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Abstract

This paper is an analysis of a two-sided search model in which agents are vertically heterogeneous and some agents do not know their own types. Agents who do not know their own types update their beliefs about their own types through the offers or rejections that they receive from others. In the belief-updating process, an agent who is unsure of her own type frequently rejects (accepts) a man whom she accepts (rejects) when she knows her own type. In this paper, we show that this *optimistic* (*pessimistic*) behavior influences both the agent's and other agents' matching behaviors. We show, specifically, that the optimism of some agents prevents the lowest-type agents from matching. However, the pessimism of some agents does not affect the matching of the lowest-type agents.

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Key Words: Imperfect self-knowledge; learning; looking-glass self; two-sided search

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1 Introduction

The "looking-glass self" has been the dominant concept in sociology and social psychology for the development of the self. The idea, attributed to Cooley (1902), is that people form their self-views by observing how others treat them. That is, others are significant as the "mirrors" that reflect images of the self. Although there is much literature on the "looking-glass self" in the field of sociology and social psychology, the topic has received little attention in economics.[1]

In this paper, we introduce the "looking-glass self" to a two-sided search model and study the implications of the "looking-glass self" in the search behavior. We construct a model in which searchers do not know their own types, although they know the types of others. They then update their beliefs about their own types when they receive offers or rejections from others. For example, workers in search of an employer are evaluated by employers on their abilities when they meet. When a worker is young in terms of experience, his or her self-assessment is based on limited experience. On the other hand, employers may have considerable experience in evaluating workers. At this time, when a young worker observes an offer or a rejection from an employer, he or she learns something about his or her own type. Of course, when an experienced worker searches for a new job that is very similar to his or her previous job, he or she may have a more accurate self-view of his or her ability than employers. However, such situations are not considered in this paper. The key feature of this study is that others have better information about agents' types than the agents themselves. Similarly, in the search for a marriage partner, a single agent is evaluated with regard to his or her marital charms by a member of the opposite sex when they meet. When an agent is young, his or her self-assessment is based on limited experience, such as academic achievement and family background. However, because marital charm is composed of various elements, an individual of the opposite sex may have better assessments of the agents' charm than the agents themselves.[2] Hence, when an agent observes an offer or a rejection from a member of the opposite sex, he or she infers something about his or her own type. In this paper, we show that this looking-glass self influences both their own and other agents' search behaviors.

We consider the basic framework of Burdett and Coles (1997), which is a two-sided search model with complete information. Although our model focuses on marriage, one could apply the ideas and techniques of the present paper to other two-sided search frameworks, such as the labor market, the housing market, and other markets in which heterogeneous buyers and sellers search for the right trading partner.[3] Using the marriage market interpretation, the

[1]As we discuss below, in economics, Bénabou and Tirole (2003), Ishida (2006), and Swank and Visser (2006) consider "the looking-glass self" in principal-agent models.

[2]Marital charm is defined by various elements, including quality, attraction, intelligence, height, age, education, income, position at work, social status, and family background, in much of the literature regarding marriage.

[3]In this paper, we assume the *non-transferable utility:* there is no bargaining for the division of the total utility. In the labor market, utility is generally transferable. However, for example, when the worker is enthusiastic about a job because of its location, or the employer is attracted by the worker because of his or her personality, their utilities can be considered to be not-transferable. Furthermore if the worker offers to

model is described as follows. Single agents are vertically heterogeneous, i.e., there exists a ranking of marital charm (types). Single men or women enter the market in order to look for a marital partner. When a man and a woman meet, an opponent's type can be recognized. The agent's optimal search strategy has the reservation-level property, i.e., he or she continues searching until he or she meets a member of the opposite sex who is at least as good as the predetermined threshold, called the "reservation level," which depends on the agent's search cost and the type distribution of agents. If a man and a woman meet and both agents propose, they marry and leave the market. If at least one of the two decides not to propose, they separate and continue to search for another partner. Given these settings, the marriage pattern (i.e., who marries whom) in the market is determined. This marriage pattern becomes a kind of positive assortative matching.[4]

Our results are as follows. When an agent (she) is unsure of her own type, she often rejects (accepts) a man whom she accepts (rejects) when she knows her own type. We call her behavior "*optimism* (*pessimism*)" in this study.[5] The optimism in an agent's learning processes generates two (pecuniary) externalities. The first is *direct externality*: the rejection from an optimistic woman delays the timing of marriage of the man who is directly rejected by her when they meet. If there are many optimistic women in the market, the second externality is generated: the men who are now rejected by the optimistic women accept another lower type of women who are rejected by these men when all agents know their own types. We call this change in an agent's behavior due to the optimism of other agents *indirect externality.* Moreover, in a two-sided search framework, the women who are now accepted by these men also reject the men whom they accept when all agents know their own types. Then, the indirect externality spreads across the market. As a result, the lowest-type agents cannot marry.

On the other hand, the pessimism does not generate the indirect externality. This is because if the *indirect externality* of pessimism occurs (in other words, if the men who are now accepted by pessimistic women reject the women whom these men accept when all agents know their own types), the offers from these men to the pessimistic women inform these pessimistic women that they are higher-type women than they think. As a result, the pessimistic women have the incentives to reject these men. Hence, even if there are many pessimistic women, the men who are now accepted by these women always accept the women whom they accept when all agents know their own types. Therefore, the pessimism has only the *direct externality*: the acceptance by a pessimistic woman makes the future partner better off because she increases the value of the match to the partner.

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work for a reduced wage, this wage might be restricted to be above some lower bound determined outside of the match, like a legislated minimum wage or an industry-wide union relationship (see Burdett and Wright (1998)). In this way, when wages and all other terms of the relationship are fixed in advance and there is nothing for the pair to negotiate after they meet, their utilities can be viewed as not-transferable utility.

[4]Positive assortative matching is said to hold if the characteristics (types and marital charm) of those who match are positively correlated. Becker (1973) found strong empirical evidence of a positive correlation between the characteristics of partners.

[5]The optimistic (pessimistic) behavior in this paper is generated due to the correct belief-updating process in the sense that there is no over-optimism, because an agent who is unsure of her own type knows the ditributions of types in the market and updates her belief by Baye's rule.

The result in which the lowest-type agents cannot marry in an optimism case is consistent with the recent data of educational assortative marriage patterns in the United States and Japan. In the United States and Japan, the percentage of never-married men or women is increasing and, in particular, the percentage of never-married men or women with a low level of education is notably high. According to the US Census Bureau data, in 2006, the proportion of never-married individuals aged 35–44 was 24% for men with a high school education or less and 14% for women with a high school education or less. On the other hand, the proportion of never married individuals aged 35–44 was 14% for men with some college education or more and 12% for women with some college education or more. In Japan, the decline in marriage has been most pronounced among less educated men aged 35–39 (Raymo and Iwasawa 2005). Our results suggest that the optimism accelerates the increase in the proportion of never-married men or women with a low level of education, as education is one of the elements of charm.

Our model can also explain the fact that the reservation level declines with the duration of search. The potential sources of declining reservation wages have received much attention in the labor market (see, Burdett and Vishwanath (1988)). Burdett and Vishwanath show that when workers learn the unknown wage distribution, the reservation wage of an unemployed worker declines with his or her unemployment spell. A high offer results in the worker getting employed. On the other hand, an offer much lower than expected leads the worker to perceive the jobs available to him or her as jobs offering low wages and then the worker revises his or her reservation wage downward. Unlike their model, ours is a two-sided search and single agents know the type distribution but do not know their own types. In the two-sided search with imperfect self-knowledge, receiving an offer lower than the reservation level is likely to lead to an increase in the reservation level. The results in this paper show that if a woman with imperfect self-knowledge starts to search with the strategy of accepting only the highest-type man, she revises her reservation level downward whenever the woman receives a rejection that has some information about her own type. On the other hand, a high offer results in the agent getting married similarly to the finding by Burdett and Vishwanath. However, an offer lower than the reservation level of a woman with imperfect self-knowledge does not affect her decision. An offer much lower than the reservation level is proposed by a man whom the woman with imperfect self-knowledge rejects. Thus, her decision regarding whether to accept or not another type of man is not affected by that offer. An offer from a man whom she decides whether to accept or not also does not affect her decision in equilibrium because the man chooses his strategy so as not to raise her reservation level.

The results in this study also show that, when there are agents with an imperfect self-knowledge under the cloning assumption and the assumption of non-transferable utility, multiple equilibria can arise in some parameter ranges. In contrast, when all agents know their own types under the cloning assumption and the assumption of non-transferable utility, a unique equilibrium always arises (see Burdett and Coles (1997)). In our multiple equilibrium case, marriage patterns are determined by all agents' expectations about the behaviors of agents with imperfect self-knowledge.

1.1 Related literature

Early psychologists and sociologists thought that the self was built on reflected assessments—people form their self-views by observing how others treat them. James (1890), who set the stage for the idea of "looking-glass self," argued that the self was a product and reflection of social life. Cooley (1902) introduced the idea of the "looking-glass self." He expanded the idea that the self develops by referencing other people in the social environment. Cooley maintained that the person observes how others view himself or herself and then incorporates those views into the self-view. Mead (1934) further developed the idea of Cooley (1902).[6] Following this long tradition, most researchers in psychology and sociology accepted that others are significant as the "mirrors" that construct and modify the self-view (see Tice and Wallace (2003)).

In economics, recent work has introduced the idea of "looking-glass self" (for example, Bénabou and Tirole (2003), Ishida (2006), and Swank and Visser (2006)). Bénabou and Tirole (2003), who present the principal-agent model, assume that, whereas the principal knows the agent's type, an agent has imperfect knowledge about his or her own type. Because the principal prefers to offer the challenging task when facing an agent with high ability, this offer becomes the signal that the principal trusts the agent. In contrast with these studies, we apply the idea of the "looking-glass self" to the two-sided search model and not to the principal-agent model.

In search literature, there are few studies that have given attention to the imperfect self-knowledge. In Gonzalez and Shi (2010), agents learn their own job-finding abilities by observing offers or rejections from firms. In the directed search model with two types of agents, they show that learning from search can induce the desired wages (the wage in the chosen submarket) and reservation wages to decline with unemployment duration. In particular, the value function of an unemployed worker strictly increases in the worker's belief in their model because a worker's (or a firm's) search decision is to choose the submarket to search. Hence, the reservation wage strictly decreases over search spell as the worker's beliefs about his or her own ability become progressively worse. In contrast, our model is the random two-side search model with three types of agents. An agent with imperfect self-knowledge decides the reservation utility by considering the composition of each (belief) type in the market and her future learning process fully. As a result, the value function is not monotonic with respect to agent's belief: there is a case in which even if an agent with imperfect self-knowledge receives an offer that has some information about her own type, her belief is updated but her reservation level does not increase.[7]

This paper is organized as follows. Section 2 is a description of the basic framework for

[6] In his view, people are affected not only by how they think significant others respond to them but also by how they think their entire social group does.

[7] In other search literature with Bayesian leaning, agents do not know the types of opponents and then they learn them (for example, Jovanovic (1979), MacDonald (1982), Chade (2006), and Anderson and Smith (2010)). There is also the search literature with Bayesian learning where agents are assumed to learn the unknown offer distribution (for example, Rothchild (1974), McLennan (1984), Burdett and Vishwanath (1988), and Adam (2001)).

our analysis. In Section 3, first, we derive a *perfect sorting equilibrium* (PSE) in which only persons of the same type marry if all agents know their own types as a benchmark case. In Section 4, we examine the case of a PSE with imperfect self-knowledge. Next, we investigate the influence of optimistic behavior on a marriage market. Finally, we examine the influence of pessimistic behavior. In Section 5, we discuss the extensions of the model. Section 6 is the conclusion.

2 Basic framework

In this section, we present a basic framework for our analysis in this study. Throughout this section, we restrict our attention to the steady state.

Let us assume that there are a large and equal number of men and women in a marriage market. Let N denote the participating men or women in this market. An agent in the market wishes to marry a member of the opposite sex.

Finding a marriage partner always involves a time cost. It is difficult for agents to meet someone of the opposite sex in the market. Let α denote the rate at which a single individual contacts a member of the opposite sex, where α is the parameter of the Poisson process.

It is assumed that agents are ex-ante heterogeneous and all agents have the same ranking about a potential partner in the marriage market. Let x_k denote the type (charm) of a k-type single man or woman in the market; it is assumed to be a real number.

When both sexes meet, each agent can instantly recognize the opponent's type and then decide whether or not to propose. We assume that both agents submit their offers or rejections simultaneously in order to simplify our analysis. If at least one of the two decides not to propose, they return to the marriage market and search for another partner. If both agents propose, they marry and leave the marriage market permanently. We assume that, if a couple marries, he or she obtains a utility flow equal to the spouse's type per unit of time and vice versa. That is, utilities are *non-transferable*: there is no bargaining for the division of the total marital utility. Let us assume that people live forever and there is no divorce.

Let $F_i(x), i = m, w$ denote the stationary distribution of actual types among men (m) or women (w) in the market. That is, $F_i(x, t) = F_i(x)$ for all x and all t, where t is period. Let us assume that x_k is drawn from $F_i(x)$. Let $\underline{x}_i$ and $\bar{x}_i$ indicate the infimum and supremum of F_i, respectively, where $\underline{x}_i > 0$. Here, $F_m(x)$ and $F_w(x)$ need not be symmetric among men and women. All agents know $F_m(x)$ and $F_w(x)$.

3 Perfect self-knowledge—Benchmark result

In this section, first, we derive the conditions under which the economy is at a PSE, in which only persons of the same type marry, if all agents know their own types (*perfect self-knowledge*) as a benchmark. In the later section, we study three cases with *imperfect self-knowledge* (i.e., agents do not know their own types perfectly) and compare these three cases with the benchmark case to show the influence of learning on a market.

Given (F_m, F_w) and the strategies of single agents, the set of single agents of the opposite sex who will agree to marry a k-type agent is well defined. Let $H_m(.|x_k)$ denote the distribution of type among men who will agree to marry a k-type woman. Further, the arrival rate of such proposals faced by a k-type woman, $\alpha_w(x_k)$, is also well defined.

Let $V_w(x_k)$ denote a k-type woman's expected discounted lifetime utility when single. Standard dynamic programming arguments imply

$$V_w(x_k) = \frac{1}{1+rdt}\left[\alpha_w(x_k)\,dtE\left[\max\left\{\frac{\tilde{x}_k}{r}, V_w(x_k)\right\}|x_k\right] + (1-\alpha_w(x_k)\,dt)\,V_w(x_k)\right]$$

where, given that an offer is made, $\tilde{x}_k$ has distribution $H_m(\tilde{x}_k|x_k)$. Manipulating and then letting $dt \to 0$ yields

$$rV_w(x_k) = \alpha_w(x_k)\left[E\left[\max\left\{\frac{\tilde{x}_k}{r}, V_w(x_k)\right\}|x_k\right] - V_w(x_k)\right]. \tag{1}$$

The strategy that maximizes a single agent's expected discounted lifetime utility takes the form of a reservation match strategy—a k-type woman will marry a man on contact if and only if his type is at least as great as $R_w(x_k) \equiv rV_w(x_k)$.

As the situation is the same for men, the expected discounted lifetime utility of a single k-type man, $V_m(x_k)$, satisfies

$$rV_m(x_k) = \alpha_m(x_k)\left[E\left[\max\left\{\frac{\tilde{x}_k}{r}, V_m(x_k)\right\}|x_k\right] - V_m(x_k)\right]. \tag{2}$$

where $\tilde{x}_k$ has distribution $H_w(\tilde{x}_k|x_k)$. From this equation, we can obtain the reservation match strategy of a k-type man $R_m(x_k) \equiv rV_m(x_k)$.

The equilibrium concept for this model with perfect self-knowledge is as follows.

Definition 1 *When all agents know their own types, an equilibrium is a* Market Equilibrium with perfect self-knowledge (ME)*:*

(ME-i) all agents maximize their expected discounted utilities given that they have correct expectations about the strategies of all others in the market;

(ME-ii) the inflow of each type and the outflow of each type are balanced.

Condition (ME-ii) requires finding a steady state number and distribution of types in the market so that the corresponding equilibrium strategies defined in condition (ME-i) generate an exit flow for each type equal to the inflow of that type.

To simplify the analysis, we assume that, if a pair marries and leaves the market, two identical agents enter the market at once (see, for example, MacNamara and Collins (1990), Morgan (1994), Burdett and Coles (2001), Bloch and Ryder (2000), Cornelius (2003), and Chade (2006)).[8]

[8] Burdett and Coles (1999) give four typical assumptions of "inflow" in search literature. However, our attention is focused not on the change of the type distribution, which is derived under an assumption of inflow, but on the change of matching strategies of others when there are agents with imperfect self-knowledge. In order to illustrate the main findings of this paper, one does not need other assumptions of inflow, such as ex-

In this paper, let us assume that there are three types of men or women according to charm: high (H), middle (M), and low (L).[9] A participant in a marriage market belongs to one of these types. Let x_k/r denote the (discounted) utility of marrying a k-type agent ($k = H, M, L$), where $r > 0$ is the discount rate. We assume that $x_H > x_M > x_L > 0$. That is, in any equilibrium, all agents would like to marry an H-type agent. Both sexes are assumed to obtain zero utility flow while they are single.

