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# Optimal Policy Analysis in a New Keynesian Economy with Credit Market Search \*

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## Abstract

To reveal a policy mandate for financial stability, we introduce a frictional credit market with a search and matching process into a standard New Keynesian model with nominal rigidities in the goods market, and then investigate optimal policy under financial frictions. We show that a second-order approximation of social welfare includes terms for credit, in addition to terms for inflation and consumption, so that any optimal policy must hold responsibility for financial and price stabilities. We highlight this issue by considering several tools for monetary and macroprudential policy. We find that optimal monetary policy requires keeping the credit market countercyclical against the real economy. Also, optimal macroprudential policy, which poses constraints on supply and demand sides of credit, reduces excessive variations in lending and contributes to both financial and price stabilities.

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# 1 Introduction

The serious economic disruptions caused by financial crises reveal the critical roles played by financial markets in the U.S. and the euro area. Acknowledging that the current policy framework cannot fully mitigate nor avoid financial crises, policymakers have begun to shed light on roles of financial markets on policy measures and vice versa. Two policy measures are particularly focused on. The first is monetary policy, which aims to achieve, in addition to traditional policy goals, stability of the financial system. The second is macroprudential policy geared toward financial stability.

A growing number of publications stresses this new role of monetary policy for financial stability. The Bank for International Settlements (BIS) emphasizes that central banks need to tighten monetary policy against accumulation of financial imbalances, such as overheating of mortgage, stock, and bond markets, even when the real economy seems to be stable in the near-term.<sup>1</sup> Taylor (2008) argues that in the U.S., the Federal Reserve Board appears to adjust the policy rate in response to credit spread to stimulate the economy and maintain financial stability.<sup>2</sup>

The role of macroprudential policy, which is independent from monetary policy, in sustaining financial stability is also highlighted in the literature. Borio (2011) empirically shows the difference between financial and business cycles, and justifies the necessity of the coexistence of macroprudential policy and other policies such as monetary policy.<sup>3</sup> In practice, international organizations have begun to introduce macroprudential policy, for example, the Basel III framework, as set forth by the Basel Committee on Banking Supervision (BCBS, 2010) and BCBS (2014). Such macroprudential policy includes, among others, bank regulations that constrain supply of credit according to capital base and/or economic situations.<sup>4</sup>

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<sup>1</sup>See BIS (2009) and Caruana (2010).

<sup>2</sup>See also Christiano et al. (2010), Gertler and Karadi (2011), Teranishi (2015), and Cúrdia and Woodford (2016). In particular, Gertler and Karadi (2011) build a model with a borrowing constraint for banks as in Bernanke, Gertler, and Gilchrist (1999), and evaluate the quantitative easing policy.

<sup>3</sup>Drehmann, Borio, and Tsatsaronis (2012) also show such empirical results.

<sup>4</sup>For example, in the Basel III, banks are required to meet a particular base level of capital ratio

In this paper, we build a model with a banking sector and a frictional credit market that is suitable for analyzing financial instability (or equivalently, inefficient fluctuations in variables related to credit) and then analytically examine the optimal policy in such an environment. As in Wasmer and Weil (2000) and Den Haan, Ramey, and Watson (2003), we introduce financial market frictions by assuming search and matching process in the credit market.<sup>5</sup> Unlike these authors, however, we incorporate the credit market by following the existing studies, in particular Ravenna and Walsh (2011), that embed a frictional labor market into a standard New Keynesian model with nominal price rigidities.<sup>6</sup> Our approach enables expression of the welfare function in an intuitive form, and allows tractable analyses of the optimal policy in a model that explicitly formulates the supply side of credit. To elaborate, in our model, the aggregate loan volume is determined by the formation and destruction of borrower-lender relationships, and the loan interest rate varies with tightness of the credit market. These financial variables are related to business cycles, and vice versa. Thus, we can analyze policies that directly focus on the supply side, rather than the demand side of credit.

We begin our analysis of the model by, as in Woodford (2003), approximating the representative household's welfare in the second order. A novel finding is that the approximated welfare function includes, in addition to terms involving inflation and consumption, terms related to credit such as credit market tightness, credit growth, and the upper bound of loan volume. This result provides a theoretical justification for including financial stability among the goals of optimal policy.

We then explore the properties of the optimal policy, starting from the monetary policy that controls a deposit (policy) rate. We find that by taking a financial friction into account, monetary policy should contribute to financial stability and thus perform

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against risk assets, where this base is changed according to economic and financial conditions. Several countries have also introduced different types of macroprudential policies, including total credit control and capital control, as described in Lim et al. (2011) and Nier et al. (2011).

<sup>5</sup>Search and matching frictions are widely assumed in analyses of labor markets. See, e.g., Mortensen and Pissarides (1994) and Rogerson, Shimer, and Wright (2005).

<sup>6</sup>Earlier studies that feature such a model include Walsh (2003), Thomas (2008), and Trigari (2009).

the macroprudential role. When equipped with this additional role, optimal monetary policy must keep the credit market countercyclical against the real economy by setting the policy rate to induce disinflation against positive credit growth. We then turn to macroprudential policy, such as taxation or a subsidy on the cost of searching for credit in the demand side of credit, i.e., firms, and total credit control that poses lending limits on the supply side of credit, i.e., banks. We show that optimal macroprudential policy reduces excessive variations of credit. A key finding is that optimal macroprudential policy, primarily linked with financial stability, is indeed closely associated with price stability, and consequently with monetary policy.

This paper is related to three strands of literature, but differs from the existing studies in important ways. First, our paper is related to the existing work that adopts search-theoretic models of the credit market, such as Wasmer and Weil (2000) and Den Haan, Ramey, and Watson (2003). This framework enables elaborating the supply side of credit, and is thus suitable for conducting policy analyses for financial stability. Furthermore, Den Haan, Ramey, and Watson (2003) show that search and matching frictions in the loan market substantially amplify business cycle shocks. This provides an interpretation of Bernanke (1983), who shows that financial disruptions through credit misallocation induced the unusual length and depth of the Great Depression. Credit market search is therefore an appropriate mechanism for explaining financial disruptions against which policy measures should play a role. Our paper, however, substantially differs from the studies above in embedding credit market search into the standard New Keynesian model, as well as in revealing the optimality criteria for policy measures.

Second, our paper is related to studies that follow Woodford (2003) and analyze the optimal policy through a linear-quadratic approach, which makes use of the first order approximation of the structural equations of the model and the second order approximation of welfare. Extending this approach, Teranishi (2015) and Cúrdia and Woodford (2016) introduce financial frictions into a standard New Keynesian model and derive the optimal policy. They show that stabilizing credit spread and loan interest rate is a principle for optimal policy, but they do not address bank behavior that induces se-

rious financial disruptions. In contrast, our analysis shows that variables closely linked to bank behavior, such as credit market tightness, credit growth, and over- and under-lending, should be stabilized. Moreover, the former studies focus only on monetary policy, whereas this paper investigates also macroprudential policy. Thomas (2008) and Ravenna and Walsh (2011) pursue a linear-quadratic approach in models with search and matching frictions in the labor market, and show that the objective function of the monetary authority includes unemployment gaps. These authors, however, do not discuss the role of financial market frictions in the conduct of optimal monetary policy and exclude roles of macroprudential policy.

Third, our paper is related to studies that explore the roles of macroprudential policy in models with credit constraints. Bianchi and Mendoza (2013) build a model with a pecuniary externality and a collateral constraint following Kiyotaki and Moore (1997) and Bernanke, Gertler, and Gilchrist (1999), and derive optimal and time-consistent macroprudential policy without commitment. Farhi and Werning (2016) introduce a borrowing constraint into a model with nominal rigidities in goods and labor markets, and show optimal interventions that are justified by an aggregate demand externality.<sup>7</sup> These authors, however, do not focus on the supply side of credit and exclude direct regulations on credit creation, and also, they do not analytically show the optimality criteria for macroprudential policy.<sup>8</sup> While we focus only on business cycles, defined as deviations from an efficient steady state, and do not deal with types of externalities discussed by these authors, we analytically show the stabilization of financial variables as a principle of optimal macroprudential policy.

The rest of the paper is organized as follows. In Section 2, we set up the model. In Section 3, we derive the second-order approximation of the social welfare function and analyze an economy without policy response. In Section 4 and 5, we discuss optimal monetary and macroprudential policy. Finally, in Section 6, we conclude the paper.

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<sup>7</sup>Also see Bianchi (2010) and Korinek and Simsek (2015).

<sup>8</sup>An exception to this is where Gertler and Kiyotaki (2010) build a model which features, in addition to the liquidity constraint, the borrowing constraint for banks as in Bernanke, Gertler, and Gilchrist (1999), and evaluate macroprudential policies.

## 2 Model

The model economy is populated by four types of private agents: a single representative household (consumer), and large numbers of wholesale firms, banks, and retail firms. We explain the problems faced by these agents in turn, then describe the credit market, which is characterized by search and matching frictions, as well as the goods market.

### 2.1 Household

An infinitely lived representative household derives utility only from consumption, and discounts the future with discount factor  $\beta \in (0, 1)$ . In period  $t$ , the household enjoys total real consumption  $C_t$  and receives  $\Pi_t$  as a real lump-sum profit from firms and banks, and  $T_t$  as a real lump-sum transfer from the government. In addition, the household deposits  $D_t$  into a bank account, to be repaid at the end of period  $t$  with a nominal interest rate  $R_t^D - 1$ , where  $R_t^D$  is a policy variable of monetary policy.

Letting  $P_t$  denote the price of  $C_t$ , the household's problem is

$$\max_{\{C_{t+i}, D_{t+i}\}_{i=0}^{\infty}} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \xi_t u(C_{t+i}), \quad (1)$$

subject to the budget constraint

$$C_t = \Pi_t + T_t + \frac{R_{t-1}^D D_{t-1} - D_t}{P_t}. \quad (2)$$

The household's period utility function is

$$\xi_t u(C_t) \equiv \xi_t \frac{C_t^{1-\sigma}}{1-\sigma}, \quad (3)$$

where  $\xi_t$  is an intertemporal preference shock which follows a known stochastic process, and  $\sigma > 0$  is the coefficient of relative risk aversion.

This optimization problem leads to

$$\lambda_t = \xi_t C_t^{-\sigma}, \quad (4)$$

$$1 = \beta \mathbb{E}_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} R_t^D \right], \quad (5)$$

where  $\lambda_t$  is the Lagrange multiplier on the budget constraint, (2).

Total consumption  $C_t$  is an aggregate of differentiated retail goods, labeled by  $j \in [0, 1]$ . Consumption of each good  $c_t(j)$  is related to  $C_t$  by

$$C_t \equiv \left[ \int_0^1 c_t(j)^{\frac{\varepsilon_t-1}{\varepsilon_t}} dj \right]^{\frac{\varepsilon_t}{\varepsilon_t-1}}, \quad (6)$$

where  $\varepsilon_t \in (1, \infty)$  is the elasticity of substitution among retail goods, which follows a known stochastic process. In what follows, random fluctuations of  $\xi_t$  and  $\varepsilon_t$  are the only sources of aggregate uncertainty.

The household chooses each  $c_t(j)$  to minimize cost  $\int_0^1 p_t(j)c_t(j)dj$ , given the level of  $C_t$  and the price of each good,  $p_t(j)$ . This minimization yields

$$c_t(j) = \left[ \frac{p_t(j)}{P_t} \right]^{-\varepsilon_t} C_t, \quad (7)$$

where

$$P_t \equiv \left[ \int_0^1 p_t(j)^{1-\varepsilon_t} dj \right]^{\frac{1}{1-\varepsilon_t}}. \quad (8)$$

## 2.2 Wholesale Firms

In any period, a wholesale firm can be either a productive firm or a credit seeker firm. A productive firm produces  $Z_t$  units of wholesale goods. To be productive, a firm must obtain  $a$  real units of credit from a bank.

The credit market is characterized by search frictions, and the flow cost of posting a vacancy is  $\kappa > 0$  in real units. In addition, the government imposes tax  $\tau_t^c$  on the search cost  $\kappa$  as a tool of macroprudential policy for demand side of credit, and rebates the tax revenue to the household as a lump-sum transfer  $T_t$ .<sup>9</sup> Thus, the total flow cost of searching for a credit is  $(1 + \tau_t^c) \kappa$ . When  $\tau_t^c < 0$ , the policy amounts to a subsidy for the firms' search cost, financed by a lump-sum tax on the household. Thus, a credit seeker firm must buy retail goods  $(1 + \tau_t^c) \kappa_t(j)$ ,  $j \in (0, 1)$ , to satisfy

$$\left[ \int_0^1 \kappa_t(j)^{\frac{\varepsilon_t-1}{\varepsilon_t}} dj \right]^{\frac{\varepsilon_t}{\varepsilon_t-1}} \geq \kappa. \quad (9)$$

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<sup>9</sup>For example, Farhi and Werning (2016) also assume a financial tax on the demand side of credit.



The cost minimization for  $\kappa_t(j)$  parallels that for  $c_t(j)$  in the household's problem. For simplicity, we assume that firms finance the cost of searching for credit by issuing stocks to the household.<sup>10</sup>

In period  $t$ , with probability  $p_t^F$ , a credit seeker firm is matched with a bank and engages in a credit contract. The firm then receives  $a$  real units of credit and becomes productive, sells the produced goods to retail firms, and repays  $R_t^L a$  to the bank, where the loan interest rate  $R_t^L - 1$  is determined in equilibrium. Finally, at the end of period  $t$ , a credit contract is terminated with probability  $\rho \in (0, 1)$ , in which case the firm and the bank separate and search for new matches in period  $t + 1$ . With probability  $1 - \rho$ , a credit contract is sustained and the firm again receives credit in period  $t + 1$ . We call  $\rho$  the credit separation rate.

There is free entry into the wholesale goods industry. Thus, in equilibrium, the value of a credit seeker firm is zero, and hence the cost of searching for credit must equal the expected revenue, or

$$(1 + \tau_t^c) \kappa = p_t^F W_t. \quad (10)$$

Here,  $W_t$  is the value of a productive wholesale firm, written as

$$W_t = \frac{Z_t}{\mu_t} - (R_t^L - 1) a + \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} (1 - \rho) W_{t+1} \right], \quad (11)$$

where

$$\mu_t \equiv \frac{P_t}{P_t^w} \quad (12)$$

is the price markup by retail firms, and  $P_t^w$  is the price of a wholesale good. The first two terms on the right-hand side (RHS) of equation (11) show the net current profit from production, while the third term is the discounted present value of future profit.

