A Representative Consumer Framework for Discrete Choice Models with Endogenous Total Demand

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Abstract: Standard discrete choice models correspond to ‘partial’ utility maximization in which the controlled total is determined exogenously; typically, consumers are assumed to demand at most one unit. The purpose of this paper is to formulate a model in which discrete choice models are incorporated consistently into the full utility maximization framework and to establish a theoretical foundation for discrete choice models that assume no a priori controlled total. We derive the form of the corresponding indirect utility function of a representative consumer and the own-price and cross-price elasticities, and develop a method for measuring welfare, clarifying their implications.

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1. Introduction

Discrete choice models are suitable for representing a consumer's micro behavior. For example, consider a problem in which a consumer chooses between one unit of brands A and B of a particular good. Comparing the utility levels obtained from brands A and B, the consumer chooses the brand that yields higher utility. This setup is consistent with utility maximization by the consumer. However, this problem represents only part of the consumer's behavior. This is because the reason for the consumer choosing a unit of the good is not explained. The third option of 'not buying', which involves not purchasing either brand, does not completely solve the problem because, in this case, one needs to make the a priori assumption that the consumer's total demand is restricted to at most one unit. Thus, typical discrete choice models explain only part of consumer behavior because their use involves making an a priori assumption about the controlled total, which is exogenous. This has been repeatedly argued by McFadden (1999, p. 273), Nevo (2000, footnote 14), and Nevo (2001, footnote 13), but no clear theoretical foundation for the argument has been developed.[2]

The purpose of this paper is to formulate a model in which discrete choice models are incorporated consistently into the full utility maximization framework and to establish a theoretical foundation for the discrete choice model that assumes no a priori controlled total. In our framework, the results from discrete choice models are explained consistently whether a controlled total exists or not: the case in which there is a controlled total is a special case. The implications of discrete choice

[2] Nevo (2001, footnote 13) states: "A comment is in place about the realism of the assumption that consumers choose no more than one brand. Many households buy more than one brand of cereal in each super market trip but most people consume only one brand of cereal at a time, which is the relevant fact for this modeling assumption. Nevertheless, if one is still unwilling to accept that this is a negligible phenomenon, then this model can be viewed as an approximation to the true choice model." Nevo (2000, footnote 14) makes the same point. McFadden (1999, p. 273) raises the possibility that an alternative can be interpreted "as a 'portfolio' of decisions made in sequence, or as one of the multiple decisions."

models are also clarified, because the form of the utility function of the representative consumer, the own-price and cross-price elasticities, and the method of measuring welfare are derived in such a way that our results are directly comparable with standard microeconomic utility maximization.

We focus on the generalized extreme value (GEV) model, and its mixed form, because it generates analytically closed-form demand functions. Given that Dagsvik (1995) and McFadden and Train (2000) show that the GEV model and the mixed logit model can approximate any random utility model, our analysis is quite general. For illustrative purposes, we focus on the logit and mixed logit models as special cases before fully analyzing the GEV and mixed GEV models. As we show subsequently, analyzing the GEV and mixed GEV models is similar to analyzing the logit and mixed logit models.

Our main results are as follows. First, if a representative consumer's choice is represented by the logit model, his or her demand function for a good has the form of the market demand function for a group of goods multiplied by the choice probability of the good. (The market demand function for a group of goods is the sum of the market demands across all goods.) This form of the demand function is obtained when the indirect utility function of the representative consumer follows Gorman's (1961) framework and incorporates its restriction, and when the log-sum term is incorporated as the price index. Making the market demand for a group endogenous affects the own-price and cross-price elasticities of the market demand for a good. The price elasticity of the market demand for a group of goods is added to the usual own-price and cross-price elasticities in 'classical' logit models. Throughout the paper, we use the word 'classical' to represent a situation in which a consumer is assumed to make a single selection among a set of mutually exclusive alternatives. The change in welfare can be measured by using any of the four levels of demand: a consumer's demand for a good, the market demand for a good, a consumer's demand for a group of goods, or the market demand for a group of goods. If we measure the change in welfare by using a consumer's demand for a good or the market demand for a good, the corresponding price is the price of the good. If we measure the welfare change by using a consumer's demand for a group of goods or the market demand for a group of goods, the corresponding price is the log-sum term. The change in welfare in the classical logit model is typically calculated as the difference in the log-sum term multiplied by the total demand; this method is a special case of our analysis.

Second, our analysis can easily be extended to cases in which goods are classified into multiple groups. In this case, we can construct a model in which the choice within each group, such as between food and clothes, is represented by the logit model but the choice among groups is subject to any relationship. Not restricting relationships between groups is an advantage of our formulation. For example, the nested-logit model incorporates the grouping of goods, but the relationship between groups is limited to the logit.

Third, analyzing the mixed logit model requires modification. In the mixed logit model, each consumer has his or her own parameters. This implies that the log-sum term, which represents his or her price index, differs among consumers. Hence, the indirect utility function of the representative consumer must be quasi-linear because all consumers must have the same income coefficient for their Gorman-form indirect utility function. Similar results are obtained with regard to elasticities: the price elasticity of the market demand for a group is added to the usual own-price and cross-price elasticities. The change in welfare can be measured by using a consumer's demand for a good or a consumer's demand for a group. A difference from the logit model is that one cannot calculate the change in welfare by using the market demand for a good or the market demand for a group. The reason for this is that because each consumer's demand depends on his or her own parameters, the market demand cannot be derived without integrating out these terms. As in the case of the logit model, our analysis can easily be extended to the case in which goods are classified into multiple groups and to the case of the mixed GEV model.

Next, we briefly relate our analysis to the existing literature. The first line of research related to our paper is analysis of the relationship between discrete choice models and representative consumer models. Anderson et al. (1988, 1992, Ch. 3) and Verboven (1996) derive direct utility functions for the representative consumer that are consistent with the logit and nested logit models. Not only does our analysis correspond to the more general GEV model, which includes their models as special cases, but also we derive a utility maximization problem that is consistent with the mixed-GEV model. Moreover, we formulate a utility maximization problem that corresponds to the GEV and mixed-GEV models in a more realistic and more general framework: goods are classified into multiple groups, and a consumer can choose any number of goods from any groups.

The second line of research is the analysis of welfare measurement for discrete choice models. First, welfare measurement for discrete choice models is theoretically analyzed by, among others, Small and Rosen (1981) in a general form, but the market demand for a group is assumed exogenous.[3] Our analysis is an extension of theirs because we make the market demand for a group endogenous and develop a method of measuring welfare that is applicable to the case of endogenous demand. Second, Herriges and Kling (1999), McFadden (1999), De Palma and Kilani (2003), and Dagsvik and Karlstrom (2005) analyze welfare measurement for discrete choice models in which an indirect utility function is nonlinear in income.[4] In these analyses, the focus is on a one-consumer economy, in which the individual consumer's indirect utility function coincides with the representative consumer's indirect utility function, and the change in welfare is derived. In a many-consumer economy, with heterogeneous consumers, these analyses are inapplicable. This is because the aggregated compensating variation may not be consistent with the compensation test and thus may be of limited use. This is known as the Boadway paradox.[5] Our analysis is complementary to

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[3] See Jong et al. (2005) for a review of the literature on welfare measurement in discrete choice models.

[4] See Pakes et al. (1993), Berry et al. (1999), and Petrin (2002) for empirical research in which it is assumed that the utility obtained from a good is nonlinear with respect to income.

[5] See Boadway (1974).

existing analyses in that we can aggregate each consumer's welfare change consistently but the form of the indirect utility functions is limited to the Gorman form.[6]

The third line of research is recent empirical applications of discrete choice models, which range from models for durable goods such as housing (Earnhart 2002) and automobiles (Berry et al. 1995, Goldberg 1995, and Petrin 2002), to daily consumables such as ready-to-eat cereal (Nevo 2002) and tuna (Nevo and Hatzitaskos 2005). The point is whether it is appropriate to assume that consumers choose no more than one unit of a good. The validity of this assumption depends on the characteristics of the good. Arguably, the assumption is reasonable for housing and automobiles but not for ready-to-eat cereal and tuna because different consumers demand different amounts. We establish a theoretical basis for applying discrete choice models to cases in which consumers choose multiple units and in which different consumers demand different amounts.[7]

The rest of the paper is structured as follows. In Section 2, we focus on the logit model. In Section 3, the analysis is extended to the GEV model. In Section 4, we examine the mixed logit model. In Section 5, we examine the mixed GEV model. Section 6 concludes the paper.

2. The Logit Model

We begin with the logit model. The GEV model, which includes the logit model as a special case, is discussed in the next section.

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[6] Blackorby and Donaldson (1990) show that the Boadway paradox is resolved by assuming that the representative consumer has an indirect utility function of the Gorman form.

[7] Dubin and McFadden (1984) and Hanemann (1984) assume that a consumer first chooses a brand and then decides how many units of that brand to buy. Our analysis differs because a consumer is free to choose multiple brands as well as multiple units. Hendel (1999) focuses on the situation in which a firm buys multiple computers of multiple brands depending on the tasks that need performing. Although his analysis takes into account that multiple brands are chosen, it is based on profit maximization by a firm.

Consider an N-consumer economy with $M+1$ goods. The goods are numbered consecutively from 0 to M. The price of good 0, whose market demand is X_0, is normalized at unity. The market demand and price of good j are X_j and p_j ($j=1,...,M$), respectively. The income of consumer i is y^i ($i=1,...,N$).

Utility maximization by consumer i yields the indirect utility function, $v^i(p_1,...,p_M,y^i)$. In this paper, we assume that each consumer's preference can be aggregated to a representative consumer's preference. Without this assumption, there is no clear relationship between the sum of consumers' compensating variations and the compensation principle, as Blackorby and Donaldson (1990) show.