Let λ_k^i $(i = m, w,\ k = H, M, L)$ denote the share of k-type men (m) or women (w) in the marriage market, where $\lambda_H^i + \lambda_M^i + \lambda_L^i = 1$. Here, λ_k^i of each sex $(i = m, w)$ need not be symmetric among men and women.

We restrict our attention to the next equilibrium in this paper in order to show the influences of the learning process on a marriage market.

Definition 2 *In the* perfect sorting equilibrium (PSE), *H-type agents marry within their group, as do M-type agents and L-type agents.*

In the PSE, men and women of the same type marry. Therefore, we can consider that H-type agents who marry within their group form the first cluster of marriages, M-type agents who marry within their group form the second cluster of marriages, and L-type agents who marry within their group form the third cluster of marriages in this equilibrium. We now define the following situation as a benchmark case: if all agents know their own types, the ME is the PSE. Proposition 1 shows the sufficient conditions for the PSE when all agents know their own types.

Proposition 1 *(PSE) Let us assume that all agents recognize their own types. The economy is at the PSE if*

$$x_M < R_i^*(x_H) \equiv \frac{\alpha\lambda_H^j x_H}{\alpha\lambda_H^j + r}, \quad i = m, w,\ j = w, m \tag{3}$$

and

$$x_L < R_i^*(x_M) \equiv \frac{\alpha\lambda_M^j x_M}{\alpha\lambda_M^j + r}, \quad i = m, w,\ j = w, m. \tag{4}$$

Proof. See Appendix A.2. ■

Proposition 1 means that, with constant α, if the share of H-type agents of the opposite sex j is large enough or if the difference between x_H and x_M is large enough ($\alpha\lambda_H^j > \frac{rx_M}{(x_H - x_M)}$), an H-type agent turns down an M- and an L-type opposite sex agent in the market. Conversely, if there are sufficiently few H-type opposite sex agents or if $(x_H - x_M)$ is small enough ($\alpha\lambda_H^j \leq \frac{rx_M}{(x_H - x_M)}$), an H-type agent accepts an M-type opposite sex agent j. A similar discussion can be held for parameter λ_M^j by inequality (4). If λ_H^j and λ_M^j are small enough

ogenous inflow, which complicates matters without adding intuition. Thus, we apply the "cloning assumption" to our model for technical simplicity.

[9] As we will discuss in detail in Appendix B, we assume not two but three types of agents in order to show the indirect effect (indirect externality) of the learning process: even if the agents with perfect self-knowledge do not directly meet agents with imperfect self-knowledge, these agents with perfect self-knowledge may change their marriage behavior due to the existence of agents with imperfect self-knowledge in the market.

$(j = w, m)$ to satisfy $R_i^*(x_H) \leq x_L$ and $R_i^*(x_M) \leq x_L$, all agents i $(= m, w)$ obtain the same expected discounted lifetime utility: $V_i(x_L) = V_i(x_M) = V_i(x_H) < \frac{x_L}{r}$.[10]

If $r = 0$, then $x_M < R_i^*(x_H)$ and $x_L < R_i^*(x_M)$ $(i = m, w)$ hold. Therefore, the equilibrium is the PSE when $r = 0$.

In the next section, we introduce the imperfect knowledge about agents' own types into the benchmark case. To investigate the influence of the learning on a market, in the following sections, we consider the case in which the conditions in Proposition 1 are satisfied: $x_M < R_i^*(x_H)$ and $x_L < R_i^*(x_M)$ $(i = m, w)$ hold.[11]

4 Imperfect self-knowledge

In this section, we introduce the imperfect self-knowledge into the benchmark case. Let us assume that all agents understand the (actual) type distributions $F_m(x)$ and $F_w(x)$ and that all men know their own types. However, no women initially know their own types when they have just entered the marriage market.[12] A woman with imperfect self-knowledge may learn something about her actual type by observing the offers or rejections by men. Thus, a woman's belief about her own type depends on the men whom she met in the past. As a result, there are different kinds of women with different beliefs even if they belong to the same actual type.

For the explanation, suppose that a k-type woman is at the start of period $t = \{0, 1, ...\}$ of single. Let Ξ denote a set of all (actual) types of women and $b_k^0 \in \Delta(\Xi)$ denote a prior belief for a k-type woman, where $\Delta(\Xi)$ is the set of probability distributions over Ξ. Let $b_k^t\left(b_k^0, h_k^t\right)$ denote a k-type woman's belief at the start of period t about her own type after history h_k^t given the prior belief b_k^0, where $b_k^0\left(b_k^0, h_k^0\right) = b_k^0$. In this paper, we assume that b_k^0 is the distribution of new single women over the levels of charm.[13] Thus, $b_0 \equiv b_k^0$. Moreover, $h_k^t = \left(\left(\tilde{x}_k^0, a_m^0(x_k)\right), ..., \left(\tilde{x}_k^{t-1}, a_m^{t-1}(x_k)\right)\right)$ is the k-type woman's history up to, but not including, period t and $a_m^t(x_k) \in A = \{a, a^-\}$ is the action of a man observed by the k-type woman as the result of a search outcome in period t, where a indicates that the man proposed the k-type woman and a^- indicates that he rejected her. If a k-type woman observes $\left(\tilde{x}_k^t, a\right)$ (or $\left(\tilde{x}_k^t, a^-\right)$), she knows that a k-type man accepted (rejected) her. In this paper, we use

[10] When $F_m(x)$ and $F_w(x)$ are symmetric, four possible steady-state equilibrium outcomes can be considered when all agents have perfect self-knowledge: Equilibrium (i) agents of the same type marry (PSE); Equilibrium (ii): agents of the H-type and the M-type form the first cluster of marriages, and agents of the L-type form the second cluster of marriages; Equilibrium (iii) agents of the H-type form the first cluster of marriages, and agents of the middle- and L-type form the second cluster of marriages; and Equilibrium (iv) all agents marry the first person of the opposite sex they meet. From the proof of Proposition 1, Equilibrium (ii) occurs when $R_i^*(x_H) \leq x_M$, $R_i^*(x_M) > x_L$. Equilibrium (iii) occurs when $R_i^*(x_H) > x_M$, $R_i^*(x_M) \leq x_L$.Equilibrium (iv) occurs when $R_i^*(x_H) \leq x_M$, $R_i^*(x_M) \leq x_L$.

[11] For other parameter ranges, it is difficult to show the indirect effect (indirect externality) of the learning process. We will discuss this in detail in Appendix B.

[12] This one-sided imperfect knowledge assumption can make the influence of imperfect self-knowledge clearer than the two-sided imperfect knowledge assumption. We discuss this in detail in Section 5.

[13] Gonzalez and Shi (2009) assume that the initial prior expectation of ability for a new worker is calculated from the distribution of new workers over the levels of ability. In our model, the initial prior expectation of charm for a new single woman also depends on the distribution of new single women over the levels of charm. This distribution consists of the shares of women with belief b_k^0 in the market.

the term "action" to distinguish it from the reservation "strategy." Specifically, in our model with discrete types, even if an agent lowers his (her) reservation utility strategy, this does not guarantee that he (she) accepts a woman (man) whom he (she) has rejected previously. Therefore, in the following analysis, the statement that an agent changes his (her) *action* means that he (she) changes the type of women (men) whom he (she) is willing to accept.

Let $b_k^t\left(b_0,h_k^t\right)(x_k)$ denote a k-type woman's probability assigned to the particular type $x_k\in\Xi$. This probability is determined by Bayes' rule. The k-type woman's posterior belief $b_k^{t+1}\left(b_0,h_k^t|\left(\tilde{x}_k^t,a_m^t(x_k)\right)\right)(x_k)$ after observing $\left(\tilde{x}_k^t,a_m^t(x_k)\right)$ at period t given her current belief $b_k^t\left(b_0,h_k^t\right)$ is given by

$$b_k^{t+1}\left(b_0,h_k^t|\left(\tilde{x}_k^t,a_m^t(x_k)\right)\right)(x_k)=\frac{b_k^t\left(b_0,h_k^t\right)(x_k)\Pr\left(\left(\tilde{x}_k^t,a_m^t(x_k)\right)|x_k\right)}{\sum_{x_k\in\Xi}b_k^t\left(b_0,h_k^t\right)(x_k)\times\Pr\left(\left(\tilde{x}_k^t,a_m^t(x_k)\right)|x_k\right)},$$

where $b_k^{t+1}\left(b_0,h_k^{t+1}\right)(x_k)=b_k^{t+1}\left(b_0,h_k^t|\left(\tilde{x}_k^t,a_m^t(x_k)\right)\right)(x_k)$. In what follows, because b_0 is fixed in this paper, we omit writing explicitly b_0.

Because we consider only pure strategies when self-knowledge is perfect in the model presented here, $\Pr\left(\left(\tilde{x}_k^t,a_m^t(x_k)\right)|x_k\right)=0$ or 1 when a k-type woman observes $\left(\tilde{x}_k^t,a_m^t(x_k)\right)$ given the strategies of men. Because $\Pr\left(\left(\tilde{x}_k^t,a_m^t(x_k)\right)|x_k\right)=0$ or 1, given $\left(\tilde{x}_k^t,a_m^t(x_k)\right)$, h_k^t and strategies of men, a k-type woman knows that her actual type does not belong to a type set. Let $\Phi_k^t\left(\left(\tilde{x}_k^t,a_m^t(x_k)\right),h_k^t\right)$ denote the impossible type set of a k-type woman, which she recognizes by observing $\left(\tilde{x}_k^t,a_m^t(x_k)\right)$ given h_k^t and strategies of men. Then, we can define the set of the k-type woman's remaining possible types at the period $t+1$ recursively. Let $\Xi_k^{t+1}=\Xi_k^t\backslash\Phi^t\left(\left(\tilde{x}_k^t,a_m^t(x_k)\right),h_k^t\right)$ denote the set of the k-type woman's remaining possible types at the start of period $t+1$, where $\Xi_k^0=\Xi$. It is noteworthy that the set Ξ_k^t can be interpreted as an information set in a sequential-move game. Because Ξ_k^t depends on h_k^t, to simplify notation we write $b_k^t\left(h_k^t\right)$ as $b_k^t\left(\Xi_k^t\right)$ in the following analysis.

Let $V_w\left(b_k^t\left(\Xi_k^t\right)\right)$ denote the lifetime expected discounted utility of a k-type woman at the start of period t conditional on her belief $b_k^t\left(\Xi_k^t\right)$. Thus,

$$V_w\left(b_k^t\left(\Xi_k^t\right)\right)=\sum\nolimits_{x_k\in\Xi_k^t}b_k^t\left(\Xi_k^t\right)(x_k)V_w\left(b_k^t\left(\Xi_k^t\right)\right)$$
$$=\frac{1}{1+rdt}\sum\nolimits_{x_k\in\Xi_k^t}b_k^t\left(\Xi_k^t\right)(x_k)\left[\begin{array}{l}\left(1-\alpha_w\left(b_k^t\left(\Xi_k^t\right)\right)dt\right)V_w\left(b_k^t\left(\Xi_k^t\right)\right)\\+\alpha_w\left(b_k^t\left(\Xi_k^t\right)\right)dtE\left(\max\left\{\frac{\tilde{x}_k}{r},V_w\left(b_k^{t+1}\left(\Xi_k^t|\left(\tilde{x}_k^t,a_m^t(x_k)\right)\right)\right)\right\}|x_k\right)\end{array}\right].$$

where $\tilde{x}_k$ has distribution $H_m(\tilde{x}_k|x_k)$. Manipulating and then letting $dt\to0$ yields

$$rV_w\left(b_k^t\left(\Xi_k^t\right)\right)=\sum\nolimits_{x_k\in\Xi_k^t}b_k^t\left(\Xi_k^t\right)(x_k)\alpha_w\left(b_k^t\left(\Xi_k^t\right)\right)\left[E\left(\max\left\{\frac{\tilde{x}_k}{r},V_w\left(b_k^{t+1}\left(\Xi_k^{t+1}\right)\right)\right\}|x_k\right)-V_w\left(b_k^t\left(\Xi_k^t\right)\right)\right].\tag{5}$$

where $b_k^{t+1}\left(\Xi_k^{t+1}\right)=b_k^{t+1}\left(\Xi_k^t|\left(\tilde{x}_k^t,a_m^t(x_k)\right)\right)$. Because $V_w\left(b_k^t\left(\Xi_k^t\right)\right)$ depends only on $b_k^t\left(\Xi_k^t\right)$, women in the same information set Ξ_k^t at period t face the same decision problem regardless of their actual types.

In this paper, because we consider the three-types case, a woman learns about her own type at most three times and there are at most six kinds of information sets: $\Xi_0 \equiv \Xi = \{x_H, x_M, x_L\}$, $\Xi_{HM} \equiv \{x_H, x_M\}$, $\Xi_{ML} \equiv \{x_M, x_L\}$, $\Xi_H \equiv \{x_H\}$, $\Xi_M \equiv \{x_M\}$, and $\Xi_L \equiv \{x_L\}$. Therefore, from now on, we remove the superscript t from the notations of the state variables for the sake of simplicity because the changes of belief over time can be represented by the elements of Ξ_k^t. In the following analysis, let us call a "k_l-type woman" and a "k-type woman" as a woman whose actual type is $k \in \{H, M, L\}$ with belief $b_k(\Xi_l), l \in \{0, HM, ML, H, M, L\}$, and a woman whose actual type is $k \in \{H, M, L\}$ with any belief, respectively. Moreover, we write b_l and $b_{l|(\tilde{x}_k^t, a_m(x_k))}$ instead of $b_k(\Xi_l)$ and $b_k\left(\Xi_l | \left(\tilde{x}_k^t, a_m(x_k)\right)\right)$, respectively.[14]

Let $G_m(x)$ and $G_w(x)$ denote the stationary distribution of men's belief and that of women's belief, respectively. That is, $G_i(x,t) = G_i(x)$ for all x and all t. Let us assume that all agents also know $G_m(x)$ and $G_w(x)$ (we later show that $G_m(x)$ and $G_w(x)$ depend on α and $F_i(x)$, which are common knowledge among all agents) and believe the market is characterized by (G_m, G_w). Because all men know their own types, $G_m(x) = F_m(x)$.

We introduce an equilibrium concept for our model with imperfect self-knowledge. Although each woman's belief (state) changes over time, we now focus on the market in a steady state.

Definition 3 *In a* Market Equilibrium with imperfect self-knowledge (MEI)$(R(x_k), R(b_l), G_m, G_w)$*:*

(MEI-i) the agent strategies satisfy sequential rationality;

(MEI-ii) the agent beliefs at the sets of remaining possible types (the information sets) along the equilibrium path are consistent with Bayesian updating given the equilibrium strategies;

(MEI-iii) an exit flow of each belief in each actual type equals the entry rate of that belief in that type.

We consider the following three MEI in the following subsections. First, we consider the case of PSE with imperfect self-knowledge. Next, we find the Type 1 equilibrium to show that the optimism by some women has two externalities. Finally, we find the Type 2 equilibrium to show that the pessimism by some women has an externality. These equilibria satisfy (MEI-i)–(MEI-iii).

4.1 PSE with imperfect self-knowledge

In this subsection, we discuss the PSE with imperfect self-knowledge (PSEI) in which agents of the same type marry, when there are women with imperfect self-knowledge. Therefore, k_0-type ($k = H, M, L$) women always reject M-type men.[15] Moreover, H-type women always reject M-type men, M-type women always reject L-type men, and some L-type women always accept L-type men.

Distribution of beliefs of women in the PSEI

[14] When $t = 0$, $b^0 = b_0$.

[15] Otherwise, the PSEI does not occur as men and women of different types marry.

To obtain the MEI, first, we derive the distribution of beliefs of women $G_w^P(x)$ in the PSEI. This distribution is derived in Appendix A.1.1.

Matching strategies and a market equilibrium

Men decide their optimal strategies given G_i^P $(i = m, w)$ and women's actions (who accepts (or rejects) whom) in the market. Because all agents know $G_i^P(x)$, the strategy of each type is common knowledge among all agents.

An H-type man has the same reservation level as an H-type man in the PSE because all women want to marry H-type men. Given the strategies of H-type men, the decision of an M-type man is as follows. Because a fraction $\eta \in (0,1)$ of M-type women accept M-type men and a fraction $\zeta \in (0,1)$ of L-type women accept M-type men, the following lemma applies to an M-type man.