Given these assumptions, the demand for retail good  $j$  and total demand are

$$y_t^d(j) \equiv c_t(j) + \kappa_t(j) u_t, \quad (13)$$

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<sup>10</sup>In an older version of our paper, Munakata, Nakamura, and Teranishi (2013) pursue an alternative setup in which wholesale firms costlessly search for credit and banks pay the cost of posting vacancies. The form of the approximated welfare function under this setup is identical to that obtained below.

and

$$Y_t^d \equiv C_t + \kappa u_t, \quad (14)$$

respectively, where  $u_t$  is the number of credit seeker firms. Note that the tax  $\tau_t^c$  does not enter these equations, since the entire tax revenue is rebated to the household. Also note that  $y_t^d$  is related to  $Y_t^d$  by the following equation:

$$y_t^d(j) = \left[ \frac{p_t(j)}{P_t} \right]^{-\varepsilon_t} Y_t^d. \quad (15)$$

## 2.3 Banks

Banks collect money from the household as deposits, and lend it to wholesale firms. To search for credit seeker firms, banks must post credit offers, which we call “credit vacancies”. Posting credit vacancies is costless, but total funds available for lending is capped at  $aL_t^*$ , such that the upper limit of the number of credit contracts is  $L_t^*$ .<sup>11</sup>

Therefore, the number of credit vacancies  $v_t$  is expressed as

$$v_t = L_t^* - (1 - \rho)L_{t-1}, \quad (16)$$

where  $L_t$  is the number of productive wholesale firms. In period  $t$ , a credit vacancy is filled with probability  $q_t^B$ . Thus,  $L_t$  evolves according to

$$L_t = (1 - \rho)L_{t-1} + q_t^B v_t. \quad (17)$$

In such settings, the value of a credit match for banks is

$$J_t^1 = a(R_t^L - 1) + \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \{ (1 - \rho)J_{t+1}^1 + \rho [q_{t+1}^B J_{t+1}^1 + (1 - q_{t+1}^B)J_{t+1}^0] \} \right). \quad (18)$$

The first term on the RHS of the equation shows current profit from lending, while the second term represents discounted present value of future profit. On the other hand, the value of a credit vacancy for banks is

$$J_t^0 = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} [q_{t+1}^B J_{t+1}^1 + (1 - q_{t+1}^B)J_{t+1}^0] \right\}. \quad (19)$$

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<sup>11</sup>For simplicity, we assume that  $aL_t^*$  is less than the amount of deposit.

Since a credit vacancy yields no current profit, it has only discounted future values. These two equations imply that the bank's surplus from a credit match is

$$J_t \equiv J_t^1 - J_t^0 = a(R_t^L - 1) + \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} (1 - \rho)(1 - q_{t+1}^B) J_{t+1} \right]. \quad (20)$$

We conclude the description of banks by explaining the nature of the upper limit of credit contracts  $L_t^*$ . In the real economy, when banks are allowed to extend ample loans, the screening criteria become less strict, resulting in the deterioration of the average productivity of projects funded by loans. To examine such a feature, we assume the following relationship between  $L_t^*$  and the productivity of wholesale firms  $Z_t$ :

$$Z_t = f(L_t^*). \quad (21)$$

Here, the function  $f$  is strictly positive, strictly decreasing, strictly concave, and continuously differentiable.

Furthermore,  $L_t^*$  works as the macroprudential regulation of lending limit for supply side of credit. This policy corresponds to a total credit control that is implemented in some countries as explained in Lim et al. (2011) and Nier et al. (2011).<sup>12</sup>

## 2.4 Retail Firms

Retail firms produce differentiated retail goods from wholesale goods, which are then sold to the household in a monopolistically competitive market. One unit of wholesale goods is converted into one unit of retail good  $j$ . To introduce price stickiness, we assume that a firm can adjust its price each period with probability  $1 - \omega$ , as in Calvo (1983) and Yun (1996). Since the demand for good  $j$  is given by equation (15), the profit maximization problem of a retail firm that has a chance to adjust its price  $P_t^*$  becomes

$$\max_{P_t^*} E_t \sum_{i=0}^{\infty} (\omega\beta)^i \left[ \left( \frac{\lambda_{t+i}}{\lambda_t} \right) \left( \frac{(1 + \tau)P_t^* - P_{t+i}^w}{P_{t+i}} \right) \left( \frac{P_t^*}{P_{t+i}} \right)^{-\varepsilon_{t+i}} Y_{t+i}^d \right]. \quad (22)$$

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<sup>12</sup>This policy is also interpreted as a capital adequacy ratio regulation that poses a constraint on banks to supply credit as explained in Lim et al. (2011) and Nier et al. (2011).

We here assume that the subsidy for retails firms  $\tau$  is set to ensure that price flexibility is achieved at the efficient steady-state equilibrium defined below. Note that  $P_t$  is related to  $P_{t-1}$  and  $P_t^*$  as

$$P_t^{1-\varepsilon_t} = (1 - \omega) (P_t^*)^{1-\varepsilon_t} + \omega P_{t-1}^{1-\varepsilon_t}. \quad (23)$$

## 2.5 Credit Market

The number of new credit matches in a period is given by a Cobb-Douglas matching function

$$m(u_t, v_t) = \chi u_t^{1-\alpha} v_t^\alpha, \quad \chi, \alpha \in (0, 1). \quad (24)$$

Defining credit market tightness as<sup>13</sup>

$$\theta_t = \frac{u_t}{v_t}, \quad (25)$$

we obtain

$$p_t^F = \chi \theta_t^{-\alpha}, \quad (26)$$

$$q_t^B = \chi \theta_t^{1-\alpha}, \quad (27)$$

$$L_t = (1 - \rho)L_{t-1} + \chi \theta_t^{1-\alpha} v_t. \quad (28)$$

The loan interest rate is determined according to generalized Nash bargaining between the matched wholesale firm and bank. Thus,  $R_t^L$  solves

$$\max_{R_t^L} W_t^{1-b} J_t^b, \quad (29)$$

where  $b \in (0, 1)$  is the bargaining power for banks. The first-order condition with respect to  $R_t^L$  yields

$$bW_t = (1 - b)J_t. \quad (30)$$

Using equations (10), (26), (27), and (30) to eliminate  $p_t^F$ ,  $q_t^B$ ,  $W_t$ , and  $J_t$  from (11) and (20), we obtain

$$(1 + \tau_t^c) \frac{\kappa}{\chi} \theta_t^\alpha = \frac{Z_t}{\mu_t} - (R_t^L - 1)a + \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} (1 - \rho) \frac{(1 + \tau_{t+1}^c) \kappa}{\chi} \theta_{t+1}^\alpha \right] \quad (31)$$

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<sup>13</sup>Note that in our environment,  $u_t$  and  $v_t$  correspond, respectively, to the demand and the supply of credit. Thus, market tightness is defined as  $u_t/v_t$ , rather than its inverse.

and

$$\frac{b}{1-b} \frac{(1 + \tau_t^c) \kappa}{\chi} \theta_t^\alpha = (R_t^L - 1)a + \beta \mathbb{E}_t \left[ \frac{\lambda_{t+1}}{\lambda_t} (1 - \rho) (1 - \chi \theta_{t+1}^{1-\alpha}) \frac{b}{1-b} \frac{(1 + \tau_{t+1}^c) \kappa}{\chi} \theta_{t+1}^\alpha \right]. \quad (32)$$

By further eliminating  $R_t^L$  from these equations and using equation (4), we obtain the following condition, which relates the markup  $\mu_t$  to credit market tightness  $\theta_t$ :

$$\frac{Z_t}{\mu_t} = \frac{1 + \tau_t^c}{1-b} \frac{\kappa}{\chi} \theta_t^\alpha - \beta \mathbb{E}_t \left[ \frac{\xi_{t+1} (C_{t+1})^{-\sigma}}{\xi_t (C_t)^{-\sigma}} (1 - \rho) \frac{1 + \tau_{t+1}^c}{1-b} \left( \frac{\kappa}{\chi} \theta_{t+1}^\alpha - b \kappa \theta_{t+1} \right) \right]. \quad (33)$$

Equation (33) shows that the credit market affects the real economy, that is, the price setting behavior, through the cost channel.

## 2.6 Goods Market Clearing Condition

Since one unit of wholesale goods is needed as an input to produce one unit of each retail good  $j$ , the market clearing condition for wholesale goods is

$$Z_t L_t = \int_0^1 y_t^d(j) dj. \quad (34)$$

Together with the demand equation for retail goods (15), the following goods market clearing condition is obtained:

$$\frac{Z_t L_t}{Q_t} = C_t + \kappa u_t. \quad (35)$$

Here,

$$Q_t \equiv \int_0^1 \left[ \frac{p_t(j)}{P_t} \right]^{-\varepsilon_t} dj \quad (36)$$

represents the dispersion of prices of retail goods due to price stickiness for retail firms.

## 3 Economy without Policy Response

We first analyze an economy without any policy response. Thus, we set policy variables at constant values, such as  $R_t^D = \bar{R}^D$ ,  $L_t^* = \bar{L}^*$  (and thus  $Z_t = \bar{Z} \equiv f(\bar{L}^*)$ ), and  $\tau_t^c = 0$ .<sup>14</sup> This analysis serves to clarify the effect of optimal policy on the social welfare and economic structure.

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<sup>14</sup>Throughout, a bar above each variable implies its value in the efficient steady-state equilibrium.

### 3.1 Welfare Criteria

We examine the optimal policy under the linear-quadratic approximation framework of Woodford (2003) and Benigno and Woodford (2003, 2012). In this section, we first introduce the concepts of efficient stochastic equilibrium and efficient steady-state equilibrium. The household's utility function is expanded around the efficient steady-state equilibrium to derive the second-order approximation of the welfare function. We then comment on the implications of the approximated welfare function.

#### 3.1.1 Efficient Stochastic Equilibrium

The efficient stochastic equilibrium is defined as the equilibrium of the economy without the cost-push shock (i.e.,  $\varepsilon_t = \bar{\varepsilon}$  for all  $t$ ), whose allocation coincides with that of a benevolent social planner who maximizes the discounted lifetime utility of the representative household. Such an equilibrium can be achieved only when the model exhibits neither credit matching inefficiency nor price markup. Specifically, we assume throughout that (1) the Hosios (1990) condition holds, that is, the bargaining power of banks ( $b$ ) equals the elasticity of the matching function with respect to credit vacancies ( $\alpha$ ), and (2) the subsidy for retail firms  $\tau$  is chosen to ensure  $\bar{\mu} = \frac{\bar{\varepsilon}}{(\bar{\varepsilon}-1)(1+\tau)} = 1$ .

By definition, the allocation in the efficient stochastic equilibrium is obtained by solving the following optimization problem of the social planner:

$$\begin{aligned} \max_{\{C_{t+i}^e, L_{t+i}^e, v_{t+i}^e, \theta_{t+i}^e\}_{i=0}^{\infty}} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left\{ \frac{\xi_{t+i} (C_{t+i}^e)^{1-\sigma}}{1-\sigma} + \phi_{t+i}^e [\bar{Z} L_{t+i}^e - \kappa \theta_{t+i}^e v_{t+i}^e - C_{t+i}^e] \right. & (37) \\ & + \nu_{t+i}^e [(1-\rho)L_{t+i-1}^e + \chi(\theta_{t+i}^e)^{1-\alpha} v_{t+i}^e - L_{t+i}^e] \\ & \left. + s_{t+i}^e [v_{t+i}^e - \bar{L}^* + (1-\rho)L_{t+i-1}^e] \right\}, \end{aligned}$$

where the superscript  $e$  represents the value of each variable in the efficient stochastic equilibrium. This problem yields the following condition that characterizes the allocation in the efficient stochastic equilibrium.

$$\bar{Z} = \frac{1}{1-\alpha} \frac{\kappa}{\chi} (\theta_t^e)^\alpha - \beta \mathbb{E}_t \left[ \frac{\xi_{t+1} (C_{t+1}^e)^{-\sigma}}{\xi_t (C_t^e)^{-\sigma}} (1-\rho) \frac{1}{1-\alpha} \left( \frac{\kappa}{\chi} (\theta_{t+1}^e)^\alpha - \alpha \kappa \theta_{t+1}^e \right) \right]. \quad (38)$$

### 3.1.2 Efficient Steady-State Equilibrium

The efficient steady-state equilibrium is defined as a steady-state equilibrium of the deterministic (i.e.,  $\xi_t = \bar{\xi} = 1$  and  $\varepsilon_t = \bar{\varepsilon}$  for all  $t$ ) model, whose allocation coincides with that of a benevolent social planner. By setting  $\xi_t = \xi_{t+1} = 1$  in (38) and removing the expectation operator as well as subscripts and superscripts, we obtain the following condition that characterizes the efficient steady-state equilibrium:

$$\bar{Z} - \frac{1}{1-\alpha} \frac{\kappa \bar{\theta}^\alpha}{\chi} = -\beta(1-\rho) \frac{1}{1-\alpha} \frac{\kappa \bar{\theta}^\alpha}{\chi} (1 - \alpha \chi \bar{\theta}^{1-\alpha}). \quad (39)$$

For later convenience, let

$$\delta_1 \equiv \bar{Z} - \frac{1}{1-\alpha} \frac{\kappa \bar{\theta}^\alpha}{\chi}, \quad (40)$$

$$\delta_2 \equiv (1-\rho) \frac{1}{1-\alpha} \frac{\kappa \bar{\theta}^\alpha}{\chi} (1 - \alpha \chi \bar{\theta}^{1-\alpha}), \quad (41)$$

to simplify (39) as

$$\delta_1 = -\beta \delta_2. \quad (42)$$

Since  $\bar{q}^B = \chi \bar{\theta}^{1-\alpha} \leq 1$ , it follows that  $\delta_2 \geq 0$  and thus  $\delta_1 \leq 0$ .

### 3.1.3 Policy Objective Function

Below, we express the log-deviation of a variable (e.g.,  $C_t$ ) from its efficient steady-state value ( $\bar{C}$ ) by placing a hat ( $\hat{\cdot}$ ) over its lower case ( $\hat{c}_t$ ); the difference, or the *gap*, between such variable with a hat from its value in the efficient stochastic equilibrium is denoted by placing a tilde ( $\tilde{\cdot}$ ). We call  $\tilde{c}_t \equiv \hat{c}_t - \hat{c}_t^e = \ln C_t - \ln C_t^e$  the consumption gap, and similarly for other variables with a tilde.

As shown in Appendix A, the second-order expansion of the household's utility function around the efficient steady-state equilibrium yields

$$\text{E}_t \sum_{i=0}^{\infty} \beta^i \xi_{t+i} u(C_{t+i}) \simeq V_{\max} - \frac{1}{2} \text{E}_t \sum_{i=0}^{\infty} \beta^i N_{t+i}, \quad (43)$$

where the maximum achievable welfare  $V_{\max}$  is given by

$$V_{\max} = \text{E}_t \sum_{i=0}^{\infty} \beta^i \xi_{t+i} u(C_{t+i}^e) - \frac{1}{2} \lambda_\pi (1-\omega) \Delta_{t-1}^V, \quad (44)$$

while the period loss function  $N_{t+i}$  is given by

$$N_{t+i} = \lambda_\pi \pi_{t+i}^2 + \lambda_c \tilde{c}_{t+i}^2 + \lambda_\theta \tilde{\theta}_{t+i}^2. \quad (45)$$

Here,  $\pi_t \equiv \hat{p}_t - \hat{p}_{t-1}$  is the inflation rate,  $\lambda_\pi \equiv u_c \bar{Z} \bar{L} \bar{\varepsilon} / \delta$ ,  $\lambda_c \equiv \sigma u_c \bar{C}$ ,  $\lambda_\theta \equiv u_c \kappa \bar{u} \alpha$ ,  $u_c \equiv u'(\bar{C})$ ,  $\delta \equiv (1 - \omega)(1 - \omega\beta) / \omega$ , and  $\Delta_t^V \equiv \text{Var}_j \hat{p}_t(j) \geq 0$ . On the RHS of equation (43),  $V_{\max}$  represents, as observed from (44), the social welfare achieved by the social planner less the welfare cost due to inherited price dispersion.<sup>15</sup> Since these terms are independent of policy, welfare maximization amounts to minimizing  $E_t \sum_{i=0}^{\infty} \beta^i N_{t+i}$ ; since  $N_{t+i}$  is nonnegative and equals zero if and only if all gaps are zero, the social welfare under the optimal policy is bounded above by  $V_{\max}$ .

Equation (45) shows that optimal policy faces a trade-off between variations in the inflation rate, consumption, and credit market tightness. The presence of the market tightness gap  $\tilde{\theta}_t$  in the approximated welfare function has a novel implication for optimal policy – even when the real economy is perfectly stable, with zero gaps in consumption and inflation, optimal policy should respond to an inefficient state of the credit market. Thus, introduction of a frictional credit market provides a theoretical justification for including financial stability as among the goals of optimal policy. We now explore in more detail the economic wedge represented by each of the terms in  $N_{t+i}$ .

