Optimal Indirect Tax Design for a Developing Country

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Abstract

Given that tariffs continue to serve as a primary source of government revenue in many developing countries, we analyze the optimal indirect tax problem, consisting of commodity taxes and tariffs, under a revenue constraint. This study derives the revenue-constrained optimal commodity taxes and tariffs in both a small and a large country and then examines their structure and properties. We show that the optimal commodity tax structure follows the Ramsey rule regardless of whether a country is small or large, which implies that the same optimal commodity tax rules are applied across a range of situations. We also show that the optimal tariffs are not zero, but negative, even in the small country case, which implies stronger support for the World Bank’s recommendation of tariff reductions for a country facing a revenue constraint. In addition, this study analyzes the optimal commodity taxation when tariffs cannot be fully adjusted. Numerical examples demonstrate some of our major findings and the welfare gain of the optimal taxation for a few developing countries.

Keywords: Commodity Tax, Tariff, Revenue Constraint, Terms-of-Trade Effect

JEL Classification: F11, F13, H21

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1 Introduction

The reconciliation of the trade-offs between efficiency losses from indirect tax imposition and stable tax revenue raising has long been a major real-world policy issue. This is especially difficult to resolve in developing countries, which have weak tax systems and low taxable incomes and thus tend to rely heavily on trade taxes to raise revenues to meet their fiscal demand. Figure 1 shows the clear downward trend in import tariff dependency along with per capita GDP, showing that many countries rely heavily on customs and other import duties for their tax revenue.

![Figure 1. Dependence on customs and other import duties in 2012](image)

X: per capita GDP (current USD), Y: Revenues from customs and other import duties (% of tax revenue)
(Source: World Development Indicators)

While being heavily dependent on tariff revenues, developing countries began to use value added taxes (VATs) in the 1990s (Ebrill et al., 2001). Crowe Horwath International (2016) reports that all but six of the 54 countries in Africa levy VAT.
Further, according to the International Monetary Fund (2011), VAT revenue has increased over the past two decades, while trade tax revenue has declined in low-income, lower middle-income, and upper middle-income countries. Nonetheless, import tariffs still constitute a significant proportion of their tax revenues and cannot be fully replaced with domestic taxes, especially in low-income countries.

The above-mentioned reality implies that the optimal tax structure of commodity taxes and tariffs ought to be investigated with an explicit consideration of a revenue constraint. However, most previous studies have investigated them separately. The theory of optimal commodity taxation, which was initiated by Ramsey (1927), is to find the tax structure that minimizes inevitable tax-induced price distortions subject to a revenue constraint.\(^1\) Although the theory has been developed mainly in a closed economy case by subsequent works such as Diamond and Mirrlees (1971a, 1971b), Dixit (1985) extends the optimal tax theory to the open-economy case.\(^2\) Hatta and Ogawa (2007) examine the optimal tariffs for collecting revenue (without commodity taxes) in a small open economy.\(^3\) These studies do not analyze the optimal tax mix of commodity taxes and tariffs.

In addition to the optimal tax mix problem of commodity taxes and tariffs, this study has another challenge, namely the incorporation of the manipulation of terms-of-trade effects into a revenue-constrained optimal tax framework. This is particularly important for a large country (Kaldor, 1940; Johnson, 1953–54; Bond, 1990; Syropoulos, 2002; Ogawa, 2007b, 2012). That is, tariffs are a device for a large country to manipulate the terms of trade for exploiting monopoly power in trade, rather than for revenue raising. As the terms-of-trade effect complicates the problem, the optimal tax problem under a revenue constraint and the optimal tariff problem in a large country have also been examined separately in most previous studies. As a notable exception, Keen and Wildasin (2004) allow commodity taxes and tariffs to be adjusted cooperatively to achieve the global Pareto-efficient allocation in a multicounty economy where

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\(^1\)Boadway (2012) provides a useful survey of the theory and policy in this field.

\(^2\)Dixit (1985, Section 3.2) considers the case where the government directly manipulates consumer and producer prices and all of the rent generated in the production side (i.e., producer price minus world price) is shared by private producers. Hence, his model is not the revenue-constrained tax mix of commodity taxes and tariffs.

\(^3\)No terms-of-trade effects are generated in their model since they consider a small open economy.
each country has a distinct revenue constraint. In their framework, tariffs serve as a device for transferring tax revenue across countries. Their tax coordination across countries for the Pareto-efficient allocation is fascinating in theory but practically difficult and empirically inconsistent with the reality. As Broda et al. (2008) show, for example, the actual tariff structure in the United States is consistent with the optimal tariff that exploits the terms-of-trade effects rather than international coordination. Allowing for this backdrop, we analyze the optimal indirect tax design jointly using commodity taxes and tariffs for a tax-imposing country that arbitrarily acts only in its own interest, in contrast to Keen and Wildasin (2004).

Our paper considers a small country case in which world prices are constant and a large country case where the terms of trade can be manipulated, and compares the optimal indirect tax structure between these two cases. We also analyze the constrained optimal indirect tax problem where the country optimizes only commodity taxes while keeping tariffs at the given levels. This is because under the WTO regime, the governments of developing countries may be unable to adjust import tariffs fully and thus can use only commodity taxes as policy instruments. In these contexts, we examine the optimal indirect tax policy not only as general as possible but also numerically demonstrate the size of the possible gains by optimizing taxes and tariffs, using a computable general equilibrium (CGE) model and data for developing countries.

By solving the truly comprehensive optimal tax combination problems for commodity taxes and tariffs, we find that even in an open economy, the optimal commodity taxes follow the Ramsey tax rule originally derived in an optimal tax framework in a closed economy and that the Ramsey tax rule holds consistently both in a small and in a large country. They imply that the same policy implications for commodity taxation hold across a wide range of situations. In addition, the optimal commodity tax rules are found to be independent of the income effects and the foreign country’s substitution effects. This finding can make our optimal tax design more practical.

In contrast to optimal commodity taxes, we find that the optimal tariff vector

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4 Hatzipanayotou et al. (1994), Keen and Ligthart (2002), and Emran and Stiglitz (2005) examine tax and tariff reforms jointly only for a small country but do not analyze the optimal tax problem.

5 In this study, the Ramsey tax rule is not confined to the inverse elasticity rule, but rather indicates the optimal commodity tax expression provided in Propositions 1 and 2 herein.
takes different forms in the small and large country cases. This implies that tariff policies should take the type of country into consideration. As an interesting and suggestive result, we show that the optimal tariffs are not zero, but negative, in a small country,\textsuperscript{6} despite commodity taxes being available. International institutions such as the World Bank and WTO recommend tariff reductions to raise the efficiency of resource allocation. This finding supports their standard policy recommendation of trade reform from positive to zero tariffs that achieve better welfare and a more rapid transition to a free-trade regime.\textsuperscript{7}

The remainder of the paper is structured as follows. Section 2 describes our general model of optimal indirect taxation. Sections 3 and 4 analyze the optimal taxation in the small country and large country cases, respectively. We derive the general rules of optimal indirect taxation by highlighting the sign of optimal taxes and tariffs and the relative size of the optimal commodity tax rates and tariff rates between commodities. Section 5 numerically demonstrates the optimal indirect tax structure and welfare gains through the optimal indirect taxation. Section 6 concludes.

2 The Model

We employ the framework of a general equilibrium model of international trade. There are $N+1$ tradable commodities, which are indexed as $0, 1, \ldots, N$, where commodity 0 denotes the numeraire. We consider commodity 0 to be an exported commodity and non-numeraire commodities to be imported commodities. Production factors, fixed in supply, are internationally immobile and fully utilized in production sectors. The markets for commodities and factors are perfectly competitive. The home country imposes commodity taxes and tariffs, without using a lump-sum tax and a profit tax, to collect revenue, while the foreign country engages in free trade.

The home commodity taxes ($t$) and tariffs ($\tau$) imposed on the non-numeraire

\textsuperscript{6} Developing counties often provide import subsidies for necessities such as gasoline in Iran and barley in Saudi Arabia.

\textsuperscript{7} Our numerical results, provided in Section 5 and the Appendix, show that free trade (zero tariffs) enhances welfare in the small country case, compared with the status quo, even under a revenue constraint.
commodities are defined by

\[ \mathbf{t} \equiv \mathbf{q} - \mathbf{p}, \quad \mathbf{r} \equiv \mathbf{p} - \mathbf{w}, \]

where \( \mathbf{q}, \mathbf{p}, \) and \( \mathbf{w} \) denote the home consumer, producer, and world price vectors of non-numeraire commodities, respectively. We assume without loss of generality that no tax or tariff is imposed on the numeraire: \( t_0 = r_0 = 0 \). By setting \( w_0 = 1 \), we have \( q_0 = p_0 = w_0 = 1 \).

