \int_0^N \frac{\mathrm{d}p_i}{\mathrm{d}t} q_i \mathrm{d}i - \int_0^N p_i \frac{\mathrm{d}q_i}{\mathrm{d}t} \mathrm{d}i,$$

where we set $q_i = q_i(p_i, \mathbf{p}, N, U, A(N), b(N))$ for simplicity. Hence,

$$\frac{\mathrm{dCV}}{\mathrm{d}t} = -N p_N q_N \frac{\mathrm{d}\ln N}{\mathrm{d}t} - \int_0^N p_i q_i \frac{\mathrm{d}\ln p_i}{\mathrm{d}t} \mathrm{d}i - \int_0^N p_i q_i \frac{\mathrm{d}\ln q_i}{\mathrm{d}t} \mathrm{d}i.$$

Noting that $e = 1$ by choice of numeraire, when all arguments are evaluated at time $t = 0$, the above expression is equivalent to that of $-\mathrm{d}\ln e$ as given by (37) in Appendix C. We can then derive the welfare change to obtain Proposition 2.

Turning next to EV, it is given by

$$\begin{aligned}
\text{EV} &= e(\mathbf{p}^0, N^0, U^t, A(N^0), b(N^0)) - e(\mathbf{p}^0, N^0, U^0, A(N^0), b(N^0)) \\
&= e(\mathbf{p}^0, N^0, U^t, A(N^0), b(N^0)) - e(\mathbf{p}^t, N^t, U^t, A(N^t), b(N^t)) \\
&= \int_0^{N^0} p_i^0 q_i(p_i^0, \mathbf{p}^0, N^0, U^t, A(N^0), b(N^0))\mathrm{d}i \\
&\quad - \int_0^{N^t} p_i^t q_i(p_i^t, \mathbf{p}^t, N^t, U^t, A(N^t), b(N^t))\mathrm{d}i.
\end{aligned}$$

Note that U is evaluated at time t in both terms of the last equation. To obtain a marginal change in EV, we differentiate this expression with respect to t and evaluate it at $t = 0$ as follows:

$$\begin{aligned}
\left.\frac{\mathrm{dEV}}{\mathrm{d}t}\right|_{t=0} &= -\left.\frac{\mathrm{d}N^t}{\mathrm{d}t} p_{N^t}^t q_{N^t}(p_{N^t}^t, \mathbf{p}^t, N^t, U^t, A(N^t), b(N^t))\right|_{t=0} \\
&\quad - \left.\int_0^{N^t} \frac{\mathrm{d}p_i^t}{\mathrm{d}t} q_i(p_i^t, \mathbf{p}^t, N^t, U^t, A(N^t), b(N^t))\mathrm{d}i\right|_{t=0} \\
&\quad - \left.\int_0^{N^t} p_i^t \frac{\mathrm{d}q_i(p_i^t, \mathbf{p}^t, N^t, U^t, A(N^t), b(N^t))}{\mathrm{d}t}\mathrm{d}i\right|_{t=0}.
\end{aligned}$$

Since all variables are evaluated at $t = 0$, we can again suppress the time index by letting $N^t = N^0 = N$ as follows:

$$\frac{\mathrm{dEV}}{\mathrm{d}t} = -\frac{\mathrm{d}N}{\mathrm{d}t} p_N q_N - \int_0^N \frac{\mathrm{d}p_i}{\mathrm{d}t} q_i \mathrm{d}i - \int_0^N p_i \frac{\mathrm{d}q_i}{\mathrm{d}t}\mathrm{d}i,$$

where we set $q_i = q_i(p_i, \mathbf{p}, N, U, A(N), b(N))$ for simplicity. Hence,

$$\frac{\mathrm{dEV}}{\mathrm{d}t} = -N p_N q_N \frac{\mathrm{d}\ln N}{\mathrm{d}t} - \int_0^N p_i q_i \frac{\mathrm{d}\ln p_i}{\mathrm{d}t}\mathrm{d}i - \int_0^N p_i q_i \frac{\mathrm{d}\ln q_i}{\mathrm{d}t}\mathrm{d}i.$$

Noting that $e = 1$ by choice of numeraire, when all arguments are evaluated at time $t = 0$, the above expression is equivalent to that of $-\mathrm{d}\ln e$ as given by (37) in Appendix C. We can then derive the welfare change to obtain Proposition 2.

To summarize, marginal changes in both CV and EV are equivalent to $-\mathrm{d}\ln e$ so that they can be used interchangeably.

Appendix E. The "love of variety" condition

As mentioned in Section 4, when the transformation Φ depends on N, we need to consider the "love of variety" condition to ensure that all available varieties are consumed. To derive that condition, we consider the following optimization problem:

$$\begin{aligned}
\max_{\{q_i,\ i\in[0,N],\ N\}} \quad & U = G\left(\int_0^N \Phi\left(u(q_i), N\right) \mathrm{d}i\right) \\
\text{s.t.} \quad & \int_0^N p_i q_i \mathrm{d}i = w, \\
& q_i \geq 0,\ \forall i \in [0, N], \\
& N \leq N^p,
\end{aligned}$$

where we have applied the equality constraint $q_i = 0$ and thus $\Phi\left(u(q_i), N\right) = 0$ for all $i \in (N, \overline{N}]$ to the original problem. Note that the consumers choose not only the quantity of each variety consumed but also the range of varieties consumed to take into account the impact of a new variety on the existing ones. Without loss of generality, we order varieties such that p_i is non-decreasing in i. Let λ and μ be the multipliers associated with the budget constraint and with the constraint on the range of varieties. The first-order condition with respect to N is given by

$$\Phi(u(q_N), N) + \int_0^N \frac{\partial \Phi(u(q_i), N)}{\partial N} \mathrm{d}i = \frac{\lambda p_N q_N + \mu}{G'(\cdot)}.$$