The first two terms in  $N_{t+i}$  are present also in the standard New Keynesian model without credit market frictions. The inflation variation term  $\pi_{t+i}^2$  results from price dispersion due to price stickiness. Even when the aggregate consumption is at an efficient level, price dispersion distorts the composition of differentiated retail goods that are produced and consumed. The resulting welfare loss is captured by this term  $\pi_{t+i}^2$ . Note that when prices are flexible, i.e.,  $\omega = 0$ , we have  $\delta = \infty$  and thus  $\lambda_\pi = 0$ , such that the inflation variation term disappears from the objective function. The term  $\tilde{c}_{t+i}^2$ , on the other hand, appears here because consumption variation induces welfare loss due to the concavity of the consumer's utility function.

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<sup>15</sup>The term representing welfare cost due to inherited price dispersion arises because we assume the absence of such inherited price dispersion in the planner's problem.



The third term in  $N_{t+i}$ ,  $\tilde{\theta}_{t+i}^2$ , represents welfare loss due to inefficient variations in credit market tightness. The weight on this term,  $\lambda_\theta$ , depends on the model parameters in a complicated fashion through the steady state number of credit seeker firms,  $\bar{u}$ . As shown below in Section 3.1.4, however, the relative importance of the variations in market tightness to that in consumption,  $\lambda_\theta/\lambda_c$ , increases with the search cost  $\kappa$ . This result is intuitive since the presence of the term  $\tilde{\theta}_{t+i}^2$  here is a result of introducing a frictional credit market. In fact, without the cost of searching for credit, i.e.,  $\kappa = 0$ , the term  $\tilde{\theta}_{t+i}^2$  is absent from the policy objective function, just like in a standard New Keynesian model.<sup>16</sup>

Note that the approximated welfare function can be transformed as

$$\text{E}_t \sum_{i=0}^{\infty} \beta^i \xi_{t+i} u(C_{t+i}) \simeq V^{\max} - \frac{1}{2} \sum_{i=0}^{\infty} \beta^i \left[ \lambda_\pi \pi_{t+i}^2 + \lambda_c \tilde{c}_{t+i}^2 + \frac{\lambda_\theta}{(1-\alpha)^2 \rho^2} \left( \tilde{l}_{t+i} - \rho_u \tilde{l}_{t+i-1} \right)^2 \right], \quad (46)$$

where

$$\rho_u \equiv (1-\rho) \left( 1 - \rho \frac{\bar{L}}{\bar{v}} \right). \quad (47)$$

Equation (46) clearly shows that the optimal policy should respond to the volume of credit. Note that  $\rho_u \in (0, 1)$ , since equations (27) and (28) yield  $\rho \bar{L}/\bar{v} = \bar{q}^B \in (0, 1)$ . In particular, as the separation rate  $\rho$  approaches 1,  $\rho_u$  approaches zero. This implies that the optimal policy should focus on the current volume of credit, because the history of the credit market is irrelevant when all matches are replaced each period. In contrast, as  $\rho$  approaches zero,  $\rho_u$  approaches 1. In this limit, all existing loans continue to the next period, so the optimal policy should focus on the volume of new loans, or equivalently, on the growth of credit.

The result that the criteria for optimal policy directly includes the volume of credit is a nontrivial finding. It justifies that optimal policy works for eliminating inefficient dynamics of lending, such as an over- and under-lending. This is quite consistent with

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<sup>16</sup>When  $\kappa = 0$ , the zero profit condition for credit seeker firms, equation (10), implies  $p_t^F = 0$ . That is, there will be infinite number of credit seeker firms, and thus  $\theta_{t+1} = \infty$ . To deal with such a situation, we need to redefine the matching function as  $m(u_t, v_t) = \min \{ \chi u_t^{1-\alpha} v_t^\alpha, u_t, v_t \}$ . Then,  $q_t^B = 1$  and  $L_t = \bar{L}^*$  for all  $t$ , such that all funds available for lending are lent out in all periods.

the aim of macroprudential policy to stabilize the volume of loans.<sup>17</sup>

Furthermore, the approximated welfare includes both financial variables, such as credit, and real economic variables, such as inflation and consumption. This finding clearly contrasts with recent argument that insists that the macroprudential authority should focus only on the financial variables.<sup>18</sup>

### 3.1.4 Analysis for Welfare Criteria

We now analyze the dependence of the welfare function on several parameters, especially focusing on the weight of the term corresponding to financial frictions.

Straightforward calculation in Appendix B yields

$$\frac{\partial}{\partial \kappa} \left( \frac{\lambda_\theta}{\lambda_c} \right) > 0, \quad (48)$$

where the parameters (e.g.,  $\alpha$ ) except  $\kappa$  are fixed, but the efficient steady-state value of the variables (e.g.,  $\bar{\theta}$ ) are allowed to vary with  $\kappa$ . This relationship (48) implies that, as the cost of searching for credit  $\kappa$  increases, the relative weight for credit to that for consumption in the approximated welfare function ( $\lambda_\theta/\lambda_c$  of equation (45)) increases. This is because as  $\kappa$  increases, the degree of financial friction and thus welfare improvement from addressing inefficiency in the credit market becomes greater.

A similar relationship holds for the credit separation rate  $\rho$ , namely:

$$\frac{\partial}{\partial \rho} \left( \frac{\lambda_\theta}{\lambda_c} \right) > 0. \quad (49)$$

Again, this is because the increase in  $\rho$  raises the cost of holding credit.

These results imply that the relative weight for the credit term increases when the cost of obtaining credit increases. In other words, as the degree of market imperfection increases, the optimal policy should react more strongly to the credit market condition.

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<sup>17</sup>For example, see BCBS (2010, 2014).

<sup>18</sup>For example, Drehmann, Borio, and Tsatsaronis (2012) show that the credit–GDP ratio is a good predictive indicator of financial crisis and emphasize that policymakers should use this ratio as the criterion for implementing macroprudential policy.

### 3.2 Linearization

In this subsection, we log-linearize the structural equations around the efficient steady-state equilibrium. For a non-efficient stochastic equilibrium, the Calvo-type price stickiness introduced in the retail sector leads to the standard Phillips curve with a cost-push shock  $\widehat{\varepsilon}_t$ ,

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} - \delta \left( \frac{1}{\bar{\varepsilon} - 1} \widehat{\varepsilon}_t + \widehat{\mu}_t \right). \quad (50)$$

The retail price markup term  $\widehat{\mu}_t$  in this equation can be obtained from the log-linearized version of equation (33),

$$\bar{Z} \widehat{\mu}_t = -\frac{\alpha}{1 - \alpha} \frac{\kappa}{\chi} \bar{\theta}^\alpha \left( \widehat{\theta}_t - \beta \rho_u \mathbb{E}_t \widehat{\theta}_{t+1} \right) - \beta \delta_2 \left( \sigma \mathbb{E}_t \widehat{c}_{t+1} - \mathbb{E}_t \widehat{\xi}_{t+1} - \sigma \widehat{c}_t + \widehat{\xi}_t \right), \quad (51)$$

where equation (4) and the Hosios condition  $b = \alpha$  is used.

From equations (4) and (5), the IS relation is given as

$$\widehat{c}_t = \mathbb{E}_t \widehat{c}_{t+1} + \frac{1}{\sigma} \left( \mathbb{E}_t \pi_{t+1} - \mathbb{E}_t \widehat{\xi}_{t+1} + \widehat{\xi}_t \right). \quad (52)$$

On the other hand, by linearizing equations (16) and (28), we can express the credit market tightness term  $\widehat{\theta}_t$  by utilizing the loan volume term  $\widehat{l}_t$  in

$$\widehat{\theta}_t = \frac{1}{(1 - \alpha)\rho} \left( \widehat{l}_t - \rho_u \widehat{l}_{t-1} \right). \quad (53)$$

By combining equation (53) with the linearized equation of the market clearing condition (equation (35)), the consumption term  $\widehat{c}_t$  is given by

$$\widehat{c}_t = \frac{\bar{L} \delta_2}{\bar{C}} \left( -\beta \widehat{l}_t + \widehat{l}_{t-1} \right). \quad (54)$$

These equations are a closed system of the linearized economy around the efficient steady-state equilibrium.<sup>19</sup>

Next, we log-linearize the structural equations of the efficient stochastic equilibrium around the efficient steady-state equilibrium. The equation similar to the markup equation (51) is obtained by the log-linearizing equation (38) as

$$0 = -\frac{\alpha}{1 - \alpha} \frac{\kappa}{\chi} \bar{\theta}^\alpha \left( \widehat{\theta}_t^e - \beta \rho_u \mathbb{E}_t \widehat{\theta}_{t+1}^e \right) - \beta \delta_2 \left( \sigma \mathbb{E}_t \widehat{c}_{t+1}^e - \mathbb{E}_t \widehat{\xi}_{t+1} - \sigma \widehat{c}_t^e + \widehat{\xi}_t \right), \quad (55)$$

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<sup>19</sup>It is noteworthy that, by using linearized equations of (4) and (32) and equations (52)–(54), we obtain a loan curve showing a relationship between the loan interest rate and credit volume. See a detail of a loan curve in Appendix C.

while the credit market tightness term and the consumption term are given by

$$\widehat{\theta}_t^e = \frac{1}{(1-\alpha)\rho} \left( \widehat{l}_t^e - \rho_u \widehat{l}_{t-1}^e \right), \quad (56)$$

$$\widehat{c}_t^e = \frac{\bar{L}\delta_2}{\bar{C}} \left( -\beta \widehat{l}_t^e + \widehat{l}_{t-1}^e \right). \quad (57)$$

In addition, the deposit interest rate term for the efficient stochastic equilibrium  $\widehat{r}_t^e$  is implicitly defined by the the following equation that corresponds to the IS equation (52):<sup>20</sup>

$$\widehat{c}_t^e = \mathbf{E}_t \widehat{c}_{t+1}^e + \frac{1}{\sigma} \left( -\widehat{r}_t^e - \mathbf{E}_t \widehat{\xi}_{t+1} + \widehat{\xi}_t \right). \quad (58)$$

Finally, by subtracting each structural equation of the efficient stochastic equilibrium from its counterpart in the non-efficient stochastic equilibrium, we obtain the structural equations in terms of gaps. More precisely, equations (51) and (55) yield

$$\bar{Z}\widehat{\mu}_t = -\frac{\alpha}{1-\alpha} \frac{\kappa}{\chi} \bar{\theta}^\alpha \left( \widetilde{\theta}_t - \beta \rho_u \mathbf{E}_t \widetilde{\theta}_{t+1} \right) - \beta \sigma \delta_2 \left( \mathbf{E}_t \widetilde{c}_{t+1} - \widetilde{c}_t \right), \quad (59)$$

while equations (52) and (58) lead to

$$\widetilde{c}_t = \mathbf{E}_t \widetilde{c}_{t+1} + \frac{1}{\sigma} \left( \mathbf{E}_t \pi_{t+1} + \widehat{r}_t^e \right). \quad (60)$$

Similarly, equations (53) and (56) lead to

$$\widetilde{\theta}_t = \frac{1}{(1-\alpha)\rho} \left( \widetilde{l}_t - \rho_u \widetilde{l}_{t-1} \right), \quad (61)$$

and equations (54) and (57) yield

$$\widetilde{c}_t = \frac{\bar{L}\delta_2}{\bar{C}} \left( -\beta \widetilde{l}_t + \widetilde{l}_{t-1} \right). \quad (62)$$

## 4 Optimal Policy

We now illustrate optimal policy under financial frictions by considering several monetary and macroprudential policy tools. Throughout, optimal policy refers to the optimal commitment policy under the timeless perspective.

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<sup>20</sup>Here,  $\widehat{r}_t^e = r_t^e - \bar{r}$ , where  $r_t^e$  and  $\bar{r}$  are, respectively, the interest rate in the efficient stochastic equilibrium and the efficient steady-state equilibrium. We do not refer to  $\widehat{r}_t^e$  as the gap in natural interest rate since the flexible price equilibrium does not coincide with the efficient stochastic equilibrium due to the presence of cost-push shocks.

## 4.1 Monetary Policy

In this subsection, we investigate optimal monetary policy when the central bank is the unique authority responsible for financial as well as real economic stability.<sup>21</sup> Following Woodford (2003), we assume that the central bank controls the nominal interest rate on deposits,  $R_t^D$ , to maximize social welfare. In this case, equation (60) includes the deposit interest rate gap,  $\tilde{r}_t^D \equiv \hat{r}_t^D - \hat{r}_t^e$ , as

$$\tilde{c}_t = \mathbb{E}_t \tilde{c}_{t+1} - \frac{1}{\sigma} (\tilde{r}_t^D - \mathbb{E}_t \pi_{t+1}). \quad (63)$$

The central bank controls  $\tilde{r}_t^D$  and thus the real deposit interest rate gap  $\tilde{r}_t^D - \mathbb{E}_t \pi_{t+1}$  by varying  $\hat{r}_t^D$ . Accordingly, the central bank can affect consumption, and thus the entire economy, through the IS relation given by equation (63). This is the typical transmission channel of monetary policy in the literature.

It is noteworthy that by linearizing equations (5) and (32), the loan interest rate gap  $\tilde{r}_t^L$  is shown to be related to  $\tilde{\theta}_t$  and  $\tilde{r}_t^D$  as

$$a\bar{R}^L \tilde{r}_t^L = \frac{\alpha}{1-\alpha} \frac{\kappa}{\chi} \bar{\theta}^\alpha \left[ \alpha \tilde{\theta}_t - \beta(1-\rho) (\alpha - \chi \bar{\theta}^{1-\alpha}) \mathbb{E}_t \tilde{\theta}_{t+1} + \beta(1-\rho) (1 - \chi \bar{\theta}^{1-\alpha}) (\tilde{r}_t^D - \mathbb{E}_t \pi_{t+1}) \right]. \quad (64)$$

In particular, equation (64) implies that when the deposit interest rate gap and credit market tightness gap increase, so does the loan interest rate gap.

For the approximated welfare function in equation (43), the optimal policy for the central bank is obtained by solving

$$\min_{\{\pi_{t+i}, \tilde{\theta}_{t+i}, \tilde{c}_{t+i}, \tilde{l}_{t+i}, \tilde{r}_{t+i}^D\}_{i=0}^{\infty}} \mathbb{E}_t \sum_{i=0}^{\infty} \frac{1}{2} \beta^i \left( \lambda_\pi \pi_{t+i}^2 + \lambda_\theta \tilde{\theta}_{t+i}^2 + \lambda_c \tilde{c}_{t+i}^2 \right), \quad (65)$$

subject to the Phillips curve equation (50), the markup equation (59), the IS equation (63), the credit market tightness equation (61), and the consumption equation (62).<sup>22</sup>

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<sup>21</sup>To elaborate, we assume here that macroprudential policy variables are held constant ( $L_t^* = \bar{L}^*$  and  $\tau_t^c = 0$ ) such that monetary policy is expected to play a macroprudential role as well.

<sup>22</sup>Optimal criteria for monetary policy given by equation (65) implies that, under credit market fric-

The intertemporal preference shock terms  $\widehat{\xi}_t$  and  $E_t\widehat{\xi}_{t+1}$  appear in neither the objective function nor these constraints. Therefore, optimal monetary policy varies  $\widehat{r}_t^D$  one-to-one with  $\widehat{r}_t^e$  to prevent the preference shock from affecting the deposit interest rate gap  $\widehat{r}_t^D$  and accordingly the gaps of inflation, market tightness, consumption, and the loan volume. Thus, under optimal monetary policy, these gaps and consequently the welfare loss depend solely on the cost-push shock; in the absence of the cost-push shock, optimal monetary policy eliminates these gaps and achieves  $V_{\max}$ .