In the home country, there is a representative consumer with a well-behaved utility function. The expenditure function is given by \( e(q_0, \mathbf{q}, u, g) \), where \( u \) is the utility level and \( g \) is a publicly provided good. The demand vector and substitution matrix for the non-numeraire commodities are then given by \( \mathbf{e}_q(\equiv \partial e/\partial \mathbf{q}) \) and \( \mathbf{e}_{qq} \equiv \partial e/\partial \mathbf{q}' \), respectively, whose elements are \( e_i(\equiv \partial e/\partial q_i) \) and \( e_{ij}(\equiv \partial e_i/\partial q_j) \). Commodity \( i \) is a substitute for commodity \( j \) in consumption if \( e_{ij} > 0 \). Let us denote \( e_g \equiv \partial e/\partial g \) and \( \mathbf{e}_{qg} \equiv \partial \mathbf{e}_q/\partial g \), \( e_u \equiv \partial e/\partial u \), and \( \mathbf{e}_{qu} \equiv \partial \mathbf{e}_q/\partial u \). The expenditure function \( e(\cdot) \) has the following properties: (i) symmetry, \( e_{ij} = e_{ji} \) for all \( i, j \); (ii) homogeneity, \( \sum_{i=0}^{N} q_i e_{ji} = 0 \) for all \( j \); and (iii) negative semidefiniteness, \( \sum_{i=0}^{N} \sum_{j=0}^{N} h_i h_j e_{ji} = 0 \) if \( \mathbf{h} = \zeta \mathbf{q} \) for some scalar \( \zeta \) and \( \mathbf{h}' \equiv (h_0, h_1, \ldots, h_n) \), and \( \sum_{i=0}^{N} \sum_{j=0}^{N} h_i h_j e_{ji} < 0 \) otherwise.\(^8\) Hereafter, \( q_0 \) is not explicitly shown in \( e(\cdot) \) because \( q_0 = 1 \).

Given convex technology, the behavior of the production sectors is characterized by a revenue function \( r(p_0, \mathbf{p}, \mathbf{v}) \),\(^9\) where \( \mathbf{v} \) is a factor endowment vector. The supply vector and substitution matrix for the non-numeraire commodities are then given by \( \mathbf{r}_p(\equiv \partial r/\partial \mathbf{p}) \) and \( \mathbf{r}_{pp} \equiv \partial r/\partial \mathbf{p}' \), respectively, whose elements are \( r_i(\equiv \partial r/\partial q_i) \) and \( r_{ij}(\equiv \partial r_i/\partial q_j) \). Commodity \( i \) is a substitute for commodity \( j \) in production if \( r_{ij} < 0 \). The revenue function \( r(\cdot) \) has the following properties: (i) symmetry, \( r_{ij} = r_{ji} \) for all \( i, j \); (ii) homogeneity, \( \sum_{i=0}^{N} p_i r_{ji} = 0 \) for all \( j \); and (iii) positive semidefiniteness, \( \sum_{i=0}^{N} \sum_{j=0}^{N} k_i k_j r_{ji} = 0 \) if \( \mathbf{k} = v \mathbf{k} \) for some scalar \( v \) and \( \mathbf{k}' \equiv (k_0, k_1, \ldots, k_n) \), and \( \sum_{i=0}^{N} \sum_{j=0}^{N} k_i k_j r_{ji} > 0 \) otherwise.\(^{10}\) Hereafter, \( p_0 \) and \( \mathbf{v} \) are not explicitly shown in \( r(\cdot) \) because \( p_0 = 1 \) and each element of \( \mathbf{v} \) is fixed.

\(^8\)We assume that there is some substitutability between the numeraire and non-numeraire goods. See Dixit and Norman (1980).


\(^{10}\)See Footnote 8.
The budget constraint of the private sector in the home country is given by

\[ e(q, u, g) = r(p), \]  

(2)

in which the left-hand side (LHS) represents the expenditure of the consumer and the right-hand side (RHS) represents the income that the consumer receives, which is equal to factor payments plus pure profits. As the revenue function is defined under constant returns to scale (CRS) production technology, which leads to zero profit, or under decreasing returns to scale (DRS) production technology, which yields positive profit, equation (2) holds regardless of whether there is pure profit or no profit (Emran, 2005; Emran and Stiglitz, 2005). Because the equilibrium conditions including (2) and the properties of the expenditure and revenue functions are the same under CRS and DRS, our results hold regardless of whether there is pure profit or no profit.

Following Keen and Wildasin (2004), we assume that the government spends tax revenue on the purchase of the numeraire commodity and provides it to the consumer as a public good. The government budget constraint is

\[ t'e_q(q, u, g) + \tau'(e_q(q, u, g) - r_p(p)) \geq g, \]  

(3)

where the first term on the LHS represents the commodity tax revenue and the second term does the tariff revenue.\(^{11}\) We assume that the public good is required at the optimum.

The expenditure and revenue functions of the foreign country are given by \( e^*(w_0, w, u^*) \) and \( r^*(w_0, w, v^*) \), where \( u^* \) is the utility level of a representative consumer and \( v^* \) is the factor endowment vector in the foreign country. Let \( e_q^* = \partial e^*/\partial w, e_{qq}^* = \partial e^*/\partial w^2 \), \( r_p^* = \partial r^*/\partial w, r_{pp}^* = \partial r_p^*/\partial w^2 \), \( e_u^* = \partial e^*/\partial u^* \), and \( e_{qu}^* = \partial e^*/\partial u^* \). Hereafter, \( w_0 \) is not explicitly shown in \( e^*(\cdot) \) and \( r^*(\cdot) \) because \( w_0 = 1 \), and \( v^* \) is not explicitly shown in \( r^*(\cdot) \) because each element of \( v^* \) is fixed.

The budget constraint of the private sector in the foreign country is

\[ e^*(w, u^*) = r^*(w). \]  

(4)

\(^{11}\)Equations (2) and (3) with equality yield the international trade balance: \( e_0 + g - r_0 + w'(e_q - r_p) = 0 \).
The world market-clearing condition is
\[ e_q(q, u, g) - r_p(p) + e_q^*(w, u^*) - r_p^*(w) = 0_N. \] (5)

The market-clearing condition for commodity 0 is obtained by Walras’ law. Equations (2) and (3) describe a small country and equations (2)–(5) describe a large country.

3 Optimal Indirect Taxes in a Small Country

This section examines the optimal indirect tax design in a small country facing constant world prices. The government of the home country maximizes utility \( u \) subject to equations (2) and (3). The utility maximization problem is given by\(^{12}\)

\[
\begin{align*}
\max_{t, \tau, g, u} & \quad u, \\
\text{s.t.} & \quad e(q, u, g) - r(p) = 0, \\
& \quad t' e_q(q, u, g) + \tau'(e_q(q, u, g) - r_p(p)) = g.
\end{align*}
\] (6)

3.1 Optimal Indirect Tax Solution

Solving the maximization problem (6) yields the optimal commodity taxes and tariffs under the revenue constraint, which are given in the following proposition.

**Proposition 1.** The optimal commodity taxes and tariffs in a small country are given by

\[
\begin{align*}
t' &= -\alpha (e'q_{qq} - r_p'r_{pp})^{-1}, \\
\tau' &= -\alpha r_p'r_{pp}^{-1},
\end{align*}
\] (7) (8)

where \( \alpha \equiv (1 + e_g)/(e_g - e_q'q_{qq}e_{qs}) > 0. \(^{13}\)

\(^{12}\)This particular formulation of the maximization problem is also used by Munk (1978), Hatta and Ogawa (2007), and Ogawa (2012), among others.