Gorman (1961) shows that in order to define the preferences of a representative consumer by aggregating individual consumer preferences, consumer i's indirect utility function must have the so-called Gorman form:

$$v^i(p_1,...,p_M,y^i) = A^i(p_1,...,p_M) + B(p_1,...,p_M)y^i \,. \tag{1}$$

Henceforth, we refer to this requirement as the Gorman restriction. Summing indirect utility functions across consumers yields the representative consumer's indirect utility function, as follows:

$$V = \sum_{i=1}^{N} v^i(p_1,...,p_M,y^i) = \sum_{i=1}^{N} A^i(p_1,...,p_M) + B(p_1,...,p_M)Y \,, \tag{2}$$

where $Y \equiv \sum_{i=1}^{N} y^i$ is aggregate income. Applying Roy's Identity to (1) and (2) yields consumer i's demand for good j, $x_j^i(p_1,...,p_M,y^i)$, and the market demand for good j, $X_j(p_1,...,p_M,Y)$, as follows:

(3) $$x_j^i(p_1,\ldots,p_M,y^i)=\frac{-\dfrac{\partial v^i}{\partial p_j}}{\dfrac{\partial v^i}{\partial y^i}}=\frac{-\dfrac{\partial (A^i+B)}{\partial p_j}}{B},$$

(4) $$X_j(p_1,\ldots,p_M,Y)=\frac{-\dfrac{\partial V}{\partial p_j}}{\dfrac{\partial V}{\partial Y}}=\frac{-\sum_{i=1}^{N}\dfrac{\partial (A^i+B)}{\partial p_j}}{B}=\sum_{i=1}^{N}x_j^i(p_1,\ldots,p_M,y^i).$$

2.1 A utility maximization problem that yields logit-type demand functions

For now, suppose that all goods belong to the same group. The case of different groups is addressed in Section 2.4. Suppose that the market demand function for good j is consistent with the logit model:

(5) $$X_j(p_1,\ldots,p_M,Y)=C(p_1,\ldots,p_M,Y)s_j,$$

where $C(p_1,\ldots,p_M,Y)$ is the market demand for a 'group', which is the sum of demands for goods 1 to M, and s_j is the choice probability from the logit model:

(6) $$s_j=\frac{\exp(u_j(p_j,Y))}{\sum_{k=1}^{M}\exp(u_k(p_k,Y))}.$$

We obtain the following proposition about the form of the indirect utility function of the representative consumer.

Proposition 1

The necessary and sufficient condition for the market demand function for good j to have the form of (5) is that the indirect utility function of the representative consumer be:

(7) $$V = A(LS) + B(LS)Y \,,$$

where $A \equiv \sum_{i=1}^{N} A^i(LS, p_1, ..., p_M)$ and:

(8) $$LS \equiv -\frac{1}{\beta} \ln \sum_{k=1}^{M} \exp(\alpha_k - \beta p_k) \,.$$

The market demand function for good j, (5), satisfies:

(9) $$C(p_1, ..., p_M, Y) = C(LS, Y) = \frac{-\left(\frac{\partial A}{\partial LS} + \frac{\partial B}{\partial LS} Y\right)}{B} > 0 \,,$$

(10) $$s_j = \frac{\exp(\alpha_j - \beta p_j)}{\sum_{k=1}^{M} \exp(\alpha_k - \beta p_k)} = \frac{\partial LS}{\partial p_j} \,.$$

Proof

See Appendix 1.

A model in which a controlled total exists is 'partial', because the determination of the controlled total is left unexplained. Because there is no controlled total, the representative consumer's indirect utility function, (7), corresponds to complete utility maximization; the market demand for a group, (9), is endogenously determined by the log-sum term, (8) and by aggregate income. Thus, although the utility maximization problem described in Proposition 1 yields a result that is consistent with the logit model, it does not suffer from the problem associated with discrete choice models identified by McFadden (1999, p. 273), Nevo (2000, footnote 14), and Nevo (2001, footnote 13); that is, the problem of having to assume an a priori controlled total. The representative consumer's indirect utility function, (7), includes the case of the fixed controlled total as a special case; for instance, $V = -NLS + Y$ represents a representative consumer's indirect utility function that is consistent with the logit model in which the controlled total is N .

The market demand for a group, (9), is a function of the log-sum term and aggregate income only; it depends on the prices of each good only through the log-sum term. The log-sum term plays the role of the aggregate price or the price index for the market demand for the group of goods. This property is of practical use for the estimation of demand structures. Suppose that one estimates a logit model and that, as a next step, one estimates the market demand for a group by using the estimated log-sum term. In this case, the derived market demand for a group of goods and the demand for each individual good are consistent with a complete utility maximization model.

The term $u_j(p_j, Y)$, which can be interpreted as the utility obtained from consuming a unit of good j, must be linear in price and independent of income; that is, $u_j = \alpha_j - \beta p_j$. If it is nonlinear in price, the market demand for a group differs among goods, which contradicts the fact that the logit-type market demand function for a good has the form of the common market demand for a group multiplied by the choice probability for a good. If the utility from consuming a unit of good j depends on income, the log-sum term also depends on income and, consequently, the Gorman restriction is not satisfied; hence, aggregating consumers' preferences to those of the representative consumer is impossible.

Note that, in the logit model, the utility from consuming a unit of good j and, thus, the choice probability for good j, are the same for all consumers. (See Appendix 1 for details.) When the utility from consuming a unit of good j and, thus, the choice probability for good j differs among consumers, the corresponding model is the mixed logit model, which is analyzed in Section 4.

Anderson et al. (1988, 1992 Ch. 3) derive the direct utility function of a representative consumer when the market demand for a group of goods is endogenously determined. In our framework, the corresponding direct utility function is:

(11) $$U = X_0 + h\left(\frac{b}{\beta}\sum_{k=1}^{M} X_k\right) + \frac{1}{\beta}\sum_{k=1}^{M}\left[\alpha_k - \ln\left(\frac{X_k}{\sum_{k=1}^{M} X_k}\right)\right]X_k ,$$

where b is a constant, $h' > 0$, and $h'' < 0$. This direct utility function corresponds to a special case of (7), as we show below. Maximizing (11) with respect to the budget constraint:

(12) $$Y = X_0 + \sum_{k=1}^{M} p_k X_k ,$$

yields the following market demand function for good j:

(13) $$X_j = \frac{\beta}{b} h'^{-1}\left(\frac{\beta}{b} LS\right) \frac{\exp(\alpha_j - \beta p_j)}{\sum_{k=1}^{M}\exp(\alpha_k - \beta p_k)} .$$

Substituting (13) into (11) yields the representative consumer's indirect utility function:

(14) $$V = h\left(h'^{-1}\left(\frac{\beta}{b}LS\right)\right) - \frac{\beta}{b} LSh'^{-1}\left(\frac{\beta}{b}LS\right) + Y .$$

Eq. (14) is a special case of (7), in which $A = h\left(h'^{-1}\left(\frac{\beta}{b}LS\right)\right) - \frac{\beta}{b}LSh'^{-1}\left(\frac{\beta}{b}LS\right)$ and $B = 1$.

The conditions that $h' > 0$ and $h'' < 0$ are sufficient for $\frac{\partial X_j}{\partial p_j} < 0$, because:

(15) $$\frac{\partial X_j}{\partial p_j} = \frac{-\beta^2 \exp(\alpha_j - \beta p_j)\left\{bh'^{-1}(Q)\sum_{k \neq j}\exp(\alpha_k - \beta p_k) - h''^{-1}(Q)\exp(\alpha_j - \beta p_j)\right\}}{\left(b\sum_{k=1}^{M}\exp(\alpha_k - \beta p_k)\right)^2} .$$

2.2 Elasticities

From the market demand function (5), we obtain the following proposition regarding elasticities.

Proposition 2

The own-price elasticity from the logit-type market demand function for good j is:

$$\frac{\partial X_j}{\partial p_j}\frac{p_j}{X_j} = \theta_j - \beta(1-s_j)p_j, \tag{16}$$

where $\theta_j \equiv \frac{\partial C}{\partial p_j}\frac{p_j}{C}$ is the elasticity of the market demand for a group of goods, C, with respect to the price of good j, p_j. The cross-price elasticity of demand for good j is:

$$\frac{\partial X_j}{\partial p_{j'}}\frac{p_{j'}}{X_j} = \theta_{j'} + \beta s_{j'} p_{j'}, \tag{17}$$

where $j' = 1,...,M$ and $j' \neq j$.

Proof

These results follow straightforwardly from the market demand function, (5).

Both elasticities differ from those in classical discrete choice models by adding the price elasticity of market demand for the group. The cross-price elasticities are the same among all goods, and the property of independence from irrelevant alternatives (IIA) holds, even if the market demand for a group is endogenous.

With regard to the own-price elasticity, we cannot determine whether demand is more elastic if the market demand for a group is exogenous or endogenous. For example, suppose that the market demand for a group is estimated by assuming that it is exogenously fixed and includes the three choices ‘select A’, ‘select B’, or ‘select neither’. The choice probability is one-third for each alternative. For the sake of simplicity, assume that $\beta p_A = 1$. The own-price elasticity of X_A is $\frac{\partial X_A}{\partial p_A}\frac{p_A}{X_A} = -\frac{2}{3}$ from (16). When the market demand for a group is endogenous, the true own-price elasticity of X_A is $\frac{\partial X_A}{\partial p_A}\frac{p_A}{X_A} = \theta_A - \frac{1}{2}$ from $s_A = s_B = \frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{3}} = \frac{1}{2}$. $\theta_A - \frac{1}{2}$ may be larger or smaller than $-\frac{2}{3}$, depending on the value of θ_A.

With regard to the cross-price elasticity, we derive a clearer result; ignoring the price elasticity of market demand for a group may change the sign of the cross-price elasticity of each good. If the market demand for a group is exogenously fixed, the cross-price elasticity of good j is positive because $\beta s_{j'} p_{j'} > 0$. By contrast, if market demand is endogenous, the cross-price elasticity of good j may be negative because $\theta_{j'} < 0$. In reality, when the market demand for a group is endogenous, an increase in the price of a particular good has the twin effects of increasing demand for substitutes and decreasing the market demand for the group. Fixing the market demand for a group eliminates the latter effect, and, therefore, an increase in the price of a particular good cannot lower demand for other goods.

2.3 Welfare analysis

In this section, we focus on calculating equivalent variation. The same procedure applies for calculating compensating variation if V^{WO} and v^{WO} are substituted for V^W and v^W, respectively, in the following analysis. Henceforth, the superscripts WO and W denote without and with a policy, respectively. The results are summarized in Proposition 3.