To further characterize optimal monetary policy, note that the first-order conditions with respect to  $\pi_t$ ,  $\widetilde{\theta}_t$ ,  $\widetilde{c}_t$ ,  $\widetilde{l}_t$ , and  $\widetilde{r}_t^D$  are

$$\lambda_\pi \pi_t + \varphi_{1t} - \varphi_{1t-1} - \beta^{-1} \sigma^{-1} \varphi_{2t-1} = 0, \quad (66)$$

$$\lambda_\theta \widetilde{\theta}_t - \frac{\delta}{\bar{Z}} \frac{\alpha}{1-\alpha} \frac{\kappa}{\chi} \bar{\theta}^\alpha (\varphi_{1t} - \rho_u \varphi_{1t-1}) + \varphi_{3t} = 0, \quad (67)$$

$$\lambda_c \widetilde{c}_t - \frac{\delta \delta_2 \sigma}{\bar{Z}} (\varphi_{1t-1} - \beta \varphi_{1t}) + \varphi_{2t} - \beta^{-1} \varphi_{2t-1} + \varphi_{4t} = 0, \quad (68)$$

$$- \frac{1}{(1-\alpha)\rho} \varphi_{3t} + \frac{\beta \rho_u}{(1-\alpha)\rho} E_t \varphi_{3t+1} + \frac{\bar{L} \delta_2 \beta}{\bar{C}} \varphi_{4t} - \frac{\bar{L} \delta_2 \beta}{\bar{C}} E_t \varphi_{4t+1} = 0, \quad (69)$$

$$\varphi_{2t} = 0, \quad (70)$$

where  $\varphi_{1t}$ ,  $\varphi_{2t}$ ,  $\varphi_{3t}$ , and  $\varphi_{4t}$  are the Lagrange multipliers for equations (50) (combined with (59)), (63), (61), and (62), respectively. As shown in Appendix D.1, these first-order conditions yield

$$\varphi_{1t} = u_c \frac{\bar{Z} \bar{L}}{\delta} \widetilde{l}_t, \quad (71)$$

$$\varphi_{3t} = \varphi_{4t} = 0. \quad (72)$$

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tions, simple policy rules should include terms related to credit. Without a credit market, Woodford (2003) analytically shows that simple monetary policy rules, e.g, the Taylor rule, should respond to inflation rate and consumption terms, since approximated welfare includes these terms and their stabilization can improve welfare. A number of studies claim that a simple policy rule should include variables related to credit under financial frictions. For example, Christiano et al. (2010), from numerical simulations, claim that policy should respond to credit in addition to inflation and the output gap to improve welfare. Our results extend Woodford (2003) and theoretically support Christiano et al.(2010).

Substituting equations (70) and (71) into equation (66) yields the optimal targeting rule defined in Woodford (2003),

$$\pi_t + \frac{1}{\bar{\varepsilon}} \left( \tilde{l}_t - \tilde{l}_{t-1} \right) = 0. \quad (73)$$

The central bank adjusts the deposit interest rate  $R_t^D$  (and thus  $\tilde{r}_t^D$ ) to satisfy equation (73). This optimal targeting rule and the linearized structural equations (50), (59), (61), (62), and (63) define the paths of  $\pi_t$ ,  $\tilde{\theta}_t$ ,  $\tilde{c}_t$ ,  $\tilde{l}_t$ , and  $\tilde{r}_t^D$  under optimal monetary policy.

Equations (70) and (72) imply that constraints (61)–(63) do not bind under the current environment. That the IS equation (63) is slack under optimal monetary policy is not surprising, and it follows because the monetary authority can choose  $\tilde{r}_t^D$  such that the desired allocation satisfies (63). To see why the credit market tightness equation (61) and the consumption equation (62) are also slack, note that these two constraints combined represent the market clearing condition.<sup>23</sup> Now, among the paths of  $(\tilde{\theta}_t, \tilde{c}_t, \tilde{l}_t)$  that satisfy, for a given path of  $\hat{\mu}_t$ , the market clearing condition and the version of equation (59) for a general  $b \in (0, 1)$  (i.e., when the Hosios condition is not imposed), the ones that satisfy the latter equation for  $b = \alpha$  (i.e., equation (59)) maximize  $-\mathbb{E}_t \sum_{i=0}^{\infty} \frac{1}{2} \beta^i \left( \lambda_{\theta} \tilde{\theta}_{t+i}^2 + \lambda_c \tilde{c}_{t+i}^2 \right)$ . Further, when the IS equation (63) is slack, such paths of  $(\tilde{\theta}_t, \tilde{c}_t, \tilde{l}_t)$  that maximize  $-\mathbb{E}_t \sum_{i=0}^{\infty} \frac{1}{2} \beta^i \left( \lambda_{\theta} \tilde{\theta}_{t+i}^2 + \lambda_c \tilde{c}_{t+i}^2 \right)$  for some path of  $\hat{\mu}_t$  also maximize social welfare. Thus, in the central bank's problem, equation (59) already takes into account equations (61) and (62).<sup>24</sup>

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<sup>23</sup>Equation (62) is obtained by combining equation (61) with the market clearing condition. Then, since no other equations in the central bank's problem involves the loan volume term  $\tilde{l}_t$ , the only role played by equations (61) and (62) in this problem is to restrict the choices of credit market tightness and consumption gap to those satisfying the market clearing condition.

<sup>24</sup>Appendix D.2 discusses in more detail why equations (61) and (62) are slack. Key in this argument is that the central bank's problem can be split into three parts, namely, (i) choosing the path of  $(\tilde{\theta}_t, \tilde{c}_t, \tilde{l}_t)$  for a given path of  $\hat{\mu}_t$ , (ii) choosing the path of  $\pi_t$  for a given path of  $\hat{\mu}_t$ , and (iii) choosing the path of  $\hat{\mu}_t$  given (i) and (ii). When the monetary policy is not chosen optimally, the IS equation (63) imposes an additional restriction on the relationship between consumption and inflation, hence the argument breaks down and equation (72) generally fails to hold. The intuition is that imposing the Hosios condition resolves search externalities, but not the distortions caused through the IS equation.

The optimal targeting rule (73) has several important features. First, that equation (73) includes both financial variables ( $\tilde{l}_t$  and  $\tilde{l}_{t-1}$ ) and a real economic variable ( $\pi_t$ ) implies the optimal targeting rule must maintain a balance between conditions in the financial market and the real economy. By taking financial variables into account, monetary policy may contribute to financial stability and perform the macroprudential role. This result contrasts with the standard result shown in Woodford (2003): Under the model with frictions in the goods market, that is, price stickiness, the loan volume gap  $\tilde{l}_t$  is replaced by the consumption gap  $\tilde{c}_t$ , so optimal monetary policy focuses on the relationship between inflation and consumption.

Second, when monetary policy serves a macroprudential role, optimality requires keeping negative comovement between price and credit. In the optimal targeting rule, the policy rate is set to induce disinflation against positive credit growth, so as to avoid overheating or overcooling of the economy. This finding is consistent with the recent argument claiming that preventing pro-cyclicality of financial markets can reduce the occurrences of, and dampen the disruptions from, financial crises.<sup>25</sup>

## 4.2 Tax/Subsidy on the Search Cost as a Macroprudential Policy

In this subsection, we examine the macroprudential policy of varying tax  $\tau_t^c$  on the search cost  $\kappa$  of credit seeker firms. Here, we shut down the monetary policy as well as the macroprudential policy of controlling the upper bound on credit supply by keeping fixed  $R_t^D$  at  $\bar{R}^D$  and  $L_t^*$  at  $\bar{L}^*$ . Then, the linearized markup equation (59) is replaced by

$$\begin{aligned} \bar{Z}\hat{\mu}_t = & -\frac{\alpha}{1-\alpha}\frac{\kappa}{\chi}\bar{\theta}^\alpha(\tilde{\theta}_t - \beta\rho_u\mathbf{E}_t\tilde{\theta}_{t+1}) - \beta\sigma\delta_2(\mathbf{E}_t\tilde{c}_{t+1} - \tilde{c}_t) \\ & - [(\bar{Z} + \beta\delta_2)\tau_t^c - \beta\delta_2\mathbf{E}_t\tau_{t+1}^c]. \end{aligned} \quad (74)$$

The optimal tax/subsidy policy minimizes the loss function (65) subject to the Phillips curve equation (50), the markup equation (74), the IS equation (60), the credit market

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<sup>25</sup>See, e.g., BIS (2009).



tightness equation (61), and the consumption equation (62). As a result, while the first-order conditions with respect to  $\pi_t$ ,  $\tilde{\theta}_t$ ,  $\tilde{c}_t$ , and  $\tilde{l}_t$  are still given by equations (66)–(69), equation (70) is replaced by the first-order condition with respect to  $\tau_t^c$ ,

$$\varphi_{1t} - \frac{\delta_2}{\bar{Z} + \beta\delta_2} \varphi_{1t-1} = 0. \quad (75)$$

Equation (74) suggests that the tax/subsidy policy changes the marginal cost of production due to variation of the search cost for credit. As explained below, this policy plays a role that is complementary to the monetary policy. Since the term  $\tau_t^c$  does not appear in the IS equation (52), the tax/subsidy policy fails to undo the preference shock, which affects the gaps  $\pi_t$ ,  $\tilde{\theta}_t$ ,  $\tilde{c}_t$ , and  $\tilde{l}_t$  in this equation through a change in  $\hat{r}_t^c$ . However, the tax/subsidy policy is able to completely offset the cost-push shock  $\hat{\varepsilon}_t$ , which acts only on the Phillips curve equation. To see this, first substitute equation (74) into the Phillips curve equation (50) to obtain

$$\begin{aligned} (\pi_t - \beta E_t \pi_{t+1}) - \frac{\delta}{\bar{Z}} \frac{\alpha}{1 - \alpha} \frac{\kappa}{\chi} \bar{\theta}^\alpha \left( \tilde{\theta}_t - \beta \rho_u E_t \tilde{\theta}_{t+1} \right) + \frac{\delta}{\bar{Z}} \beta \delta_2 \sigma (\tilde{c}_t - E_t \tilde{c}_{t+1}) \\ = -\frac{\delta}{\bar{\varepsilon} - 1} \hat{\varepsilon}_t + \frac{\delta}{\bar{Z}} (\bar{Z} + \beta \delta_2) \left( \tau_t^c - \frac{\beta \delta_2}{\bar{Z} + \beta \delta_2} E_t \tau_{t+1}^c \right). \end{aligned} \quad (76)$$

Note that the RHS of equation (76) is freely controlled by varying  $\tau_t^c$ . Now, suppose the preference shock is absent so that  $\hat{r}_t^c = 0$  for all  $t$ . Then, since the term  $\hat{\varepsilon}_t$  does not appear in the objective function or other constraints, it follows that the optimal tax/subsidy exactly cancels the cost-push shock by setting

$$\tau_t^c = \frac{\bar{Z}}{\bar{Z} + \beta \delta_2} \frac{\hat{\varepsilon}_t}{\bar{\varepsilon} - 1} + \frac{\beta \delta_2}{\bar{Z} + \beta \delta_2} E_t \tau_{t+1}^c. \quad (77)$$

The optimal tax/subsidy policy given by equation (77) eliminates the gaps  $\pi_t$ ,  $\tilde{\theta}_t$ ,  $\tilde{c}_t$ , and  $\tilde{l}_t$  and achieves  $V_{\max}$ .

Equation (77) implies that if the cost-push shock  $\hat{\varepsilon}_t$  has no persistence or exhibits positive autocorrelation, then optimal macroprudential policy under no preference shock requires adjusting  $\tau_t^c$  in the same direction as  $\hat{\varepsilon}_t$ . To understand this, suppose the economy is hit by a cost-push shock that raises inflation ( $\hat{\varepsilon}_t < 0$ ). As observed from equation (10), lowering the tax  $\tau_t^c$  on the search cost reduces the equilibrium value of a productive

wholesale firm and thus the match surplus, and therefore lowers the loan interest rate  $R_t^L$ . This lowers the production cost of wholesale firms, causing a fall in the price of wholesale goods relative to that of retail goods. This raises the markup  $\mu_t$  of retail firms and lowers inflation, thus cancelling the cost-push shock.

### 4.3 Optimal Combination of Monetary Policy and Tax/Subsidy Policy on the Search Cost

As discussed in the previous two subsections, optimal monetary policy completely offsets the preference shock but not the cost-push shock, while the macroprudential policy is capable of fully cancelling the cost-push shock. Clearly, then, the optimal combination of monetary and macroprudential policies are given by  $\tilde{r}_t^D = 0$  and (77) and completely cancel the cost-push and preference shocks in equations (63) and (76), thereby eliminating the welfare loss.

The interpretation of these combined policy rules is straightforward. Optimal monetary policy prevents the deviation of consumption from its efficient counterpart by offsetting the effect that the preference shock has on the IS equation through  $\hat{r}_t^e$ . Optimal macroprudential policy cancels the effect that the cost-push shock has on the Phillips curve equation through varying the retail price markup. Clearly, any combinations of shocks that affect only these structural equations can be offset by the optimal combination of monetary and tax/subsidy policy.

## 5 Extended Model with Total Credit Control as a Macroprudential Policy Tool

In this section, we extend our baseline model by incorporating the macroprudential policy of controlling the upper bound on credit supply  $L_t^*$ . This policy corresponds to a total credit control. In order to focus on this form of macroprudential policy, we set  $\tau_t^c = 0$  throughout this section.

## 5.1 Total Credit Control as a Macroprudential Policy

We first inactivate the monetary policy by fixing the deposit interest rate  $R_t^D$  at  $\bar{R}^D$ .

Note that in this extended model, the social-planner problem for obtaining the condition for efficient steady-state equilibrium treats  $L_t^*$  as a choice variable. Observing that the second order expansion of equation (21) is expressed as

$$\bar{Z} \left( \hat{z}_t + \frac{1}{2} \hat{z}_t^2 \right) \simeq \bar{L}^* \left[ \bar{f}_1 \hat{l}_t^* + \frac{1}{2} (\bar{f}_1 + \bar{f}_2 \bar{L}^*) \hat{l}_t^{*2} \right], \quad (78)$$

where  $\bar{f}_1 \equiv f'(\bar{L}^*) < 0$  and  $\bar{f}_2 \equiv f''(\bar{L}^*) < 0$ , we obtain, as shown in Appendix E, in addition to equation (42), the following condition for the efficient steady-state equilibrium:

$$\bar{L} \bar{f}_1 = -\frac{\alpha}{1-\alpha} \kappa \bar{\theta}. \quad (79)$$

Furthermore, the approximated welfare function becomes

$$\begin{aligned} \text{E}_t \sum_{i=0}^{\infty} \beta^i \xi_{t+i} u(C_{t+i}) &\simeq V_{\max}^* - \frac{1}{2} \text{E}_t \sum_{i=0}^{\infty} \beta^i \left( \lambda_{\pi} \pi_{t+i}^2 + \lambda_{\theta} \tilde{\theta}_{t+i}^2 + \lambda_c \tilde{c}_{t+i}^2 + \lambda_{L^*} \tilde{l}_{t+i}^{*2} \right) \\ &\quad - \text{E}_t \sum_{i=0}^{\infty} \beta^i \psi_{L^*} \tilde{l}_{t+i}^* \tilde{l}_{t+i}, \end{aligned} \quad (80)$$

where  $\lambda_{L^*} \equiv u_c \bar{L} \bar{L}^{*2} (-\bar{f}_2) > 0$  and  $\psi_{L^*} \equiv u_c \bar{L}^* \frac{\alpha}{1-\alpha} \kappa \bar{\theta} > 0$ .<sup>26</sup>

Equation (80) has a clear implication. The second-order term  $\lambda_{L^*} \tilde{l}_{t+i}^{*2}$  on the RHS is the cost incurred by excessive variations in the limit of credit that induces, given the concavity of  $f$ , efficiency losses in productivity. The linearity of the last term in  $\tilde{l}^*$  suggests that, when the volume of credit exceeds its value in the efficient stochastic equilibrium ( $\tilde{l} > 0$ ), society is better off by restricting the supply of loans  $L^*$  to a level below that in the efficient stochastic equilibrium. This result serves as a theoretical foundation for setting a lending limit as a macroprudential policy to eliminate inefficient supply of credit.