\(^{13}\)Assume that \( e_{qg} = 0_N \) (i.e., \( e_g = e_{0g} \)). Then, \( \alpha - 1 = 1/e_g \), in which \( 1/e_g \) means the compensated effect of income on a public good.
(See the Mathematical Appendix for the proof of Proposition 1.) Noting that $e'e^{-1}$ and $-r'_{pp}r^{-1}$ relate to the price distortions in consumption and production, respectively and that a commodity tax imposes price distortion only on consumption and a tariff imposes price distortion on both consumption and production, we discuss the intuition for Proposition 1. Roughly speaking, the optimal tariffs (8) are set allowing only for price distortions on the production side because only tariffs yield price distortions on the production side. The optimal commodity taxes (7) have to be set allowing for not only the commodity tax-induced price distortions but also the optimal tariff structure, because both commodity taxes and tariffs yield price distortions on the consumption side. This is confirmed by the fact that $-\alpha r'_{p} r^{-1}$ in the optimal commodity taxes (7) represents the optimal tariffs. Thus, although the commodity taxes do not yield price distortions in production, the substitution matrix of supply appears in the optimal commodity taxes (7).

Ramsey (1927) and Munk (1978) provide the optimal commodity tax expression in which the substitution terms of both demand and supply appear in a closed economy with untaxed pure profits. The untaxed profits ensure that the supply side affects the optimal commodity taxes in their models. By contrast, our model shows that the tariffs ensure that the supply side influences the optimal commodity taxes regardless of whether there are pure profits under DRS or no profits under CRS. Incidentally, when no tariffs are imposed, the optimal commodity taxes depend only on the consumption side. This is analyzed in Section 3.3.

### 3.2 Optimal Tax Rules

This section examines the signs of the optimal commodity tax and tariff rates as well as the relative size of the optimal commodity tax rates and of the optimal tariff rates between commodities. We first determine the signs of the optimal commodity taxes and tariffs under substitution conditions. Hatta (1977) shows that $e'_{qq} < 0_{NN}$, where $0_{NN}$ denotes the $N \times N$ matrix of zeros, if all non-numeraire commodities are substitutes in consumption and that $r'_{pp} > 0_{NN}$ if all non-numeraire commodities are substitutes in production. Under these conditions, Proposition 1 shows that $t >$

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14 An import tariff (subsidy) is equivalent to a production subsidy (tax) cum consumption tax (subsidy).
0_N and \( \tau < 0_N \). From the optimal commodity taxes (7) and tariffs (8), we find that \( t + \tau = -\alpha e_q^t e_{qq}^{-1} > 0_N \) under the aforementioned substitution condition in consumption. These are summarized as follows.

**Corollary 1(a).** The optimal tax structure is such that (i) \( t + \tau > 0_N \) if all non-numeraire commodities are substitutes for each other in consumption, (ii) \( \tau < 0_N \) if all non-numeraire commodities are substitutes for each other in production, and (iii) \( t > 0_N \) if all non-numeraire commodities are substitutes for each other in both consumption and production.

Corollary 1(a) shows that taxes, but not subsidies, are imposed on both consumption and production under substitution conditions. Specifically, Corollary 1(a)–(i) means the imposition of a tax on consumption and Corollary 1(a)–(ii) means an import subsidy, which is equivalent to a production tax on the production side.\(^{15}\) If \( \tau > 0_N \), which means a combination of consumption taxes and production subsidies, the required revenue must be collected from the taxes on consumption alone to meet the fiscal demand. In this case, the economy has a smaller tax base, which naturally leads to a larger tax distortion than when \( \tau < 0_N \).

We can determine the sign of the average tax level on consumption and production without substitution conditions for \( e_{qq} \) and \( r_{pp} \).\(^{16}\) From the optimal tariffs (8), we obtain

\[
r'_p \tau = -\frac{\tau'r_{pp} \tau}{\alpha} < 0, \tag{9}
\]

where the inequality follows from \( \alpha > 0 \) and property (iii) of the revenue function. As \( \tau_i < (>)0 \) is equivalent to a production tax (subsidy) on the production side, (9) implies that a positive tax is imposed on production on average. From the optimal commodity taxes (7) and tariffs (8), we obtain

\[
e'_q(t + \tau) = -\frac{(t + \tau)'e_{qq}(t + \tau)}{\alpha} > 0, \tag{10}
\]

where the inequality follows from \( \alpha > 0 \) and property (iii) of the expenditure function. (10) shows that a positive tax is imposed on consumption on average.

\(^{15}\)See Footnote 14.

\(^{16}\)Following Ethier (1984) and Bond (1990), the tax revenue indicates an average sense of whether the tax or subsidy is imposed at a higher dimension than the model with two goods.
International institutions such as the World Bank and WTO recommend tariff reductions for many countries because they believe that free trade will be beneficial for them. Our finding that the optimal tariffs under a revenue constraint are negative implies that any positive tariff would always create a larger price distortion than a zero tariff in a country facing a revenue constraint. This finding supports their standard policy recommendation of trade reform from positive to zero tariffs and a more rapid transition to a free-trade regime. Our finding also recommends the reduction of tariffs from zero to negative levels to improve welfare further. Although negative tariffs may appear somewhat unrealistic or unconformable, they are equivalent to positive production taxes on the production side. Therefore, the optimal negative tariffs can be replaced with positive production taxes if available. Section 5 numerically demonstrates these optimal tax and tariff rates in a real context and indicates the magnitude of the welfare gains from optimal indirect taxation.

In addition to the signs of the optimal commodity tax and tariff rates, we can determine the relative size of the optimal commodity tax rates between commodities and that of the optimal tariff rate between commodities. For analytical tractability, we assume that the taxed commodities are price independent, $e_{ij} = r_{ij} = 0$ for $i, j = 1, \ldots, N$ and $i \neq j$. Let us define the total tax rate on consumption, tariff rates, and commodity tax rate as $\delta_i \equiv (t_i + \tau_i)/q_i$, $\phi_i \equiv \tau_i/p_i$, and $\gamma_i \equiv t_i/q_i$, respectively. All tax rates defined here are expressed in terms of the tax-inclusive price. Define the elasticities of compensated demand and supply as $\eta_{ii} \equiv q_i e_{ii}/e_i$ and $\sigma_{ii} \equiv p_i r_{ii}/r_i$, respectively. With these tax rate and elasticity definitions, from the optimal commodity taxes (7) and tariffs (8), when $e_{ij} = r_{ij} = 0$ for $i, j = 1, \ldots, N$ and $i \neq j$ we obtain the following equations that determine the ranking of the optimal total tax, tariff, and commodity tax rates:

$$\delta_i = \frac{\alpha}{-\eta_{ii}}, \quad i = 1, \ldots, N,$$  \hspace{1cm} (11)

$$-\phi_i = \frac{\alpha}{\sigma_{ii}}, \quad i = 1, \ldots, N,$$  \hspace{1cm} (12)

$$\gamma_i - \gamma_j = \left(\frac{1}{-\eta_{ii}} - \frac{1}{-\eta_{jj}}\right)\Omega + \left(\frac{1}{\sigma_{ii}} - \frac{1}{\sigma_{jj}}\right)\Theta, \quad i, j = 1, \ldots, N,$$  \hspace{1cm} (13)
where $\Omega \equiv \frac{1}{1 + \frac{1}{\sigma_{jj}}} > 0$, $\Theta \equiv \frac{\frac{1}{\alpha} + \frac{1}{\sigma_{ii}}}{\left(\frac{1}{\alpha} + \frac{1}{\sigma_{jj}}\right)\left(\frac{1}{\alpha} + \frac{1}{\sigma_{jj}}\right)} > 0$.

(See the Mathematical Appendix for these derivations.) (11) shows that the relative size of the optimal total tax rates is associated with the relative size of demand elasticity: $\delta_i \leq \delta_j$ if $-\eta_{ii} \geq -\eta_{jj}$. This is a standard inverse elasticity rule applied to the optimal total tax rates on consumption. Similarly, (12) shows that the relative size of the optimal tariff rates is associated with the relative size of supply elasticity: $-\phi_i \leq -\phi_j$ if $\sigma_{ii} \geq \sigma_{jj}$. Notably, this implies that the optimal tariff structure is independent of the information on the consumption side, although tariffs impose price distortions on production as well as consumption. From (13), we see that $\gamma_i < \gamma_j$ if $-\eta_{ii} \geq -\eta_{jj}$ and $\sigma_{ii} \geq \sigma_{jj}$ with at least one strict inequality. That is, the ranking of the optimal commodity tax rates depends on the elasticities of supply and compensated demand.

These results are summarized as follows.