Lemma 1 *Let us assume that* $R_i^*(x_H) > x_M$, $R_i^*(x_M) > x_L$ $(i = m, w)$, *and that* $\eta \in (0,1)$ *of* M*-type women accept* M*-type men and* $\zeta \in (0,1)$ *of* L*-type women accept* M*-type men. If*

$$x_L < (\geq)\ R_m^P(x_M) \equiv \frac{\alpha\eta\lambda_M^w x_M}{\left(r+\alpha\eta\lambda_M^w\right)}, \tag{6}$$

an M*-type man rejects (accepts) an* L*-type woman. In this case, the reservation level of an* M*-type man for an* L*-type woman decreases, in contrast with the benchmark result, i.e.,* $R_m^*(x_M) > R_m^P(x_M)$. *In the PSEI,* $\eta = \frac{\lambda_H^m}{\lambda_H^m+\lambda_M^m}$ *and* $\zeta = \frac{\lambda_H^m\lambda_L^m+\lambda_M^m\left(\lambda_H^m+\lambda_M^m\right)}{\left(\lambda_L^m+\lambda_M^m\right)\left(\lambda_H^m+\lambda_M^m\right)}$ *hold.*

Proof. See Appendix A.2. ∎

Lemma 1 means that the rejections of M-type men by some M-type women with imperfect self-knowledge lower the reservation utility level of an M-type man for an L-type woman. It is noteworthy that the result $R_m^*(x_M) > R_m^P(x_M)$ does not depend on the cloning assumption. This is because one can obtain the results of Lemma 1, except $\eta = \frac{\lambda_H^m}{\lambda_H^m+\lambda_M^m}$ and $\zeta = \frac{\lambda_H^m\lambda_L^m+\lambda_M^m\left(\lambda_H^m+\lambda_M^m\right)}{\left(\lambda_L^m+\lambda_M^m\right)\left(\lambda_H^m+\lambda_M^m\right)}$, given a stationary market environment (i.e., $F_i(x)$ and $G_i(x)$ are stationary distributions).[16]

With constant α, if $\eta\lambda_M^w$ is small enough $(\alpha\eta\lambda_M^w \leq \frac{rx_L}{(x_M-x_L)})$, an M-type man accepts an L-type woman. Conversely, if there are enough M-type women who accept M-type men $(\alpha\eta\lambda_M^w > \frac{rx_L}{(x_M-x_L)})$, an M-type man turns down an L-type woman.

Next, we investigate the optimal strategies of women. Women decide their optimal strategies given G_i^P $(i = m, w)$ and men's actions. The optimal strategies of women are obtained in the next lemma.

Lemma 2 *Let us assume that* $R_i^*(x_H) > x_M$ *and* $R_i^*(x_M) > x_L$ $(i = m, w)$. *Furthermore, let us suppose that an* M*-type man rejects an* L*-type woman and an* L*-type man accepts an* L*-type woman. If*

$$x_L < (\geq) \frac{\lambda_M^m\alpha x_M\left(r+\alpha\lambda_L^m\right)\left(b_{ML}^P(x_M)\right)}{r\left(r+\alpha\lambda_L^m+\alpha\lambda_M^m\right)+\alpha^2\lambda_L^m\lambda_M^m\left(b_{ML}^P(x_M)\right)} \equiv R_w^P(b_{ML}), \tag{7}$$

[16] At this time, the equilibrium condition (MEI-iii) is not needed.

a k_{ML}-type ($k = M, L$) woman rejects (accepts) an L-type man. If

$$x_M < (\geq) \frac{\lambda_H^m \alpha x_H \left(r+\alpha\lambda_M^m\right)\left(b_{HM}^P(x_H)\right)}{r\left(r+\alpha\lambda_H^m+\alpha\lambda_M^m\right)+\alpha^2\lambda_H^m\lambda_M^m\left(b_{HM}^P(x_H)\right)} \equiv R_w^P\left(b_{HM}\right) = R_w^P\left(b_0\right), \tag{8}$$

a k_{HM}-type ($k = H, M$) or a k_0-type ($k = H, M, L$) woman rejects (accepts) an M-type man. Moreover, $R_w^P\left(b_{ML}\right) < R_w^*\left(x_M\right) = R_w^P\left(b_M\right) \leq x_M < R_w^P\left(b_{HM}\right) = R_w^P\left(b_0\right) < R_w^*\left(x_H\right)$ holds. Here, $b_{ML}^P\left(x_M\right) = \frac{\lambda_H^m\lambda_M^w\left(\lambda_L^m+\lambda_M^m\right)}{\lambda_L^m\lambda_L^w\left(\lambda_H^m+\lambda_M^m\right)+\lambda_H^m\lambda_M^w\left(\lambda_L^m+\lambda_M^m\right)}$ and $b_{HM}^P\left(x_H\right) = \frac{\lambda_H^w\left(\lambda_H^m+\lambda_M^m\right)}{\lambda_H^w\left(\lambda_H^m+\lambda_M^m\right)+\lambda_M^m\lambda_M^w}$ in the PSEI.

Proof. See Appendix A.2. ■

Lemma 2 means that a k_{ML}-type woman rejects (accepts) an L-type man if there are enough (few enough) M-type men or if $b_{ML}^P\left(x_M\right)$ is sufficiently large (sufficiently small). Similarly, a k_{HM}- or k_0-type woman rejects (accepts) an M-type man if there are enough (few enough) H-type men or if $b_{HM}^P\left(x_H\right)$ is sufficiently large (sufficiently small).

Lemma 2 shows that the reservation level of a k_0-type woman for an M-type man is the same as that of a k_{HM}-type woman. If the actual type of a k_0-type woman is the L-type, she is rejected by H- and M-type men regardless of her own action. Therefore, the decision of a k_0-type woman regarding whether or not to accept an M-type man does not depend on the possibility that her actual type is the L-type. Hence, even if a k_0-type woman becomes a k_{HM}-type woman by meeting an M-type man, her decision does not change.

Women with imperfect self-knowledge assign probabilities to their own types. Therefore, the reservation utility levels of M_{ML}-type women, H_{HM}-type women, and H_0-type women are lowered in contrast with the benchmark results. On the other hand, the reservation utility levels of L_{ML}-type women, M_{HM}-type women, and k_0-type ($k = M, L$) women are raised in contrast with the PSE.

When $r = 0$, $R_w^P\left(b_{ML}\right) = R_i^*\left(x_M\right)\left(= x_M\right)$ holds. Therefore, a k_{ML}-type woman always prefers to meet an M-type man over accepting an L-type man in order to have the chance of confirming her actual type.[17] This is because, if the actual type of a k_{ML}-type woman is an L-type, she would marry an L-type man sooner or later regardless of her action. At this time, she obtains the same value when she is single regardless of her action due to a lack of time-consuming cost ($r = 0$). Hence, the possibility that the actual type of a k_{ML}-type woman is an L-type does not affect her own decision. Consequently, the decision of a k_{ML}-type woman is the same as that of an M-type woman with perfect self-knowledge.

If $r > 0$, the possibility that the actual type of a k_{ML}-type woman is an L-type affects her own decision. The agents with imperfect self-knowledge need to take into account the time-consuming cost due to the learning process.[18] When a k_{ML}-type woman is an L-type, she is refused by an M-type man. It is then desirable for an L_{ML}-type woman to accept an L-type man before thoroughly understanding her own type. Therefore, the reservation level

[17] An L_{ML}-type woman will be optimistic as she raises her reservation utility to reject an L-type man. We will analyze the influence of optimism on a market in Subsection 4.2.

[18] Therefore, in our model, it is possible that a woman with imperfect self-knowledge could marry before thoroughly understanding her own type.

of a k_{ML}-type $(k = M, L)$ woman is lower than that in the case of $r = 0$.[19]

A similar discussion could be presented for $R_w^P(b_{HM}) = R_w^P(b_0) = R_w^*(x_H)\,(= x_H)$ when $r = 0$.

It is noteworthy that similar to Lemma 1, the results of Lemma 2, except the value of $b_{ML}^P(x_M)$ and that of $b_{HM}^P(x_H)$, do not depend on the cloning assumption.

From Lemmas 1 and 2, we then immediately obtain sufficient conditions for the PSEI.

Proposition 2 *Let us assume that $x_M < R_i^*(x_H)$ and $x_L < R_i^*(x_M)$ $(i = m, w)$ hold. If $R_m^P(x_M) > x_L$, $R_w^P(b_{ML}) > x_L$, and $R_w^P(b_{HM}) = R_w^P(b_0) > x_M$, then there exists the PSEI in which H-type agents form the first cluster of marriages, M-type men and M_{ML}- and M_M-type women, the second cluster, and L-type men and L_{L1}- and L_{L2}-type women, the third cluster.*

Proof. Omitted. ■

The implications of Proposition 2 are as follows: if there are enough H-type men or H-type women $(R_w^P(b_{HM}) = R_w^P(b_0) > x_M)$, an M-type man is rejected by a k_0-type $(k = H, M, L)$ or k_{HM}-type $(k = H, M)$ woman. When there are enough M_M- and M_{ML}-type women $(R_m^P(x_M) > x_L)$, an M-type man rejects an L-type woman.[20] When there are enough M-type men or M_{ML}-type women $(R_w^P(b_{ML}) > x_L)$, a k_{ML}-type $(k = M, L)$ woman rejects an L-type man. However, when an L_{ML}-type woman rejects an L-type man, she becomes an L_L-type woman sooner or later because of being rejected by an M-type man. Then, an L_L-type woman accepts an L-type man. As a result, the PSEI occurs. It is noteworthy that the first cluster of marriages is not influenced by women who are unaware of their own types.

When $R_m^P(x_M) > x_L$, the M-type man's rate of contact with a woman whom he wishes to marry is $\alpha\eta\lambda_M^w$. Then, an M-type man's time (duration) until meeting such a woman is $\frac{1}{\alpha\eta\lambda_M^w}$. Therefore, his time until marriage is delayed due to the rejections from M-type women with imperfect self-knowledge because his time until marriage in the benchmark case is $\frac{1}{\alpha\lambda_M^w}$. This delay of marriage is the direct negative externality of optimism. Moreover, the marriages of all agents, except those of H-type men and some H-type women who meet H-type men in their first encounter, are delayed due to the refusals by the women who are learning their own types.[21] Therefore, the welfare of each type of women in the PSEI is always lower than that in the PSE because their marriages are delayed due to their own learning. The welfare

[19] If a k_{ML}-type woman lowers her $R_w^P(b_{ML})$ to accept an L-type man whom she rejects when she knows her own type, an M_{ML}-type woman will apparently underestimate her own type. We will analyze these effects of pessimism in Subsection 4.3.

[20] The share of M_M-type and that of M_{ML}-type women in the market depend on λ_H^m. Therefore, if there are many H-type men, there will be many M_M- and M_{ML}-type women.

[21] The duration until marriage of each agent can be obtained easily. In the PSE, the duration until marriage of k-type man (woman) is $\frac{1}{\alpha\lambda_k^i}(i = m, w,\ k = H, M, L)$. However, in the PSEI, the duration until marriage of an M-type man is $\frac{1}{\alpha\lambda_M^w}\frac{\lambda_H^m+\lambda_M^m}{\lambda_H^m}$, that of an L-type man is $\frac{1}{\alpha\lambda_L^w}\frac{\lambda_L^m+\lambda_M^m}{\lambda_M^m}$, that of an H_{HM}-type woman is $\frac{1}{\alpha\lambda_H^m\alpha\lambda_M^m}$, that of an M_{ML}-type woman is $\frac{1}{\alpha\lambda_H^m\alpha\lambda_M^m}$, that of an M_M-type woman is $\frac{1}{\alpha\lambda_H^m\alpha\lambda_M^m\alpha\lambda_L^m}$, that of an L_{L1}-type woman is $\frac{1}{\alpha\lambda_H^m\alpha\lambda_M^m\alpha\lambda_L^m}$, and that of an L_{L2}-type woman is $\frac{1}{\alpha\lambda_M^m\alpha\lambda_L^m}$. Therefore, their marriages are delayed by the learning process of women.

of each type of men, except H-type men, also decreases because of the rejections from women with imperfect self-knowledge.

4.2 Influence of optimism on a market

In this subsection, we investigate the effects of *optimism*: a woman with imperfect self-knowledge rejects a man whom, when she knows her own type, she accepts.

One effect is the direct negative externality of optimism that is obtained in Subsection 4.1. More precisely, we define the *direct externality of optimism* as the situation in which a man who is now rejected by an optimistic woman lowers his reservation level relative to the benchmark case because of the delay of marriage, but he does not change his action. In a two-sided search model, if the above man lowers his reservation level to change his action, the woman who is now accepted by him may also change her action. Therefore, in this subsection, we investigate the influence of some women's optimism on the actions of the others, including the agents who are not directly rejected by the optimistic women.

For this reason, we will find a *Type 1 equilibrium (Type 1)* in which H-type men reject M-type women, M- and L-type men accept L-type women, k_0-type ($k = H, M, L$) women reject M-type men, and k_{ML}-type ($k = M, L$) women reject L-type men. In other words, M_0-, L_0-, and L_{ML}-type women are optimistic.[22]

Distribution of beliefs of women in the Type 1

To obtain the MEI, first, we derive the distribution of beliefs of women $G_w^{T1}(x)$ in the Type 1. This is derived in Appendix A.1.2.

Matching strategies and a market equilibrium

First, we investigate the optimal strategies of men. An H-type man has the same reservation level as an H-type man in the PSE because all women want to marry H-type men. As an H_0- or M_0-type woman rejects an M-type man, the option of an M-type man is to marry or to reject an L-type woman. In the same manner as in Lemma 1, from Figure 2, we immediately obtain the reservation level of an M-type man for an L-type woman $R_m^{T1}(x_M) \equiv \frac{\alpha\tau\lambda_M^w x_M}{r+\alpha\tau\lambda_M^w} \left(< R_m^*(x_M)\right)$, where $\tau = \frac{\lambda_H^m}{\lambda_H^m+\lambda_M^m}$ in the Type 1. An L-type man also accepts an L-type woman if an M-type man accepts an L-type woman.

Next, we investigate the strategies of women given men's strategies (or actions). The optimal strategies of women are obtained in the next lemma.

Lemma 3 *Let us assume that* $x_M < R_i^*(x_H)$ *and* $x_L < R_i^*(x_M)$ $(i = m, w)$ *and that an M- or L-type man accepts an L-type woman. At this time, a k_{ML}-type woman ($k = M, L$) rejects an L-type man* $\left(R_w^{T1}(b_{ML}) = R_w^*(x_M)\right)$*. On the other hand, a k_0-type woman rejects*

[22] Of course, we can consider another equilibrium in order to describe the optimistic behavior. However, at this time, we cannot show that optimism prevents the lowest-type agents from marrying in our model with three types of agents (for the details, see Appendix B). Hence, we focus on the Type 1.

(accepts) an M-type man if

$$x_M < (\geq) \frac{\lambda_H^m \alpha x_H \left(r+\alpha\lambda_M^m\right)\left(b_0^{T1}(x_H)\right)}{r\left(r+\alpha\lambda_H^m+\alpha\lambda_M^m\right)+\alpha^2\lambda_H^m\lambda_M^m\left(b_0^{T1}(x_H)\right)} \equiv R_w^{T1}(b_0). \tag{9}$$

Moreover, $R_w^{T1}(b_0) < R_w^*(x_H)$*. In the Type 1,* $b_0^{T1}(x_H) = \frac{\lambda_H^w\left(\lambda_H^m+\lambda_M^m\right)}{\lambda_H^m\lambda_H^w+\lambda_M^m}$.

Proof. See Appendix A.2. ■

Lemma 3 suggests that a k_0-type woman rejects an M-type man if there are enough H-type men or H-type women. On the other hand, because an M-type man accepts an L-type woman, an M_{ML}-type woman and an L_{ML}-type woman face the same problem. As a result, a k_{ML}-type ($k = M, L$) woman turns down an L-type man, because there are enough M-type men ($x_L < R_w^*(x_M)$).[23]

Given matching strategies of men and women in the Type 1, we immediately obtain Proposition 3. In this equilibrium, the optimism of M_0-type women prevents the lowest-type men from marrying.

Proposition 3 *Let us assume that* $x_M < R_i^*(x_H)$ *and* $x_L < R_i^*(x_M)$ $(i = m, w)$ *hold. If* $R_w^{T1}(b_0) > x_M$ *and* $R_m^{T1}(x_M) \leq x_L$*, there exists the Type 1 in which H-type agents form the first cluster of marriages, M-type men and* M_{ML}*-type women, the second cluster, and M-type men and* L_{ML}*-type women, the third cluster. In this equilibrium, L-type men can never marry.*

Proof. Omitted. ■

The implications of Proposition 3 are as follows: when there are enough H-type men or H-type women ($R_w^{T1}(b_0) > x_M$), a k_0-type woman rejects an M-type man. If there are few enough M_{ML}-type women ($R_m^{T1}(x_M) \leq x_L$), an M-type man changes his action compared to the benchmark case; he accepts an L-type woman. This leads an M_{ML}- or L_{ML}-type woman to refuse the offer by an L-type man ($R_w^P(b_{ML}) = R_w^*(x_M) > x_L$). As a result, an L-type man can never marry.[24]

From Proposition 3, when there are many optimistic women in the market, the *indirect externality* of optimism occurs: the men who are now rejected by optimistic women change their actions; i.e., they accept another lower type of women whom, when all agents know their own types, they reject. Given this fact, in a two-sided search, the women who are now accepted by these men may also change their actions. As a result, there is a case in which someone's optimism generates the agents who cannot marry.

The welfare of M-type women in the Type 1 is always lower than that in the PSE because their marriages are delayed due to their own learning. The welfare of L-type men is also lower because they cannot get married. On the other hand, because M-type men accept

[23] Similar to Lemma 2, the results of Lemma 3, except the value of $b_0^{T1}(x_H)$, do not depend on the cloning assumption.