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<sup>26</sup>The expression for the maximum achievable welfare  $V_{\max}^*$  is given by the RHS of equation (44), just like for  $V_{\max}$ . However, since the social planner is now equipped with an extra tool of varying  $L_t^*$ , the first term on the RHS of equation (44) in this extended model is weakly greater than that in the baseline model. Thus,  $V_{\max}^* \geq V_{\max}$ , with the inequality being strict whenever varying  $L_t^*$  is optimal.

Furthermore, the time variation in productivity  $Z_t$ , which depends on the upper limit of credit  $L_t^*$ , modifies the markup equation (59) as

$$\begin{aligned} \bar{Z}\hat{\mu}_t = & -\frac{\alpha}{1-\alpha}\frac{\bar{L}^*}{\bar{L}}\kappa\tilde{\theta}_t^* - \frac{\alpha}{1-\alpha}\frac{\kappa}{\chi}\bar{\theta}^\alpha\left(\tilde{\theta}_t - \beta\rho_u\mathbf{E}_t\tilde{\theta}_{t+1}\right) \\ & - \beta\sigma\delta_2\left(\mathbf{E}_t\tilde{c}_{t+1} - \tilde{c}_t\right), \end{aligned} \quad (81)$$

and the relationship (61) between credit market tightness and the volume of credit as

$$\tilde{\theta}_t = \frac{1}{(1-\alpha)\rho}\left[\tilde{l}_t - \rho_u\tilde{l}_{t-1} - (\rho + (1-\rho)\chi\bar{\theta}^{1-\alpha})\tilde{l}_t^*\right]. \quad (82)$$

Thus, when the macroprudential authority increases the upper bound on the credit supply, tightness of the credit market loosens. Then, the marginal cost for production as in equation (81), thus real economy as well as financial economy, responds to the policy.

We observe from above that depending on the macroprudential policy variable, the form of the approximated welfare function and structural equations differ from that in the economy without policy. This is because macroprudential policies themselves become a part of the economy and change the economic structure. This simple but new finding can be demonstrated via the approach of a model-consistent welfare approximation, and is important in conducting optimal policies under new macroprudential policies.

Optimal macroprudential policy maximizes the approximated welfare in (80) subject to the Phillips curve equation (50), the modified markup equation (81), the IS equation (63), the modified credit market tightness equation (82), and the consumption equation (62). Then, the first-order condition (69) is replaced by

$$\psi_{L^*}\tilde{l}_t^* - \frac{1}{(1-\alpha)\rho}\varphi_{3t} + \frac{\beta\rho_u}{(1-\alpha)\rho}\mathbf{E}_t\varphi_{3t+1} + \frac{\bar{L}\delta_2\beta}{\bar{C}}\varphi_{4t} - \frac{\bar{L}\delta_2\beta}{\bar{C}}\mathbf{E}_t\varphi_{4t+1} = 0, \quad (83)$$

while the first-order condition with respect to  $\tilde{l}_t^*$  is given by

$$-\lambda_{L^*}\tilde{l}_t^* - \psi_{L^*}\tilde{l}_t + \frac{\delta}{\bar{Z}}\frac{\alpha}{1-\alpha}\frac{\bar{L}^*}{\bar{L}}\kappa\bar{\theta}\varphi_{1t} - \frac{1}{1-\alpha}\frac{\bar{L}^*}{\bar{v}}\varphi_{3t} = 0. \quad (84)$$

The first-order conditions (66)–(68), (83), and (84), along with equations (50), (60), (62), (81), and (82), give the optimal paths of  $\pi_t$ ,  $\tilde{\theta}_t$ ,  $\tilde{c}_t$ ,  $\tilde{l}_t$ , and  $\tilde{l}_t^*$ .

In this case, the optimal macroprudential measure of the upper bound on credit supply is determined by both financial variables, such as credit market tightness and

loan volume, and real economic variables, such as inflation and consumption. From the first-order conditions, the optimal instrumental rule defined in Woodford (2003) for the policy variable  $\tilde{l}_t^*$  is expressed as<sup>27</sup>

$$\Theta_1(F, L)\tilde{l}_t^* = \Theta_2(F, L)\tilde{c}_t + \Theta_3(F, L)\pi_t + \Theta_4(F, L)\tilde{l}_t + \Theta_5(F, L)\tilde{\theta}_t, \quad (85)$$

where  $\Theta_1$ ,  $\Theta_2$ ,  $\Theta_3$ ,  $\Theta_4$ , and  $\Theta_5$  are functions of lag operator  $L$  and forward operator  $F$ . Equation (85) implies that the macroprudential authority needs to set the level of lending limit depending on financial and real economic conditions.

As clarified in the next section, the total credit control policy turns out to be incapable of fully offsetting the cost-push shock. Thus, even in the absence of the preference shock, the total credit control policy fails to eliminate the welfare loss, unlike the case of the optimal tax/subsidy on the search cost explained in Section 4.2.<sup>28 29</sup>

## 5.2 Optimal Combination of Monetary Policy and Total Credit Control Policy

We now examine the consequence of the joint optimization of monetary and macroprudential policies. In this case, the first-order conditions include (70) in addition to those mentioned above. As shown in Appendix G, these conditions yield equation (71) and

$$\tilde{l}_t^* = \varphi_{3t} = \varphi_{4t} = 0. \quad (86)$$

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<sup>27</sup>See details of derivation in Appendix F.

<sup>28</sup>The optimal combination of monetary and total credit control policy achieves weakly greater social welfare than the optimal total credit control policy alone, since the latter case amounts to imposing an extra constraint,  $\hat{r}_t^D = 0$ . As shown below in Section 5.2, even the optimal combination of monetary and total credit control policy fails to achieve  $V_{\max}^*$ , so the same is true here.

<sup>29</sup>On the other hand, even in the absence of cost-push shocks, the total credit control policy alone is not able to fully offset the preference shock  $\xi_t$ . This observation is intuitive given that the total credit control policy  $\tilde{l}_t^*$  directly affects equations (81) and (82), while the preference shock, represented by the interest rate shock  $\hat{r}_t^e$ , appears only in a different equation (60). We have numerically confirmed this intuition.

This simple equation suggests that when the monetary policy is optimized, it is optimal to set  $\widehat{l}_t^*$  at its value in the efficient stochastic equilibrium,  $\widehat{l}_t^{*e}$ .<sup>30</sup> Since  $\widehat{l}_t^{*e}$  does not vary with the cost-push shock, the optimal total credit control policy not simply fails to eliminate, but in fact serves no role against the cost-push shock; as a result, the combination of monetary and macroprudential policy fails to eliminate all gaps and to achieve  $V_{\max}^*$ . In particular, when the only source of uncertainty is the cost-push shock, the efficient stochastic equilibrium coincides with the efficient steady-state equilibrium, hence  $\widetilde{l}_t^* = 0$  implies  $L_t^* = L_t^{*e} = \bar{L}^*$ ; in this case, the active use of total credit control policy yields no welfare improvement.

The reason why the total credit control policy plays no role against the cost-push shock here is explained as follows. Unlike the tax/subsidy policy that directly alters the markup of retail firms through its effect on the production cost of wholesale firms, the total credit control policy alters the retail price markup only indirectly by affecting the number of vacancies  $v_t$  as well as productivity  $Z_t$  and thereby varying output.<sup>31</sup> Moreover, since the monetary policy already optimizes output by taking into consideration its trade-off with inflation, there is no additional net benefit from varying output through the total credit control policy. Given the strict concavity of  $f$ , then, varying the upper limit of credit contracts  $L_t^*$  in response to the cost-push shock simply generates efficiency losses in productivity; it is thus optimal to let  $L_t^*$  exactly follow  $L_t^{*e}$ .<sup>32</sup>

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<sup>30</sup>Even when the monetary policy is absent, the same conclusion for the total credit control policy holds under flexible prices ( $\omega = 0$ ). This conclusion may be altered, however, if we introduce some additional shocks that affect the structural equations.

<sup>31</sup>Note that the term  $\tau_t^c$  appears only in the markup equation (74), while the term  $\widetilde{l}_t^*$  shows up not only in the markup equation (81), but also in the welfare function (80) and in the credit market tightness equation (82).

<sup>32</sup>In the first-order condition for  $\widetilde{l}_t^*$  (equation (84)), the second term represents the direct effect that changes in output, resulting from changes in  $\widetilde{l}_t^*$ , have on welfare. Given equation (71), this term cancels with the third term, which represents the indirect effect of changes in output on welfare that arises through its impact on inflation. Optimal monetary policy, as discussed in Section 4.3, prevents the IS equation (63) from binding, which in turn makes the credit market tightness equation (61) and the consumption equation (62) also slack. Thus, the fourth term in equation (84) is zero. This leaves

## 6 Concluding Remarks

We extend a standard New Keynesian model by introducing search and matching frictions into the credit market. In this model, the second-order approximation of social welfare includes terms related to credit, such as credit market tightness, volume of credit, and upper bound of loan volume, in addition to inflation rate and consumption. This is a new finding in the field of optimal policy. Then, we reveal several important features for monetary policy and macroprudential policy.

For future research, the following points may be of interest. Through quantitative assessment, establishing simple and optimal macroprudential and monetary policy rules with credit terms is one important extension of this paper. It would be also interesting to additionally introduce other search and matching frictions for a goods market and a labor market and examine interactive effects of search and matching frictions on conduct of monetary policy and macroprudential policy.

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the first term, which corresponds to the efficiency losses in productivity from varying  $\tilde{l}_t^*$ ; clearly, such efficiency losses are minimized when  $\tilde{l}_t^* = 0$ , or equivalently,  $\hat{l}_t^* = \hat{l}_t^{*e}$ . As this explanation makes clear, the conclusion that  $\hat{l}_t^*$  should exactly follow  $\hat{l}_t^{*e}$  does not hold when, as in Section 5.1, the monetary policy is not chosen optimally.

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# Appendix

## A Derivation of Equation (43)

Noting  $\bar{\xi} = 1$ , the second-order expansion of the household's period utility function around the efficient steady state yields

$$\xi_t u(C_t) \simeq u_0 - \frac{1}{2} \sigma u_c \bar{C} \hat{c}_t^2 + u_c \bar{C} \left( \hat{c}_t + \frac{1}{2} \hat{c}_t^2 \right) + u_c \bar{C} \hat{\xi}_t \hat{c}_t + u_0 \left( \hat{\xi}_t + \frac{1}{2} \hat{\xi}_t^2 \right) \quad (87)$$

and thus

$$\begin{aligned} \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \xi_{t+i} u(C_{t+i}) &\simeq \frac{u_0}{1-\beta} - \frac{1}{2} \sigma u_c \bar{C} \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \hat{c}_{t+i}^2 + u_0 \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \left( \hat{\xi}_{t+i} + \frac{1}{2} \hat{\xi}_{t+i}^2 \right) \\ &+ u_c \bar{C} \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \left( \hat{c}_{t+i} + \frac{1}{2} \hat{c}_{t+i}^2 \right) + u_c \bar{C} \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \hat{\xi}_{t+i} \hat{c}_{t+i}, \end{aligned} \quad (88)$$

where  $u_0 \equiv u(\bar{C}) = \bar{C}^{1-\sigma}/(1-\sigma)$  and  $u_c \equiv u'(\bar{C}) = \bar{C}^{-\sigma}$ . Below, we first eliminate the terms  $\hat{c}_{t+i}$  and  $\hat{\xi}_{t+i} \hat{c}_{t+i}$  from this expression by rewriting each of the term in the second line on the RHS. We then obtain a corresponding expression in the efficient stochastic equilibrium and combine the two expressions to obtain equation (43).

### Rewriting the Term $u_c \bar{C} \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \left( \hat{c}_{t+i} + \frac{1}{2} \hat{c}_{t+i}^2 \right)$

We first focus on the term  $u_c \bar{C} \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \left( \hat{c}_{t+i} + \frac{1}{2} \hat{c}_{t+i}^2 \right)$  in equation (88).

By using the market clearing condition (35), we obtain

$$\hat{c}_t + \frac{1}{2} \hat{c}_t^2 \simeq -\frac{\bar{Z}\bar{L}}{\bar{C}} \hat{q}_t + \frac{\bar{Z}\bar{L}}{\bar{C}} \left( \hat{l}_t + \frac{1}{2} \hat{l}_t^2 \right) - \frac{\kappa \bar{u}}{\bar{C}} \left( \hat{u}_t + \frac{1}{2} \hat{u}_t^2 \right). \quad (89)$$

Note that the efficient steady-state value of the price dispersion term  $Q_t$  is  $\bar{Q} = 1$ , and the log-deviation of this term  $\hat{q}_t$  is already in the second order, as shown below.

Expansion of equation (16) yields

$$\hat{v}_t + \frac{1}{2} \hat{v}_t^2 \simeq -\eta \left( \hat{l}_{t-1} + \frac{1}{2} \hat{l}_{t-1}^2 \right), \quad (90)$$

where  $\eta \equiv (1 - \rho)\bar{L}/\bar{v}$ , while the expansion of equation (28) yields

$$\begin{aligned} \frac{1}{\rho} \left( \hat{l}_t + \frac{1}{2} \hat{l}_t^2 \right) - \frac{1 - \rho}{\rho} \left( \hat{l}_{t-1} + \frac{1}{2} \hat{l}_{t-1}^2 \right) \\ \simeq (1 - \alpha) \left( \hat{\theta}_t + \frac{1 - \alpha}{2} \hat{\theta}_t^2 \right) + \left( \hat{v}_t + \frac{1}{2} \hat{v}_t^2 \right) + (1 - \alpha) \hat{\theta}_t \hat{v}_t. \end{aligned} \quad (91)$$

From equations (25), (90), and (91), we obtain

$$\hat{u}_t + \frac{1}{2} \hat{u}_t^2 \simeq \frac{1}{\rho(1 - \alpha)} \left( \hat{l}_t + \frac{1}{2} \hat{l}_t^2 \right) - \frac{\bar{L}}{\kappa \bar{u}} \delta_2 \left( \hat{l}_{t-1} + \frac{1}{2} \hat{l}_{t-1}^2 \right) + \frac{1}{2} \alpha \hat{\theta}_t^2. \quad (92)$$

Combining equations (89) and (92) yields

$$u_c \bar{C} \left( \hat{c}_t + \frac{1}{2} \hat{c}_t^2 \right) \simeq -u_c \bar{Z} \bar{L} \hat{q}_t - \frac{u_c \kappa \bar{u} \alpha}{2} \hat{\theta}_t^2 + u_c \bar{L} \left( \delta_1 \hat{l}_t + \delta_2 \hat{l}_{t-1} \right) + \frac{1}{2} u_c \bar{L} \left( \delta_1 \hat{l}_t^2 + \delta_2 \hat{l}_{t-1}^2 \right) \quad (93)$$

and therefore

$$\begin{aligned} u_c \bar{C} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left( \hat{c}_{t+i} + \frac{1}{2} \hat{c}_{t+i}^2 \right) \\ \simeq -u_c \bar{Z} \bar{L} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \hat{q}_{t+i} - \frac{u_c \kappa \bar{u}}{2} \alpha \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \hat{\theta}_{t+i}^2 \\ + u_c \bar{L} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left( \delta_1 \hat{l}_{t+i} + \delta_2 \hat{l}_{t+i-1} \right) + \frac{1}{2} u_c \bar{L} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left( \delta_1 \hat{l}_{t+i}^2 + \delta_2 \hat{l}_{t+i-1}^2 \right). \end{aligned} \quad (94)$$