**Corollary 1(b).** The following optimal tax rules hold: (i) when $e_{ij} = 0$ for $i, j = 1, \ldots, N$ and $i \neq j$, $\delta_i \leq \delta_j$ if $-\eta_{ii} \geq -\eta_{jj}$, (ii) when $r_{ij} = 0$ for $i, j = 1, \ldots, N$ and $i \neq j$, $-\phi_i \leq -\phi_j$ if $\sigma_{ii} \geq \sigma_{jj}$, and (iii) when $e_{ij} = r_{ij} = 0$ for $i, j = 1, \ldots, N$ and $i \neq j$, $\gamma_i < \gamma_j$ if $-\eta_{ii} \geq -\eta_{jj}$ and $\sigma_{ii} \geq \sigma_{jj}$ with at least one strict inequality.

The assumption that there are no cross-substitution effects between the taxed goods may appear somewhat strong, and with it, we may neglect an important relation between the cross-substitution effects and optimal tax structure. Thus, we consider the case where there are cross-substitution effects between commodities. Examining the optimal tax rules in the presence of cross-substitution effects leads to undue analytical complexity. Following previous studies of optimal tax theory (Harberger, 1964; Diamond and Mirrlees, 1971b), we thus consider a three-good model to avoid such complexity. In a three-good case, from the optimal commodity taxes (7)
and tariffs (8), we obtain
\[ \delta_1 - \delta_2 = (\eta_{20} - \eta_{10}) \Psi, \]
\[ (-\phi_1) - (-\phi_2) = [(-\sigma_{20}) - (-\sigma_{10})] \Lambda, \]
\[ \gamma_1 - \gamma_2 = \left( \frac{p_1}{w_1} \right) \left\{ (\eta_{20} - \eta_{10}) \Psi + [(-\sigma_{20}) - (-\sigma_{10})] \left( \frac{\Lambda p_2}{q_2} \right) \right\}, \]
where \[ \Psi \equiv \frac{\alpha e_1 e_2}{(e_{11} e_{22} - e_{12} e_{21}) q_1 q_2} > 0, \quad \Lambda \equiv \frac{\alpha r_1 r_2}{(r_{11} r_{22} - r_{12} r_{21}) p_1 p_2} > 0. \]
(See the Mathematical Appendix for these derivations.) These equations show the following Corlett–Hague optimal tax rules applied to a small open economy.18

Corollary 1(c). The optimal tax structure in a three-good case is such that (i) \( \delta_1 \geq \delta_2 \) if \( \eta_{20} \geq \eta_{10} \), (ii) \( -\phi_1 \geq -\phi_2 \) if \( -\sigma_{20} \geq -\sigma_{10} \), and (iii) \( \gamma_i > \gamma_j \) if \( \eta_{j0} \geq \eta_{i0} \) and \( -\sigma_{j0} \geq -\sigma_{i0} \) with at least one strict inequality.

Corollary 1(c)-(i) is the Corlett–Hague rule, which is originally derived in the context of an optimal commodity tax problem, applied to the total tax rate on consumption \( \delta_i \), comprising the commodity tax and tariff. Taxation on the consumption of the two non-numeraire commodities creates an incentive for the overconsumption of the untaxed commodity (i.e., the numeraire commodity 0). However, by imposing a higher tax rate on the consumption of the commodity with less substitutability for the untaxed commodity and a lower tax rate on the other non-numeraire commodity, we can partially repress the incentive for the overconsumption of the untaxed commodity (Corlett and Hague, 1953). Corollary 1(c)-(ii) is the Corlett–Hague rule for the optimal tariff rates, and its intuition is analogous to that for Corollary 1(c)-(i). Corollary 1(c)-(iii) is the generalized Corlett–Hague rule for the optimal commodity taxation that must allow for the production side. The relative commodity tax rates between commodities depend on the cross-price elasticities of both compensated demand and supply.

18 The Corlett–Hague rule is well known as the general determination rule for the relative size of the optimal tax rates in the presence of cross-substitution effects. Corlett and Hague (1953) show that in a closed economy with three goods where commodity tax rates are initially uniform, increasing the tax rate on the good that is less substitutable for leisure and decreasing the tax rate on the other good enhances welfare. The Corlett–Hague result is derived in the context of an optimal commodity tax framework by Harberger (1964), Diamond and Mirrlees (1971b), and Ogawa (2007a).
3.3 Constrained Optimum

In this section, we examine the optimal commodity taxes in a case where tariffs are not fully adjustable. This case typically arises under the WTO regime, under which countries cannot newly impose or raise tariffs in principle. This situation can be described by the maximization problem (6), except that the government takes the tariffs as given. Analogously to Proposition 1, we obtain the optimal taxes \( t \) under the given \( \tau \) as follows:

\[
t_0 = -\alpha e_0 q - 1 q q - \tau_0.
\]  

(17) (See the Mathematical Appendix for this derivation). This provides further insights into the optimal commodity tax in an open economy. If \( \tau = 0_N \), we obtain a standard Ramsey tax expression \( t' = -\alpha e_q e_{-1} q q \), which is derived in a closed economy with zero profit or with a 100% profit tax. When we depart from this special case with zero tariffs, the optimal commodity tax \( t_i \) is scaled down (up) in accordance with the given tariff level \( \tau_i \) when \( \tau_i > (>)0 \).

If tariffs are optimally set, the optimal commodity taxes (17) yield the same levels of the optimal commodity taxes implied by (7). Since the tariffs in (17) are not necessarily set at the optimal level, the tax structure (17) worsens welfare compared with the tax structure of (7) and (8). Despite this inefficiency, this constrained optimal tax structure has a practical advantage, namely the optimal tax can be determined only by the elasticities of compensated demand and observed tariff rates, without information about the production side. In Section 5, we numerically demonstrate the optimal commodity tax rates under \( \tau = 0_N \) and welfare gains compared with the status quo.

4 Optimal Indirect Taxes in a Large Country

In a large country, the government takes account of two other constraints than those in (6): the private sector’s budget constraint in the foreign country (4) and the world market-clearing condition (5). The utility maximization problem in a large country
is given by

$$\max_{t, \tau, w, g, u, u^*} u,$$

s.t.  

$$e(q, u, g) - r(p) + L = 0,$$

$$t'e_q(q, u, g) + \tau'(e_q(q, u, g) - r_p(p)) + L = g,$$

$$e^*(w, u^*) - r^*(w) = 0,$$

$$e_q(q, u, g) - r_p(p) + e_q^*(w, u^*) - r_p^*(w) = 0_N.$$  

This maximization problem introduces a lump-sum tax $L$ into the private sector’s budget constraint in the home country (2) and the government revenue constraint (3) as a device to compare our revenue-constrained optimal tariffs with the optimal tariffs that maximize welfare by improving the terms of trade. The latter is the celebrated optimal tariffs in the context of the international trade literature (Kaldor, 1940; Johnson, 1953–54; Bond, 1990).\textsuperscript{19} Note that when this device cannot be used by the government, we ignore the first-order condition (FOC) with respect to $L$ for the welfare maximization (18).

4.1 Optimal Indirect Tax Solution

Without the use of a lump-sum tax $L$, solving the problem (18) yields the optimal commodity taxes and tariffs under a revenue constraint in a large country, which are given in the following proposition.

**Proposition 2.** The optimal commodity taxes and tariffs in a large country are given by

$$t' = -\alpha(e'_q e_q^{-1} - r'_p r_p^{-1}),$$

$$\tau' = -\alpha r'_p r_p^{-1} + \theta',$$

\textsuperscript{19}Optimal tariff theory does not require the government to raise tariff revenues for public goods provision but reimburse the tariff revenues to the household in a lump-sum fashion.
where $\alpha \equiv (1 + e_g)/(e_g - e_q e_{qq} e_{qg})$, and

$$\theta' \equiv \left[ \frac{e_u^*}{e_u^* - (e_q^* - r_p^*)/(e_q^* - r_{pp}^*)^{-1} e_{qq}^*} \right] (e_q^* - r_p^*)/(e_q^* - r_{pp}^*)^{-1}. \quad (21)$$

(See the Mathematical Appendix for the proof of Proposition 2.) The important finding of this proposition is that the optimal commodity tax expression in the large country (19) is the same as that in the small country (7). This finding leads to there being no difference in the optimal commodity tax rules between small and large countries, as discussed below. It should also be apparent from (19) that a requirement for the government to estimate the optimal commodity taxes is information about the domestic substitution terms of demand $e_{ij}$, of supply $r_{ij}$, and those between public and private goods $e_{ig}$, other than the visible domestic demand and supply levels. That is, there is no need to assess the income effects and a foreign country’s substitution terms. This is a practical advantage for policymakers when they estimate the optimal commodity taxes.