If all available varieties $i \in [0, N^p]$ are consumed, the constraint on the range of varieties must be binding at $N = N^p$, i.e., $\mu \geq 0$. Since $G' > 0$, we then have

$$\Phi(u(q_N), N) + \int_0^N \frac{\partial \Phi(u(q_i), N)}{\partial N} \mathrm{d}i \geq \frac{\lambda p_N q_N}{G'(\cdot)}, \tag{40}$$

where N is evaluated at N^p. In that case, the first-order condition with respect to q_i is given by the equality

$$G'(\cdot) \frac{\partial \Phi(u(q_i), N)}{\partial u} u'(q_i) = \lambda p_i,\ \forall i \in [0, N]. \tag{41}$$

Multiplying this expression by q_i and integrating over $[0, N]$ then yield

$$\frac{\lambda}{G'(\cdot)} = \frac{1}{w} \int_0^N \frac{\partial \Phi(u(q_i), N)}{\partial u} u'(q_i) q_i \mathrm{d}i. \tag{42}$$

Plugging this expression into (40) yields

$$\Phi(u(q_N),N) + \int_0^N \frac{\partial \Phi(u(q_i),N)}{\partial N}\mathrm{d}i \geq \frac{p_N q_N}{w}\int_0^N \frac{\partial \Phi(u(q_i),N)}{\partial u}u'(q_i)q_i\mathrm{d}i. \quad (43)$$

Using the definition in (27) we obtain the "love of variety" condition (28).

Assume that Φ is independent of N, as in Sections 2 and 3. Then, the second term in the left-hand side of (43) is eliminated, so that

$$\frac{p_N q_N}{w}\int_0^N \frac{\partial \Phi(u(q_i),N)}{\partial u}u'(q_i)q_i\mathrm{d}i \leq \Phi_N.$$

which, using (42), can be rewritten as

$$\frac{p_N q_N}{G'(\cdot)}\lambda \leq \Phi_N. \quad (44)$$

The first-order condition for the marginal variety N is given by $\lambda p_N = G'(\cdot)(\Phi \circ u)'(q_N)$. Plugging this into (44) yields the "love of variety" condition (29), which always holds because $\Phi \circ u$ is assumed to be concave.

When Φ depends on N, things are more complicated. The reason is that choosing to consume a variety changes the value of all inframarginal varieties. A rational consumer will take into account this additional effect. To see this, consider an affine transformation $\Phi(u(q_i),N) = a + b(N)u(q_i)$, where b depends on the mass N of varieties. In that case, (28) becomes

$$\int_0^N \left[\frac{p_N q_N}{w}b(N)u(q_i)\eta_{u,q_i} - b'(N)u(q_i)\right]\mathrm{d}i \leq a + b(N)u(q_N).$$

Rearranging the terms, we have

$$-a + b(N)\left[\frac{p_N q_N}{w}\int_0^N u'(q_i)q_i\mathrm{d}i - u(q_N)\right] \leq b'(N)\int_0^N u(q_i)\mathrm{d}i. \quad (45)$$

Using (42) and $\partial\Phi(u(q_i),N)/\partial u = b(N)$ and then (41) for the marginal variety $i = N$, the second term of the left-hand side of (45) can be rewritten as

$$\begin{aligned}
b(N)\left[\frac{p_N q_N}{w}\int_0^N u'(q_i)q_i\mathrm{d}i - u(q_N)\right] &= b(N)\left[\frac{p_N q_N}{w}\frac{\lambda w}{G'(\cdot)b(N)} - u(q_N)\right] \\
&= b(N)\left[\frac{p_N q_N}{w}\frac{u'(q_N)w}{p_N} - u(q_N)\right] \\
&= b(N)q_N\left[u'(q_N) - \frac{u(q_N)}{q_N}\right] \leq 0.
\end{aligned}$$

Hence, when $b'(N) \geq 0$, the "love of variety" condition (45) always holds since $a \geq 0$. Thus, it is redundant for our analysis in Sections 2 and 3. Yet, it need not hold when $b'(N) < 0$.

Existing papers based on the CES subutility function allow for the case with $b'(N) < 0$ (Benassy, 1996; Blanchard and Giavazzi, 2003). Assume that $u(q_i) = q_i^{(\sigma-1)/\sigma}$. Since $q_i = p_i^{-\sigma} P(N)^{\sigma-1} w$ in the CES case, we have $p_N q_N / w = [p_N / P(N)]^{1-\sigma}$ and $u(q_i)/u(q_N) = (p_i/p_N)^{1-\sigma}$, so that the left-hand side of equation (45) becomes

$$-a + b(N)u(q_N)\left\{\frac{\sigma-1}{\sigma}\left[\frac{p_N}{P(N)}\right]^{1-\sigma}\int_0^N\left(\frac{p_i}{p_N}\right)^{1-\sigma}\mathrm{d}i - 1\right\} = -a - \frac{b(N)}{\sigma}u(q_N),$$

where we have used the definition of the price index $P(N)^{1-\sigma} = \int_0^N p_i^{1-\sigma}\mathrm{d}i$. Replacing the left-hand side of (45) with the forgoing expression and imposing symmetry on q_i, we then get (31). A sufficient condition for (31) to hold is $Nb'(N)/b(N) \geq -1/\sigma$. In that case, (31) holds for all $a \in [0, \infty)$. The greater the value of a, the more likely the "love of variety" condition (31) is satisfied. As shown in Section 4, Blanchard and Giavazzi (2003) assume that $a = 0$ and consider the borderline case with $Nb'(N)/b(N) = -1/\sigma$.