Proposition 3

Equivalent variation can be calculated from the consumer's demand for a good, the market demand for a good, each consumer's demand for a group of goods, or the market demand for a group of goods, as follows:

$$
\begin{aligned}
EV &= \sum_{i=1}^{N} \int_{p_j^W}^{p_j^{WO}} h_j^i(LS, p_1, ..., p_M, v^{iW}) dp_j \\
&= \int_{p_j^W}^{p_j^{WO}} H_j(LS, V^W) dp_j \\
&= \int_{LS^W}^{LS^{WO}} \sum_{i=1}^{N} c^i(LS, p_1, ..., p_M, v^{iW}) dLS \\
&= \int_{LS^W}^{LS^{WO}} C(LS, V^W) dLS,
\end{aligned} \tag{18}
$$

where $h_j^i(LS, p_1, ..., p_M, v^i)$ is consumer i's Hicksian demand function for good j, $H_j(LS, V)$ is the Hicksian market demand function for good j, $c^i(LS, p_1, ..., p_M, v^i)$ is consumer i's Hicksian demand function for a group of goods, and $C(LS, V)$ is the Hicksian market demand function for a group of goods.

Proof

See Appendix 2.

The transformation from the first to the second line and from the third to the fourth line in (18) follows from the fact that, under the Gorman restriction, each consumer's indirect utility function can be aggregated to the indirect utility function of the representative consumer. The distinctive feature of the logit model, which is also true of GEV models (as shown later), is that the equivalent variation can be calculated not only from the demand and price of good j but also from the demand for a group of goods, by using the log-sum term. In particular, if the market demand of a group is equal to the number of consumers, N, the result in (18) reduces to the well-known method developed by

Small and Rosen (1981), under which equivalent variation is calculated from the change in the log-sum term multiplied by the number of consumers.

2.4 Multiple groups

We have so far assumed that goods $1,...,M$ belong to the same group. In reality, goods can be classified into multiple groups, such as food and clothes. Our analysis can be readily extended to the case in which goods are classified into multiple groups and a consumer can choose multiple goods from multiple groups.

Suppose that the goods are classified into G groups and that good j belongs to group g ($g = 1,...,G$), without loss of generality. Suppose that the market demand function for good j, which belongs to group g, is consistent with the logit model, as follows:

$$X_j(p_1,..., p_M, Y) = C_g(p_1,..., p_M, Y)s_{gj}, \tag{19}$$

where $s_{gj} = \dfrac{\exp(u_j(p_j), Y)}{\sum_{k\in g}\exp(u_k(p_k), Y)}$ is the choice probability for good j within group g and $C_g(p_1,..., p_M, Y)$ is the market demand for group g.

Propositions 1 to 3 require only minor modification when there are multiple groups. To avoid repetition, our analysis of multiple groups is relegated to Appendix 3, in which we derive Propositions 1' to 3', which are modified versions of Propositions 1 to 3.

Because the relationship between groups is unrestricted, we can represent various relationships between groups in our model. This is a clear advantage over a typical discrete choice model that incorporates the grouping of goods, such as the nested logit model, which limits the relationship between groups to the logit.

As an example, suppose that there are two groups ($l = 1, 2$) and that preferences between them are represented by the CES utility function. Suppose further that the preference within each group is of the logit type. The indirect utility function of the representative consumer is:

(20)
$$V = Y\left(\sum_{l=1}^{2} LS_l^{1-\sigma}\right)^{\frac{1}{\sigma-1}},$$

where $LS_g = -\dfrac{1}{\beta}\ln\sum_{k\in g}\exp(\alpha_k - \beta_g p_k)$ and σ is the elasticity of substitution between the two groups. The demand for group g, given by C_g, has the CES form as follows:

(21)
$$C_g = \frac{YLS_g^{-\sigma}}{\sum_{l=1}^{2} LS_l^{1-\sigma}}.$$

The market demand function for good j in group g is:

(22)
$$X_j = \frac{YLS_g^{-\sigma}}{\sum_{l=1}^{2} LS_l^{1-\sigma}} \frac{\exp(\alpha_j - \beta_g p_j)}{\sum_{k\in g}\exp(\alpha_k - \beta_g p_k)}.$$

This has the form of the CES-type market demand function for a group multiplied by the logit-type choice probability.

3. The GEV Model

The analysis of Section 2 can be extended to the GEV model, which is a general form of logit model. From McFadden (1978, Theorem 1), the GEV model can be described by using the function $F(z_1, \cdots, z_M)$, where $z_j \equiv \exp(u_j(p_j, Y))$.

(GEV–1) $F(z_1, \cdots, z_M)$ is nonnegative.

(GEV–2) $F(z_1,\cdots,z_M)$ is homogenous of degree n.[8]

(GEV–3) $\lim_{z_j\to\infty} F(z_1,\cdots,z_M)=\infty$.

(GEV–4) The ς th partial derivative of $F(z_1,\cdots,z_M)$ with respect to any combination of distinct z_j s is nonnegative if ς is odd and nonpositive if ς is even. That is, $\frac{\partial F}{\partial z_j}\geq 0$ for all j, $\frac{\partial^2 F}{\partial z_j \partial z_{j'}}\leq 0$ for all $j'=1,\ldots,M$ and $j'\neq j$, $\frac{\partial^3 H}{\partial z_j \partial z_{j'} \partial z_{j''}}\geq 0$ for any distinct j, j', and j'' ($j''=1,\ldots,M$), and so on for higher-order derivatives.

Under assumptions (GEV–1) to (GEV–4), from McFadden (1978, Theorem 1), the choice probability for good j is:

(23) $$s_{GEVj}=\frac{\frac{\partial F}{\partial z_j}z_j}{nF}.$$

Extending the analysis of Section 2 to the GEV model is straightforward. The points to note are as follows.

i) Proposition 1 holds if the log-sum term and the choice probability for good j are modified from (8) and (10) to:

(24) $$LS_{GEV}\equiv -\frac{1}{n\beta}\ln F(\exp(\alpha_1-\beta p_1),\cdots,\exp(\alpha_M-\beta p_M)),$$

[8] McFadden (1978, Theorem 1) assumed homogeneity of degree one. Ben-Akiba and Francois (1983) demonstrate that H can be homogeneous of degree n. See also Ben-Akiba and Lerman (1985, p. 126).

(25) $$s_{GEVj} = \frac{\frac{\partial F}{\partial z_j}\exp(\alpha_j - \beta p_j)}{nF} = \frac{\partial LS_{GEV}}{\partial p_j}.$$

ii) The essence of Proposition 2 holds: for the GEV model, the elasticities of market demand are added to the standard own-price and cross-price elasticities when the market demand for a group is endogenous. The own-price elasticity of market demand for good j, X_{GEVj}, is:

(26) $$\frac{\partial X_{GEVj}}{\partial p_j}\frac{p_j}{X_{GEVj}} = \theta_j - \beta(1 - ns_{GEVj})p_j + \eta_{jj},$$

where $\eta_{jj} \equiv \frac{\partial\left(\frac{\partial F}{\partial z_j}\right)}{\partial p_j}\frac{p_j}{\left(\frac{\partial F}{\partial z_j}\right)}$ is the elasticity of $\frac{\partial F}{\partial z_j}$ with respect to the price of good j, p_j. The cross-price elasticity is:

(27) $$\frac{\partial X_{GEVj}}{\partial p_{j'}}\frac{p_{j'}}{X_{GEVj}} = \theta_{j'} + \beta ns_{j'}p_{j'} + \eta_{jj'}.$$

iii) With regard to welfare analysis, Proposition 3 holds, although the log-sum term is modified from (8) to (24).

iv) The extension to the case of multiple groups is analogous to that for the logit model.

4. The Mixed Logit Model

From now on, we focus on ‘mixed’ versions of the logit and GEV models. We consider the mixed logit model in this section and consider the mixed GEV model in the next section.

Suppose that each consumer derives a different level of utility from consuming a unit of good j. The differences in utility among consumers are unobservable and are treated probabilistically; we assume that each consumer has his or her own parameter, γ^i, whose probability density function is $f(\gamma^i)$.

From Train (2003), the mixed logit model has the following choice probability:

(28) $$s_{ML\,j} = \int_{\gamma^i} s^i_{MLj}(\gamma^i) f(\gamma^i) d\gamma^i ,$$

where:

(29) $$s^i_{MLj}(\gamma^i) = \frac{\exp(u^i_j(p_j, y^i, \gamma^i))}{\sum_{k=1}^{M} \exp(u^i_k(p_k, y^i, \gamma^i))} .$$

For now, suppose that all goods belong to the same group. Our analysis is easily extended to the case in which goods are classified into multiple groups: see Appendix 6.

Because indirect utility functions must satisfy Gorman's (1961) restriction, given that consumer i's parameter is γ^i, the conditional indirect utility function of consumer i is:

(30) $$v^{ci}(p_1,..., p_M, y^i, \gamma^i) = A^i(p_1,..., p_M, \gamma^i) + B(p_1,..., p_M) y^i ,$$

Because γ^i follows the probability density function $f(\gamma^i)$, the unconditional indirect utility function of consumer i is:

(31) $$v^i = \int_{\gamma^i} v^{ci} f(\gamma^i) d\gamma^i = \int_{\gamma^i} \left(A^i(p_1,..., p_M, \gamma^i) + B(p_1,..., p_M) y^i \right) f(\gamma^i) d\gamma^i .$$

The unconditional indirect utility function of the representative consumer is:

(32)
$$\begin{aligned} V &= \sum_{i=1}^{N} v^i = \sum_{i=1}^{N} \int_{\gamma^i} \left(A^i(p_1,..., p_M, \gamma^i) + B(p_1,..., p_M) y \right) f(\gamma^i) d\gamma^i \\ &= \sum_{i=1}^{N} \int_{\gamma^i} \left(A^i(p_1,..., p_M, \gamma^i) \right) f(\gamma^i) d\gamma^i + B(p_1,..., p_M) Y \end{aligned} .$$

4.1 A utility maximization problem that yields mixed-logit-type demand functions

When a change in consumer i's demand is taken into account, the market demand function for good j that is consistent with the mixed logit model is:

(33)
$$X_{MLj} = \sum_{i=1}^{N} \int_{\gamma^i} c^i_{ML}(p_1,..., p_M, y^i, \gamma^i) s^i_{MLj}(\gamma^i) f(\gamma^i) d\gamma^i = \sum_{i=1}^{N} \int_{\gamma^i} x^i_{MLj}(\gamma^i) f(\gamma^i) d\gamma^i ,$$

where X_{MLj} is the market demand for good j, $c^i_{ML}(p_1,..., p_M, y^i, \gamma^i)$ is consumer i's demand for a group of goods, and $x^i_{MLj}(\gamma^i)$ is consumer i's demand for good j.