The optimal tariff expression in a large country (20) consists of $-\alpha r_p^* r_{pp}^{-1}$ and $\theta'$. The former is nothing but the optimal tariff expression in a small country and hence reflects the price distortion effects in production generated by the revenue constraint. The latter $\theta'$ represents the optimal tariffs that maximize welfare by improving the terms of trade. This is confirmed by employing lump-sum tax $L$, that is, by considering the no revenue constraint case. If we allow the use of lump-sum tax $L$ in this economy, the optimal commodity tax and tariff can be expressed by

$$t = 0_N, \quad \tau = \theta. \quad (22)$$

(See the Mathematical Appendix for this derivation.) The second equation in (22) is the optimal tariff expression that maximizes welfare by improving the terms of trade (Bond, 1990; Ogawa, 2007b).

Before proceeding further, let us examine the sign of $\alpha$ for a large country. There is no guarantee that $\alpha$ is positive in the large country case in contrast to the small country case. Applying some manipulations to the optimal commodity taxes (19) and tariffs (20) and making use of the government budget constraint (3) with the equality
yield

\[-(t + \tau - \theta)'e_{qq}(t + \tau - \theta) + (\tau - \theta)'r_{pp}(\tau - \theta) = \alpha[g - \theta'(e_q - r_p)] > 0, \quad (23)\]

where the inequality follows from property (iii) of the expenditure and revenue functions. (See the Mathematical Appendix for the derivation of (23).) The term \(\theta'(e_q - r_p)\) represents the revenue from the optimal tariff that maximizes welfare with the terms-of-trade effects. The sign of the expression in the square brackets in (23) is ambiguous, and so is the sign of \(\alpha\). We thus make the following assumption to determine the sign of \(\alpha\).

**Assumption.** \(g > \theta'(e_q - r_p)\).

This assumption means that the required tax revenue must exceed the revenue from the optimal tariffs that maximize welfare through the manipulation of the terms of trade. Under this assumption, \(\alpha > 0\) from (23).

### 4.2 Optimal Tax Rules

Since the optimal commodity tax expression in the large country (19) is the same as that in the small country (7), we expect that the optimal commodity tax rules are the same as those in the small country case. This expectation is indeed true, although a complex proof is needed. (See the Mathematical Appendix for the proof of Corollary 2.)

**Corollary 2.** Suppose that the assumption is satisfied. Then, the following optimal commodity tax rules hold in a large country: (i) \(t > 0_N\) if all commodities are substitutes for each other in both consumption and production; (ii) when \(e_{ij} = r_{ij} = 0\) for \(i, j = 1, \ldots, N\) and \(i \neq j\), \(\gamma_i < \gamma_j\) if \(-\eta_{ii} \geq -\eta_{jj}\) and \(\sigma_{ii} \geq \sigma_{jj}\) with at least one strict inequality; and (iii) in the three-commodity case, when taxed commodities are substitutes for each other in production, \(\gamma_i < \gamma_j\) if \(\eta_{j0} \leq \eta_{i0}\) and \(-\sigma_{j0} \leq -\sigma_{i0}\) with at least one strict inequality.

The qualitative results for the optimal commodity taxes are the same in the small and large country cases. The relative size of the optimal commodity tax rates depends
on that of the elasticities of the home country, but not on the foreign country’s elasticities. While Corollary 2 for a large country with the terms-of-trade effect as well as Corollary 1(c) for a small country are limited to the three-good case or the case with no cross-substitution effects for analytical tractability, more general cases (six sectors) are numerically demonstrated in Section 5.

The optimal tariff expression (20) implies that non-zero tariffs are required at the optimum because it is not generally the case that \( \alpha r'_p r_{pp}^{-1} = \theta' \). The sign of the optimal tariff on commodity \( i \) is ambiguous in general. However, the sign turns out to be positive if the impact of tariffs on the terms of trade is greater than the price distortion effects generated by a revenue constraint, namely \( \alpha r'_p r_{pp}^{-1} \zeta < \theta_i \), where \( \theta_i \) is the \( i \)-th element in \( \theta \) and \( \zeta \) is the \( n \)-dimensional vector whose \( i \)-th element is 1 and whose other elements are zero. Our numerical analysis in Section 5 shows that the signs of the optimal tariffs in the larger country case are positive for most categories.

4.3 Constrained Optimum

Just as examined in Section 3.3 for a small country, we consider the case where tariffs are not fully adjustable in a large country. Its maximization problem is the same as (18), except that the government takes the tariffs as given. Analogously to Proposition 2, we obtain the optimal taxes \( t \) under the given \( \tau \) as

\[
t' = -\alpha e'_q e^{-1}_{qq} - \tau' + \theta'.
\]  

(See the Mathematical Appendix for this derivation.) The expression (24) shows that the optimal commodity tax \( t_i \) is scaled down (up) in accordance with the given tariff level \( \tau_i \) from the Ramsey tax when \( \tau_i > (<)0 \) and scaled up (down) in accordance with \( \theta_i \) when \( \theta_i > (<)0 \). Even if \( \tau = 0_N \), the optimal commodity taxes must allow for not only the Ramsey taxes \(-\alpha e'_q e^{-1}_{qq}\) but also the terms-of-trade effects \( \theta \). In this case, the domestic commodity tax must collect the required tax revenue while manipulating the terms of trade.
5 Numerical Example

5.1 A Numerical General Equilibrium Model

To numerically illustrate some of the optimal tax rules shown in Propositions 1 and 2 and indicate the welfare gains under the optimal tariffs and taxes, we develop single-country CGE models for small and large countries and compute the combinations of the optimal commodity taxes and import tariffs. The models are developed based on the standard CGE model by Hosoe et al. (2010) with some modifications (discussed later). The standard CGE model consists of nested-constant elasticity of substitution/transformation (CES/CET) functions, which describe the substitution between goods in production and consumption (Figure 2).

Figure 2. Description of Substitution between Commodities

Source: Hosoe et al. (2010, Figure 6.1), modified by the authors.

The models distinguish six sectors (agriculture, mining, light and heavy manufacturing, transportation, and other services) and are calibrated to data of Côte d’Ivoire.
and the elasticity of substitution/transformation for Armington’s (1969) composite good, provided by the GTAP database version 9a (Hertel, 1997). We conduct the same experiments for Madagascar and India in the Appendix. Their results are qualitatively similar to those for Côte d’Ivoire. The revenue for these countries depends strongly on tariff revenue. The ratio of tariff revenue to tax revenue and tariff rates for these countries are shown in Table 1.

<table>
<thead>
<tr>
<th>Countries</th>
<th>Côte d’Ivoire</th>
<th>Madagascar</th>
<th>India</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Ratio of Tariff Revenue (in 2002)</td>
<td>46.3%</td>
<td>49.3%</td>
<td>17.8%</td>
</tr>
<tr>
<td>Average Tariff Rate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Agriculture</td>
<td>6.6%</td>
<td>5.3%</td>
<td>16%</td>
</tr>
<tr>
<td>Mining</td>
<td>0.2%</td>
<td>3.2%</td>
<td>1.5%</td>
</tr>
<tr>
<td>Light Manufacturing</td>
<td>11.9%</td>
<td>8.6%</td>
<td>26.6%</td>
</tr>
<tr>
<td>Heavy Manufacturing</td>
<td>8.8%</td>
<td>5.9%</td>
<td>7.5%</td>
</tr>
<tr>
<td>Transportation</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Service</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Source: GTAP database version 9a

While the standard CGE model assumes ad valorem import tariffs, we use ad quantum tariffs, following the specification in the theoretical part. We newly equip the model with ad quantum commodity taxes, imposed on the consumption of Armington’s composite good, which are used for household and government consumption, investment, and intermediate. For simplicity, we fix government consumption and investment at the initial level. Other than these two taxes, the model has a production tax, a factor input tax, and an export tax; their tax rates are kept unchanged.20 In the small country model, international prices are constant. For the large country model, we assume a downward-sloping export good demand curve and an upward-

20Our numerical results show that negative tariffs (which are identical to the production taxes and consumption subsidies) should be imposed on most categories even if production taxes are imposed.
sloping import good supply curve. They are characterized by price elasticities, which are approximately equal to the elasticities of transformation/substitution provided by the GTAP database (Hosoe et al., 2010, Ch. 10). Given this setup, the commodity tax and/or import tariff rates are adjusted to maximize household utility subject to the government budget constraint, which allows the government to afford to maintain its consumption level at the status quo.