From the efficient steady-state condition (42), the second line on the RHS of equation (94) equals  $u_c \bar{L} \delta_2 \hat{l}_{t-1} + \frac{1}{2} u_c \bar{L} \delta_2 \hat{l}_{t-1}^2$ . Therefore, we obtain

$$\begin{aligned} u_c \bar{C} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left( \hat{c}_{t+i} + \frac{1}{2} \hat{c}_{t+i}^2 \right) \\ \simeq -u_c \bar{Z} \bar{L} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \hat{q}_{t+i} - \frac{u_c \kappa \bar{u} \alpha}{2} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \hat{\theta}_{t+i}^2 + u_c \bar{L} \delta_2 \hat{l}_{t-1} + \frac{1}{2} u_c \bar{L} \delta_2 \hat{l}_{t-1}^2. \end{aligned} \quad (95)$$

We now rewrite the first term on the RHS of equation (95). From equation (36),

$$\begin{aligned} \hat{q}_t &= \int_0^1 dj \exp[-\varepsilon_t (\hat{p}_t(j) - \hat{p}_t)] - 1 \\ &\simeq -\bar{\varepsilon} (\Delta_t^E - \hat{p}_t) (1 + \hat{\varepsilon}_t) + \frac{1}{2} \bar{\varepsilon}^2 \left[ \Delta_t^V + (\Delta_t^E - \hat{p}_t)^2 \right], \end{aligned} \quad (96)$$

where  $\Delta_t^E \equiv \mathbb{E}_j \hat{p}_t(j) = \int_0^1 \hat{p}_t(j) dj$  and  $\Delta_t^V \equiv \text{Var}_j \hat{p}_t(j) = \mathbb{E}_j \hat{p}_t(j)^2 - (\mathbb{E}_j \hat{p}_t(j))^2$ . The definition of the aggregate price  $P_t$  given by equation (23) can be used to show that

$$\Delta_t^E - \hat{p}_t \simeq -\frac{1}{2} (1 - \bar{\varepsilon}) \Delta_t^V. \quad (97)$$

Thus, up to the second order in  $\widehat{p}_t$ , we can rewrite  $\widehat{q}_t$  as

$$\widehat{q}_t \simeq \frac{1}{2} \bar{\varepsilon} \Delta_t^V, \quad (98)$$

leading to

$$\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \widehat{q}_{t+i} \simeq \frac{1}{2} \bar{\varepsilon} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \Delta_{t+i}^V. \quad (99)$$

On the other hand, the expression defining  $\Delta_t^V$  is written as

$$\Delta_t^V = \mathbb{E}_j(\widehat{p}_t(j) - \Delta_{t-1}^E)^2 - (\Delta_t^E - \Delta_{t-1}^E)^2. \quad (100)$$

Here, recall that only fraction  $1 - \omega$  of all firms adjust their prices to  $P_t^*$ , while other firms do not change their prices  $p_{t-1}(j)$ . This condition yields

$$P_t^{1-\varepsilon_t} = (1 - \omega)(P_t^*)^{1-\varepsilon_t} + \omega P_{t-1}^{1-\varepsilon_t}. \quad (101)$$

Also note that, by using the same condition, equation (100) can be restated as

$$\Delta_t^V = \omega \mathbb{E}_j(\widehat{p}_{t-1}(j) - \Delta_{t-1}^E)^2 + (1 - \omega)(\widehat{p}_t^* - \Delta_{t-1}^E)^2 - (\Delta_t^E - \Delta_{t-1}^E)^2, \quad (102)$$

where  $\widehat{p}_t^*$  is the log-deviation of  $P_t^*$ .

By taking the log-deviation of both sides of equation (101),  $\widehat{p}_t^*$  can be expressed by  $\widehat{p}_t$  and  $\widehat{p}_{t-1}$ . Substituting this equation into equation (102) yields

$$\Delta_t^V \simeq \omega \Delta_{t-1}^V + (1 - \omega) \left( \frac{1}{1 - \omega} \widehat{p}_t - \frac{\omega}{1 - \omega} \widehat{p}_{t-1} - \widehat{p}_{t-1} \right)^2 - (\widehat{p}_t - \widehat{p}_{t-1})^2, \quad (103)$$

up to the second order in  $\widehat{p}_t$ . Using  $\pi_t = \widehat{p}_t - \widehat{p}_{t-1}$ , we thus have

$$\Delta_t^V \simeq \omega \Delta_{t-1}^V + \frac{\omega}{1 - \omega} \pi_t^2. \quad (104)$$

and thus let  $\delta = (1 - \omega)(1 - \omega\beta)/\omega$ ,

$$\begin{aligned} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \Delta_{t+i}^V &\simeq \omega \Delta_{t-1}^V + \omega\beta \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \Delta_{t+i}^V + \frac{\omega}{1 - \omega} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \pi_{t+i}^2 \\ &= \frac{1}{\delta} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \pi_{t+i}^2 + \frac{\omega}{1 - \omega\beta} \Delta_{t-1}^V. \end{aligned} \quad (105)$$

Finally, substituting equations (99) and (105) into equation (95) yields

$$\begin{aligned}
u_c \bar{C} \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \left( \widehat{c}_{t+i} + \frac{1}{2} \widehat{c}_{t+i}^2 \right) &\simeq -\frac{1}{2} \lambda_\pi \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \pi_{t+i}^2 - \frac{1}{2} \lambda_\pi (1 - \omega) \Delta_{t-1}^V \\
&\quad - \frac{1}{2} \lambda_\theta \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \widehat{\theta}_{t+i}^2 + u_c \bar{L} \delta_2 \widehat{l}_{t-1} + \frac{1}{2} u_c \bar{L} \delta_2 \widehat{l}_{t-1}^2,
\end{aligned} \tag{106}$$

where  $\lambda_\pi = u_c \bar{Z} \bar{L} \bar{\varepsilon} / \delta$  and  $\lambda_\theta = u_c \kappa \bar{u} \alpha$ .

### Rewriting the Term $u_c \bar{C} \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \widehat{\xi}_{t+i} \widehat{c}_{t+i}$

We now rewrite the term  $u_c \bar{C} \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \widehat{\xi}_{t+i} \widehat{c}_{t+i}$ . Using equations (54) and (55), we have

$$\begin{aligned}
u_c \bar{C} \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \widehat{\xi}_{t+i} \widehat{c}_{t+i} &= u_c \bar{L} \delta_2 \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \left( \widehat{\xi}_{t+i} \widehat{l}_{t+i-1} - \beta \widehat{\xi}_{t+i} \widehat{l}_{t+i} \right) \\
&= u_c \bar{L} \delta_2 \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \left( \widehat{\xi}_{t+i} \widehat{l}_{t+i-1} - \beta \left[ \widehat{\xi}_{t+i+1} - \sigma (\widehat{c}_{t+i+1}^e - \widehat{c}_{t+i}^e) \right. \right. \\
&\quad \left. \left. - \frac{1}{\beta \delta_2} \frac{\alpha}{1 - \alpha} \frac{\kappa}{\chi} \bar{\theta}^\alpha (\widehat{\theta}_{t+i}^e - \beta \rho_u \widehat{\theta}_{t+i+1}^e) \right] \widehat{l}_{t+i} \right) \\
&= u_c \bar{L} \delta_2 \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \left( \widehat{\xi}_{t+i} \widehat{l}_{t+i-1} - \beta \widehat{\xi}_{t+i+1} \widehat{l}_{t+i} \right) \\
&\quad + u_c \bar{L} \delta_2 \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \beta \sigma (\widehat{c}_{t+i+1}^e - \widehat{c}_{t+i}^e) \widehat{l}_{t+i} \\
&\quad + u_c \bar{L} \frac{\alpha}{1 - \alpha} \frac{\kappa}{\chi} \bar{\theta}^\alpha \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i (\widehat{\theta}_{t+i}^e - \beta \rho_u \widehat{\theta}_{t+i+1}^e) \widehat{l}_{t+i}.
\end{aligned} \tag{107}$$

On the RHS of (107), the first term is written as

$$u_c \bar{L} \delta_2 \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \left( \widehat{\xi}_{t+i} \widehat{l}_{t+i-1} - \beta \widehat{\xi}_{t+i+1} \widehat{l}_{t+i} \right) = u_c \bar{L} \delta_2 \widehat{\xi}_t \widehat{l}_{t-1}. \tag{108}$$

The second term becomes

$$\begin{aligned}
& u_c \bar{L} \delta_2 \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \beta \sigma (\widehat{c}_{t+i+1}^e - \widehat{c}_{t+i}^e) \widehat{l}_{t+i} \\
&= u_c \bar{L} \delta_2 \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \beta \sigma (\widehat{c}_{t+i+1}^e \widehat{l}_{t+i} - \widehat{c}_{t+i}^e l_{t+i}) \\
&= u_c \bar{L} \delta_2 \sigma \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \left[ \beta \widehat{c}_{t+i+1}^e \widehat{l}_{t+i} - \widehat{c}_{t+i}^e \left( \widehat{l}_{t+i-1} - \frac{\bar{C}}{\bar{L} \delta_2} \widehat{c}_{t+i} \right) \right] \\
&= u_c \bar{L} \delta_2 \sigma \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \left( \beta \widehat{c}_{t+i+1}^e \widehat{l}_{t+i} - \widehat{c}_{t+i}^e \widehat{l}_{t+i-1} \right) + u_c \sigma \bar{C} \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \widehat{c}_{t+i}^e \widehat{c}_{t+i} \\
&= -u_c \bar{L} \delta_2 \sigma \widehat{c}_t^e \widehat{l}_{t-1} + \sigma u_c \bar{C} \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \widehat{c}_{t+i}^e \widehat{c}_{t+i}, \tag{109}
\end{aligned}$$

where we have used equation (54) to obtain the second equality.

Finally, the third term on the RHS of (107) is rewritten as

$$\begin{aligned}
& u_c \bar{L} \frac{\alpha}{1-\alpha} \frac{\kappa}{\chi} \bar{\theta}^\alpha \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i (\widehat{\theta}_{t+i}^e - \beta \rho_u \widehat{\theta}_{t+i+1}^e) \widehat{l}_{t+i} \\
&= u_c \bar{L} \frac{\alpha}{1-\alpha} \frac{\kappa}{\chi} \bar{\theta}^\alpha \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i (\widehat{\theta}_{t+i}^e \widehat{l}_{t+i} - \beta \rho_u \widehat{\theta}_{t+i+1}^e \widehat{l}_{t+i}) \\
&= u_c \bar{L} \frac{\alpha}{1-\alpha} \frac{\kappa}{\chi} \bar{\theta}^\alpha \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \left[ \widehat{\theta}_{t+i}^e \left( (1-\alpha) \rho \widehat{\theta}_{t+i} + \rho_u \widehat{l}_{t+i-1} \right) - \beta \rho_u \widehat{\theta}_{t+i+1}^e \widehat{l}_{t+i} \right] \\
&= u_c \bar{L} \alpha \frac{\kappa}{\chi} \bar{\theta}^\alpha \rho \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \widehat{\theta}_{t+i}^e \widehat{\theta}_{t+i} + u_c \bar{L} \frac{\alpha}{1-\alpha} \frac{\kappa}{\chi} \bar{\theta}^\alpha \rho_u \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \left( \widehat{\theta}_{t+i}^e \widehat{l}_{t+i-1} - \beta \widehat{\theta}_{t+i+1}^e \widehat{l}_{t+i} \right) \\
&= \lambda_\theta \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \widehat{\theta}_{t+i}^e \widehat{\theta}_{t+i} + u_c \bar{L} \frac{\alpha}{1-\alpha} \frac{\kappa}{\chi} \bar{\theta}^\alpha \rho_u \widehat{\theta}_t^e \widehat{l}_{t-1}, \tag{110}
\end{aligned}$$

where we have used equation (54) to obtain the second equality.

Using equations (108)–(110) and letting  $\lambda_c = \sigma u_c \bar{C}$ , equation (107) becomes:

$$\begin{aligned}
u_c \bar{C} \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \widehat{\xi}_{t+i} \widehat{c}_{t+i} &= \lambda_\theta \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \widehat{\theta}_{t+i}^e \widehat{\theta}_{t+i} + \lambda_c \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \widehat{c}_{t+i}^e \widehat{c}_{t+i} \\
&\quad + u_c \bar{L} \delta_2 \widehat{\xi}_t \widehat{l}_{t-1} - u_c \bar{L} \delta_2 \sigma \widehat{c}_t^e \widehat{l}_{t-1} + u_c \bar{L} \frac{\alpha}{1-\alpha} \frac{\kappa}{\chi} \bar{\theta}^\alpha \rho_u \widehat{\theta}_t^e \widehat{l}_{t-1}. \tag{111}
\end{aligned}$$



## Combining the Results

Substituting equations (106) and (111) into equation (88) yields

$$\begin{aligned}
& \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \xi_{t+i} u(C_{t+i}) \tag{112} \\
& \simeq -\frac{1}{2} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i (\lambda_{\pi} \pi_{t+i}^2 + \lambda_c \widehat{c}_{t+i}^2 + \lambda_{\theta} \widehat{\theta}_{t+i}^2) + \lambda_{\theta} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \widehat{\theta}_{t+i}^e \widehat{\theta}_{t+i} + \lambda_c \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \widehat{c}_{t+i}^e \widehat{c}_{t+i} \\
& + u_0 \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i (\widehat{\xi}_{t+i} + \frac{1}{2} \widehat{\xi}_{t+i}^2) + u_c \bar{L} \delta_2 \widehat{\xi}_t \widehat{l}_{t-1} - u_c \bar{L} \delta_2 \sigma \widehat{c}_t^e \widehat{l}_{t-1} + u_c \bar{L} \frac{\alpha}{1-\alpha} \frac{\kappa}{\chi} \bar{\theta}^{\alpha} \rho_u \widehat{\theta}_t^e \widehat{l}_{t-1} \\
& + \frac{u_0}{1-\beta} + u_c \bar{L} \delta_2 \widehat{l}_{t-1} + \frac{1}{2} u_c \bar{L} \delta_2 \widehat{l}_{t-1}^2 - \frac{1}{2} \lambda_{\pi} (1-\omega) \Delta_{t-1}^V.
\end{aligned}$$

Note that in the efficient stochastic equilibrium, there is neither inherited price dispersion nor price changes over time. Thus, the counterpart of equation (112) in the efficient stochastic equilibrium is obtained by setting  $\pi_{t+i}^2 = \Delta_{t-1}^V = 0$  and putting superscript  $e$  on endogenous variables (except  $\widehat{l}_{t-1}$ , which is pretermined at date  $t$ ) as

$$\begin{aligned}
& \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \xi_{t+i} u(C_{t+i}^e) \\
& \simeq -\frac{1}{2} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i (\lambda_c (\widehat{c}_{t+i}^e)^2 + \lambda_{\theta} (\widehat{\theta}_{t+i}^e)^2) + \lambda_{\theta} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i (\widehat{\theta}_{t+i}^e)^2 + \lambda_c \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i (\widehat{c}_{t+i}^e)^2 \tag{113} \\
& + u_0 \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i (\widehat{\xi}_{t+i} + \frac{1}{2} \widehat{\xi}_{t+i}^2) + u_c \bar{L} \delta_2 \widehat{\xi}_t \widehat{l}_{t-1} - u_c \bar{L} \delta_2 \sigma \widehat{c}_t^e \widehat{l}_{t-1} + u_c \bar{L} \frac{\alpha}{1-\alpha} \frac{\kappa}{\chi} \bar{\theta}^{\alpha} \rho_u \widehat{\theta}_t^e \widehat{l}_{t-1} \\
& + \frac{u_0}{1-\beta} + u_c \bar{L} \delta_2 \widehat{l}_{t-1} + \frac{1}{2} u_c \bar{L} \delta_2 \widehat{l}_{t-1}^2.
\end{aligned}$$

Subtracting equation (113) from (112) yields the approximated welfare function given by equation (43), which is accurate up to the second order in the log-linearized variables.