We can now state the following proposition about the form of the indirect utility function of the representative consumer.

Proposition 4

The necessary and sufficient condition for the market demand function for good j to have the form of (33) is that the indirect utility function of the representative consumer be:

(34)
$$V = \sum_{i=1}^{N} \int_{\gamma^i} \left(A^i(LS^i_{ML}(\gamma^i), \gamma^i) \right) f(\gamma^i) d\gamma^i + \overline{B} Y ,$$

where $\overline{B}$ is a fixed constant and:

(35)
$$LS^i_{ML}(\gamma^i) \equiv -\frac{1}{\beta^i(\gamma^i)} \ln \sum_{k=1}^{M} \exp(\alpha^i_k(\gamma^i) - \beta^i(\gamma^i) p_k) .$$

The market demand function for good j, (33), satisfies:

(36) $$c^i_{ML}(LS^i_{ML}(\gamma^i),\gamma^i) = \frac{-\dfrac{\partial A^i}{\partial LS^i_{ML}(\gamma^i)}}{\overline{B}} = \frac{-\dfrac{\partial v^{ci}}{\partial LS^i_{ML}(\gamma^i)}}{\overline{B}} > 0,$$

(37) $$s^i_{MLj}(\gamma^i) = \frac{\exp(\alpha^i_j(\gamma^i) - \beta^i(\gamma^i)p_j)}{\sum_{k=1}^{M}\exp(\alpha^i_k(\gamma^i) - \beta^i(\gamma^i)p_k)} = \frac{\partial LS^i_{ML}(\gamma^i)}{\partial p_j}.$$

Proof

See Appendix 4.

The major difference from the logit model is that the indirect utility function of the representative consumer is quasi-linear. The reason is as follows. In the mixed logit model, the log-sum term, $LS^i_{ML}(\gamma^i)$, differs among consumers. From Gorman's (1961) restriction, the coefficient of income, $B(p_1,...,p_M)$, must be the same for all consumers. This implies that the coefficient of income, $B(p_1,...,p_M)$, does not include the log-sum term, $LS^i_{ML}(\gamma^i)$. Moreover, one can obtain the market demand function for a group of goods and the choice probability for the mixed logit model only when the coefficient of income, $B(p_1,...,p_M)$, does not depend on the prices, p_j. Thus, $B(p_1,...,p_M)$ is the fixed constant $\overline{B}$, and the indirect utility function of the representative consumer is quasi-linear. Hence, the market demand function for good j is independent of income. The mixed logit model, if it is formulated to be consistent with standard microeconomic utility maximization, can deal with differences in parameters among consumers and can thereby incorporate consumer differences in utility obtained from the consumption of goods. However, the mixed logit model cannot deal with the income effect.

4.2 Elasticities

From the mixed logit-type market demand function, (33), we obtain the following proposition about elasticities.

Proposition 5

The own-price elasticity of the market demand for good j is:

$$(38)\qquad \frac{\partial X_{MLj}}{\partial p_j}\frac{p_j}{X_{MLj}} = \sum_{i=1}^{N}\int_{\gamma^i}\left\{\frac{c^i_{ML}(LS^i_{ML}(\gamma^i),\gamma^i)s^i_{MLj}(\gamma^i)}{X_{MLj}}\left(\varphi^i_j(\gamma^i)-\beta(1-s^i_{MLj}(\gamma^i))p_j\right)\right\}f(\gamma^i)d\gamma^i \ ,$$

where $\varphi^i_j(\gamma^i)=\frac{\partial c^i_{ML}(LS^i_{ML}(\gamma^i),\gamma^i)}{\partial p_j}\frac{p_j}{c^i_{ML}(LS^i_{ML}(\gamma^i),\gamma^i)}$ is the price elasticity of consumer i ’s group demand. The cross-price elasticity is:

$$(39)\qquad \frac{\partial X_{MLj}}{\partial p_{j'}}\frac{p_{j'}}{X_{MLj}} = \sum_{i=1}^{N}\int_{\gamma^i}\left\{\frac{c^i_{ML}(LS^i_{ML}(\gamma^i),\gamma^i)s^i_{MLj}(\gamma^i)}{X_{MLj}}\left(\varphi^i_{j'}(\gamma^i)+\beta s^i_{MLj'}(\gamma^i)p_{j'}\right)\right\}f(\gamma^i)d\gamma^i \ .$$

Proof

The results follow straightforwardly from the market demand function, (33).

If the parameter γ^i is a fixed constant, the results in (38) and (39) are consistent with the corresponding results for the logit model, given by (16) and (17), respectively. Otherwise, the own-price and cross-price elasticities are more flexible. Thus, the mixed logit model can deal with more complex substitution and complementarity patterns. In particular, the IIA property does not hold because the cross-price elasticities in (39) depend on $\frac{c^i_{ML}(LS^i_{ML}(\gamma^i),\gamma^i)s^i_{MLj}(\gamma^i)}{X_{MLj}}$, which differs among goods.

4.3 Welfare analysis

The indirect utility function of the representative consumer that yields the mixed logit model, (34), is quasi-linear. Thus, the Hicksian and the Marshallian demand curves coincide, and equivalent variation, compensating variation, and the change in the consumer surplus also coincide. We can state the following proposition about welfare measurement.

Proposition 6

Equivalent variation can be calculated from consumer i's demand for a good or from consumer i's demand for a group of goods; that is:

$$
\begin{aligned}
EV &= \sum_{i=1}^{N} \int_{\gamma^i} \left(\int_{p_j^{iW}}^{p_j^{iWO}} x_{MLj}^i (LS_{ML}^i(\gamma^i), \gamma^i) dp_j \right) f(\gamma^i) d\gamma^i \\
&= \sum_{i=1}^{N} \int_{\gamma^i} \left(\int_{LS_{ML}^{iW}(\gamma^i)}^{LS_{ML}^{iWO}(\gamma^i)} c_{ML}^i (LS_{ML}^i(\gamma^i), \gamma^i) dLS_{ML}^i(\gamma^i) \right) f(\gamma^i) d\gamma^i .
\end{aligned} \tag{40}
$$

Proof

See Appendix 5.

In the mixed logit model, there are two ways of calculating equivalent variation. The first method is to calculate equivalent variation for consumer i by using his or her demand function for good j, and then sum over all consumers. The second method is to calculate equivalent variation by using his or her demand function for a group of goods and the log-sum term, $LS_{ML}^i(\gamma^i)$. Note that equivalent variation cannot be calculated by using the market demand for good j or the market demand for a group of goods. This represents a difference from Proposition 3 for the logit model. The reason for this is that because consumer i's demand depends on his or her own parameter, it cannot be summed without integrating over this parameter. This result is not surprising given that the mixed logit model explicitly considers differences in parameters among consumers.

As explained in Section 4.1, the mixed logit model cannot deal with the income effect. By contrast, the logit model can deal with the income effect under the Gorman restriction. Thus, welfare measurement based on the logit model, (18), is not generally a special case of that based on the mixed logit model, (40). This is only the case for a quasi-linear indirect utility function. In this case, if all consumers have the same parameter ($\gamma^i = \gamma$), the first and second lines in (40) coincide with the first and third lines in (18).

5. The Mixed GEV Model

McFadden and Train (2000) demonstrate that the mixed logit model can approximate any random utility model. However, the mixed GEV model is probably more suitable because of its analytical properties (Bhat et al. 2007). Therefore, in this section, we derive the properties of the mixed GEV model.

The extension from the mixed logit model to the mixed GEV model is analogous to that from the logit model to the GEV model. The following points should be noted.

i) Proposition 4 holds if the log-sum term and the choice probability for good j are modified from (35) and (37) to:

$$LS^i_{MGEV}(\gamma^i) \equiv -\frac{1}{n\beta^i(\gamma^i)} \ln F(\exp(\alpha^i_1(\gamma^i) - \beta^i(\gamma^i)p_1), \cdots, \exp(\alpha^i_M(\gamma^i) - \beta^i(\gamma^i)p_M)), \tag{41}$$

$$s^i_{MGEVj}(\gamma^i) = \frac{\frac{\partial F}{\partial z_j}\exp(\alpha^i_j(\gamma^i) - \beta^i(\gamma^i)p_j)}{nG} = \frac{\partial LS^i_{MGEV}(\gamma^i)}{\partial p^i_j}. \tag{42}$$

ii) The own-price and cross-price elasticities from the GEV model are mixed in the same way as are those from the mixed logit model. The own-price elasticity of the market demand for good j, X_{MGEVj}, is:

$$\begin{aligned} &\frac{\partial X_{MGEVj}}{\partial p_j}\frac{p_j}{X_{MGEVj}} \\ &= \sum_{i=1}^{N}\int_{\gamma^i}\left\{\frac{c^i_{MGEV}(LS^i_{MGEV}(\gamma^i),\gamma^i)s^i_{MGEVj}(\gamma^i)}{X_{MGEVj}}\left(\varphi^i_j(\gamma^i) - \beta(1-ns^i_{MGEVj}(\gamma^i))p_j + \psi^i_{jj}(\gamma^i)\right)\right\} f(\gamma^i)d\gamma^i, \end{aligned} \tag{43}$$

where $\psi^i_{jj}(\gamma^i) \equiv \dfrac{\partial\left(\dfrac{\partial F(\gamma^i)}{\partial z^i_j(\gamma^i)}\right)}{\partial p_j} \dfrac{p_j}{\left(\dfrac{\partial F(\gamma^i)}{\partial z^i_j(\gamma^i)}\right)}$ is the elasticity of $\dfrac{\partial F(\gamma^i)}{\partial z^i_j(\gamma^i)}$, where $z^i_j(\gamma^i) \equiv \alpha^i_j(\gamma^i) - \beta^i(\gamma^i) p_j$,

with respect to the price of good j, p_j. The cross-price elasticity is:

$$\begin{aligned} &\frac{\partial X_{MGEVj}}{\partial p_{j'}} \frac{p_{j'}}{X_{MGEVj}} \\ &= \sum_{i=1}^{N} \int_{\gamma^i} \left\{ \frac{c^i_{MGEV}(LS^i_{MGEV}(\gamma^i), \gamma^i) s^i_{MGEVj}(\gamma^i)}{X_{MGEVj}} \left(\varphi^i_{j'}(\gamma^i) + \beta n s^i_{MGEVj'}(\gamma^i) p_{j'} + \psi^i_{jj'}(\gamma^i) \right) \right\} f(\gamma^i) d\gamma^i. \end{aligned} \tag{44}$$

iii) Welfare analysis is the same as in the mixed logit model if $LS^i_{ML}(\gamma^i)$ is replaced by $LS^i_{MGEV}(\gamma^i)$ in (41).

iv) The extension to the case of multiple groups is analogous to that for the mixed logit model.