[24] The share of k_{ML}-type women ($k = M, L$) depends on the share of H-type men λ_H^m. However, even if $F_m(x)$ and $F_w(x)$ are symmetric, $R_m^{T1}(x_M) \leq x_L$ and $R_w^{T1}(b_0) > x_M$ hold in some parameter ranges (see Example 1).

both M- and L-type women, their welfare increases or decreases as a whole, depending on $x_k(k = M, L)$ and the shapes of $F_m(x)$ and $F_w(x)$. Although L-type women marry M-type men, their welfare also increases or decreases as a whole, depending on $F_m(x)$, $F_w(x)$, and $x_k(k = M, L)$.

In the next subsection, we investigate the effects of pessimism.

4.3 Influence of pessimism on a market

In this subsection, we investigate the effects of *pessimism*: a woman with imperfect self-knowledge accepts a man whom, when she knows her own type, she rejects. From Lemma 2, if $r > 0$ is large, a woman with imperfect knowledge would tend to be pessimistic.

The pessimism of a woman makes the future partner better off because she increases the value of the match to the partner. Therefore, the pessimism increases the reservation level of the men who are now accepted by pessimistic women. If these men raise their reservation levels but do not change their actions, we call this effect the *direct* (positive) *externality of pessimism.*

On the other hand, the pessimism has no indirect externality. To establish this fact by proof by contradiction, we have to investigate all cases in which *indirect externality occurs*: if there are enough pessimistic women, the men who are now accepted by these pessimistic women reject the women whom they accept when all agents know their own types.

The next Lemma shows that the indirect externality of the pessimism does not occur.

Lemma 4 *Let us assume that* $x_M < R_i^*(x_H)$ *and* $x_L < R_i^*(x_M)$ $(i = m, w)$.

- *When there are pessimistic M_{ML}-type women who accept L-type men, an L-type man always accepts an L-type woman.*
- *When there are pessimistic H_0- or H_{HM}-type women who accept M-type men, an M-type man always accepts an M-type woman.*
- *When there are pessimistic k_0- or k_{HM}-type women $(k = H, M)$ who accept L-type men, an L-type man always accepts an L-type woman.*

Proof. See Appendix A.2. ■

Lemma 4 shows that the pessimism does not have the indirect externality. For example, if an L-type man rejects an L-type woman because of the expectation to marry a pessimistic M_{ML}-type woman, the proposal from him to an M_{ML}-type woman informs her that she is the M-type. As a result, a k_{ML}-type woman has the incentive to reject an L-type man because she prefers to have the chance of learning her actual type than to accept an L-type man.[25] This contradicts the assumption that a k_{ML}-type woman accepts an L-type man. Therefore, an L-type man does not change his action even if there are enough pessimistic M_{ML}-type

[25] If the actual type of a k_{ML}-type woman is the L-type, she is rejected by an M-type or an L-type man regardless of her actions. Hence, the possibility that she is an L-type does not affect her decision regarding whether to accept or not an L-type man.

women for L-type men to reject L-type women. Similarly, the other pessimistic behaviors by women do not have the indirect externality.

Lemma 4 also shows that the reservation level of a woman with imperfect self-knowledge never rises even if she receives an offer that has some information about her own type. An offer from a man whom she decides whether to accept or not does not affect her decision in equilibrium, because the man chooses his strategy so as not to raise her reservation level. An offer much lower than her reservation level is proposed by a man whom the woman with imperfect self-knowledge rejects. At this time, her decision is also not affected by that offer because she decides whether to accept or not another type of man. A high offer results in the agent getting married similarly to the finding by Burdett and Vishwanath (1988). However, in their model, workers learn the unknown wage distribution. In contrast, our model is a two-sided search and single agents know the type distribution but do not know their own types.

On the other hand, the rejections from a man decrease the reservation utility level of a woman with imperfect self-knowledge when k_0-type women reject M-type men.[26] From Lemmas 2 and 4, we obtain the next lemma.

Lemma 5 *Let us assume that $R_i^*(x_H) > x_M$ and $R_i^*(x_M) > x_L$ $(i = m, w)$ and that k_0-type women reject M-type men. If a rejection from a man has some information about a woman with imperfect self-knowledge, the rejection from him for her decreases her reservation utility level.*

Proof. See Appendix A.2. ■

Burdett and Vishwanath (1988) show that an offer much lower than expected leads the worker to revise his or her reservation wage downward. In contrast, this result shows that if women with imperfect self-knowledge start to search with the strategies of accepting only the highest type of men, a woman revises her reservation level downward whenever she receives a rejection that has some information about her own type.

The results of Lemmas 4 and 5 can be obtained when $F_i(x)$ and $G_i(x)$ are stationary distributions. Hence, these results do not depend on the cloning assumption.

Next, we show that the pessimism has direct externality. To reuse the framework of the PSEI in what follows, let us consider a *Type 2 equilibrium (Type 2)* in which H-type men reject M-type women, M-type men reject L-type women, L-type men accept L-type women, k_0-type $(k = H, M, L)$ and k_{HM}-type $(k = H, M)$ women reject M-type men, and k_{ML}-type $(k = M, L)$ women accept L-type men. In other words, M_{ML}-type women are pessimistic.[27]

[26] When a k_0-type woman accepts an M-type man and rejects an L-type man, there exists a case in which a rejection from a H-type man increases the reservation level of a woman with imperfect self-knowledge. In this case, we only have to investigate whether $R_w(b_0)$ is higher than $R_w(b_{ML})$ (and whether $R_w(b_{ML})$ is higher than $R_w(b_L)$) because there are no k_{HM}-type women. However, if a k_{ML}-type woman rejects an L-type man, $R_w(b_0)$ is higher or lower than $R_w(b_{ML})$, depending on the actual type distributions $F_i(x)$, $(i = m, w)$.

Note that if $F_i(x)$ is the discrete uniform distribution (i.e., $\lambda_k^i = \frac{1}{3}$ $(i = m, w,\ k = H, M, L)$), $R_w(b_0) > R_w(b_{ML})$ holds.

[27] Of course, we can consider another equilibrium in order to describe the pessimistic behavior. At this time, we can qualitatively obtain the same results as in Proposition 4 below.

Distribution of beliefs of women in the Type 2

We then obtain the distribution of beliefs of women in the Type 2 ($G_w^{T2}(x)$) in Appendix A.1.3.

Matching strategies and a market equilibrium

Let us investigate the strategies of men. An H-type man has the same reservation level as an H-type man in the PSE. In the same manner as in Lemma 1, the reservation level of an M-type man for an L-type woman $R_m^{T2}(x_M) = \frac{\alpha(\mu_2+\gamma_2\phi_2)\lambda_M^w x_M}{(r+\alpha(\mu_2+\gamma_2\phi_2)\lambda_M^w)}$ $(< R_m^*(x_M))$, where $(\mu_2+\gamma_2\phi_2) = \frac{\lambda_H^m}{\lambda_H^m+(\lambda_M^m+\lambda_L^m)(\lambda_H^m+\lambda_M^m)}$ in the Type 2, is obtained. From Lemma 4, an L-type man accepts an L-type woman, even if there are enough pessimistic M_{ML}-type women.

Next, the optimal strategies of women are obtained in the next lemma.

Lemma 6 *Let us assume that $x_M < R_i^*(x_H)$ and $x_L < R_i^*(x_M)$ $(i = m, w)$ and that an M-type man rejects an L-type woman. If*

$$x_L < (\geq) R_w^{T2}(b_{ML}) \equiv \frac{\lambda_M^m \alpha x_M \left(r+\alpha\lambda_L^m\right)\left(b_{ML}^{T2}(x_M)\right)}{r\left(r+\alpha\lambda_L^m+\alpha\lambda_M^m\right)+\alpha^2\lambda_L^m\lambda_M^m\left(b_{ML}^{T2}(x_M)\right)}, \tag{10}$$

a k_{ML}-type woman $(k = M, L)$ rejects (accepts) an L-type man. If

$$x_M < (\geq) R_w^{T2}(b_{HM}) \equiv \frac{\lambda_H^m \alpha x_H \left(r+\alpha\lambda_M^m\right)\left(b_{HM}^{T2}(x_H)\right)}{r\left(r+\alpha\lambda_H^m+\alpha\lambda_M^m\right)+\alpha^2\lambda_H^m\lambda_M^m\left(b_{HM}^{T2}(x_H)\right)} = R_w^{T2}(b_0), \tag{11}$$

a k_{HM}-type $(k = H, M)$ and a k_0-type $(k = H, M, L)$ woman reject (accept) an M-type man. Moreover, $R_w^{T2}(b_{ML}) < R_w^*(x_M) = R_w^{T2}(b_M)$ and $R_w^{T2}(b_{HM}) = R_w^{T2}(b_0) < R_w^*(x_H)$ hold. Here, $b_{ML}^{T2}(x_M) = \frac{\lambda_H^m\lambda_M^w(\lambda_L^m+\lambda_M^m)}{\lambda_L^w\lambda_L^m(\lambda_H^m+\lambda_H^m\lambda_L^m+\lambda_M^m)+\lambda_H^m\lambda_M^w(\lambda_L^m+\lambda_M^m)}$ and $b_{HM}^{T2}(x_H) = \frac{\lambda_H^w(\lambda_H^m+(\lambda_H^m+\lambda_M^m)(\lambda_L^m+\lambda_M^m))}{\lambda_H^m\lambda_H^w+(\lambda_L^m+\lambda_M^m)(\lambda_H^m+\lambda_M^m)(\lambda_H^w+\lambda_M^w)}$ in the Type 2.

Proof. See Appendix A.2. ■

Lemma 6 implies the following: except the value of $b_{ML}^{T2}(x_M)$ and that of $b_{HM}^{T2}(x_H)$, Equations (10) and (11) have the same form as Equations (7) and (8), respectively. However, the difference between the PSEI and the Type 2 is whether or not k_{ML}-type women accept L-type men. As a result, $G_w^P(x)$ and $G_w^{T2}(x)$ are different and then $b_{ML}^{T2}(x_M)$ and $b_{HM}^{T2}(x_H)$ are different.

From $R_m^{T2}(x_M)$ and Lemma 6, we immediately have Proposition 4 for the Type 2. In this equilibrium, pessimism has direct externality.

Proposition 4 *(Type 2) Let us assume that $x_M < R_i^*(x_H)$ and $x_L < R_i^*(x_M)$ $(i = m, w)$. If $x_M < R_w^{T2}(b_{HM}) = R_w^{T2}(b_0)$ and $R_w^{T2}(b_{ML}) \leq x_L < R_m^{T2}(x_M)$, then there exists the Type 2 in which H-type agents form the first cluster of marriages, M-type men and M_{ML}-type and M_M-type women, the second cluster, L-type men and M_{ML}-type women, the third cluster, and L-type men and L_{ML}-type, L_{L1}-type, and L_{L2}-type women, the fourth cluster.*

Proof. Omitted. ■

The implications of Proposition 4 are as follows: there is the direct externality of pessimism in the Type 2, in which all agents can marry. If there are enough H-type men and H-type women ($x_M < R_w^{T2}(b_{HM})$), an M-type man is rejected by a k_0-type and a k_{HM}-type woman. Moreover, a large λ_H^m implies a large share of M_M- and M_{ML}-type women in M-type women. If there are enough M_M- and M_{ML}-type women ($x_L < R_m^{T2}(x_M)$), an M-type man rejects an L-type woman. If there are few enough M_{ML}-type women satisfying $R_w^{T2}(b_{ML}) \leq x_L \left(< R_m^{T2}(x_M)\right)$, a k_{ML}-type woman accepts an L-type man. This is because she assigns low probability to being an M-type woman. However, as an L-type man accepts an L-type woman, all agents can marry sooner or later.

The welfare of L-type women in the Type 2 is always lowered due to their own learning, relative to those in the PSE. The welfare of M-type men also decreases because these men are rejected by M_0- and M_{HM}-type women. The M_{ML}-type women who marry L-type men obtain lower utilities than M-type women with perfect self-knowledge. However, as M_{ML}-type women in the Type 2 can marry earlier than M-type women in the PSE, the welfare of M-type women increases or decreases as a whole, depending on $F_m(x)$, $F_w(x)$, and x_k $(k = M, L)$. Because L-type men marry M- and L-type women, their welfare also increases or decreases depending on $F_m(x)$, $F_w(x)$, and x_k $(k = M, L)$.

Propositions 3, 4 and Lemma 4 suggest that, whereas optimism has an indirect externality, pessimism does not have indirect externality. This difference depends on the agents whom the indirect externality first affects. In the case of pessimism, some M-type women with imperfect self-knowledge accept L-type men. Given this, if an L-type man rejects an L-type woman, his offer to a pessimistic M-type woman informs her that she belongs to a higher type than she believes. Hence, the indirect externality of pessimism does not occur. On the other hand, in the case of optimism, even if an M-type man accepts an L-type woman due to the existence of many optimistic M-type women, the acceptance of an L-type woman by an M-type man makes an L-type woman better off. Therefore, the indirect externality of optimism remains.[28]

When all agents know their own types under the cloning assumption and the assumption of non-transferable utility, a unique equilibrium always occurs (see Burdett and Coles (1997)). However, if there are agents with imperfect self-knowledge under the cloning assumption and the non-transferable utility, it is possible that multiple equilibria occur. From Propositions 2, 3, and 4, in some parameter ranges, multiple equilibria can occur; both the PSEI and the Type 2 can exist. To clarify this point, we consider the next example.

Example 1 *Let us assume that $2\alpha > 3r$ and that $F_m(x)$ and $F_w(x)$ are discrete uniform distributions: $\lambda_k^i = \frac{1}{3}$, $(i = m, w,\ k = H, M, L)$. At this time, the sufficient conditions*

[28] In the Type 1, as M-type men accept the lowest type of women (that is, L-type women), his offer carries no information about types of women. Then, an L- or M-type woman learns nothing from his offer. However, if there are a lower type than "L-type" and if an M-type man accepts an "L-type" woman, an L- or M-type woman will learn something about her own type from the acceptance by an M-type man. However, this acceptance will still make an L-type woman better off. Then, the indirect externality of optimism would remain in this case.

for the PSEI become $R_w^P(b_0) = \frac{(3r+\alpha)2\alpha x_H}{18r\alpha+2\alpha^2+27r^2} > x_M$ *and* $R_m^P(x_M) = \frac{\alpha x_M}{6r+\alpha} > R_w^P(b_{ML}) = \frac{(3r+\alpha)\alpha x_M}{12r\alpha+\alpha^2+18r^2} > x_L$ *from Proposition 2. Similarly, the sufficient conditions for the Type 1 are* $R_w^{T1}(b_0) = \frac{(3r+\alpha)\alpha x_H}{12r\alpha+\alpha^2+18r^2} > x_M$ *and* $R_m^{T1}(x_M) = \frac{\alpha x_M}{6r+\alpha} \leq x_L$ *from Proposition 3. The sufficient conditions for the Type 2 are* $x_M < R_w^{T2}(b_{HM}) = R_w^{T2}(b_0) = \frac{(3r+\alpha)7\alpha x_H}{66r\alpha+7\alpha^2+99r^2}$ *and* $R_m^{T2}(x_L) = \frac{\alpha x_M}{21r+\alpha} < R_w^{T2}(b_{ML}) = \frac{(3r+\alpha)2\alpha x_M}{26r\alpha+2\alpha^2+39r^2} \leq x_L < R_m^{T2}(x_M) = \frac{\alpha x_M}{7r+\alpha}$ *from Proposition 4. The Type 2 and the Type 1 do not hold because* $R_m^{T2}(x_M) - R_m^{T1}(x_M) = -\frac{r\alpha x_M}{(7r+\alpha)(6r+\alpha)} < 0$*. Moreover, as* $R_m^{T1}(x_M) = R_m^P(x_M)$*, the PSEI and the Type 1 do not hold either.*[29] *However, because* $x_M < R_w^{T2}(b_0)\,(< R_w^P(b_0))$ *and* $R_w^{T2}(b_{ML}) < x_L < R_w^P(b_{ML})\,(< R_m^{T2}(x_M))$*, the PSEI and the Type 2 hold.*

Figures 4 and 5 show how the outcomes in Example 1 depend on x_H, x_M and x_L while holding all other parameters constant. There is an overlap between the two equilibria for $x_M < R_w^{T2}(b_0)$ and $R_w^{T2}(b_{ML}) < x_L < R_w^P(b_{ML})$.

The intuition of multiple equilibria is as follows: marriage patterns are determined by all agents' expectations about the actions of agents with imperfect self-knowledge. If all agents expect that k_{ML}-type ($l = M, L$) women will accept L-type men, these expectations form $G_w^{T2}(x)$. Then, the marriage pattern of the Type 2 arises. On the other hand, if all agents expect that k_{ML}-type women will reject L-type men, the marriage pattern of the PSEI arises through $G_w^P(x)$.[30]

5 Discussion—two-sided imperfect self-knowledge

In this paper, we assume one-sided imperfect self-knowledge: no women initially know their own types, whereas all men know their own types.[31] This one-sided imperfect self-knowledge assumption is important in order to clarify the influence of imperfect self-knowledge. From Lemma 2, the uncertainty of an agent's own type affects her own expected life utility. Moreover, the existence of others with imperfect self-knowledge also affects agents' expected life utilities from Lemma 1. We can analyze these two influences on the expected life utility of an agent separately, under the assumption of the one-sided imperfect self-knowledge. The one-sided imperfect self-knowledge assumption describes the situations given below. In the context of the labor market, a firm has more information about its own type than a worker because the firm will generally have more experience than the worker. In the context of the marriage market, when more men work outside the home than women, it will be easier for men than for women to get the objective data on their own charm, such as income, position at work, and social status.