## B Proof of Inequalities (48) and (49)

The first step for the proof is to express

$$\frac{\lambda_{\theta}}{\lambda_c} = \frac{\alpha \kappa \bar{u}}{\sigma \bar{C}} \tag{114}$$

in terms of  $\bar{\theta}$ . By evaluating equations (16), (25), (28), and (35) at the efficient steady-state equilibrium, we obtain

$$\bar{u} = \frac{\rho\bar{\theta}}{\rho + (1 - \rho)\chi\bar{\theta}^{1-\alpha}}L^* \quad (115)$$

and

$$\bar{C} = \frac{\bar{Z}\chi\bar{\theta}^{1-\alpha} - \kappa\rho\bar{\theta}}{\rho + (1 - \rho)\chi\bar{\theta}^{1-\alpha}}L^*. \quad (116)$$

From equations (114)–(116), we have

$$\frac{\lambda_\theta}{\lambda_c} = \frac{\alpha}{\sigma} \frac{\rho\frac{\kappa}{\chi}\bar{\theta}^\alpha}{\bar{Z} - \rho\frac{\kappa}{\chi}\bar{\theta}^\alpha}. \quad (117)$$

By taking the partial derivative with respect to  $\kappa$ , we obtain

$$\begin{aligned} \frac{\partial}{\partial\kappa} \left( \frac{\lambda_\theta}{\lambda_c} \right) &= \frac{\alpha}{\sigma} \frac{\bar{Z}}{(\bar{Z} - \rho\frac{\kappa}{\chi}\bar{\theta}^\alpha)^2} \frac{\partial}{\partial\kappa} \left( \rho\frac{\kappa}{\chi}\bar{\theta}^\alpha \right) \\ &= \frac{\alpha}{\sigma} \frac{\bar{Z}}{(\bar{Z} - \rho\frac{\kappa}{\chi}\bar{\theta}^\alpha)^2} \frac{\rho}{\chi} \left( \kappa\alpha\bar{\theta}^{\alpha-1} \frac{\partial\bar{\theta}}{\partial\kappa} + \bar{\theta}^\alpha \right). \end{aligned} \quad (118)$$

On the other hand, taking the partial derivative of equation (39) with respect to  $\kappa$  yields the following expression for  $\partial\bar{\theta}/\partial\kappa$ :

$$\begin{aligned} \left[ \frac{\kappa}{\chi} \alpha \bar{\theta}^{\alpha-1} (1 - \beta(1 - \rho)) + \alpha\beta(1 - \rho)\kappa \right] \frac{\partial\bar{\theta}}{\partial\kappa} \\ + \frac{1}{\chi} \bar{\theta}^\alpha (1 - \beta(1 - \rho)) + \alpha\beta(1 - \rho)\bar{\theta} = 0. \end{aligned} \quad (119)$$

Finally, by eliminating  $\partial\bar{\theta}/\partial\kappa$  from equations (118) and (119), we prove inequality (48). Inequality (49) can be shown in a similar way.

## C Loan Curve

From linearized equations of (4) and (32) and equations (52)–(54), we can derive a loan curve as

$$\hat{r}_t^L = h_1 E_t \hat{l}_{t+1} + h_2 \hat{l}_t + h_3 \hat{l}_{t-1} - h_4 \left( E_t \hat{\xi}_{t+1} - \hat{\xi}_t \right), \quad (120)$$

where

$$\begin{aligned}
h_1 &\equiv \frac{1}{h_5} \left[ -\beta \frac{(1-\rho)(\alpha - \chi \bar{\theta}^{1-\alpha})}{\rho(1-\alpha)} - \beta^2 \sigma \rho_u \frac{\bar{L}}{\bar{C}} \delta_2 \right], \\
h_2 &\equiv \frac{1}{h_5} \left[ \frac{\alpha}{(1-\alpha)\rho} + \beta \frac{(1-\rho)(\alpha - \chi \bar{\theta}^{1-\alpha})}{\rho(1-\alpha)} \rho_u + \beta \sigma \rho_u \frac{\bar{L}}{\bar{C}} \delta_2 (1+\beta) \right], \\
h_3 &\equiv \frac{1}{h_5} \left[ -\frac{\alpha}{1-\alpha} \frac{\rho_u}{\rho} - \beta \sigma \rho_u \frac{\bar{L}}{\bar{C}} \delta_2 \right], \\
h_4 &\equiv \frac{1}{h_5} \beta \rho_u, \\
h_5 &\equiv \frac{a(1-\alpha)\chi \bar{R}^L}{\alpha \kappa \bar{\theta}^\alpha}.
\end{aligned}$$

Thus, the loan interest rate and credit volume are closely related. By using equation (120), it is possible to include the loan rate term in the approximated welfare function. The welfare function that includes the loan interest rate is consistent with those in Teranishi (2015) and Cúrdia and Woodford (2016). Teranishi (2015) shows that under the staggered cost channel model, an approximated welfare function includes growth of the loan interest rate. Cúrdia and Woodford (2016) show that an approximated welfare function includes the credit spread term under a model where households face financial market frictions.

## D Derivation and Intuition of the Optimal Monetary Policy Rules

### D.1 Derivation of Equations (71) and (72)

Let us first eliminate  $\tilde{\theta}_t$  and  $\tilde{c}_t$  from equations (67) and (68) by using equations (61), (62), and (70). This leads to

$$\varphi_{3t} = \frac{\delta}{\bar{Z}} \frac{\alpha}{1-\alpha} \frac{\kappa}{\chi} \bar{\theta}^\alpha (h_t - \rho_u h_{t-1}), \tag{121}$$

$$\varphi_{4t} = \frac{\delta \delta_2 \sigma}{\bar{Z}} (-\beta h_t + h_{t-1}), \tag{122}$$

where

$$h_t \equiv \varphi_{1t} - \frac{\lambda_\theta \bar{Z} \chi}{\delta \alpha \rho \kappa \bar{\theta}^\alpha} \tilde{l}_t = \varphi_{1t} - u_c \frac{\bar{Z} \bar{L}}{\delta} \tilde{l}_t. \quad (123)$$

By substituting equations (121) and (122) into equation (69), we obtain

$$E_t h_{t+1} - k_1 h_t - k_2 h_{t-1} = 0, \quad (124)$$

where

$$k_1 \equiv \frac{1 + \beta \rho_u^2 + \zeta (1 + \beta)}{\beta (\rho_u + \zeta)}, \quad (125)$$

$$k_2 \equiv -\frac{1}{\beta}, \quad (126)$$

and

$$\zeta \equiv \beta \frac{(1 - \alpha)^2}{\alpha} \rho \sigma \delta_2^2 \frac{\bar{L}}{\bar{C}} \frac{\chi}{\kappa \bar{\theta}^\alpha}. \quad (127)$$

The path of  $h_t$  described by the second difference equation (124) is dynamically stable if and only if both roots of the characteristic equation

$$h^2 - k_1 h - k_2 = 0 \quad (128)$$

are inside the unit circle, which is equivalent to  $|k_1| < 1 - k_2$  and  $k_2 > -1$ .<sup>33</sup> Since  $\beta < 1$ , however,  $k_2 = -1/\beta < -1$  and thus the stability condition is not satisfied. The optimal policy therefore requires  $h_t = 0$  for all  $t$ , because otherwise, the path of  $h_t$  will be divergent, which is clearly suboptimal. Thus, equation (71) follows from equation (123), while equation (72) follows from equations (121) and (122).

## D.2 Intuition for Equation (72)

In this section, we provide an intuition for equation (72), that is, why the constraints (61) and (62), which together serve as the restriction from the market clearing condition, do not bind. As explained below, this result follows because in the presence of optimal monetary policy, the markup equation (51) incorporates the market clearing condition.

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<sup>33</sup>See, e.g., Enders (2004, Chapter 1). Enders, W., 2004. Applied Econometric Time Series, 2nd ed. John Wiley and Sons, New York, NY.

Note that in the central bank's problem in Section 4.1, the IS equation (63) is slack because it can always be satisfied by an adequate choice of  $\tilde{r}_{t+i}^D$ . We can thus remove the IS equation (63) from the constraint and consider the problem of choosing  $\left\{ \pi_{t+i}, \tilde{\theta}_{t+i}, \tilde{c}_{t+i}, \tilde{l}_{t+i} \right\}_{i=0}^{\infty}$ . Then, clearly, for any path of  $\hat{\mu}_t$ , the problem of choosing the path of  $(\tilde{\theta}_t, \tilde{c}_t, \tilde{l}_t)$  is independent of the problem of choosing the path of  $\pi_t$ . More precisely, the former problem is of maximizing  $-(1/2)\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left( \lambda_c \tilde{c}_{t+i}^2 + \lambda_{\theta} \tilde{\theta}_{t+i}^2 \right)$  with respect to  $\left\{ \tilde{\theta}_{t+i}, \tilde{c}_{t+i}, \tilde{l}_{t+i} \right\}_{i=0}^{\infty}$  subject to equations (59), (61), and (62), taking  $\{\hat{\mu}_{t+i}\}_{i=0}^{\infty}$  as given. Let us explore this problem, ignoring for now the latter two constraints.

The Lagrangian for the simplified problem of maximizing  $-(1/2)\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left( \lambda_c \tilde{c}_{t+i}^2 + \lambda_{\theta} \tilde{\theta}_{t+i}^2 \right)$  subject to equation (59) is written as

$$\begin{aligned} \mathcal{L} = & \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left\{ -\frac{1}{2} \left( \lambda_c \tilde{c}_{t+i}^2 + \lambda_{\theta} \tilde{\theta}_{t+i}^2 \right) \right. \\ & \left. + \phi_{t+i} \left[ \bar{Z} \hat{\mu}_{t+i} + \frac{\alpha}{1-\alpha} \frac{\kappa}{\chi} \bar{\theta}^{\alpha} \left( \tilde{\theta}_{t+i} - \beta \rho_u \mathbb{E}_t \tilde{\theta}_{t+i+1} \right) + \beta \sigma \delta_2 \left( \mathbb{E}_t \tilde{c}_{t+i+1} - \tilde{c}_{t+i} \right) \right] \right\}. \end{aligned} \quad (129)$$

Taking the first-order conditions with respect to  $\tilde{c}_{t+i}$  and  $\tilde{\theta}_{t+i}$  and rearranging yields

$$\tilde{\theta}_t = \frac{1}{\lambda_{\theta}} \frac{\alpha}{1-\alpha} \frac{\kappa}{\chi} \bar{\theta}^{\alpha} (\phi_t - \rho_u \phi_{t-1}) = \frac{1}{(1-\alpha) \rho u_c L} (\phi_t - \rho_u \phi_{t-1}), \quad (130)$$

$$\tilde{c}_t = \frac{\delta_2 \sigma}{\lambda_c} (-\beta \phi_t + \phi_{t-1}) = \frac{L \delta_2}{\bar{C}} \frac{1}{u_c L} (-\beta \phi_t + \phi_{t-1}). \quad (131)$$

Equations (130) and (131) reveal that the solution to this simplified problem can always be made to satisfy equations (61) and (62) by setting  $\tilde{l}_t = \phi_t / (u_c L)$ .<sup>34</sup> Since this argument holds for any path of  $\hat{\mu}_t$ , equations (61) and (62) do not bind in the original problem of the central bank.

To understand the role played by the Hosios condition in the result above, let the banks' bargaining power  $b$  take on a general value in  $[0, 1]$ , in which case equation (59)

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<sup>34</sup>This condition can be rewritten as  $\phi_t = u_c \tilde{l}_t = u_c (L_t - L_t^e)$ , whose intuition is explained as follows. Relaxing constraint (59) by one unit enables increasing each wholesale firm's output by 1 unit, or equivalently, the aggregate wholesale good production by  $L_t$  units; these wholesale goods are then converted into  $L_t$  units of final goods, whose utility value is approximately  $u_c L_t$ . The same explanation implies that the corresponding increase in utility in the efficient stochastic state is  $u_c L_t^e$ , hence the value of relaxing constraint (59) is  $\phi_t = u_c (L_t - L_t^e)$ .

is replaced by

$$\begin{aligned} \bar{Z}\hat{\mu}_t &= -\frac{b}{1-b}\frac{\kappa}{\chi}\bar{\theta}^\alpha\left(\tilde{\theta}_t - \beta\rho_u\mathbb{E}_t\tilde{\theta}_{t+1}\right) - \beta\sigma\delta_2\left(\mathbb{E}_t\tilde{c}_{t+1} - \tilde{c}_t\right) \\ &+ \left(\frac{\alpha}{1-\alpha} - \frac{b}{1-b}\right)\frac{\kappa}{\chi}\bar{\theta}^\alpha\left(\hat{\theta}_t^e - \beta\rho_u\mathbb{E}_t\hat{\theta}_{t+1}^e\right). \end{aligned} \quad (132)$$

The Hosios condition makes the loan interest rate determined by the bargaining between the bank and the wholesale firm, and the resulting equation (132), socially efficient in the following sense. Among the paths of  $(\tilde{\theta}_t, \tilde{c}_t, \tilde{l}_t)$  that satisfy, for a given path of  $\hat{\mu}_t$ , the market clearing condition and equation (132) for a general  $b \in (0, 1)$ , the ones that satisfy equation (132) for  $b = \alpha$  (i.e., equation (59)) maximize  $-\mathbb{E}_t \sum_{i=0}^{\infty} \frac{1}{2}\beta^i \left(\lambda_\theta \tilde{\theta}_{t+i}^2 + \lambda_c \tilde{c}_{t+i}^2\right)$ . Thus, in the central bank's problem, equation (59) already takes into account equations (61) and (62) that correspond to the market clearing condition.

The argument above hinges on the fact that the paths of  $(\tilde{\theta}_t, \tilde{c}_t, \tilde{l}_t)$  that maximize, for some path of  $\hat{\mu}_t$ , the function  $-(1/2)\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left(\lambda_c \tilde{c}_{t+i}^2 + \lambda_\theta \tilde{\theta}_{t+i}^2\right)$  subject to equations (61), (62), (132) coincide with the optimal paths of  $(\tilde{\theta}_t, \tilde{c}_t, \tilde{l}_t)$  in the original problem of the central bank. This is no longer true when optimal monetary policy is absent as in Section 4.2. In such a case, satisfaction of the Hosios condition does not make equation (132) socially efficient, since the resulting equation (59) pays no consideration to the restriction imposed by the IS equation (60) on the relation between  $\tilde{c}_t$  and  $\pi_t$ . In other words, unlike the search externality, the distortion arising through the IS equation (60) is not addressed by the Hosios condition.