6. Concluding Remarks

In this paper, we formulated a structure for utility maximization problems that are consistent with demand functions derived from the generalized extreme value (GEV) model and the associated mixed models. We also clarified the characteristics of the form of the utility function, the elasticities, and the measurement of welfare. The results of the paper demonstrate that GEV and mixed GEV models that incorporate endogenous demands for groups of goods; that is, those models without a controlled total, are consistently formulated as standard microeconomic utility maximization problems of a representative consumer. Before concluding our analysis, we comment on three issues.

First, to be consistent with our analysis, in the GEV and the mixed GEV models, the utility gained from consuming a good should not depend on income, in which case, the choice probability does not depend on income. This result is a consequence of the Gorman restriction. Unfortunately, the three elements of income nonlinearities, the existence of many consumers, and practical benefit estimation cannot be incorporated simultaneously in the current framework of microeconomics. Our analysis can only be used for practical benefit estimation in the context of a many-consumer economy if the Gorman restriction is imposed, but this is incompatible with income nonlinearity. When the indirect utility function is nonlinear in income, practical benefit estimation is only possible in the case of a one-consumer economy. When indirect utility functions are nonlinear in income in a many-consumer economy, although one can use a social welfare function, practical benefit estimation is almost impossible.

Second, another restriction we derived is that the utility gained from consuming a good is linear in price. Without this restriction, one cannot relate GEV and mixed GEV models that incorporate endogenous demand for groups of goods to a complete utility maximization problem for a representative consumer. Empirically, it would be easy to construct a model in which utility is nonlinear in price. However, our analysis cannot be applied if there is nonlinearity in price; in this case, one could resolve the situation by using a random utility maximization framework that incorporates an a priori controlled total, but this would be inconsistent with complete utility maximization.

Third, Berry (1994), Berry et al. (1995, 1999), and Nevo (2000, 2001) estimate a mixed logit model by using market data. Although an advantage of this is that there is no need to collect individual data, the approach is not consistent with complete utility maximization because an a priori controlled total must be incorporated. Hence, the estimated own-price and cross-price elasticities would be biased. Although our formulation circumvents these problems, it is relatively costly to apply empirically because one requires data on individual consumers.

Appendix 1 Proof of Proposition 1

Suppose that each consumer values good j differently, $u_j^i(p_j, y^i)$. Define $\widetilde{LS^i}(p_1,..., p_M, y^i)$ as:

(A1) $$\widetilde{LS^i}(p_1,..., p_M, y^i) \equiv -\ln \sum_{k=1}^{M} \exp(u_k^i(p_k, y^i)).$$

By solving (A1) with respect to p_1 and then substituting the resulting expression into (1), we can rewrite (1) as:

(A2) $$v^i = A^i(p_1(\widetilde{LS^i}, p_2,..., p_M, y^i),..., p_M) + B(p_1(\widetilde{LS^i}, p_2,..., p_M, y^i),..., p_M)y^i.$$

Because the Gorman restriction implies that $A^i(p_1(\widetilde{LS^i}, p_2,..., p_M, y^i),..., p_M)$ is independent of y^i and that $B(p_1(\widetilde{LS^i}, p_2,..., p_M, y^i),..., p_M)$ is independent of y^i and the same for all consumers, it follows that:

(A3) $$v^i = A^i(\widetilde{LS^i}(p_1,..., p_M), p_2,..., p_M) + B(p_2,..., p_M)y^i,$$

where:

(A4) $$\widetilde{LS^i}(p_1,..., p_M) = -\ln \sum_{k=1}^{M} \exp(u_k^i(p_k)).$$

By applying Roy's Identity to (A3), the market demand function for good 1 is derived as:

(A5) $$X_1 = \sum_{i=1}^{N} x_1^i = \sum_{i=1}^{N} \left(\frac{\partial A^i}{\partial \widetilde{LS^i}} u_1^{i\prime} \frac{\exp(u_1^i(p_1))}{\sum_{k=1}^{M} \exp(u_k^i(p_k))} \right).$$

Because $u_1^i(p_1)$ differs among consumers, for (A5) to be consistent with (5):

(A6) $$X_1 = \frac{\partial A^i}{\partial \widetilde{LS}^i} u_1^{i\prime} \sum_{i=1}^{N} \left(\frac{\exp(u_1^i(p_1))}{\sum_{k=1}^{M} \exp(u_k^i(p_k))} \right),$$

where $C(p_1,..., p_M) = \frac{\partial A^i}{\partial \widetilde{LS}^i} u_1^{i\prime}$ is constant and $s_{L1} = \sum_{i=1}^{N} \left(\frac{\exp(u_1^i(p_1))}{\sum_{k=1}^{M} \exp(u_k^i(p_k))} \right)$.

This implies that the market demand for a group of goods must be fixed; that is, the determination of market demand for a group of goods cannot be incorporated into a utility maximization problem. Thus, if the market demand for a group is endogenously determined and is consistent with the market demand function of the logit model, (5), all consumers place the same valuation on good j.

We prove the necessary condition first. Define $\widetilde{LS}(p_1,..., p_M, y^i)$ as:

(A7) $$\widetilde{LS}(p_1,..., p_M, y^i) \equiv -\ln \sum_{k=1}^{M} \exp(u_k(p_k, y^i)) .$$

By solving (A7) with respect to p_1 and then substituting the resulting expression for p_1 into (2), we can rewrite (2) as:

(A8) $$\begin{aligned} V &= \sum_{i=1}^{N} A^i(p_1(\widetilde{LS}, p_2,..., p_M, y^i),..., p_M) + B(p_1(\widetilde{LS}, p_2,..., p_M, y^i),..., p_M)Y \\ &= \sum_{i=1}^{N} A^i(\widetilde{LS}(p_1,..., p_M, y^i), p_2,..., p_M) + B(\widetilde{LS}(p_1,..., p_M, y^i), p_2,..., p_M)Y. \end{aligned}$$

To satisfy the condition that consumer i 's indirect utility function have the Gorman form, $A^i(\widetilde{LS}(p_1,..., p_M, y^i), p_2,..., p_M)$ must be independent of y^i and $B(\widetilde{LS}(p_1,..., p_M, y^i), p_2,..., p_M)$ must be independent of y^i and the same for all consumers. Thus:

(A9) $$V = \sum_{i=1}^{N} A^i(\widetilde{LS}(p_1,..., p_M), p_2,..., p_M) + B(\widetilde{LS}(p_1,..., p_M), p_2,..., p_M)Y,$$

where:

(A10) $$\widetilde{LS}(p_1,..., p_M) = -\ln \sum_{k=1}^{M} \exp(u_k(p_k)).$$

Eq. (A10) implies that the utility obtained from consuming a unit of good j is independent of income. Applying Roy's Identity to (A9) yields the following market demand function for good 1:

(A11) $$X_1(p_1,..., p_M, Y) = \frac{\left(\sum_{i=1}^{N} \frac{\partial A^i}{\partial \widetilde{LS}} + \frac{\partial B}{\partial \widetilde{LS}} Y\right) u_1'(p_1)}{B} \frac{\exp(u_1(p_1))}{\sum_{k=1}^{M} \exp(u_k(p_k))}.$$

Comparing (5) and (A11) when $j = 1$ reveals:

(A12) $$C(p_1,..., p_M, Y) = \frac{\left(\sum_{i=1}^{N} \frac{\partial A^i}{\partial \widetilde{LS}} + \frac{\partial B}{\partial \widetilde{LS}} Y\right) u_1'(p_1)}{B}.$$

In the same way, the market demand function for good m ($m = 2,..., M$) is derived as:

(A13) $$X_m(p_1,..., p_M, Y) = \frac{\left(\sum_{i=1}^{N} \frac{\partial A^i}{\partial \widetilde{LS}} + \frac{\partial B}{\partial \widetilde{LS}} Y\right) u_m'(p_m) \frac{\exp(u_m(p_m))}{\sum_{k=1}^{M} \exp(u_k(p_k))} - \left(\sum_{i=1}^{N} \frac{\partial A^i}{\partial p_m} + \frac{\partial B}{\partial p_m} Y\right)}{B}.$$

Comparing (5) and (A13) when $j = m$ reveals:

(A14) $$C(p_1,..., p_M, Y) = \frac{\left(\sum_{i=1}^{N} \frac{\partial A^i}{\partial \widetilde{LS}} + \frac{\partial B}{\partial \widetilde{LS}} Y\right) u_m'(p_m)}{B},$$

(A15) $$\sum_{i=1}^{N} \frac{\partial A^i}{\partial p_m} + \frac{\partial B}{\partial p_m} Y = 0 .$$

Because (A15) holds for any Y , we obtain:

(A16) $$\sum_{i=1}^{N} \frac{\partial A^i}{\partial p_m} = \frac{\partial B}{\partial p_m} = 0 .$$

The above analysis applies when solving (A7) with respect to any p_j and then substituting the resulting expression into (2). Thus, for all cases, we can write the indirect utility function of the representative consumer as:

(A17) $$V = \sum_{i=1}^{N} A^i (\widetilde{LS}, p_1, ..., p_M) + B(\widetilde{LS})Y ,$$

where, from (A16):

(A18) $$\frac{\partial \left(\sum_{i=1}^{N} A^i (\widetilde{LS}, p_1, ..., p_M) \right)}{\partial p_j} = 0 \text{ for any } p_j .$$

Thus, we can rewrite (A17) as:

(A19) $$V = A(\widetilde{LS}) + B(\widetilde{LS})Y ,$$

where $A(\widetilde{LS}) = \sum_{i=1}^{N} A^i (\widetilde{LS}, p_1, ..., p_M)$.