5.2 Application to Côte d’Ivoire

The optimal commodity tax and tariff structure and welfare impacts in the small country case are given in Tables 2 and 3, respectively and those in the large country case are given in Tables 4 and 5, respectively. We optimize (i) the commodity taxes with zero tariffs and (ii) both the commodity taxes and the tariffs in the small and large country cases. The optimal commodity taxes with zero tariffs would indeed improve welfare compared with the status quo under the import tariffs shown in Table 1 (Table 3-(i)), as free trade advocates expect. When we further optimize the import tariff as well as the commodity tax, the optimized tariffs become negative for most categories (Table 2-(ii)), as predicted by Corollary 1(a).21 The joint optimization of tariffs and taxes would nearly double the welfare gains (Table 3).

21Strictly speaking, our numerical CGE model slightly differs from our theoretical model. In the theoretical model, commodity taxation on a commodity creates the incentive for the overconsumption of other commodities, as discussed in the interpretation of Corollary 1(c). In our CGE model, it creates the incentive not only for the overconsumption but also for the overuse of its intermediate input. In addition, although our CGE model is a six-sector model, it is made of CES/CET functions for practical model estimation by the calibration method and thus cannot assume no cross-substitution effects as in Corollary 1(b). Therefore, in our numerical examples, we do not demonstrate the optimal tax ranking but focus on demonstrating the sign of the optimal tariffs in a small and a large country and the welfare impacts of the tax and tariff optimization.
Table 2. Optimal Tariff and Tax Structure in the Small Country Case

<table>
<thead>
<tr>
<th>Sectors</th>
<th>(i) Zero Tariff and Optimal Tax</th>
<th>(ii) Optimal Tariff and Tax</th>
<th>Optimal Tariff Rate</th>
<th>Optimal Tax Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>0</td>
<td>0.136</td>
<td>−0.141</td>
<td>0.121</td>
</tr>
<tr>
<td>Mining</td>
<td>0</td>
<td>0.264</td>
<td>0.152</td>
<td>0.228</td>
</tr>
<tr>
<td>Light Manufacturing</td>
<td>0</td>
<td>0.114</td>
<td>−0.251</td>
<td>0.165</td>
</tr>
<tr>
<td>Heavy Manufacturing</td>
<td>0</td>
<td>−0.01</td>
<td>−0.061</td>
<td>−0.015</td>
</tr>
<tr>
<td>Transportation</td>
<td>0</td>
<td>0.076</td>
<td>−0.22</td>
<td>0.120</td>
</tr>
<tr>
<td>Service</td>
<td>0</td>
<td>−0.185</td>
<td>−0.201</td>
<td>−0.121</td>
</tr>
</tbody>
</table>

Note: The tariff and tax rates are expressed by the ad quantum tax rate divided by the tax-inclusive price as defined in Section 3.2.

Table 3. Impact on Welfare in the Small Country Case

(Change from the Base in Equivalent Variations [% of GDP])

<table>
<thead>
<tr>
<th></th>
<th>(i) Zero Tariff and Optimal Tax</th>
<th>(ii) Optimal Tariff and Tax</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) Zero Tariff and Optimal Tax</td>
<td>0.00843</td>
<td>0.01418</td>
</tr>
</tbody>
</table>

By contrast, the optimal tariffs in the large country case turn positive for all categories (Table 4-(ii)) because positive tariffs can improve the terms of trade to gain more than the loss from the price distortion effect as discussed in Section 4. In contrast to the small country case, commodity tax optimization with zero tariffs would deteriorate welfare slightly because the zero tariffs lose exploiting rents from abroad under the initial tariffs (Table 5-(i)). The joint optimization of both taxes and tariffs would improve welfare (Table 5-(ii)).
Table 4. Optimal Tariff and Tax Structure in the Large Country Case

<table>
<thead>
<tr>
<th>Sectors</th>
<th>(i) Zero Tariff and Optimal Tax</th>
<th>(ii) Optimal Tariff and Tax</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tariff Rate</td>
<td>Optimal Tax Rate</td>
</tr>
<tr>
<td>Agriculture</td>
<td>0</td>
<td>0.013</td>
</tr>
<tr>
<td>Mining</td>
<td>0</td>
<td>0.034</td>
</tr>
<tr>
<td>Light Manufacturing</td>
<td>0</td>
<td>0.119</td>
</tr>
<tr>
<td>Heavy Manufacturing</td>
<td>0</td>
<td>0.071</td>
</tr>
<tr>
<td>Transportation</td>
<td>0</td>
<td>0.090</td>
</tr>
<tr>
<td>Service</td>
<td>0</td>
<td>−0.101</td>
</tr>
</tbody>
</table>

Note: The tariff and tax rates are expressed by the ad quantum tax rate divided by the tax-inclusive price as defined in Section 3.2

Table 5. Impact on Welfare in the Large Country Case

(Change from the Base in Equivalent Variations [% of GDP])

<table>
<thead>
<tr>
<th></th>
<th>(i) Zero Tariff and Optimal Tax</th>
<th>(ii) Optimal Tariff and Tax</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>−0.00109</td>
<td>0.01504</td>
</tr>
</tbody>
</table>

6 Conclusion

This study analyzes the optimal mix of commodity taxes and tariffs under a revenue constraint in an open economy. The optimal commodity taxes and tariffs create an interaction for each other via the common tax base of consumption and the terms-of-trade effects. This led us to expect the optimal tax structure under a revenue constraint to be complex. This study, however, provides simple and intuitive expressions for the optimal commodity tax and tariff vectors. They help us understand the structure and properties that can make tax policymaking practical.

In particular, we obtain a robust result that the optimal commodity taxes follow the Ramsey rule relating to the elasticities of compensated demand and supply. This
finding is consistent with the Ramsey rule in a closed economy, and the rule holds regardless of whether a country is small or large. The Ramsey rule is likely to hold across a wide range of situations. By contrast, the optimal tariffs take different forms, which depend crucially on the country’s size (small or large). This result implies that tariff policies should take the type of country and domestic commodity taxation system into consideration.

In this study, we examine optimal taxes and tariffs in the context of less developed countries, where few tax policy options are available. As those countries grow, however, more policy devices become available for them, such as corporate tax and labor income tax. These devices could be examined in future research.

Mathematical Appendix

Proof of Proposition 1

By using (1), the problem (6) yields the FOCs with respect to $t$, $\tau$, $g$, and $u$:

\[-\mu e_q' - \lambda(e_q' + t'e_{qq} + \tau'e_{qq}) = 0_N',\]

\[-\mu(e_q' - r_p') - \lambda[e_q' - r_p' + t'e_{qq} + \tau'(e_{qq} - r_{pp})] = 0_N',\]

\[-\mu e_g - \lambda(t'e_{qg} + \tau'e_{qg} - 1) = 0,\]

\[1 - \mu e_u - \lambda(t' + \tau')e_{qu} = 0,\]

where $\mu$ and $\lambda$ are Lagrange multipliers. From (A1), we have

\[t' + \tau' = -\left(1 + \frac{\mu}{\lambda}\right)e_q'e_{qq}^{-1}.\]

From (A1) and (A2), we obtain

\[\tau' = -\left(1 + \frac{\mu}{\lambda}\right)r_p'r_{pp}^{-1}.\]

From (A3) and (A5), we obtain

\[1 + \frac{\mu}{\lambda} = \frac{1 + e_g}{e_g - e_q'e_{qq}^{-1}e_{qs}}.\]
We finally obtain the optimal commodity tax expression (7) from (A5)–(A7) and the optimal tariff expression (8) from (A6) and (A7).22

Multiplying (A5) by $e_{qq}(t + \tau)$ and (A6) by $r_{pp}\tau$, adding up the results, and making use of (3) with the equality and (A7) yields

$$-(t + \tau)'e_{qq}(t + \tau) + \tau' r_{pp}\tau = \alpha g.$$  

The LHS is positive from property (iii) of the expenditure and revenue functions. Thus, $\alpha > 0$ since $g > 0$.

**Derivations of (11), (12), and (13)**

When $e_{ij} = 0$ for $i, j = 1, \ldots, N$, using the definitions of tax rates and elasticities, (A5) leads to (11). When $r_{ij} = 0$ for $i, j = 1, \ldots, N$, using the definitions of tax rates and elasticities, (A6) leads to (12).