## E Derivation of Equations (79) and (80)

We here adopt the upper bound of the credit supply  $L_t^*$  as an additional policy tool and assume the relation (21) between this variable and the productivity of wholesale firms

$Z_t$ . The social planner then solves the following problem:

$$\begin{aligned} \max_{\{C_{t+i}^e, L_{t+i}^e, v_{t+i}^e, \theta_{t+i}^e, L_{t+i}^{*e}\}_{i=0}^{\infty}} \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \left\{ \frac{\xi_{t+i} (C_{t+i}^e)^{1-\sigma}}{1-\sigma} + \phi_{t+i}^e [f(L_{t+i}^{*e}) L_{t+i}^e - \kappa \theta_{t+i}^e v_{t+i}^e - C_{t+i}^e] \right. \\ \left. + \nu_{t+i}^e [(1-\rho)L_{t+i-1}^e + \chi(\theta_{t+i}^e)^{1-\alpha} v_{t+i}^e - L_{t+i}^e] + s_{t+i}^e [v_{t+i}^e - L_{t+i}^{*e} + (1-\rho)L_{t+i-1}^e] \right\}. \end{aligned} \quad (133)$$

By taking the first-order conditions and rearranging them, we obtain

$$f(L_t^{*e}) = \frac{1}{1-\alpha} \frac{\kappa}{\chi} (\theta_t^e)^\alpha - \beta \mathbf{E}_t \left[ \frac{\xi_{t+1} (C_{t+1}^e)^{-\sigma}}{\xi_t (C_t^e)^{-\sigma}} (1-\rho_t) \frac{1}{1-\alpha} \left( \frac{\kappa}{\chi} (\theta_{t+1}^e)^\alpha - \alpha \kappa \theta_{t+1}^e \right) \right], \quad (134)$$

and

$$L_t^e f'(L_t^{*e}) = -\frac{\alpha}{1-\alpha} \kappa \theta_t^e. \quad (135)$$

The latter equation becomes equation (79) by evaluating it at the steady state.

To obtain the second-order approximation of welfare, equation (80), we only need to allow the time variation of  $L_t^*$  and  $Z_t$  and conduct the calculation similar to that explained in Appendix A for the baseline model. This time, the equation corresponding to equation (89) is given by

$$\widehat{c}_t + \frac{1}{2} \widehat{c}_t^2 \simeq \frac{\bar{Z}\bar{L}}{\bar{C}} (\widehat{z}_t + \frac{1}{2} \widehat{z}_t^2 + \widehat{z}_t \widehat{l}_t - \widehat{q}_t) + \frac{\bar{Z}\bar{L}}{\bar{C}} \left( \widehat{l}_t + \frac{1}{2} \widehat{l}_t^2 \right) - \frac{\kappa \bar{u}}{\bar{C}} \left( \widehat{u}_t + \frac{1}{2} \widehat{u}_t^2 \right), \quad (136)$$

while the expansion of the number of credit seeker firms (92) is modified as:

$$\begin{aligned} \widehat{u}_t + \frac{1}{2} \widehat{u}_t^2 \simeq \frac{1}{\rho(1-\alpha)} \left( \widehat{l}_t + \frac{1}{2} \widehat{l}_t^2 \right) - \frac{\bar{L}}{\kappa \bar{u}} \delta_2 \left( \widehat{l}_{t-1} + \frac{1}{2} \widehat{l}_{t-1}^2 \right) \\ + \frac{1}{2} \frac{\alpha}{\rho^2 (1-\alpha)^2} \left( \widehat{l}_t - \rho_u \widehat{l}_{t-1} - (\rho + (1-\rho)\chi \bar{\theta}^{1-\alpha}) \widehat{l}_t^* \right)^2 \\ - \frac{\alpha}{1-\alpha} \frac{\bar{L}^*}{\bar{v}} \left( \widehat{l}_t^* + \frac{1}{2} \widehat{l}_t^{*2} \right). \end{aligned} \quad (137)$$

These two equations yield

$$\begin{aligned} \xi_t u(C_t) \simeq u_0 + u_c \bar{Z}\bar{L} \left( \widehat{z}_t + \frac{1}{2} \widehat{z}_t^2 + \widehat{z}_t \widehat{l}_t - \widehat{q}_t \right) \\ + u_c \frac{\alpha}{1-\alpha} \kappa \bar{\theta} \bar{L}^* \left( \widehat{l}_t^* + \frac{1}{2} \widehat{l}_t^{*2} \right) + u_c \bar{C} \left[ \frac{\bar{L}}{\bar{C}} \left( \delta_1 \widehat{l}_t + \delta_2 \widehat{l}_{t-1} \right) + \frac{\bar{L}}{2\bar{C}} \left( \delta_1 \widehat{l}_t^2 + \delta_2 \widehat{l}_{t-1}^2 \right) \right] \\ - \frac{u_c \kappa \bar{u}}{2} \alpha \widehat{\theta}_t^2 - \frac{1}{2} \sigma u_c \bar{C} \widehat{c}_t^2 + u_c \bar{C} \widehat{\xi}_t \widehat{c}_t + u_0 \left( \widehat{\xi}_t + \frac{1}{2} \widehat{\xi}_t^2 \right). \end{aligned} \quad (138)$$

Some of the terms in equation (138) can be rearranged by using equations (78) and (79) as follows:

$$\begin{aligned}
& u_c \bar{Z} \bar{L} \left( \hat{z}_t + \frac{1}{2} \hat{z}_t^2 + \hat{z}_t \hat{l}_t \right) + u_c \frac{\alpha}{1-\alpha} \kappa \bar{\theta} \bar{L}^* \left( \hat{l}_t^* + \frac{1}{2} \hat{l}_t^{*2} \right) \\
&= u_c \bar{L} \left[ \bar{f}_1 \bar{L}^* \hat{l}_t^* + \frac{1}{2} \bar{L}^* (\bar{f}_1 + \bar{f}_2 \bar{L}^*) \hat{l}_t^{*2} + \bar{f}_1 \bar{L}^* \hat{l}_t^* \hat{l}_t \right] + u_c \frac{\alpha}{1-\alpha} \kappa \bar{\theta} \bar{L}^* \left( \hat{l}_t^* + \frac{1}{2} \hat{l}_t^{*2} \right) \\
&= u_c \bar{L}^* \left( \bar{L} \bar{f}_1 + \frac{\alpha}{1-\alpha} \kappa \bar{\theta} \right) \left( \hat{l}_t^* + \frac{1}{2} \hat{l}_t^{*2} \right) + \frac{1}{2} u_c \bar{L} \bar{L}^{*2} \bar{f}_2 \hat{l}_t^{*2} + u_c \bar{f}_1 \bar{L} \bar{L}^* \hat{l}_t^* \hat{l}_t \\
&= 0 + \frac{1}{2} u_c \bar{L} \bar{L}^{*2} \bar{f}_2 \hat{l}_t^{*2} - u_c \bar{L}^* \frac{\alpha}{1-\alpha} \kappa \bar{\theta} \hat{l}_t^* \hat{l}_t \\
&= -\frac{1}{2} \lambda_{L^*} \hat{l}_t^{*2} - \psi_{L^*} \hat{l}_t^* \hat{l}_t.
\end{aligned} \tag{139}$$

We therefore see that equation (138) is simplified as

$$\begin{aligned}
\xi_t u(C_t) &\simeq u_0 - u_c \bar{Z} \bar{L} \hat{q}_t - \frac{1}{2} \lambda_{L^*} \hat{l}_t^{*2} - \psi_{L^*} \hat{l}_t^* \hat{l}_t \\
&\quad + u_c \bar{C} \left[ \frac{\bar{L}}{\bar{C}} \left( \delta_1 \hat{l}_t + \delta_2 \hat{l}_{t-1} \right) + \frac{\bar{L}}{2\bar{C}} \left( \delta_1 \hat{l}_t^2 + \delta_2 \hat{l}_{t-1}^2 \right) \right] \\
&\quad - \frac{u_c \kappa \bar{u}}{2} \alpha \hat{\theta}_t^2 - \frac{1}{2} \sigma u_c \bar{C} \hat{c}_t^2 + u_c \bar{C} \hat{\xi}_t \hat{c}_t + u_0 (\hat{\xi}_t + \frac{1}{2} \hat{\xi}_t^2).
\end{aligned} \tag{140}$$

The rest of the argument closely follows that in Appendix A, the only difference being that the linearized version of equation (135) is additionally used.

## F Derivation of Equation (85)

From the first-order conditions (66)–(68), (83), and (84), we can eliminate the Lagrange multipliers and express  $\tilde{l}_t^*$  as a function of other endogenous variables as

$$\begin{aligned}
& \left[ 1 + \frac{\lambda_{L^*} \sigma \delta_2 \beta \bar{Z} \bar{L}^2}{\psi_{L^*} \alpha \kappa \delta \bar{C} \bar{L}^*} (1-F) \frac{1-\alpha}{\bar{\theta}} \text{ST}^{-1} - \frac{\lambda_{L^*} \bar{L} \bar{\theta}^{\alpha-1}}{\psi_{L^*} \chi (1-\alpha) \rho \bar{L}^*} (1-\beta \rho_u F) \text{T}^{-1} (1-\rho_u L) \right] \tilde{l}_t^* \\
&= \frac{\lambda_c \delta_2 \beta \bar{L}}{\psi_{L^*} \bar{C}} (1-F) \tilde{c}_t - \frac{\lambda_\pi \delta_2 \beta \sigma \bar{L}}{\psi_{L^*} \bar{C}} (1-F) (1-\beta F) \pi_t \\
&+ \frac{\bar{L}}{\bar{L}^*} \left[ \frac{\bar{\theta}^{\alpha-1}}{(1-\alpha) \rho \chi} (1-\beta \rho_u F) \text{T}^{-1} (1-\rho_u L) - \frac{\sigma \delta_2 \beta \bar{Z} \bar{L}}{\bar{C} \alpha \kappa \delta} (1-F) \frac{1-\alpha}{\bar{\theta}} \text{ST}^{-1} \right] \tilde{l}_t \\
&+ \frac{\lambda_\theta}{\psi_{L^*}} \left[ \frac{\sigma \delta_2 \beta \bar{Z} \bar{L}^2}{\delta \alpha \kappa \bar{\theta} \bar{v} \bar{C}} (1-F) \text{ST}^{-1} - \frac{1}{(1-\alpha) \rho} (1-\beta \rho_u F) \text{T}^{-1} \right] \tilde{\theta}_t,
\end{aligned} \tag{141}$$



where

$$S \equiv -\beta F + \left( -\beta \frac{\delta \delta_2}{\bar{Z}} + \beta + 1 \right) + \left( \frac{\delta \delta_2}{\bar{Z}} - 1 \right) L, \quad (142)$$

$$T \equiv \left( 1 - \frac{1}{1-\alpha} \frac{\bar{\theta}^{\alpha-1} \bar{L}}{\chi \bar{v}} \right) + \frac{\rho_u}{1-\alpha} \frac{\bar{\theta}^{\alpha-1} \bar{L}}{\chi \bar{v}} L. \quad (143)$$

Clearly, this equation can be rewritten in the form of equation (85).

## G Derivation of Equation (86)

Let us first eliminate  $\tilde{\theta}_t$  and  $\tilde{c}_t$  from equations (67) and (68) by using equations (62), (70), and (82). This leads to

$$\varphi_{3t} = \frac{\delta}{\bar{Z}} \frac{\alpha}{1-\alpha} \frac{\kappa}{\chi} \bar{\theta}^\alpha (h_t - \rho_u h_{t-1}) + \frac{\alpha}{1-\alpha} u_c \kappa \bar{\theta} \bar{L}^* \tilde{l}_t^*, \quad (144)$$

$$\varphi_{4t} = \frac{\delta \delta_2 \sigma}{\bar{Z}} (-\beta h_t + h_{t-1}), \quad (145)$$

where

$$h_t \equiv \varphi_{1t} - \frac{\lambda_\theta \bar{Z} \chi}{\delta \alpha \rho \kappa \bar{\theta}^\alpha} \tilde{l}_t = \varphi_{1t} - u_c \frac{\bar{Z} \bar{L}}{\delta} \tilde{l}_t. \quad (146)$$

In addition, equation (84) is rewritten using  $h_t$  as

$$\lambda_{L^*} \tilde{l}_t^* - \frac{\delta}{\bar{Z}} \frac{\alpha}{1-\alpha} \frac{\bar{L}^*}{\bar{L}} \kappa \bar{\theta} h_t + \frac{1}{1-\alpha} \frac{\bar{L}^*}{\bar{v}} \varphi_{3t} = 0. \quad (147)$$

By eliminating  $\tilde{l}_t^*$ ,  $\varphi_{3t}$ , and  $\varphi_{4t}$ , we can obtain a second-order difference equation for  $h_t$  that involves only the model parameters. To achieve this, first substitute equation (144) into (147) to obtain

$$\tilde{l}_t^* = \frac{\delta}{\bar{Z}} \frac{\bar{v} \gamma}{u_c \bar{L} \bar{L}^*} (-\alpha h_t + \rho_u h_{t-1}), \quad (148)$$

where

$$\gamma \equiv \frac{\frac{\alpha}{(1-\alpha)^2} u_c \kappa \bar{\theta} \frac{\bar{L}^{*2}}{\bar{v}}}{\lambda_{L^*} + \frac{\alpha}{(1-\alpha)^2} u_c \kappa \bar{\theta} \frac{\bar{L}^{*2}}{\bar{v}}} \in (0, 1). \quad (149)$$

By substituting equations (144), (145), and (148) into (83), we obtain

$$E_t h_{t+1} - k_3 h_t - k_4 h_{t-1} = 0, \quad (150)$$

where

$$k_3 \equiv \frac{\alpha(1-\alpha)\rho^2\gamma + (1-\alpha\rho\gamma) + \beta\rho_u^2(1-\rho\gamma) + \zeta(1+\beta)}{\beta[\rho_u(1-\alpha\rho\gamma) + \zeta]}, \quad (151)$$

$$k_4 \equiv -\frac{\rho^2\gamma(1-\alpha)\rho_u + (1-\rho\gamma)\rho_u + \zeta}{\beta[\rho_u(1-\alpha\rho\gamma) + \zeta]}, \quad (152)$$

and  $\zeta$  is as defined by equation (127).

As discussed in Appendix D.1, the path of  $h_t$  described by equation (150) is dynamically stable if and only if  $|k_3| < 1 - k_4$  and  $k_4 > -1$ . Now, note that we can express  $k_3 + k_4 - 1$  in two ways as

$$\begin{aligned} & k_3 + k_4 - 1 \\ &= \frac{(1 - \beta\rho_u)(1 - \alpha) + (1 - \beta\rho_u)(\alpha - \rho_u)(1 - \rho\gamma) + \rho^2\gamma(1 - \alpha)(\alpha - \rho_u)}{\beta[\rho_u(1 - \alpha\rho\gamma) + \zeta]} \end{aligned} \quad (153)$$

$$= \frac{(1 - \beta\rho_u)(1 - \rho_u) + \rho\gamma(\rho_u - \alpha) [(1 - \rho)(1 - \beta(1 - \bar{q}^B)) + \alpha\rho]}{\beta[\rho_u(1 - \alpha\rho\gamma) + \zeta]}. \quad (154)$$

Here,  $\beta$ ,  $\rho_u$ ,  $\alpha$ ,  $\rho$ ,  $\gamma$ , and  $\bar{q}^B$  all lie in  $(0, 1)$ , and  $\zeta > 0$ . Then, if  $\alpha \geq \rho_u$ , we have  $k_3 + k_4 \geq 1$  as observed from equation (153), while if  $\rho_u > \alpha$ , then  $k_3 + k_4 \geq 1$  as seen from equation (154). The stability condition is therefore never satisfied, and thus the same argument as in Appendix D.1 implies that under the optimal policy,  $h_t = 0$  for all  $t$ . Therefore, equation (71) follows from equation (146). Further,  $\varphi_{4t} = \tilde{l}_t^* = 0$  from equations (145) and (148), which in turn implies  $\varphi_{3t} = 0$  from equation (147).