Because $C(p_1, ..., p_M, Y)$ is the same for any good j , from (A12) and (A14), we obtain:

(A20) $$u_1'(p_1) = ... = u_M'(p_M) .$$

This implies that $u_j(p_j)$ is linear in p_j . Thus, we can express $u_j(p_j)$ as:

(A21) $$u_j(p_j) = \alpha_j - \beta p_j \,.$$

The choice probability, (10), follows from (6) and (A21). From (A21), $\widetilde{LS}(p_1,..., p_M)$ can be written as:

(A22) $$\widetilde{LS}(p_1,..., p_M) = -\ln \sum_{k=1}^{M} \exp(\alpha_k - \beta p_k)\,.$$

Eq. (8) is derived by defining:

(A23) $$LS \equiv \frac{1}{\beta}\widetilde{LS}(p_1,..., p_M)\,.$$

Substituting (A23) into (A19) yields (7).

Because the indirect utility function of the representative consumer is decreasing in p_j, we obtain:

(A24) $$\frac{\partial V}{\partial p_j} = \frac{\partial V}{\partial LS}\frac{\partial LS}{\partial p_j} = \frac{\partial V}{\partial LS}\frac{\exp(\alpha_j - \beta p_j)}{\sum_{k=1}^{M}\exp(\alpha_k - \beta p_k)} < 0\,.$$

This implies:

(A25) $$\frac{\partial V}{\partial LS} = \sum_{i=1}^{N}\frac{\partial A^i}{\partial LS} + \frac{\partial B}{\partial LS}Y < 0\,.$$

From (A14), (A23), (A25), and $u_1{}'(p_1) = ... = u_M{}'(p_M) = -\beta < 0$, the market demand for a group of goods, (9), is:

(A26) $$C(LS, Y) = \frac{\left(\sum_{i=1}^{N}\frac{\partial A^i}{\partial \widetilde{LS}} + \frac{\partial B}{\partial \widetilde{LS}}Y\right)u_m{}'(p_m)}{B} = \frac{-\left(\sum_{i=1}^{N}\frac{\partial A^i}{\partial LS} + \frac{\partial B}{\partial LS}Y\right)}{B} > 0\,.$$

By using Roy's Identity, proving sufficiency is straightforward. Applying Roy's Identity to (7) yields:

(A27) $$X_j = \frac{-\dfrac{\partial V}{\partial p_j}}{\dfrac{\partial V}{\partial Y}} = \frac{-\left(\displaystyle\sum_{i=1}^{N} \frac{\partial A^i}{\partial LS} + \frac{\partial B}{\partial LS} Y\right)}{B} \frac{\exp(\alpha_j - \beta p_j)}{\displaystyle\sum_{k=1}^{M} \exp(\alpha_k - \beta p_k)},$$

where:

(A28) $$C(LS, Y) = \frac{-\left(\displaystyle\sum_{i=1}^{N} \frac{\partial A^i}{\partial LS} + \frac{\partial B}{\partial LS} Y\right)}{B}.$$

From (A25), this expression is positive. In addition:

(A29) $$s_j = \frac{\exp(\alpha_j - \beta p_j)}{\displaystyle\sum_{k=1}^{M} \exp(\alpha_k - \beta p_k)} = \frac{\partial LS}{\partial p_j}.$$

Eq. (5) is derived from (A27) to (A29).

Appendix 2 Proof of Proposition 3

From (7), the expenditure functions of consumer *i* and the representative consumer are, respectively:

(A30) $$e^i(LS, p_1, ..., p_M, v) = \frac{v^i - A^i(LS, p_1, ..., p_M)}{B(LS)},$$

(A31) $$E(LS, V) = \frac{V - \displaystyle\sum_{i=1}^{N} A^i(LS, p_1, ..., p_M)}{B(LS)}.$$

From (A30) and (A31), we obtain consumer i's Hicksian demand function for good j and the Hicksian market demand function for good j:

(A32) $$h_j^i(LS, p_1,..., p_M, v) \equiv \frac{\partial e^i}{\partial p_j} = \frac{-\left(\frac{\partial A^i}{\partial LS}\frac{\partial LS}{\partial p_j} + \frac{\partial A^i}{\partial p_j}\right)B - \left(v^i - A^i\right)\frac{\partial B}{\partial LS}\frac{\partial LS}{\partial p_j}}{(B(LS))^2},$$

and:

(A33) $$\begin{aligned} H_j(LS,V) &\equiv \frac{\partial E}{\partial p_j} = \frac{-\left(\sum_{i=1}^N \left(\frac{\partial A^i}{\partial LS}\frac{\partial LS}{\partial p_j} + \frac{\partial A^i}{\partial p_j}\right)\right)B - \left(V - \sum_{i=1}^N A^i\right)\frac{\partial B}{\partial LS}\frac{\partial LS}{\partial p_j}}{(B(LS))^2} \\ &= \frac{-\left(\sum_{i=1}^N \left(\frac{\partial A^i}{\partial LS}\frac{\partial LS}{\partial p_j}\right)\right)B - \left(V - \sum_{i=1}^N A^i\right)\frac{\partial B}{\partial LS}\frac{\partial LS}{\partial p_j}}{(B(LS))^2} \\ &= \sum_{i=1}^N h_j^i(LS, p_1,..., p_M, v). \end{aligned}$$

Denoting consumer i's equivalent variation by ev^i, from (A30) to (A33), we can derive:

(A34) $$\begin{aligned} EV &= \sum_{i=1}^N ev^i \\ &= \sum_{i=1}^N \int_{p_j^W}^{p_j^{WO}} \left(\frac{\partial e^i}{\partial p_j}\right) dp_j \\ &= \sum_{i=1}^N \int_{p_j^W}^{p_j^{WO}} h_j^i(LS, p_1,..., p_M, v^{iw}) dp_j \\ &= \int_{p_m^W}^{p_m^{WO}} \sum_{i=1}^N h_j^i(LS, p_1,..., p_M, v^{iw}) dp_j \\ &= \int_{p_j^W}^{p_j^{WO}} H_j(LS,V^W) dp_j. \end{aligned}$$

Substituting (A33) into (A34) and rearranging yields:

(A35)

$$
\begin{aligned}
EV &= \int_{p_j^{W}}^{p_j^{WO}} H_j(LS, V^W) dp_j \\
&= \int_{p_j^{W}}^{p_j^{WO}} \frac{-\left(\sum_{i=1}^{N}\left(\frac{\partial A^i}{\partial LS}\frac{\partial LS}{\partial p_j}\right)\right)B - \left(V - \sum_{i=1}^{N} A^i\right)\frac{\partial B}{\partial LS}\frac{\partial LS}{\partial p_j}}{(B(LS,w))^2} dp_j \\
&= \int_{p_j^{W}}^{p_j^{WO}} \sum_{i=1}^{N}\left(\frac{-\left(\frac{\partial A^i}{\partial LS}\frac{\partial LS}{\partial p_j}\right)B - \left(v^i - A^i\right)\frac{\partial B}{\partial LS}\frac{\partial LS}{\partial p_j}}{(B(LS))^2}\right) dp_j \\
&= \int_{LS^{W}}^{LS^{WO}} \sum_{i=1}^{N}\left(\frac{-\left(\frac{\partial A^i}{\partial LS}\right)B - \left(v^i - A^i\right)\frac{\partial B}{\partial LS}}{(B(LS))^2}\right) dLS \\
&= \int_{LS^{W}}^{LS^{WO}} \sum_{i=1}^{N} c^i(LS, p_1, \ldots, p_M, v^{iW}) dLS \\
&= \int_{LS^{W}}^{LS^{WO}} C(LS, V^W) dLS,
\end{aligned}
$$

where:

(A36)

$$
c^i(LS, p_1, \ldots, p_M, v^{iW}) = \frac{-\left(\frac{\partial A^i}{\partial LS}\right)B - \left(v^i - A^i\right)\frac{\partial B}{\partial LS}}{(B(LS))^2},
$$

(A37)

$$
C(LS, V^W) = \frac{-\left(\sum_{i=1}^{N}\frac{\partial A^i}{\partial LS}\right)B - \left(V - \sum_{i=1}^{N} A^i\right)\frac{\partial B}{\partial LS}}{(B(LS))^2}.
$$

Appendix 3 Analysis for Multiple Groups

Taking into account the classification into multiple groups, Propositions 1 to 3 are modified as follows.