Although two-sided imperfect self-knowledge—all men and women initially lack knowledge of their own types—is a nontrivial extension, our results suggest that, if two-sided imperfect

[29] $R_m^{T1}(x_M) = R_m^P(x_M)$ always holds from Propositions 2 and 3.

[30] The welfare implication of these two steady states is obtained in Example 2 in Appendix C.

[31] In our analysis, a man and a woman are assumed to propose or reject a member of the opposite sex simultaneously. As we assume that all men know their own types and no women initially know their own types, similar results can also be obtained in the case of a sequential move in which a man proposes to a woman in the first move and she proposes or rejects him in the next move.

self-knowledge is assumed in the optimism case, the reservation level of any agent ("he") will be simultaneously affected by the following two factors: (i) the large share of optimistic women who now reject his type; and (ii) the uncertainty of his own type. The first element always lowers his reservation level from Lemma 1. For the second element, as we show in Lemma 2, his reservation level decreases or increases relative to that in the case of perfect self-knowledge. On the other hand, in the case of pessimism, only the uncertainty of his own type will affect his reservation level. Hence, two-sided imperfect self-knowledge will make the analysis more complex. Such work is left for future research.

6 Concluding remarks

We analyzed a two-sided search model in which we presumed that no women initially know their own types and then learn their own types from offers or rejections by men. With this learning process, the two-sided aspect of a search problem generated a significant interest. In particular, we showed that the optimism of some agents prevents the lowest-type agents from matching in an equilibrium. However, the pessimism of some agents does not affect the matching of the lowest-type agents.

Our model can also explain that the reservation level declines with the duration of search. Burdett and Vishwanath (1988) showed that when workers learn the unknown wage distribution, the reservation wage of an unemployed worker declines with his or her unemployment spell. Unlike their model, ours is a two-sided search and single agents know the type distribution but do not know their own types. The results in this paper showed that a high offer results in the agent getting married similarly to the finding by Burdett and Vishwanath (1988). However, if women with imperfect self-knowledge start to search with the strategies of accepting only the highest type of men, a woman revises her reservation level downward whenever she receives a rejection that has some information about her own type. On the other hand, an offer lower than the reservation level does not affect the decision of a woman with imperfect self-knowledge. An offer much lower than the reservation level is proposed by a man whom the woman with imperfect self-knowledge rejects. Thus, her decision regarding whether to accept or not another type of men is not affected by that offer. An offer from a man whom she decides whether to accept or not also does not affect her decision in equilibrium because the man chooses his strategy so as not to raise her reservation level.

We conclude with a discussion of some possible further extensions of this model. First, this paper assumes that there is no divorce. However, when a woman marries a man before thoroughly understanding her own type, she may learn about her actual type after she gets married. In this case, the divorce rate will be influenced by this learning in marriage.

Next, we assume three types of agents. If we consider a model in which there are n types of agents and many clusters of marriages, the learning process about one's own type will be more complex. However, if there are n types of agents and three clusters of marriages are generated by a large enough α, our results also apply to this case.

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Appendix A.1

Appendix A.1.1. Beliefs and the distribution of beliefs of women in the PSEI As actions of each type of men are the same as those in the PSE, learning processes of women are described as in Figure 1. The outline box for each type in Figure 1 represents the share of each type of women $\lambda_k^w, (k = H, M, L)$. For example, if an H_0-type woman meets an H-type man, then she learns that she is an H-type, leaving the market with him. After another H_0-type woman meets an M-type man, she becomes an H_{HM}-type woman. As a result, there are two kinds of H-type women according to different beliefs: H_0-type and H_{HM}-type. Here, let $\theta \in (0, 1)$ denote the share of H_{HM}-type women in H-type women. Similarly, we can consider the learning process of M-type and L-type women (see Figure 1). Here, for the M-type women, let $\mu \in (0, 1)$, $\gamma(1-\phi) \in (0, 1)$, and $\gamma\phi \in (0, 1)$ denote the share of M_{ML}-type women, M_{HM}-type women, and M_M-type women in M-type women, respectively. For the L-type women, let $\psi(1-\nu) \in (0, 1)$, $\psi\nu \in (0, 1)$, and $\kappa \in (0, 1)$ denote the share of L_{ML}-type women, L_{L1}-type women, and L_{L2}-type women in L-type women, respectively. From these shares $(\theta, \mu, \gamma, \phi, \psi, v, k)$ and λ_k^w, the stationary distribution $G_w^P(x)$ is given.

Moreover, in a steady state, (MEI-iii) is required. That is, in Figure 1,

$$\alpha\lambda_M^m (1-\theta) \lambda_H^w = \alpha\lambda_H^m \theta \lambda_H^w, \tag{12}$$

$$\alpha\lambda_H^m (1-\mu-\gamma) \lambda_M^w = \alpha\lambda_M^m \mu \lambda_M^w, \tag{13}$$

$$\alpha\lambda_M^m (1-\mu-\gamma) \lambda_M^w = \alpha\lambda_H^m \gamma (1-\phi) \lambda_M^w = \alpha\lambda_M^m \gamma\phi\lambda_M^w, \tag{14}$$

$$\alpha\lambda_H^m (1-\psi-\kappa) \lambda_L^w = \alpha\lambda_M^m \psi (1-\nu) \lambda_L^w = \alpha\lambda_L^m \psi v \lambda_L^w, \tag{15}$$

$$\alpha\lambda_M^m (1-\psi-\kappa) \lambda_L^w = \alpha\lambda_L^m \kappa \lambda_L^w. \tag{16}$$

From (12) to (16), we obtain

$$\theta = \gamma = \frac{\lambda_M^m}{\lambda_H^m + \lambda_M^m}, \tag{17}$$

$$\mu = \frac{(\lambda_H^m)^2}{(\lambda_H^m + \lambda_M^m)^2}, \ \phi = \psi = \frac{\lambda_H^m}{(\lambda_H^m + \lambda_M^m)}, \tag{18}$$

$$\nu = \frac{\lambda_M^m}{(\lambda_L^m + \lambda_M^m)}, \ \kappa = \frac{(\lambda_M^m)^2}{(\lambda_L^m + \lambda_M^m)(\lambda_H^m + \lambda_M^m)}. \tag{19}$$

The (actual) type distribution of the new single women consists of the shares of k_0-type women ($k = H, M, L$). From Figure 1, the prior belief of a woman consists of $b_0^P(x_H) = \frac{(1-\theta)\lambda_H^w}{(1-\theta)\lambda_H^w + (1-\mu-\gamma)\lambda_M^w + (1-\psi-\kappa)\lambda_L^w}$, $b_0^P(x_M) = \frac{(1-\mu-\gamma)\lambda_M^w}{(1-\theta)\lambda_H^w + (1-\mu-\gamma)\lambda_M^w + (1-\psi-\kappa)\lambda_L^w}$, and $b_0^P(x_L) = \frac{(1-\psi-\kappa)\lambda_L^w}{(1-\theta)\lambda_H^w + (1-\mu-\gamma)\lambda_M^w + (1-\psi-\kappa)\lambda_L^w}$. Substituting (17)–(19) into these three equations, we obtain

$$b_0^P(x_H) = \frac{\lambda_H^m \lambda_H^w (\lambda_H^m + \lambda_M^m)(\lambda_L^m + \lambda_M^m)}{\lambda_H^m (\lambda_H^m \lambda_H^w + \lambda_H^w \lambda_M^m + \lambda_M^m \lambda_M^w)(\lambda_L^m + \lambda_M^m) + (\lambda_H^m + \lambda_M^m)\lambda_L^m \lambda_L^w \lambda_M^m},$$

$$b_0^P(x_M) = \frac{(\lambda_L^m + \lambda_M^m)\lambda_H^m \lambda_M^m \lambda_M^w}{\lambda_H^m (\lambda_H^m \lambda_H^w + \lambda_H^w \lambda_M^m + \lambda_M^m \lambda_M^w)(\lambda_L^m + \lambda_M^m) + (\lambda_H^m + \lambda_M^m)\lambda_L^m \lambda_L^w \lambda_M^m},$$

$$b_0^P(x_L) = \frac{(\lambda_H^m+\lambda_M^m)\lambda_L^m\lambda_L^w\lambda_M^m}{\lambda_H^m(\lambda_H^m\lambda_H^w+\lambda_H^w\lambda_M^m+\lambda_M^m\lambda_M^w)(\lambda_L^m+\lambda_M^m)+(\lambda_H^m+\lambda_M^m)\lambda_L^m\lambda_L^w\lambda_M^m}.$$

Given the prior belief, by Bayes' rule, we obtain

$$b_{HM}^P(x_H) = \frac{\lambda_H^w(\lambda_H^m+\lambda_M^m)}{\lambda_H^w(\lambda_H^m+\lambda_M^m)+\lambda_M^m\lambda_M^w}, \tag{20}$$

$$b_{HM}^P(x_M) = \frac{\lambda_M^m\lambda_M^w}{\lambda_H^m\lambda_H^w+\lambda_H^w\lambda_M^m+\lambda_M^m\lambda_M^w},$$

$$b_{ML}^P(x_M) = \frac{\lambda_H^m\lambda_M^w(\lambda_L^m+\lambda_M^m)}{\lambda_L^m\lambda_L^w(\lambda_H^m+\lambda_M^m)+\lambda_H^m\lambda_M^w(\lambda_L^m+\lambda_M^m)}, \tag{21}$$

$$b_{ML}^P(x_L) = \frac{\lambda_L^m\lambda_L^w(\lambda_H^m+\lambda_M^m)}{\lambda_L^m\lambda_L^w(\lambda_H^m+\lambda_M^m)+\lambda_H^m\lambda_M^w(\lambda_L^m+\lambda_M^m)},$$

$$b_{HM}^P(x_L) = b_{ML}^P(x_H) = 0,$$

$$b_M^P(x_M) = b_L^P(x_L) = 1.$$

It is noteworthy that from Figure 1, we can confirm that these beliefs are consistent with $G_w^P(x)$.[32]

Appendix A.1.2. Beliefs and the distribution of beliefs of women in the Type 1 As an M- or L-type man accepts any woman, the woman who receives his proposal learns nothing about her own type. Therefore, only when a woman meets an H-type man does she learn something about her own type. Figure 2 describes the women's learning processes. From Figure 2, there are five kinds of women according to different beliefs: k_0-type women ($k = H, M, L$) and k_{ML}-type women ($k = M, L$). Here, $\tau \in (0,1)$ and $\varpi \in (0,1)$ denote the share of M_{ML}-type women in M-type women and that of L_{ML}-type women in L-type women, respectively.

(MEI-iii) requires that

$$\alpha\lambda_H^m(1-\tau)\lambda_M^w = \alpha\lambda_M^m\tau\lambda_M^w, \tag{22}$$

$$\alpha\lambda_H^m(1-\varpi)\lambda_L^w = \alpha\lambda_M^m\varpi\lambda_L^w, \tag{23}$$

hold in Figure 2. From these equations, we obtain

$$\tau = \varpi = \frac{\lambda_H^m}{\lambda_H^m+\lambda_M^m}. \tag{24}$$

Therefore, the prior belief of a woman consists of $b_0^{T1}(x_H) = \frac{\lambda_H^w}{\lambda_H^w+(1-\tau)(\lambda_M^w+\lambda_L^w)}$, $b_0^{T1}(x_M) = \frac{(1-\tau)\lambda_M^w}{\lambda_H^w+(1-\tau)(\lambda_M^w+\lambda_L^w)}$, and $b_0^{T1}(x_L) = \frac{(1-\tau)\lambda_L^w}{\lambda_H^w+(1-\tau)(\lambda_M^w+\lambda_L^w)}$. If we substitute (22) and (23) into

[32] As we employ the cloning assumption, one can confirm that $b_{HM}^P(x_H) = \frac{\theta\lambda_H^w}{\theta\lambda_H^w+\gamma(1-\phi)\lambda_M^w}$, $b_{HM}^P(x_M) = \frac{\gamma(1-\phi)\lambda_M^w}{\theta\lambda_H^w+\gamma(1-\phi)\lambda_M^w}$, $b_{ML}^P(x_M) = \frac{\mu\lambda_M^w}{\mu\lambda_M^w+\psi(1-\nu)\lambda_L^w}$, $b_{ML}^P(x_L) = \frac{\psi(1-\nu)\lambda_L^w}{\mu\lambda_M^w+\psi(1-\nu)\lambda_L^w}$, $b_M^P(x_M) = \frac{\gamma\phi\lambda_M^w}{\gamma\phi\lambda_M^w}$, and $b_L^P(x_L) = \frac{(\psi\upsilon+\kappa)\lambda_L^w}{(\psi\upsilon+\kappa)\lambda_L^w}$ hold.

these three equations, we obtain:

$$\begin{aligned} b_0^{T1}(x_H) &= \frac{\lambda_H^w(\lambda_H^m+\lambda_M^m)}{\lambda_H^m\lambda_H^w+\lambda_M^m}, \\ b_0^{T1}(x_M) &= \frac{\lambda_M^w\lambda_M^m}{\lambda_H^m\lambda_H^w+\lambda_M^m}, \\ b_0^{T1}(x_L) &= \frac{\lambda_L^w\lambda_M^m}{\lambda_H^m\lambda_H^w+\lambda_M^m} \end{aligned} \tag{25}$$

From Figure 2, we can confirm that these beliefs are consistent with $G_w^{T1}(x)$.

Appendix A.1.3. Beliefs and the distribution of beliefs of women in the Type 2 The information about women's types from proposals or rejections by men is also the same as that in the PSEI because the actions of men are the same as those in the PSEI. Then, the learning processes of women are also the same as those in the PSEI (see Figure 3). Let $\theta_2 \in (0,1)$, $\gamma_2(1-\phi_2) \in (0,1)$, $\gamma_2\phi_2 \in (0,1)$, $\mu_2 \in (0,1)$, $\psi_2(1-\nu_2) \in (0,1)$, $\psi_2\nu_2 \in (0,1)$, and $\kappa_2 \in (0,1)$ denote the share of H_{HM}-type women, that of M_{HM}-type women, that of M_M-type women, that of M_{ML}-type women, that of L_{ML}-type women, that of L_{L1}-type women, and that of L_{L2}-type women, respectively.

(MEI-iii) requires that in Figure 3,

$$\alpha\lambda_M^m(1-\theta_2)\lambda_H^w = \alpha\lambda_H^m\theta_2\lambda_H^w, \tag{26}$$
$$\alpha\lambda_H^m(1-\mu_2-\gamma_2)\lambda_M^w = \alpha(\lambda_M^m+\lambda_L^m)\mu_2\lambda_M^w, \tag{27}$$
$$\alpha\lambda_M^m(1-\mu_2-\gamma_2)\lambda_M^w = \alpha\lambda_H^m\gamma_2(1-\phi_2)\lambda_M^w = \alpha\lambda_M^m\mu_2\phi_2\lambda_M^w, \tag{28}$$
$$\alpha\lambda_H^m(1-\psi_2-\kappa_2)\lambda_L^w = \alpha(\lambda_M^m+\lambda_L^m)\psi_2(1-\nu_2)\lambda_L^w \tag{29}$$
$$\alpha\lambda_M^m\psi_2(1-\nu_2)\lambda_L^w = \alpha\lambda_L^m\psi_2\nu_2\lambda_L^w, \tag{30}$$
$$\alpha\lambda_M^m(1-\psi_2-\kappa_2)\lambda_L^w = \alpha\lambda_L^m\kappa_2\lambda_L^w. \tag{31}$$

All equations, except (27) and (29), have the same forms as (12) and (14)–(16) in the PSEI. From (26) to (31), we obtain

$$\theta_2 = \frac{\lambda_M^m}{\lambda_H^m+\lambda_M^m}, \tag{32}$$
$$\mu_2 = \frac{(\lambda_H^m)^2}{\lambda_H^m+(\lambda_M^m+\lambda_L^m)(\lambda_H^m+\lambda_M^m)}, \tag{33}$$
$$\gamma_2 = \frac{(\lambda_M^m+\lambda_L^m)(\lambda_H^m+\lambda_M^m)}{\lambda_H^m+(\lambda_M^m+\lambda_L^m)(\lambda_H^m+\lambda_M^m)}, \phi_2 = \frac{\lambda_H^m}{(\lambda_M^m+\lambda_H^m)}, \tag{34}$$
$$\psi_2 = \lambda_H^m,\ \nu_2 = \frac{\lambda_M^m}{(\lambda_M^m+\lambda_L^m)},\ \kappa_2 = \lambda_M^m. \tag{35}$$

Then, the prior belief of a woman consists of $b_0^{T2}(x_H) = \frac{(1-\theta_2)\lambda_H^w}{(1-\theta_2)\lambda_H^w+(1-\mu_2-\gamma_2)\lambda_M^w+(1-\psi_2-\kappa_2)\lambda_L^w}$, $b_0^{T2}(x_M) = \frac{(1-\mu_2-\gamma_2)\lambda_M^w}{(1-\theta_2)\lambda_H^w+(1-\mu_2-\gamma_2)\lambda_M^w+(1-\psi_2-\kappa_2)\lambda_L^w}$, and $b_0^{T2}(x_L) = \frac{(1-\psi_2-\kappa_2)\lambda_L^w}{(1-\theta_2)\lambda_H^w+(1-\mu_2-\gamma_2)\lambda_M^w+(1-\psi_2-\kappa_2)\lambda_L^w}$.