When $e_{ij} = r_{ij} = 0$ for $i, j = 1, \ldots, N$ and $i \neq j$, from (7) we have

$$\gamma_i = \alpha \left[ -\frac{1}{\eta_{ii}} + (1 - \gamma_i) \frac{1}{\sigma_{ii}} \right],$$

where we use $p_i/q_i = 1 - \gamma_i$. Solving this equation with respect to $\gamma_i$ yields

$$\gamma_i = \left( \frac{1}{-\eta_{ii}} + \frac{1}{\sigma_{ii}} \right) / \left( \frac{1}{\alpha} + \frac{1}{\sigma_{ii}} \right).$$  \hspace{1cm} (A8)

After some manipulation, (A8) leads to (13). As $\alpha > 0$ and $\sigma_{ii} > 0$ for $i = 1, \ldots, N$, we have $\Omega > 0$. From (A8) and the fact that $p_i/q_i = 1 - \gamma_i$, it follows that

$$\frac{1}{\alpha} + \frac{1}{\eta_{ii}} = \frac{p_i}{q_i} \left( \frac{1}{\alpha} + \frac{1}{\sigma_{ii}} \right) > 0,$$  \hspace{1cm} (A9)

where the inequality follows from $\alpha > 0$ and $\sigma_{ii} > 0$. (A9), together with $\sigma_{ii} > 0$ and $\alpha > 0$, proves that $\Theta > 0$.

22 The FOC (A4) does not affect the optimal tax and tariff expressions in Proposition 1. From (A4), we find that $\mu = [1 - \lambda(t + \tau)'e_{qq}] / e_u$, which shows that $\mu$ is the social marginal utility of income (Diamond, 1975, Equation 6, p. 338).
Derivations of (14), (15), and (16)

In the three-good case, using the definitions of tax rates and elasticities, (A5) is reduced to $\delta_i = (-\eta_{jj} + \eta_{ij})\Psi$ for $i, j = 1, 2$ and $i \neq j$. Since $\eta_{j0} + \eta_{jj} + \eta_{ji} = 0$ for $i, j = 1, 2$ and $i \neq j$, this can be rewritten as

$$\delta_1 = (\eta_{20} + \eta_{12} + \eta_{21})\Psi, \quad \delta_2 = (\eta_{10} + \eta_{12} + \eta_{21})\Psi,$$

which yields (14). The positivity of $\Psi$ follows from property (iii) of the expenditure function.

By using the definitions of tax rates and elasticities, from (5) we find that $\phi_i = (-\sigma_{jj} + \sigma_{ij})\Lambda$. Since $\sigma_{j0} + \sigma_{jj} + \sigma_{ji} = 0$ for $i, j = 1, 2$ and $i \neq j$, this can be rewritten as

$$-\phi_1 = -(\sigma_{20} + \sigma_{12} + \sigma_{21})\Lambda, \quad \phi_2 = -(\sigma_{10} + \sigma_{12} + \sigma_{21})\Lambda,$$

which yields (15). The positivity of $\Lambda$ follows from property (iii) of the revenue function.

From the definitions of $\gamma_i$ and $\phi_i$, we obtain $\delta_i = t_i/q_i + (p_i/q_i)(\tau_i/p_i)$. From this and $p_i/q_i = 1 - \gamma_i$, we have $\delta_i = (1 - \phi_i)\gamma_i + \phi_i$. By using this, (14), and (15), we obtain (16).

Derivation of (17)

Since the tariffs are taken at a given level, we ignore the FOC (A2). From (A5) and (A7), we obtain (17).

Proof of Proposition 2

By using the definition of commodity taxes and tariffs (1), we can write the FOCs of the maximization problem (18) as follows.

$$-\mu e_q' - \lambda (e_q' + t' e_{qq} + \tau' e_{qq}) - \psi' e_{qq} = 0_N', \quad (A10)$$

$$-\mu (e_q' - r_p') - \lambda [e_q' - r_p' + t' e_{qq} + \tau' (e_{qq} - r_{pp})] - \psi' (e_{qq} - r_{pp}) = 0_N', \quad (A11)$$

23 This follows from property (ii) of the expenditure function.

24 This follows from property (ii) of the revenue function.
\[-\pi (e^*_q - r^*_p)' + \lambda (e_q - r_p)' - \psi'(e^*_{qq} - r^*_{pp}) = 0_N,' \tag{A12}
\]
\[-\mu - \lambda = 0, \tag{A13}\]
\[-\mu e_g - \lambda (t' e_{qg} + \tau' e_{qg} - 1) - \psi' e_{qg} = 0, \tag{A14}\]
\[1 - \mu e_u - \lambda (t' + \tau') e_{qu} - \psi' e_{qu} = 0, \tag{A15}\]
\[-\pi e^*_u - \psi' e^*_{qu} = 0, \tag{A16}\]

where \(\mu, \lambda, \pi,\) and \(\psi' \equiv (\psi_1, \ldots, \psi_N)\) are Lagrange multipliers. Note that we ignore the FOC with respect to \(L\) (A13) because the lump-sum tax is not used in the proof of Proposition 2.

From (A10), we have
\[-t' - \tau' - \frac{\psi'}{\lambda} = \left(1 + \frac{\mu}{\lambda}\right) e_q e_{qq}^{-1}. \tag{A17}\]

From (A10) and (A11), we obtain
\[-\tau' - \frac{\psi'}{\lambda} = \left(1 + \frac{\mu}{\lambda}\right) r_p r_{pp}^{-1}. \tag{A18}\]

From (A14) and (A17), we obtain
\[1 + \frac{\mu}{\lambda} = \frac{1 + e_g}{e_g - e_q e_{qq}^{-1} e_{qg}}. \tag{A19}\]

By using (5), (A12) leads to
\[-\frac{\psi'}{\lambda} = \left(1 + \frac{\pi}{\lambda}\right) (e^*_q - r^*_p)' (e^*_{qq} - r^*_{pp})^{-1}. \tag{A20}\]

From (A16) and (A20), we obtain
\[1 + \frac{\pi}{\lambda} = \frac{e^*_u}{e^*_u - (e^*_q - r^*_p)' (e^*_{qq} - r^*_{pp})^{-1} e^*_{qu}}. \tag{A21}\]

By combining (A17)–(A19), we obtain the optimal commodity tax expression (19), and with (A18)–(A21), we obtain the tariff expression (20).\(^{25}\)

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\(^{25}\)The FOC (A15) does not affect the optimal tax and tariff expressions (19) and (20). From (A15), we find that \(\mu = [1 - \lambda (t + \tau') e_{qu} + \lambda \theta' e_{qu}] / e_u\), which shows that \(\mu\) is the social marginal utility of
Derivation of (22)

By applying the FOC (A13) to (A17) and (A18), we obtain the first equation in (22) from (A17) and (A18) and the second equation from (A18), (A20), and (A21).

Derivation of (23)

From the optimal tariff expression (20), we obtain

\[-\alpha r_p' r_{pp}^{-1} = \tau' - \theta'.\]

By substituting this equation into \[-\alpha r_p' r_{pp}^{-1}\] in the optimal commodity tax expression (19), we find that \[-(t + \tau - \theta)' e_{qq} = \alpha e_q'.\] The optimal tariff expression immediately yields \[(\tau - \theta)' r_{pp} = -\alpha r_p'.\] Multiplying the former by \[(t + \tau - \theta)\] and the latter by \[(\tau - \theta),\]

adding up the results, and making use of the government budget constraint (3) with the equality yields (23).

Proof of Corollary 2

By applying the proofs of Corollaries 1(a)-(iii) and (b)-(iii), we can immediately prove Corollaries 2-(i) and 2-(ii), respectively. The proof of Corollary 2-(iii) is as follows.