Proposition 1'

The necessary and sufficient condition for the market demand function for good j to have the form of (19) is that the indirect utility function of the representative consumer be:

(A38) $$V = A(LS_1,...,LS_G) + B(LS_1,...,LS_G)Y\ ,$$

where $A = \sum_{i=1}^{N} A^i(LS_1,...,LS_G, p_1,..., p_M)$ and:

(A39) $$LS_g \equiv -\frac{1}{\beta_g} \ln \sum_{k \in g} \exp(\alpha_k - \beta_g p_k)\ .$$

The market demand function for good j, (19), satisfies:

(A40) $$C_g(LS_1,...,LS_G,Y) = \frac{-\left(\sum_{i=1}^{N} \frac{\partial A^i}{\partial LS_g} + \frac{\partial B}{\partial LS_g} Y \right)}{B} = \frac{-\frac{\partial V}{\partial LS_g}}{B} > 0\ ,$$

(A41) $$s_{gj} = \frac{\exp(\alpha_j - \beta_g p_j)}{\sum_{k \in g} \exp(\alpha_k - \beta_g p_k)} = \frac{\partial LS_g}{\partial p_j}\ .$$

Proposition 2'

The own-price elasticity of the market demand for good j, X_j, is:

(A42) $$\frac{\partial X_j}{\partial p_j} \frac{p_j}{X_j} = \theta_{gj} - \beta_g (1 - s_{gj}) p_j\ ,$$

where $\theta_{gj} \equiv \frac{\partial C_g}{\partial p_j} \frac{p_j}{C_g}$ is the elasticity of the market demand for group g, C_g, with respect to the price of good j, p_j. The cross-price elasticity is:

(A43) $$\frac{\partial X_j}{\partial p_{j'}}\frac{p_{j'}}{X_j}=\theta_{gj'}+\beta_g s_{gj'}p_{j'}\text{, where }\theta_{gj'}\equiv\frac{\partial C_g}{\partial p_{j'}}\frac{p_{j'}}{C_g},$$

when both goods j and j' belong to the same group. When these goods belong to different groups, the corresponding cross-price elasticity is:

(A44) $$\frac{\partial X_j}{\partial p_{j'}}\frac{p_{j'}}{X_j}=\theta_{gj'}.$$

Proposition 3'

Equivalent variation can be calculated from the consumer's demand for a good, the market demand for a good, the consumer's demand for a group of goods, or the market demand for a group of goods, as follows:

(A45) $$\begin{aligned}EV&=\sum_{i=1}^{N}\int_{p_j^W}^{p_j^{WO}}h_j^i(LS_1,...,LS_G,p_1,...,p_M,v^{iW})dp_j\\&=\int_{p_j^W}^{p_j^{WO}}H_j(LS_1,...,LS_G,V^{iW})dp_j\\&=\int_{LS_g^W}^{LS_g^{WO}}\sum_{i=1}^{N}c_g^i(LS_1,...,LS_G,p_1,...,p_M,v^{iW})dLS_g\\&=\int_{LS_g^W}^{LS_g^{WO}}C_g(LS_1,...,LS_G,V^{iW})dLS_g.\end{aligned}$$

where $c_g^i(LS_1,...,LS_G,p_1,...,p_M,v^{iW})$ is consumer i 's Hicksian demand for group g , and $C_g(LS_1,...,LS_G,V^{iW})$ is the Hicksian market demand for group *g*.

The derivation of the above results is a straightforward extension of the analysis of Sections 2.1 to 2.3. A different result is that goods j and j' may belong to different groups, in which case, (A44) holds: the cross-price elasticity depends only on the elasticity of the market demand for group g with respect to the price of good j , $\theta_{gj'}$.

Appendix 4 Proof of Proposition 4

We prove the necessary condition first. Define $\widetilde{LS^i_{ML}}$ as:

(A46) $$\widetilde{LS^i_{ML}}(p_1,...,p_M,\gamma^i,y^i) \equiv -\ln\sum_{k=1}^{M}\exp(u^i_k(p_k,\gamma^i,y^i)).$$

Solving (A46) with respect to p_1 and then substituting this expression for p_1 into $A^i(p_1,...,p_M,\gamma^i)$ and $B(p_1,...,p_M)$ in (31) yields:

(A47)

$$\begin{aligned} v^i &= \int_{\gamma^i}\Big(A^i(p_1(\widetilde{LS^i_{ML}},p_2,...,p_M,y^i),p_2,...,p_M,\gamma^i) + B(p_1(\widetilde{LS^i_{ML}},p_2,...,p_M,y^i),p_2,...,p_M)y^i\Big)f(\gamma^i)d\gamma^i \\ &= \int_{\gamma^i}\Big(A^i(\widetilde{LS^i_{ML}}(p_1,...,p_M,\gamma^i,y^i),p_2,...,p_M,\gamma^i) + B(\widetilde{LS^i_{ML}}(p_1,...,p_M,\gamma^i,y^i),p_2,...,p_M)y^i\Big)f(\gamma^i)d\gamma^i. \end{aligned}$$

To satisfy the condition that consumer i 's indirect utility function have the Gorman form, $A^i(\widetilde{LS^i_{ML}}(p_1,...,p_M,\gamma^i,y^i),p_2,...,p_M,\gamma^i)$ must be independent of y^i and $B(\widetilde{LS^i_{ML}}(p_1,...,p_M,\gamma^i,y^i),p_2,...,p_M)$ must be independent of y^i and the same for all consumers. Thus, consumer i 's unconditional indirect utility function is:

(A48) $$v^i = \int_{\gamma^i}\Big(A^i(\widetilde{LS^i_{ML}}(p_1,...,p_M,\gamma^i),p_2,...,p_M,\gamma^i) + B(p_2,...,p_M,\gamma^i)y^i\Big)f(\gamma^i)d\gamma^i,$$

where:

(A49) $$\widetilde{LS^i_{ML}}(p_1,...,p_M,\gamma^i) \equiv -\ln\sum_{k=1}^{M}\exp(u^i_k(p_k,\gamma^i)).$$

Applying Roy's Identity to (A48) yields consumer i 's demand function for good 1:

(A50) $$x^i_{ML1}(p_1,\ldots,p_M)=\int_{\gamma^i}\left(\frac{\dfrac{\partial A^i}{\widetilde{\partial LS^i_{ML}}}u^{i\prime}_1(p^i_1)}{B(p_2,\ldots,p_M)}\left(\frac{\exp(u^i_1(p_1))}{\sum_{k=1}^{M}\exp(u^i_k(p_k))}\right)\right)f(\gamma^i)d\gamma^i\ .$$

Summing (A50) across consumers yields the market demand function for good 1:

(A51) $$X_{ML1}=\sum_{i=1}^{N}\int_{\gamma^i}\left(\frac{\dfrac{\partial A^i}{\widetilde{\partial LS^i_{ML}}}u^{i\prime}_1(p^i_1)}{B(p_2,\ldots,p_M)}\left(\frac{\exp(u^i_1(p_1))}{\sum_{k=1}^{M}\exp(u^i_k(p_k))}\right)\right)f(\gamma^i)d\gamma^i\ .$$

Comparing (33) and (A51) when $j=1$ reveals:

(A52) $$c^i(p_1,\ldots,p_M,\gamma^i)=\frac{\dfrac{\partial A^i}{\widetilde{\partial LS^i_{ML}}}u^{i\prime}_1(p^i_1)}{B(p_2,\ldots,p_M)}\ .$$

Analogously, the market demand function for good m ($m=2,\ldots,M$) is:

(A53) $$X_{MLm}=\sum_{i=1}^{N}\int_{\gamma^i}\left(\frac{\dfrac{\partial A^i}{\widetilde{\partial LS^i_{ML}}}u^{i\prime}_m(p^i_m)\left(\dfrac{\exp(u^i_m(p_m))}{\sum_{k=1}^{M}\exp(u^i_k(p_k))}\right)-\left(\dfrac{\partial A^i}{\partial p_m}+\dfrac{\partial B}{\partial p_m}y^i\right)}{B(p_2,\ldots,p_M)}\right)f(\gamma^i)d\gamma^i\ .$$

Comparing (35) and (A53) when $j=m$ reveals:

(A54) $$c^i(p_1,\ldots,p_M,\gamma^i)=\frac{\dfrac{\partial A^i}{\widetilde{\partial LS^i_{ML}}}u^{i\prime}_m(p^i_m)}{B(p_2,\ldots,p_M)}\ ,$$

(A55) $$\frac{\partial A^i}{\partial p_m} + \frac{\partial B}{\partial p_m} y^i = 0 .$$

Because (A55) holds for any y^i, we obtain:

(A56) $$\frac{\partial A^i}{\partial p_m} = \frac{\partial B}{\partial p_m} = 0 .$$

The above analysis applies when solving (A46) with respect to any p_j and then substituting the resulting expression into (33). Thus, from (A56), we know that consumer i's conditional indirect utility function is independent of p_j, as follows:

(A57) $$v^i = \int_{\gamma^i} \left(A^i(\widetilde{LS^i_{ML}}(p_1,..., p_M, \gamma^i), \gamma^i) + \overline{B} y^i \right) f(\gamma^i) d\gamma^i ,$$

where $\overline{B}$ is constant. The indirect utility function of the representative consumer is:

(A58) $$V = \sum_{i=1}^{N} \int_{\gamma^i} \left(A^i(\widetilde{LS^i_{ML}}(p_1,..., p_M, \gamma^i), \gamma^i) \right) f(\gamma^i) d\gamma^i + \overline{B} Y .$$

Because $c^i(p_1,..., p_M, \gamma^i)$ is the same for all j, from (A52) and (A54) we obtain:

(A59) $$u_1^{i\prime}(p_1) = ... = u_M^{i\prime}(p_M) .$$

This expression indicates that $u_j^{i\prime}(p_j)$ is linear in p_j. Thus, we can express $u_j^i(p_j)$ as:

(A60) $$u_j^i(p_j) = \alpha_j^i(\gamma^i) - \beta^i(\gamma^i) p_j .$$

The choice probability, (37), follows from (29) and (A60). Given (A60), $\widetilde{LS^i_{ML}}(p_1,..., p_M, \gamma^i)$ can be written as:

(A61) $$\widetilde{LS^i_{ML}}(p_1,...,p_M,\gamma^i) = -\ln\sum_{k=1}^{M}\exp(\alpha^i_k(\gamma^i)-\beta^i(\gamma^i)p_k)\,.$$

Eq. (35) is derived by defining:

(A62) $$LS^i_{ML}(\gamma^i) \equiv \frac{1}{\beta^i(\gamma^i)}\widetilde{LS^i_{ML}}(p_1,...,p_M,\gamma^i)\,.$$

Substituting (A62) into (A58) yields (34).