If we substitute (32)–(35) into these three equations, we obtain:

$$b_0^{T2}(x_H) = \frac{\lambda_H^w \lambda_H^m (\lambda_H^m + \lambda_H^m \lambda_L^m + \lambda_M^m)}{(\lambda_H^m + \lambda_M^m + \lambda_H^m \lambda_L^m)(\lambda_H^m \lambda_H^w + (\lambda_H^m \lambda_L^w + \lambda_L^m \lambda_M^m)\lambda_L^w) + \lambda_M^w \lambda_H^m (\lambda_M^m + \lambda_L^m)(\lambda_H^m + \lambda_M^m)},$$
$$b_0^{T2}(x_M) = \frac{\lambda_M^w \lambda_H^m (\lambda_M^m + \lambda_L^m)(\lambda_H^m + \lambda_M^m)}{(\lambda_H^m + \lambda_M^m + \lambda_H^m \lambda_L^m)(\lambda_H^m \lambda_H^w + (\lambda_H^m \lambda_L^w + \lambda_L^m \lambda_M^m)\lambda_L^w) + \lambda_M^w \lambda_H^m (\lambda_M^m + \lambda_L^m)(\lambda_H^m + \lambda_M^m)},$$
$$b_0^{T2}(x_L) = \frac{\lambda_L^w \lambda_L^m (\lambda_H^m + \lambda_M^m)(\lambda_H^m + \lambda_H^m \lambda_L^m + \lambda_M^m)}{(\lambda_H^m + \lambda_M^m + \lambda_H^m \lambda_L^m)(\lambda_H^m \lambda_H^w + (\lambda_H^m \lambda_L^m + \lambda_L^w \lambda_M^m)\lambda_L^w) + \lambda_M^w \lambda_H^m (\lambda_M^m + \lambda_L^m)(\lambda_H^m + \lambda_M^m)}.$$

Given the prior belief, we obtain

$$b_{HM}^{T2}(x_H) = \frac{\lambda_H^w (\lambda_H^m + (\lambda_H^m + \lambda_M^m)(\lambda_L^m + \lambda_M^m))}{\lambda_H^m \lambda_H^w + (\lambda_L^m + \lambda_M^m)(\lambda_H^m + \lambda_M^m)(\lambda_H^w + \lambda_M^w)}, \tag{36}$$
$$b_{HM}^{T2}(x_M) = \frac{\lambda_M^w (\lambda_L^m + \lambda_M^m)(\lambda_H^m + \lambda_M^m)}{\lambda_H^m \lambda_H^w + (\lambda_L^m + \lambda_M^m)(\lambda_H^m + \lambda_M^m)(\lambda_H^w + \lambda_M^w)},$$
$$b_{ML}^{T2}(x_M) = \frac{\lambda_H^m \lambda_M^w (\lambda_L^m + \lambda_M^m)}{\lambda_L^w \lambda_L^m (\lambda_H^m + \lambda_H^m \lambda_L^m + \lambda_M^m) + \lambda_H^m \lambda_M^w (\lambda_L^m + \lambda_M^m)}, \tag{37}$$
$$b_{ML}^{T2}(x_L) = \frac{\lambda_L^w \lambda_L^m (\lambda_H^m + \lambda_H^m \lambda_L^m + \lambda_M^m)}{\lambda_L^w \lambda_L^m (\lambda_H^m + \lambda_H^m \lambda_L^m + \lambda_M^m) + \lambda_H^m \lambda_M^w (\lambda_L^m + \lambda_M^m)},$$
$$b_{HM}^{T2}(x_L) = b_{ML}^{T2}(x_H) = 0,$$
$$b_M^{T2}(x_M) = b_L^{T2}(x_L) = 1.$$

From Figure 3, we can confirm that these beliefs are consistent with $G_w^{T2}(x)$.

Appendix A.2

Proof of Proposition 1: First, we consider the decision of an H-type agent. He (she) decides whether to accept or not a woman (a man) of the M- or L-type. From (2), the expected discounted lifetime utility of a single H-type agent $V_i(x_H), (i = m, w)$ becomes

$$rV_i(x_H) = \alpha\lambda_H^j \left(\tfrac{x_H}{r} - V_i(x_H)\right) + \alpha\lambda_M^j \left[\max\left(V_i(x_H), \tfrac{x_M}{r}\right) - V_i(x_H)\right] + \alpha\lambda_L^j \left[\max\left(V_i(x_H), \tfrac{x_L}{r}\right) - V_i(x_H)\right]. \tag{38}$$

An H-type agent $(i = m, w)$ meets an H-type agent of the opposite sex j $(= w, m)$ with probability $\alpha\lambda_H^j$, and they marry. However, if an H-type agent meets an M- (L-) type agent of the opposite sex with probability $\alpha\lambda_M^j$ ($\alpha\lambda_L^j$), he or she compares x_M/r (x_L/r) and $V_i(x_H)$ and then decides whether or not to propose.

If an H-type agent turns down an M-type agent of the opposite sex j, $V_i(x_H) > \frac{x_M}{r}$. From (38), this H-type agent's discounted lifetime utility when he or she is single becomes

$$rV_i^r(x_H) = \alpha\lambda_H^j \left(\tfrac{x_H}{r} - V_i^r(x_H)\right).$$

On the other hand, when he or she accepts an M-type agent j and turns down an L-type agent j, i.e., $\frac{x_M}{r} \geq V_i(x_H) > \frac{x_L}{r}$, his or her value function is[33]

[33] If an H-type agent proposes to an M-type agent but turns down an L-type agent ($x_L/r < V_H \leq x_M/r$),

$$rV_i^a(x_H) = \alpha\lambda_H^j\left(\tfrac{x_H}{r} - V_i^a(x_H)\right) + \alpha\lambda_M^j\left(\tfrac{x_M}{r} - V_i^a(x_H)\right).$$

If $V_i^r(x_H) > V_i^a(x_H)$ is satisfied, an H-type agent refuses an M-type opposite sex agent j. This inequality $V_i^r(x_H) > V_i^a(x_H)$ means that

$$x_M < R_i^*(x_H) \equiv \tfrac{\alpha\lambda_H^j x_H}{\alpha\lambda_H^j + r}.$$

If $x_M \geq R_i^*(x_H)$, an H-type agent proposes to an M-type agent i.

Under inequality (3), we can obtain the condition for an M-type agent to reject an L-type opposite sex agent j by the same process as that described above. Consequently, we have

$$x_L < R_i^*(x_M) \equiv \tfrac{\alpha\lambda_M^j x_M}{\alpha\lambda_M^j + r}.$$

■

Proof of Lemma 1: The reservation level of an M-type man for an L-type woman can be calculated as follows: now, a fraction $\eta \in (0,1)$ of M-type women and a fraction $\zeta \in (0,1)$ of L-type women accept M-type men. If an M-type man turns down an L-type woman ($V_m^r(x_M) > x_L/r$), his value function becomes

$$rV_m^r(x_M) = \alpha\eta\lambda_M^w\left(\tfrac{x_M}{r} - V_m^r(x_M)\right).$$

Conversely, when an M-type man proposes to an L-type woman ($V_m^a(x_M) \leq x_L/r$)

$$rV_m^a(x_M) = \alpha\eta\lambda_M^w\left(\tfrac{x_M}{r} - V_m^a(x_M)\right) + \alpha\zeta\lambda_L^w\left(\tfrac{x_L}{r} - V_m^a(x_M)\right).$$

Hence, we have his reservation utility level for declining an L-type woman, $\frac{\alpha\eta\lambda_M^w x_M}{\left(r+\alpha\eta\lambda_M^w\right)} \equiv R_m^P(x_M)$. Compared to the PSE, we have

$$R_m^P(x_M) - R_m^*(x_M) = -\tfrac{r\alpha\lambda_M^w x_M(1-\eta)}{\left(r+\alpha\lambda_M^w\right)\left(r+\alpha\eta\lambda_M^w\right)} < 0.$$

From (17) to (19) and Figure 1, $\eta = \mu + \gamma\phi = \frac{\lambda_H^m}{\lambda_H^m + \lambda_M^m}$ and $\zeta = \psi + \kappa = \frac{\lambda_H^m\lambda_L^m + \lambda_H^m\lambda_M^m + \lambda_M^{2m}}{\left(\lambda_L^m + \lambda_M^m\right)\left(\lambda_H^m + \lambda_M^m\right)}$ hold in inequality (6) in the PSEI. ■

Proof of Lemma 2: The reservation level of a k_{ML}-type woman for an L-type man can be calculated as follows. The expected discounted lifetime utility of a k_{ML}-type woman can be written as

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the high- and M-type agents receive at least the same number of offers. Hence, $V_H \geq V_M$, and then we have $V_M \leq x_M/r$. Namely, an M-type agent wishes to marry another M-type agent. Similarly, if an M-type agent accepts an L-type agent ($V_M \leq x_L/r$), the middle- and L-type agents receive at least the same number of offers. Then, an L-type agent also wants to marry another L-type agent.

$$rV_w(b_{ML}) = b^P_{ML}(x_M)\left[\alpha\lambda^m_M\left(\tfrac{x_M}{r} - V_w(b_{ML})\right) + \alpha\lambda^m_L\left(\max\left\{\tfrac{x_L}{r}, V_w(b_{ML})\right\} - V_w(b_{ML})\right)\right]$$
$$+ b^P_{ML}(x_L)\left[\alpha\lambda^m_M(V_w(b_L) - V_w(b_{ML})) + \alpha\lambda^m_L\left(\max\left\{\tfrac{x_L}{r}, V_w(b_{ML})\right\} - V^r_w(b_{ML})\right)\right] \tag{39}$$

$$rV_w(b_L) = \alpha\lambda^m_L\left(\tfrac{x_L}{r} - V_w(b_L)\right). \tag{40}$$

The first term in the second square bracket in Equation (39) means that, if the actual type of a k_{ML}-type woman is an L-type, she learns that she is an L-type by meeting an M-type man. She then changes her value function to (40) because she is accepted by an L-type man.

From (39) and (40), the reservation level of a k_{ML}-type woman for an L-type man is

$$R^P_w(b_{ML}) \equiv \frac{\lambda^m_M\alpha x_M\left(r+\alpha\lambda^m_L\right)\left(b^P_{ML}(x_M)\right)}{r\left(r+\alpha\lambda^m_L+\alpha\lambda^m_M\right)+\alpha^2\lambda^m_L\lambda^m_M\left(b^P_{ML}(x_M)\right)}.$$

Compared to the benchmark case, we have

$$R^P_w(b_{ML}) - R^*_w(x_M) = -\frac{r\lambda^m_M\alpha x_M\left(r+\alpha\lambda^m_L+\alpha\lambda^m_M\right)(1-\left(b^P_{ML}(x_M)\right))}{\left(r+\alpha\lambda^m_M\right)(r\left(r+\alpha\lambda^m_L+\alpha\lambda^m_M\right)+\alpha^2\lambda^m_L\lambda^m_M\left(b^P_{ML}(x_M)\right))} < 0.$$

On the other hand, the reservation level of a k_{HM}-type woman for an M-type man can be calculated as follows. The expected discounted lifetime utility of a k_{HM}-type woman becomes

$$rV_w(b_{HM}) = b^P_{HM}(x_H)\left[\alpha\lambda^m_H\left(\tfrac{x_H}{r} - V_w(b_{HM})\right) + \alpha\lambda^m_M\left(\max\left\{\tfrac{x_M}{r}, V_w(b_{HM})\right\} - V_w(b_{HM})\right)\right]$$
$$+b^P_{HM}(x_M)\left[\alpha\lambda^m_H(V_w(b_M) - V_w(b_{HM})) + \alpha\lambda^m_M\left(\max\left\{\tfrac{x_M}{r}, V_w(b_{HM})\right\} - V_w(b_{HM})\right)\right] \tag{41}$$
$$rV_w(b_M) = \alpha\lambda^m_M\left(\tfrac{x_M}{r} - V_w(b_M)\right). \tag{42}$$

The first term in the second square bracket in (41) implies that, if the actual type of a k_{HM}-type woman is the M-type, she learns that she is an M-type after a meeting with an H-type man. She then changes her value function to (42) as M-type men accept M-type women.

Therefore, the reservation level of a k_{HM}-type woman for an M-type man is

$$R^P_w(b_{HM}) \equiv \frac{\lambda^m_H\alpha x_H\left(r+\alpha\lambda^m_M\right)\left(b^P_{HM}(x_H)\right)}{r\left(r+\alpha\lambda^m_H+\alpha\lambda^m_M\right)+\alpha^2\lambda^m_H\lambda^m_M\left(b^P_{HM}(x_H)\right)}.$$

Compared to the benchmark case, we have

$$R^P_w(b_{HM}) - R^*_w(x_H) = -\frac{r\lambda^m_H\alpha x_H\left(r+\alpha\lambda^m_H+\alpha\lambda^m_M\right)(1-\left(b^P_{HM}(x_H)\right))}{\left(r+\alpha\lambda^m_H\right)\left(r^2+r\alpha\lambda^m_H+r\alpha\lambda^m_M+\alpha^2\lambda^m_H\lambda^m_M\left(b^P_{HM}(x_H)\right)\right)} < 0.$$

Given the strategies of all agents except k_0-type women, we can obtain the lifetime utility of a k_0-type woman. The expected discounted lifetime utility of a k_0-type woman becomes

$$
\begin{aligned}
rV_w(b_0) &= b_0^P(x_H)\left[\alpha\lambda_H^m\left(\tfrac{x_H}{r}-V_w(b_0)\right)+\alpha\lambda_M^m\left(\max\left\{\tfrac{x_M}{r},V_w(b_{HM})\right\}-V_w(b_0)\right)\right] \\
&\quad +b_0^P(x_M)\left[\alpha\lambda_H^m\left(V_w(b_{ML})-V_w(b_0)\right)+\alpha\lambda_M^m\left(\max\left\{\tfrac{x_M}{r},V_w(b_{HM})\right\}-V_w(b_0)\right)\right] \\
&\quad +b_0^P(x_L)\left[\alpha\lambda_H^m\left(V_w(b_{ML})-V_w(b_0)\right)+\alpha\lambda_M^m\left(V_w(b_L)-V_w(b_0)\right)\right]. \qquad (43)
\end{aligned}
$$

The third term in the first (or the second) square bracket in Equation (43) means that, if a k_0-type woman is actually the H- (or M-) type and if she rejects an M-type man, she learns that she is the H- or the M-type after meeting an M-type man. She then changes her optimal strategy to $R_w^P(b_{HM})$. On the other hand, if a k_0-type woman is actually the H- (or M-) type and if a k_0-type woman accepts an M-type man, the third term in the first (or the second) square bracket in Equation (43) becomes $\frac{x_M}{r}$.

Therefore, the reservation level of a k_0-type woman for an M-type man is

$$R_w^P(b_0)=R_w^P(b_{HM})>(\leq)\,x_M.$$

If $R_w^P(b_0)>(\leq)\,x_M$, a k_0-type woman rejects (accepts) an M-type man.

From (20) and (21), $b_{ML}^P(x_M)=\frac{\lambda_H^m\lambda_M^w(\lambda_L^m+\lambda_M^m)}{\lambda_L^m\lambda_L^w(\lambda_H^m+\lambda_M^m)+\lambda_H^m\lambda_M^w(\lambda_L^m+\lambda_M^m)}$ and $b_{HM}^P(x_H)=\frac{\lambda_H^w(\lambda_H^m+\lambda_M^m)}{\lambda_H^w(\lambda_H^m+\lambda_M^m)+\lambda_M^m\lambda_M^w}$ hold in steady state.■

Proof of Lemma 3: As an M-type man accepts an L-type woman, an M_{ML}-type woman and an L_{ML}-type woman face the same problem. They decide whether to accept or not L-type men. Therefore, the value function of a k_{ML}-type ($k=M,L$) woman is

$$rV_w(b_{ML})=\alpha\lambda_M^m\left(\tfrac{x_M}{r}-V_w(b_{ML})\right)+\alpha\lambda_L^m\left(\max\left\{\tfrac{x_L}{r},V_w(b_{ML})\right\}-V_w(b_{ML})\right). \qquad (44)$$

Therefore, the reservation level of a k_{ML}-type woman for an L-type man is

$$R_w^{T1}(b_{ML})\equiv\frac{\alpha\lambda_M^m x_M}{r+\alpha\lambda_M^m}=R_w^*(x_M).$$

As $x_L<R_w^*(x_M)$, $x_L<R_w^P(b_{ML})$ holds. Hence, a k_{ML}-type woman turns down an L-type man.