From (A17) and (A19)–(A21), we find that \(\kappa_i = (-\eta_{jj} + \eta_{ij}) \Psi\) for \(i, j = 1, 2\) and \(i \neq j\), where \(\kappa_i \equiv (t_i + \tau_i - \theta_i)/q_i\) and \(\theta_i\) is the \(i\)-th element in \(\theta\). By using this and \(\eta_{j0} + \eta_{jj} + \eta_{ji} = 0\), we obtain

\[\kappa_1 - \kappa_2 = (\eta_{20} - \eta_{10}) \Psi.\] (A22)

From (A18)–(A21), we obtain

\[\nu_i = (-\sigma_{jj} + \sigma_{ij}) \Psi, \quad i, j = 1, 2 \text{ and } i \neq j,\] (A23)

where \(\nu_i \equiv (\tau_i - \theta_i)/p_i\). This and \(\sigma_{j0} + \sigma_{jj} + \sigma_{ji} = 0\) lead to

\[(-\nu_1) - (-\nu_2) = [(-\sigma_{20}) - (-\sigma_{10})] \Lambda.\] (A24)

From the definitions of \(\kappa_i, \gamma_i,\) and \(\nu_i\), we obtain \(\kappa_i = \gamma_i + (1-\gamma_i) \nu_i\). This equation, income in a country with monopoly power in trade. The last term on the RHS is the welfare impact through the terms-of-trade effects.
(A22), and (A24) yield

\[(\gamma_1 - \gamma_2)(1 - \nu_2) = (\eta_{20} - \eta_{10})\Psi + [(-\sigma_{20}) - (-\sigma_{10})](\Lambda p_1/q_1).\]  \hspace{1cm} (A25)

As \(\alpha > 0\) from the assumption, \(\Psi > 0\) and \(\Lambda > 0\) hold. It follows from (A23) that \(\nu_i < 0\) as \(\sigma_{ij} < 0\) from the substitution condition between the taxed commodities. Thus, \(1 - \nu_i > 0\).\(^{26}\) As \(1 - \gamma_i > 0\), \(1 - \nu_i > 0\), \(\Psi > 0\), and \(\Lambda > 0\), (A25) proves (iii).

Derivation of (24)

Since the tariffs are taken at a given level, we ignore the FOC (A11). From (A17) and (A19)-(A21), we obtain (24).

Appendix: Numerical Examples for Other Countries

We carry out the same numerical experiments for Madagascar and India that we did for Côte d’Ivoire in Section 5. Tables 6 and 10 show that their optimal tariffs in a small country case are consistently negative for most categories in Madagascar and India, as we find for Côte d’Ivoire in Table 2. Tables 8 and 12 consistently show that the optimal tariffs in a large country turn positive for all categories. Tables 7, 9, 11, and 13 show that, in the small and large country cases, welfare would be improved by commodity tax optimization and further by the joint optimization of both taxes and tariffs. In contrast to the results of the Côte d’Ivoire case, in the large country case of Madagascar and India, welfare would be improved by commodity tax optimization. This result implies that the initial tariffs in Madagascar and India would be far from the optimal tariff structure that improves the terms-of-trade effects.

\(^{26}\)This substitution condition in production, which is not required to prove Corollary 1(c)-(iii), is used to satisfy \(1 - \nu_i > 0\).
Application to Madagascar

Table 6. Optimal Tariff and Tax Structure in the Small Country Case for Madagascar

<table>
<thead>
<tr>
<th>Sectors</th>
<th>(i) Zero Tariff and Optimal Tax</th>
<th>(ii) Optimal Tariff and Tax</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tariff Rate</td>
<td>Optimal Tax Rate</td>
</tr>
<tr>
<td>Agriculture</td>
<td>0</td>
<td>0.069</td>
</tr>
<tr>
<td>Mining</td>
<td>0</td>
<td>0.000</td>
</tr>
<tr>
<td>Light Manufacturing</td>
<td>0</td>
<td>0.001</td>
</tr>
<tr>
<td>Heavy Manufacturing</td>
<td>0</td>
<td>0.069</td>
</tr>
<tr>
<td>Transportation</td>
<td>0</td>
<td>−0.004</td>
</tr>
<tr>
<td>Service</td>
<td>0</td>
<td>−0.105</td>
</tr>
</tbody>
</table>

Note: The tariff and tax rates are expressed by the ad quantum tax rate divided by the tax-inclusive price as defined in Section 3.2.

Table 7. Impact on Welfare in the Small Country Case for Madagascar

(Change from the Base in Equivalent Variations [% of GDP])

<table>
<thead>
<tr>
<th></th>
<th>(i) Zero Tariff and Optimal Tax</th>
<th>(ii) Optimal Tariff and Tax</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.01871</td>
<td>0.02729</td>
</tr>
</tbody>
</table>

Table 8. Optimal Tariff and Tax Structure in the Large Country Case for Madagascar

<table>
<thead>
<tr>
<th>Sectors</th>
<th>(i) Zero Tariff and Optimal Tax</th>
<th>(ii) Optimal Tariff and Tax</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tariff Rate</td>
<td>Optimal Tax Rate</td>
</tr>
<tr>
<td>Agriculture</td>
<td>0</td>
<td>−0.048</td>
</tr>
<tr>
<td>Mining</td>
<td>0</td>
<td>0.000</td>
</tr>
<tr>
<td>Light Manufacturing</td>
<td>0</td>
<td>0.043</td>
</tr>
<tr>
<td>Heavy Manufacturing</td>
<td>0</td>
<td>0.013</td>
</tr>
<tr>
<td>Transportation</td>
<td>0</td>
<td>0.063</td>
</tr>
<tr>
<td>Service</td>
<td>0</td>
<td>0.015</td>
</tr>
</tbody>
</table>

Note: The tariff and tax rates are expressed by the ad quantum tax rate divided by the tax-inclusive price as defined in Section 3.2.
Table 9. Impact on Welfare in the Small Country Case for Madagascar
(Change from the Base in Equivalent Variations [\% of GDP])

<table>
<thead>
<tr>
<th></th>
<th>(i) Zero Tariff and Optimal Tax</th>
<th>(ii) Optimal Tariff and Tax</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.00075</td>
<td>0.00416</td>
</tr>
</tbody>
</table>

Application to India

Table 10. Optimal Tariff and Tax Structure in the Small Country Case for India

<table>
<thead>
<tr>
<th>Sectors</th>
<th>(i) Zero Tariff and Optimal Tax</th>
<th>(ii) Optimal Tariff and Tax</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>0 −0.232</td>
<td>0.223 −0.257</td>
</tr>
<tr>
<td>Mining</td>
<td>0 −0.219</td>
<td>0.174 0.000</td>
</tr>
<tr>
<td>Light Manufacturing</td>
<td>0 0.026</td>
<td>−0.013 0.019</td>
</tr>
<tr>
<td>Heavy Manufacturing</td>
<td>0 0.008</td>
<td>−0.110 0.107</td>
</tr>
<tr>
<td>Transportation</td>
<td>0 0.025</td>
<td>−0.004 0.015</td>
</tr>
<tr>
<td>Service</td>
<td>0 0.092</td>
<td>−0.105 0.104</td>
</tr>
</tbody>
</table>

Note: The tariff and tax rates are expressed by the ad quantum tax rate divided by the tax-inclusive price as defined in Section 3.2

Table 11. Impact on Welfare in the Small Country Case for India
(Change from the Base in Equivalent Variations [\% of GDP])

<table>
<thead>
<tr>
<th></th>
<th>(i) Zero Tariff and Optimal Tax</th>
<th>(ii) Optimal Tariff and Tax</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.00394</td>
<td>0.01130</td>
</tr>
</tbody>
</table>
Table 12. Optimal Tariff and Tax Structure in the Large Country Case for India

<table>
<thead>
<tr>
<th>Sectors</th>
<th>(i) Zero Tariff and Optimal Tax</th>
<th>(ii) Optimal Tariff and Tax</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tariff Rate</td>
<td>Optimal Tax Rate</td>
</tr>
<tr>
<td>Agriculture</td>
<td>0</td>
<td>−0.258</td>
</tr>
<tr>
<td>Mining</td>
<td>0</td>
<td>−0.068</td>
</tr>
<tr>
<td>Light Manufacturing</td>
<td>0</td>
<td>0.043</td>
</tr>
<tr>
<td>Heavy Manufacturing</td>
<td>0</td>
<td>0.027</td>
</tr>
<tr>
<td>Transportation</td>
<td>0</td>
<td>0.017</td>
</tr>
<tr>
<td>Service</td>
<td>0</td>
<td>0.062</td>
</tr>
</tbody>
</table>

Note: The tariff and tax rates are expressed by the ad quantum tax rate divided by the tax-inclusive price as defined in Section 3.2.

Table 13. Impact on Welfare in the Large Country Case for India

<table>
<thead>
<tr>
<th>Change from the Base in Equivalent Variations [% of GDP]</th>
<th>(i) Zero Tariff and Optimal Tax</th>
<th>(ii) Optimal Tariff and Tax</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.00117</td>
<td>0.01394</td>
</tr>
</tbody>
</table>

References