Because consumer i 's conditional indirect utility function is decreasing in p_j, we obtain:

(A63) $$\frac{\partial v^{ci}}{\partial p_j} = \frac{\partial v^{ci}}{\partial LS^i_{ML}(\gamma^i)}\frac{\partial LS^i_{ML}(\gamma^i)}{\partial p_j} = \frac{\partial v^{ci}}{\partial LS^i_{ML}(\gamma^i)}\frac{\exp(\alpha^i_j(\gamma^i)-\beta^i(\gamma^i)p_j)}{\sum_{k=1}^{M}\exp(\alpha^i_k(\gamma^i)-\beta^i(\gamma^i)p_k)} < 0\,.$$

This implies:

(A64) $$\frac{\partial v^{ci}}{\partial LS^i_{ML}(\gamma^i)} = \frac{\partial A^i}{\partial LS^i_{ML}(\gamma^i)} < 0\,.$$

Given (A54), (A62), (A64), and $u^{i\prime}_1(p_1) = ... = u^{i\,\prime}_M(p_M) = -\beta^i(\gamma^i)$, consumer i 's demand for a group of goods, (36), is:

(A65) $$c^i(LS^i_{ML}(\gamma^i),\gamma^i) = \frac{\dfrac{\partial A^i}{\partial \widetilde{LS^i_{ML}}}u^{i\,\prime}_m(p^i_m)}{\overline{B}} = \frac{-\dfrac{\partial A^i}{\partial LS^i_{ML}(\gamma^i)}}{\overline{B}} > 0\,.$$

Sufficiency is straightforward to prove by using Roy's Identity. Applying Roy's Identity to (34) and rearranging yields:

(A66) $$X_{MLj} = \sum_{i=1}^{N} x_{MLj}^{i} = \sum_{i=1}^{N} \frac{-\dfrac{\partial v^i}{\partial p_j}}{\dfrac{\partial v^i}{\partial y^i}} = \sum_{i=1}^{N} \int_{\gamma^i} \left(\frac{-\dfrac{\partial A^i}{\partial LS_{ML}^{i}(\gamma^i)}}{\overline{B}} \frac{\exp(\alpha_j^i(\gamma^i) - \beta^i(\gamma^i) p_j)}{\sum_{k=1}^{M} \exp(\alpha_k^i(\gamma^i) - \beta^i(\gamma^i) p_k)} \right) f(\gamma^i) d\gamma^i ,$$

where:

(A67) $$c^i(LS_{ML}^i(\gamma^i), \gamma^i) = \frac{-\dfrac{\partial A^i}{\partial LS_{ML}^{i}(\gamma^i)}}{\overline{B}} ,$$

(A68) $$s_{MLj}^i(\gamma^i) = \frac{\exp(\alpha_j^i(\gamma^i) - \beta^i(\gamma^i) p_j)}{\sum_{k=1}^{M} \exp(\alpha_k^i(\gamma^i) - \beta^i(\gamma^i) p_k)} = \frac{\partial LS_{ML}^i(\gamma^i)}{\partial p_j} .$$

Eq. (67) is positive from (A64). From (A66) to (A68), we derive (33).

Appendix 5 Proof of Proposition 6

From (30), consumer i 's conditional expenditure function is:

(A69) $$e^i(LS_{ML}^i(\gamma^i), \gamma^i, v^i) = \frac{v^{ci} - A^i(LS_{ML}^i(\gamma^i), \gamma^i)}{\overline{B}} .$$

From (A69), we obtain consumer i 's conditional Hicksian demand function for good j as follows:

(A70) $$h_j^i(LS_{ML}^i(\gamma^i), \gamma^i) = \frac{\partial e^i}{\partial p_j} = \frac{-\dfrac{\partial A^i}{\partial LS_{ML}^i(\gamma^i)} \dfrac{\partial LS_{ML}^i(\gamma^i)}{\partial p_j}}{\overline{B}} .$$

Because the Hicksian and Marshallian demand functions are the same when the indirect utility function is quasi-linear, from (36), (A69), and (A70), we obtain:

$$\begin{aligned} EV &= \sum_{i=1}^{N} \int_{\gamma^i} ev^i f(\gamma^i) d\gamma^i \\ &= \sum_{i=1}^{N} \int_{\gamma^i} \left(\int_{p_j^{iW}}^{p_j^{iWO}} h_j^i (LS_{ML}^i(\gamma^i), \gamma^i) dp_j \right) f(\gamma^i) d\gamma^i \\ &= \sum_{i=1}^{N} \int_{\gamma^i} \left(\int_{p_j^{iW}}^{p_j^{iWO}} x_j^i (LS_{ML}^i(\gamma^i), \gamma^i) dp_j \right) f(\gamma^i) d\gamma^i \\ &= \sum_{i=1}^{N} \int_{\gamma^i} \left(\int_{p_j^{iW}}^{p_j^{iWO}} \left(\frac{-\dfrac{\partial A^i}{\partial LS_{ML}^i(\gamma^i)}}{\overline{B}} \frac{\partial LS_{ML}^i(\gamma^i)}{\partial p_j} \right) dp_j \right) f(\gamma^i) d\gamma^i \\ &= \sum_{i=1}^{N} \int_{\gamma^i} \left(\int_{LS_{ML}^{iW}}^{LS_{ML}^{iWO}} c^i (LS_{ML}^i(\gamma^i), \gamma^i) dLS_{ML}^i(\gamma) \right) f(\gamma^i) d\gamma^i. \end{aligned} \tag{A71}$$

Appendix 6 Multiple Groups in the Case of the Mixed Logit Model

Because the analysis of multiple groups in the mixed logit model is similar to that in the logit model described in Appendix 3, in this appendix, we simply state results.

When there are multiple groups, the market demand function that is consistent with the mixed logit model is:

$$X_{MLj} = \sum_{i=1}^{N} \int_{\gamma^i} c_{MLg}^i(\gamma^i) s_{MLgj}^i(\gamma^i) f(\gamma^i) d\gamma^i \ , \tag{A72}$$

where $c_{MLg}^i(\gamma^i)$ is consumer i's demand for group g and $s_{MLgj}^i(\gamma^i)$ is consumer i's logit-type choice probability for group g, which is:

$$s_{MLgj}^i(\gamma^i) = \frac{\exp(u_j^i(p_j, y^i, \gamma^i))}{\sum_{k \in g} \exp(u_k^i(p_k, y^i, \gamma^i))}. \tag{A73}$$

The necessary and sufficient condition for the market demand function for good j to have the form of (A72) is that the indirect utility function of the representative consumer be given by:

(A74) $$V = \sum_{i=1}^{N} \int_{\gamma^i} \left(A^i (LS^i_{ML1}(\gamma^i),..., LS^i_{MLG}(\gamma^i), \gamma^i) \right) f(\gamma^i) d\gamma^i + \overline{B}Y \ ,$$

which satisfies:

(A75) $$LS^i_{MLg}(\gamma^i) \equiv -\frac{1}{\beta^i_g(\gamma^i)} \ln \sum_{k \in g} \exp(\alpha^i_k(\gamma^i) - \beta^i_g(\gamma^i) p_k) \ .$$

The market demand function for good j , (A72), satisfies:

(A76) $$c^i_{MLg}(LS^i_{ML1}(\gamma^i),..., LS^i_{MLM}(\gamma^i), \gamma^i) = \frac{-\dfrac{\partial A^i}{LS^i_{MLg}(\gamma^i)}}{\overline{B}} > 0 \ ,$$

(A77) $$s^i_{MLgj}(\gamma^i) = \frac{\exp(\alpha^i_j(\gamma^i) - \beta^i_g(\gamma^i) p_j)}{\sum_{k \in g} \exp(\alpha^i_k(\gamma^i) - \beta^i_g(\gamma^i) p_k)} \ .$$

The own-price elasticity of the market demand for good j is:

(A78) $$\frac{\partial X_{MLj}}{\partial p_j} \frac{p_j}{X_{MLj}} = \sum_{i=1}^{N} \int_{w^i} \left\{ \frac{c^i_{MLg}(LS^i_{ML1}(\gamma^i),..., LS^i_{MLG}(\gamma^i), \gamma^i) s^i_{MLgj}(\gamma^i)}{X_{MLj}} \left(\varphi^i_{gj}(\gamma^i) - \beta_g (1 - s^i_{MLgj}(\gamma^i)) p_j \right) \right\} f(\gamma^i) d\gamma^i \ ,$$

where $\varphi^i_{gj}(\gamma^i) = \dfrac{\partial c^i_{MLg}(LS^i_{ML1}(\gamma^i),..., LS^i_{MLG}(\gamma^i), \gamma^i)}{\partial p_j} \dfrac{p_j}{c^i_{MLg}(LS^i_{ML1}(\gamma^i),..., LS^i_{MLG}(\gamma^i), \gamma^i)}$ is the elasticity of consumer i 's demand for group g with respect to the price of good j , p_j . When goods j and j' belong to the same group, the cross-price elasticity is:

(A79)
$$\begin{aligned}&\frac{\partial X_{MLj}}{\partial p_{j'}}\frac{p_{j'}}{X_{MLj}}\\&=\sum_{i=1}^{N}\int_{w^i}\left\{\frac{c^i_{MLg}(LS^i_{ML1}(\gamma^i),...,LS^i_{MLG}(\gamma^i),\gamma^i)s^i_{MLgj}(\gamma^i)}{X_{MLj}}\left(\varphi^i_{gj'}(\gamma^i)+\beta_g s^i_{MLgj'}(\gamma^i)p_{j'}\right)\right\}f(\gamma^i)d\gamma^i\ .\end{aligned}$$

Otherwise, the cross-price elasticity is:

(A80)
$$\frac{\partial X_{MLj}}{\partial p_{j'}}\frac{p_{j'}}{X_{MLj}}=\sum_{i=1}^{N}\int_{w^i}\left\{\frac{c^i_{MLg}(LS^i_{ML1}(\gamma^i),...,LS^i_{MLG}(\gamma^i),\gamma^i)s^i_{MLgj}(\gamma^i)}{X_{MLj}}\varphi^i_{gj'}(\gamma^i)\right\}f(\gamma^i)d\gamma^i\ .$$

Equivalent variation can be calculated from consumer i 's demand for good j or from consumer i 's demand for group g ; that is:

(A81)
$$\begin{aligned}EV&=\sum_{i=1}^{N}\int_{\gamma^i}\left(\int_{p^{iW}}^{p^{iWO}}x^i_j(LS^i_{ML1}(\gamma^i),...,LS^i_{MLG}(\gamma^i),\gamma^i)dp_j\right)f(\gamma^i)d\gamma^i\\&=\sum_{i=1}^{N}\int_{\gamma^i}\left(\int_{LS^{iW}_{MLg}(\gamma^i)}^{LS^{iWO}_{MLg}(\gamma^i)}c^i_{MLg}(LS^i_{ML1}(\gamma^i),...,LS^i_{MLG}(\gamma^i),\gamma^i)dLS^i_{MLg}(\gamma^i)\right)f(\gamma^i)d\gamma^i.\end{aligned}$$

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