Given the strategies of all agents except a k_0-type woman, we can obtain the lifetime utility of a k_0-type ($k=H,M,L$) woman. As an M-type man accepts an L-type woman, an M_0-type woman and an L_0-type woman face the same problem. Then, the value function of a k_0-type woman is

$$
\begin{aligned}
rV_w(b_0) &= b_0^{T1}(x_H)\left[\alpha\lambda_H^m\left(\tfrac{x_H}{r}-V_w(b_0)\right)+\alpha\lambda_M^m\left(\max\left\{\tfrac{x_M}{r},V_w(b_0)\right\}-V_w(b_0)\right)\right] \\
&\quad +\left(1-b_0^{T1}(x_H)\right)\left[\alpha\lambda_H^m\left(V_w^r(b_{ML})-V_w(b_0)\right)+\alpha\lambda_M^m\left(\max\left\{\tfrac{x_M}{r},V_w(b_0)\right\}-V_w(b_0)\right)\right] \qquad (45)
\end{aligned}
$$

The second term in Equation (45) means that, if the actual type of a k_0-type woman is the M- or L-type, she becomes a k_{ML}-type woman after meeting an H-type man. She then changes her optimal strategy to $R_w^{T1}(b_{ML}) > x_L$.

Therefore, the reservation level of a k_0-type woman for an M-type man is

$$R_w^{T1}(b_0) \equiv \frac{\lambda_H^m \alpha x_H \left(r+\alpha\lambda_M^m\right) b_0^{T1}(x_H)}{r\left(r+\alpha\lambda_H^m+\alpha\lambda_M^m\right)+\alpha^2\lambda_H^m\lambda_M^m b_0^{T1}(x_H)}.$$

Compared to the benchmark case, we have

$$R_w^{T1}(b_0) - R_w^*(x_H) = -\frac{\left(r\lambda_H^m \alpha x_H \left(r+\alpha\lambda_H^m+\alpha\lambda_M^m\right)\right)(1-b_0^{T1}(x_H))}{\left(r+\alpha\lambda_H^m\right)(r^2+r\alpha\lambda_H^m+r\alpha\lambda_M^m+\alpha^2\lambda_H^m\lambda_M^m b_0^{T1}(x_H))} < 0.$$

From (25), $b_0^{T1}(x_H) = \frac{\lambda_H^w\left(\lambda_H^m+\lambda_M^m\right)}{\lambda_H^m\lambda_H^w+\lambda_M^m}$ holds in the Type 1.■

Proof of Lemma 4: We can consider the following three cases for the pessimistic behavior of a woman with imperfect self-knowledge: (I) an M_{ML}-type woman accepts an L-type man; (II) an H_0- or H_{HM}-type woman rejects an L-type man; and (III) a k_0- or k_{HM}-type ($k = H, M$) woman accepts an L-type man.

First, we consider the case of (I). Let us assume that k_{ML}-type ($k = M, L$) women accept L-type men. Furthermore, because there are enough pessimistic M_{ML}-type women for an L-type man, L-type men reject L-type women. From $R_m^*(x_H) > x_M$, H-type men reject M-type women. In addition, to clear the influence of pessimism, we assume that M-type men reject L-type women.

Given the strategies of all men, the expected discounted lifetime utility of a k_{ML}-type woman can be written as

$$\begin{aligned} rV_w(b_{ML}) &= b_{ML}(x_M)\left[\alpha\lambda_M^m\left(\tfrac{x_M}{r} - V_w(b_{ML})\right) + \alpha\lambda_L^m\left(\max\left\{\tfrac{x_L}{r}, V_w(b_M)\right\} - V_w(b_{ML})\right)\right] \\ &\quad + b_{ML}(x_L)\left[\alpha\left(\lambda_M^m + \lambda_L^m\right)\left(0 - V_w(b_{ML})\right)\right]. \end{aligned}$$

As $R_w^*(b_M) = R_w^*(x_M) > x_L$, $\frac{x_L}{r} < V_w(b_M)$. That is, a k_{ML}-type woman rejects an L-type man when he rejects an L-type woman. This contradicts the assumption that an M_{ML}-type woman accepts an L-type man. Therefore, even if there are enough M_{ML}-type women who accept L-type men, an L-type man always accepts an L-type woman.

Next, we consider the case of (II). Let us assume that M-type men reject M-type women because there are enough pessimistic H_0-type women who accept M-type men. From $R_m^*(x_H) > x_M$, H-type men reject M-type women. At this time, from (5), the expected discounted lifetime utility of a k_0-type woman can be written as

$$\begin{aligned} rV_w(b_0) &= b_0(x_H)\left[\begin{array}{c} \alpha\lambda_H^m\left(\frac{x_H}{r} - V_w(b_0)\right) + \alpha\lambda_M^m\left(\max\left\{\frac{x_M}{r}, V_w(b_H)\right\} - V_w(b_0)\right) \\ +\alpha\lambda_L^m\left(V_w\left(b_{l|(x_L,a_m^t(x_H))}\right) - V_w(b_0)\right) \end{array}\right] \\ &\quad + b_0(x_M)\left[\alpha\left(\lambda_H^m + \lambda_M^m\right)\left(V_w(b_{ML}) - V_w(b_0)\right) + \alpha\lambda_L^m\left(V_w\left(b_{l|(x_L,a_m^t(x_M))}\right) - V_w(b_0)\right)\right] \\ &\quad + b_0(x_L)\left[\alpha\left(\lambda_H^m + \lambda_M^m\right)\left(V_w(b_{HL}) - V_w(b_0)\right) + \alpha\lambda_L^m\left(V_w\left(b_{l|(x_L,a_m^t(x_L))}\right) - V_w(b_0)\right)\right], \end{aligned}$$

where $V_w\left(b_{l|(x_L,a_m^t(x_k))}\right)$ depends on the strategy of an L-type man.[34] However, from the

[34] If an L-type man rejects an L-type woman, then

above equation, learning by the action of an L-type man does not affect her decision regarding whether to accept or not an M-type man. Therefore, the problem, whether a k_0-type woman accepts an M-type man or not, reduces to the comparison between $\frac{x_M}{r}$ and $V_w(b_H)$.[35] As $x_M < R_i^*(x_H)$, $\frac{x_M}{r} < V_w(b_H) = V(x_H)$, i.e., a k_0-type woman rejects an M-type man. This contradicts the assumption that a k_0-type woman accepts an M-type man.

Similarly, let us assume that M-type men reject M-type women because there are enough pessimistic H_{HM}-type women for an M-type man. In this case, L-type men reject L-type women and accept M-type women. Otherwise, there are no k_{HM}-women in the market. The expected discounted lifetime utility of an H_{HM}-type woman becomes

$$\begin{aligned} rV_w(b_{HM}) &= b_{HM}(x_H)\left[\alpha\lambda_H^m\left(\tfrac{x_H}{r} - V_w(b_{HM})\right) + \alpha\lambda_M^m\left(\max\left\{\tfrac{x_M}{r}, V_w(b_H)\right\} - V_w(b_{HM})\right)\right] \\ &\quad + b_{HM}(x_M)\left[\alpha\left(\lambda_H^m + \lambda_M^m\right)\left(V_w(b_M) - V_w(b_{HM})\right)\right]. \end{aligned}$$

Therefore, the decision of a k_{HM}-type woman is the same as that of a k_0-type woman. Hence, a k_{HM}-type woman rejects an M-type man. However, this contradicts the assumption that a k_{HM}-type woman accepts an M-type man. Therefore, when there are pessimistic H_0- or H_{HM}-type women who accept M-type men, M-type men always accept M-type women.

Finally, we consider the case of (III). Let us assume that k_0-type women accept L-type men. Furthermore, L-type men reject L-type women because there are enough pessimistic k_0-type ($k = H, M$) women for an L-type man. From $R_m^*(x_H) > x_M$, H-type men reject M-type women. From the results in the case of (II), M-type men always accept M-type women even if there are enough pessimistic H_0-type women.[36] Because L-type men reject L-type women, M-type men also reject L-type women.

At this time, the expected discounted lifetime utility of a k_0-type woman can be written as

$$\begin{aligned} rV_w(b_0) &= b_0(x_H)\left[\begin{array}{c} \alpha\lambda_H^m\left(\frac{x_H}{r} - V_w(b_0)\right) + \alpha\lambda_M^m\left(\frac{x_M}{r} - V_w(b_0)\right) \\ +\alpha\lambda_L^m\left(\max\left\{\frac{x_L}{r}, V_w(b_{HM})\right\} - V_w(b_0)\right) \end{array}\right] \\ &\quad + b_0(x_M)\left[\begin{array}{c} \alpha\lambda_H^m\left(V_w(b_{ML}) - V_w(b_0)\right) + \alpha\lambda_M^m\left(\frac{x_M}{r} - V_w(b_0)\right) \\ +\alpha\lambda_L^m\left(\max\left\{\frac{x_L}{r}, V_w(b_{HM})\right\} - V_w(b_0)\right) \end{array}\right] \\ &\quad + b_0(x_L)\left[\alpha\lambda_H^m\left(V_w(b_{ML}) - V_w(b_0)\right) + \alpha\left(\lambda_M^m + \lambda_L^m\right)\left(0 - V_w(b_0)\right)\right]. \end{aligned}$$

Therefore, the problem, whether a k_0-type woman accepts an L-type man or not, reduces to the comparison between $\frac{x_L}{r}$ and $V_w(b_{HM})$. The expected discounted lifetime utility of a

$$V_w\left(b_{l|\left(x_L, a_m^t(x_k)\right)}\right) = \begin{cases} V_w(b_{HM}) & \text{if } \left(x_L, a_m^t(x_k)\right) = (x_L, a) \\ 0 & \text{if } \left(x_L, a_m^t(x_k)\right) = \left(x_L, a^-\right) \end{cases}.$$

If an L-type man accepts an L-type woman, then $V_w\left(b_{l|\left(x_L, a_m^t(x_k)\right)}\right) = V_w(b_0)$.

[35] In the above equation, the number of terms in the case where a k_0-type woman accepts an M-type man equals that of terms in the case where a k_0-type woman rejects an M-type man because $V(b_H) \neq V(b_0)$.

[36] M-type men know their own types. Hence, their decisions are the same as those in the case of (II).

k_{HM}-type woman can be written as

$$rV_w(b_{HM}) = b_{HM}(x_H)\left[\begin{array}{c}\alpha\lambda_H^m\left(\frac{x_H}{r} - V_w(b_{HM})\right) + \alpha\lambda_M^m\left(\frac{x_M}{r} - V_w(b_{HM})\right)\\ +\alpha\lambda_L^m\left(\max\left\{\frac{x_L}{r}, V_w(b_{HM})\right\} - V_w(b_{HM})\right)\end{array}\right]$$
$$+ b_{HM}(x_M)\left[\begin{array}{c}\alpha\lambda_H^m\left(V_w(b_M) - V_w(b_{HM})\right) + \alpha\lambda_M^m\left(\frac{x_M}{r} - V_w(b_{HM})\right)\\ +\alpha\lambda_L^m\left(\max\left\{\frac{x_L}{r}, V_w(b_{HM})\right\} - V_w(b_{HM})\right)\end{array}\right].$$

Hence, we have

$$\frac{\lambda_M^m\alpha x_M\left(r+\alpha\lambda_H^m b(x_M)+\alpha\lambda_M^m\right)+\lambda_H^m\alpha x_H\left(r+\alpha\lambda_M^m\right)b(x_H)}{\left(r+\alpha\lambda_M^m\right)\left(r+\alpha\lambda_H^m+\alpha\lambda_M^m\right)} = R_w(b_{HM}).$$

From $R_w^*(x_M) \leq x_H$

$$R_w(b_{HM}) - R_w^*(x_M) = \alpha\lambda_H^m\frac{b(x_H)\left(rx_H+\alpha\lambda_M^m(x_H-x_M)\right)}{\left(r+\alpha\lambda_M^m\right)\left(r+\alpha\lambda_H^m+\alpha\lambda_M^m\right)} > 0.$$

As $R_w^*(x_M) > x_L$, $R_w(b_{HM}) > x_L$. Therefore, a k_{HM}- and k_0-type woman always rejects an L-type man. This contradicts the assumption that a k_0- or k_{HM}-type woman accepts an L-type man when an L-type man rejects an L-type woman. Therefore, when there are pessimistic k_0- or k_{HM}-type women who accept L-type men, L-type men always accept L-type women.■

Proof of Lemma 5: From Lemma 4, M- (L-)type men always accept M- (L-)type women, even if there are many H- (H- or M-)type women with imperfect self-knowledge, who accept M- (L-)type men. Moreover, H-type men always reject M-type women from $R_i^*(x_H) > x_M$. Therefore, we only have to consider the following cases: (i) M-type men accept L-type women, and (ii) M-type men accept M-type women, and L-type men accept L-type women.

When a k_0-type woman rejects an M-type man, the case of (i) is the same as Lemma 3. At this time, learning occurs only when H-type men reject M- or L-type women. As a k_{ML}-type woman accepts an M-type man, $R_w^{T1}(b_0) > x_M \geq R_w^{T1}(b_{ML}) = R_w^*(x_M)$. Hence, the rejection from an H-type man decreases the reservation level of a woman with imperfect self-knowledge.

Next, we investigate the case of (ii). At this time, $R_w^P(b_{ML}) < R_w^*(x_M) = R_w(b_M) < R_w^P(b_{HM}) = R_w^P(b_0)$ holds because this case is the same as the case in Lemma 2. Therefore, a rejection from a man decreases the reservation utility level of the woman with imperfect self-knowledge.

Proof of Lemma 6: As $R_m^{T2}(x_M) > x_L$, the information about types of women from the proposals by men and the learning processes of women are the same as those in the PSEI. Hence, the reservation levels of women are obtained in the same manner as those in the PSEI:

$$R_w^{T2}(b_{ML}) \equiv \frac{\lambda_M^m\alpha x_M\left(r+\alpha\lambda_L^m\right)b_{ML}^{T2}(x_M)}{r\left(r+\alpha\lambda_L^m+\alpha\lambda_M^m\right)+\alpha^2\lambda_L^m\lambda_M^m b_{ML}^{T2}(x_M)} < R_w^*(x_M) = R_w^{T2}(b_M),$$

$$R_w^{T2}\left(b_{HM}\right) = R_w^{T2}\left(b_0\right) \equiv \frac{\lambda_H^m \alpha x_H \left(r+\alpha\lambda_M^m\right) b_{HM}^{T2}\left(x_H\right)}{r\left(r+\alpha\lambda_H^m+\alpha\lambda_M^m\right)+\alpha^2\lambda_H^m\lambda_M^m b_{HM}^{T2}\left(x_H\right)} \left(< R_w^*\left(x_H\right)\right).$$

From (36) and (37), $b_{ML}^{T2}\left(x_M\right) = \frac{\lambda_H^m\lambda_M^w\left(\lambda_L^m+\lambda_M^m\right)}{\lambda_L^w\lambda_L^m\left(\lambda_H^m+\lambda_H^m\lambda_L^m+\lambda_M^m\right)+\lambda_H^m\lambda_M^w\left(\lambda_L^m+\lambda_M^m\right)}$ and $b_{HM}^{T2}\left(x_H\right) = \frac{\lambda_H^w\left(\lambda_H^m+\left(\lambda_H^m+\lambda_M^m\right)\left(\lambda_L^m+\lambda_M^m\right)\right)}{\lambda_H^m\lambda_H^w+\left(\lambda_L^m+\lambda_M^m\right)\left(\lambda_H^m+\lambda_M^m\right)\left(\lambda_H^w+\lambda_M^w\right)}$ hold in the Type 2.■

Appendix B

Two types In this paper, we assume not two but three types of agents in order to show the influence of the indirect externality of optimism on the market. If we consider two types of agents in the case of optimism, the indirect externality does not occur, and we cannot find a case in which the indirect externality of optimism prevents the lowest type of agents from marrying. To see this, now, let us assume two types of agents: good and bad. Let us assume that, when all agents have perfect self-knowledge, the PSE occurs. To describe the optimism, let us assume that i_0-type women $(i = g, b)$ reject bad-type men. Therefore, there are two kinds of bad-type women with respect to differences in their beliefs: bad-type women who are optimistic and bad-type women who know their own types. However, good-type men do not change their reservation utility levels relative to the case of perfect self-knowledge because all women want to marry them. Then, the indirect externality of optimism does not occur. Hence, bad-type men always marry bad-type women with perfect self-knowledge.

On the other hand, in the case of pessimism with two types of agents, we obtain, qualitatively, the same results as those of Lemma 4 and Proposition 4. That is, the indirect externality of pessimism does not occur, and, as a result, all agents can then marry.

The assumption of optimism We adopt the assumption that a k_0-type woman rejects an M-type man in the optimism case because we focus on the case in which M-type women are optimistic in order to show that the indirect externality of optimism affects the marriage behaviors of lower-type agents.

If k_0-type women accept M-type men (or reject L-type men), all agents can marry. To see this, let us assume that k_0-type women accept M-type men. At this time, L_0-type women are optimistic, and H_0-type women are pessimistic. The pessimism of H_0-type women does not change the actions of M-type men for the same reason as in Lemma 4. Moreover, the indirect externality of optimism by L-type women does not arise, similarly to that in two types of agents.

Let us consider another optimism case in which because there are enough M_{HM}-type women who reject M-type men, M-type men accept L-type women. However, this case does not arise. This is because the existence of M_{HM}-type women requires that an M- or L-type man rejects an L-type woman (now, H-type men reject M-type women as $x_M < R_m^*\left(x_H\right)$).

Benchmark case In our analysis, we consider the case in which $x_M < R_i^*\left(x_H\right)$ and $x_L < R_i^*\left(x_M\right)$ $(i = m, w)$ hold as the benchmark case. If we consider the case in which

$x_M \geq R_i^*(x_H)$ and $x_L < R_i^*(x_M)$ hold as the benchmark case, the result is the same as the case of the two types. Thus, the indirect externality of optimism does not occur. To see this, let us define the next situation as a benchmark case: if all agents know their own types perfectly, $x_M \geq R_i^*(x_H)$ and $x_L < R_i^*(x_M)$ hold. Let us assume that, under $x_M \geq R_i^*(x_H)$ and $x_L < R_i^*(x_M)$, k_0-type women reject M-type men, and then M-type men accept L-type women due to the rejections from M_0-type women. That is, the indirect externality of optimism occurs. However, the reservation level of a k_0-type woman is always lower than $R_i^*(x_H)$ because she assigns probabilities to her own types, similarly to the case of Lemma 2. This contradicts the assumption that, under $x_M \geq R_i^*(x_H)$ and $x_L < R_i^*(x_M)$, k_0-type women reject M-type men. Therefore, when $x_M \geq R_i^*(x_H)$ and $x_L < R_i^*(x_M)$, k_0-type women always accept M-type men.[37]

Under $x_M < R_i^*(x_H)$ and $x_L \geq R_i^*(x_M)$, the indirect externality of optimism does not occur. When $x_M < R_i^*(x_H)$ and $x_L \geq R_i^*(x_M)$, there are few enough M-type agents. Therefore, even if there are some M-type women who reject M-type men due to imperfect self-knowledge, M-type men do not change their actions: they accept L-type women. Now, because there are few enough M-type men ($x_L \geq R_i^*(x_M)$), some L-type women (at least, the L-type women who were rejected by H-type men) always accept L-type men. Therefore, in this case, the indirect externality of optimism does not occur.[38]

Appendix C—welfare and the number of marriages

In this section, we investigate whether the existence of women with imperfect self-knowledge improves the welfare of the economy relative to the benchmark case. To do so, let us examine the overall number of marriages and the overall welfare from new marriages that take place in the marriage market at any point in time.

First, we investigate the number of marriages at the PSE as a benchmark. In the PSE, an H-type man meets an H-type woman with probability $\alpha\lambda_H^w$, and there are $\lambda_H^m N$ number of H-type men in the market. Then, the number of marriages among H-type agents is $\alpha\lambda_H^w\lambda_H^m N$. In the same way, we obtain the number of marriages of M-type $\alpha\lambda_M^m\lambda_M^w N$ and L-type $\alpha\lambda_L^w\lambda_L^m N$. Therefore, the overall number of marriages in the marriage market T^* is

$$T^* = \alpha\lambda_H^m\lambda_H^w N + \alpha\lambda_M^m\lambda_M^w N + \alpha\lambda_L^m\lambda_L^w N. \tag{46}$$

[37] The assumption of $x_M \geq R_i^*(x_H)$, $i = m, w$ means that there are a few H-type men and women in the market. If $x_M \geq R_i^*(x_H)$ and $x_L < R_i^*(x_M)$, any H-type woman accepts an M-type man. In this case, even if H-type women with imperfect self-knowledge accept M-type men, the behavior of these women is the same as that of the H-type with perfect self-knowledge. Then, an M-type man does not change his behavior: he accepts an M-type woman. If some M-type women with imperfect self-knowledge accept L-type men under $x_M \geq R_i^*(x_H)$ and $x_L < R_i^*(x_M)$, the indirect externality of pessimism does not occur in a steady state for the same reason as in Lemma 4 qualitatively.

[38] When $x_M < R_i^*(x_H)$ and $x_L \geq R_i^*(x_M)$, the following cases of pessimism can be considered. If H-type women with imperfect knowledge accept M-type men under $x_M < R_i^*(x_H)$ and $x_L \geq R_i^*(x_M)$, the indirect externality of pessimism does not occur for the same reason as Lemma 4. If M-type women with imperfect knowledge accept L-type men under $x_M < R_i^*(x_H)$ and $x_L \geq R_i^*(x_M)$, L-type men do not change their behavior: they accept L-type women. Then, in this case, there is no influence of the pessimism of M-type women.

The number of marriages in the PSEI (T^p), the Type 1 $\left(T^{T1}\right)$, and the Type 2 $\left(T^{T2}\right)$ can be derived similarly (see also Figure 1–3). Therefore, we obtain

$$T^p = \alpha\lambda_H^m\lambda_H^w N + \alpha\left(\frac{\lambda_H^m}{\lambda_H^m+\lambda_M^m}\right)\lambda_M^w\lambda_M^m N + \alpha\left(\frac{\lambda_M^m}{\lambda_L^m+\lambda_M^m}\right)\lambda_L^w\lambda_L^m N, \quad (47)$$

$$T^{T1} = \alpha\lambda_H^m\lambda_H^w N + \alpha\left(\frac{\lambda_H^m}{\lambda_H^m+\lambda_M^m}\right)\lambda_M^m\lambda_M^w N + \alpha\left(\frac{\lambda_H^m}{\lambda_H^m+\lambda_M^m}\right)\lambda_M^m\lambda_L^w N, \quad (48)$$

and

$$T^{T2} = \alpha\lambda_H^m\lambda_H^w N + \alpha\left(\frac{\lambda_H^m}{\lambda_H^m+\left(\lambda_H^m+\lambda_M^m\right)\left(\lambda_L^m+\lambda_M^m\right)}\right)\lambda_M^m\lambda_M^w N$$
$$+\alpha\left(\frac{\left(\lambda_H^m\right)^2}{\lambda_H^m+\left(\lambda_M^m+\lambda_L^m\right)\left(\lambda_H^m+\lambda_M^m\right)}\right)\lambda_M^w\lambda_L^m N + \alpha\left(\lambda_H^m+\lambda_M^m\right)\lambda_L^m\lambda_L^w N. \quad (49)$$

Next, we explore overall welfare. If an H-type man marries an H-type woman, each of them obtains the utility of marriage x_H. Hence, the aggregation of H-type agents' utilities from marriage is $2\alpha\lambda_H^m\lambda_H^w x_H N$ in the PSE. Similarly, we obtain $2\alpha\lambda_M^m\lambda_M^w x_M N$ for the M-type and $2\alpha\lambda_L^m\lambda_L^w x_L N$ for the L-type. As a result, the welfare in the PSE (W^*) is

$$W^* = \alpha\lambda_H^m\lambda_H^w\left(2x_H\right)N + \alpha\lambda_M^m\lambda_M^w\left(2x_M\right)N + \alpha\lambda_L^m\lambda_L^w\left(2x_L\right)N. \quad (50)$$

The welfare in the PSEI (W^p), the Type 1 $\left(W^{T1}\right)$, and the Type 2 $\left(W^{T2}\right)$ can be derived similarly. Hence

$$W^p = \alpha\lambda_H^m\lambda_H^w\left(2x_H\right)N + \alpha\left(\frac{\lambda_H^m}{\lambda_H^m+\lambda_M^m}\right)\lambda_M^w\lambda_M^m\left(2x_M\right)N + \alpha\left(\frac{\lambda_M^m}{\lambda_L^m+\lambda_M^m}\right)\lambda_L^w\lambda_L^m\left(2x_L\right)N, \quad (51)$$

$$W^{T1} = \alpha\lambda_H^m\lambda_H^w\left(2x_H\right)N$$
$$+\alpha\left(\frac{\lambda_H^m}{\lambda_H^m+\lambda_M^m}\right)\lambda_M^m\lambda_M^w\left(2x_M\right)N + \alpha\left(\frac{\lambda_H^m}{\lambda_H^m+\lambda_M^m}\right)\lambda_M^m\lambda_L^w\left(x_M+x_L\right)N, \quad (52)$$

and

$$W^{T2} = \alpha\lambda_H^m\lambda_H^w\left(2x_H\right)N + \alpha\left(\frac{\lambda_H^m}{\lambda_H^m+\left(\lambda_H^m+\lambda_M^m\right)\left(\lambda_L^m+\lambda_M^m\right)}\right)\lambda_M^m\lambda_M^w\left(2x_M\right)N$$
$$+\alpha\left(\frac{\left(\lambda_H^m\right)^2}{\lambda_H^m+\left(\lambda_M^m+\lambda_L^m\right)\left(\lambda_H^m+\lambda_M^m\right)}\right)\lambda_M^w\lambda_L^m\left(x_M+x_L\right)N + \alpha\left(\lambda_H^m+\lambda_M^m\right)\lambda_L^m\lambda_L^w\left(2x_L\right)N, \quad (53)$$

hold. From these equations, the next lemma is immediately obtained.

Proposition 5 *The number of marriages and the welfare in the PSE are higher than those in the PSEI, i.e.,* $T^* > T^p$ *and* $W^* > W^p$*, respectively.*

Proof. From (46) and (47),

$$T^* - T^p = N\alpha\left(\left(1-\left(\mu+\gamma\phi\right)\right)\lambda_M^m\lambda_M^w + \left(1-\left(\kappa+\upsilon\psi\right)\right)\lambda_L^m\lambda_L^w\right) > 0$$

holds. From (50) and (51), we also have

$$W^p - W^* = -\lambda_L^m\lambda_L^w x_L\left(1-\left(\kappa+\upsilon\psi\right)\right) - \lambda_M^m\lambda_M^w x_M\left(1-\left(\mu+\gamma\phi\right)\right) < 0.$$

■

From Proposition 5, the welfare and the number of marriages in the PSEI are lower than those in the PSE under any $F_i(x)$, $i = m, w$. This is because the marriages of all agents, except those of H-type men and some H-type women who meet H-type men in their first encounter, are delayed due to the refusals by the women who are learning their own types.[39] As a result, their marriages decrease, and the overall welfare then decreases. From Proposition 5, in the PSEI, the policy that promotes women's learning can improve the overall welfare.

On the other hand, the comparisons of the overall welfare and the overall number of marriages between the PSE and Type 1 are ambiguous.

The comparisons of the overall welfare and the overall number of marriages between the PSE and the Type 2 are also ambiguous.

It is noteworthy that, in both the Type 1 and the Type 2, the number of marriages and the welfare of H-type men or women are not influenced by the imperfect self-knowledge of women.

Finally, we compare the number of marriages and the welfare in the PSEI with those in the Type 2. As we show in Example 1, the PSEI and the Type 2 hold in some parameter ranges.

Example 2 *Let us assume that* $\lambda_k^i = \frac{1}{3}$, $(i = m, w,\ k = H, M, L)$. *At this time, the number of marriages in the PSE, the PSEI, and the Type 2 are* $T^* = \frac{1}{3}N\alpha$, $T^p = \frac{2}{9}N\alpha$, *and* $T^{T2} = \frac{47}{189}N\alpha$, *respectively, from (46), (47), and (49). Then,* $T^* > T^{T2} > T^p$. *The welfare of marriages in the PSE, the PSEI, and the Type 2 is* $W^* = \frac{2}{9}N\alpha(x_H + x_L + x_M)$, $W^p = \frac{1}{9}N\alpha(2x_H + x_L + x_M)$, *and* $W^{T2} = \frac{1}{189}N\alpha(42x_H + 31x_L + 21x_M)$, *respectively, from (50), (51), and (53). Hence,* $W^* > W^{T2} > W^p$.

When $3\alpha > 2r$ and $F_m(x)$ and $F_w(x)$ are discrete uniform distributions, multiple equilibria arise. At this time, the PSEI and the Type 2 are not Pareto-rankable; pessimistic women prefer the PSEI, and L-type men prefer the Type 2. However, because pessimistic women accept M- and L-type men in the Type 2, the overall number of marriages in the Type 2 increases relative to that in the PSEI. As a result, the overall welfare in the Type 2 also increases relative to that in the PSEI. However, because $\lambda_k^i = \frac{1}{3}$, the welfare and the number of marriages in the PSE are larger than those in the Type 2. Then, in the Type 2, the policy that informs pessimistic women of their own types can improve the overall welfare when $\lambda_k^i = \frac{1}{3}$ $(i = m, w,\ k = H, M, L)$.

[39] The duration until marriage of each agent can be obtained easily. In the PSE, the duration until marriage of k-type man (woman) is $\frac{1}{\alpha\lambda_k^i}(i = m, w,\ k = H, M, L)$. However, in the PSEI, the duration until marriage of an M-type man is $\frac{1}{\alpha\lambda_M^w}\frac{\lambda_H^m+\lambda_M^m}{\lambda_H^m}$, that of an L-type man is $\frac{1}{\alpha\lambda_L^w}\frac{\lambda_L^m+\lambda_M^m}{\lambda_M^m}$, that of an H_{HM}-type woman is $\frac{1}{\alpha\lambda_H^m\alpha\lambda_M^m}$, that of an M_{ML}-type woman is $\frac{1}{\alpha\lambda_H^m\alpha\lambda_M^m}$, that of an M_M-type woman is $\frac{1}{\alpha\lambda_H^m\alpha\lambda_M^m\alpha\lambda_L^m}$, that of an L_{L1}-type woman is $\frac{1}{\alpha\lambda_H^m\alpha\lambda_M^m\alpha\lambda_L^m}$, and that of an L_{L2}-type woman is $\frac{1}{\alpha\lambda_M^m\alpha\lambda_L^m}$. Therefore, their marriages are delayed by the learning process of women.

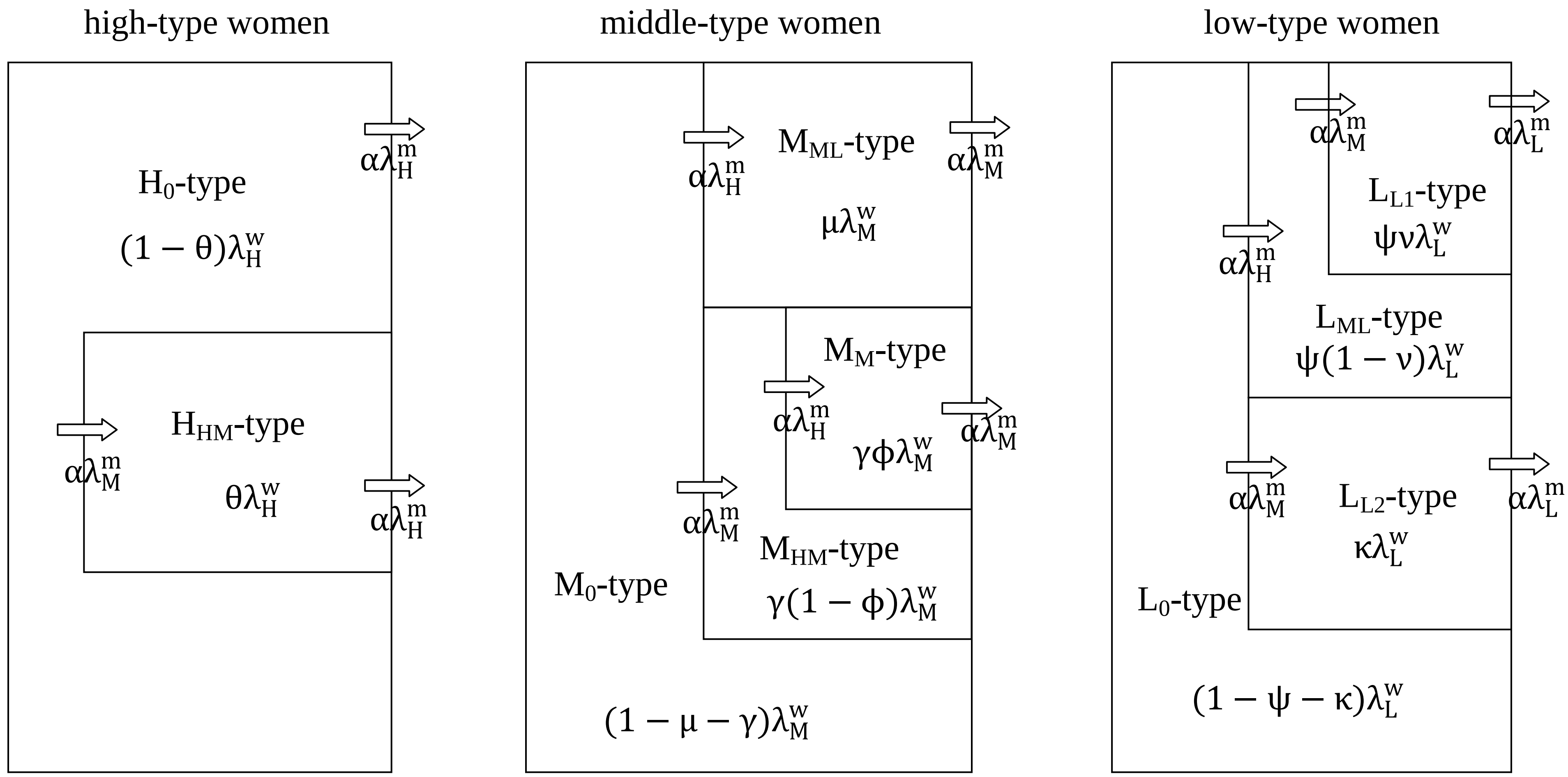

Figure 1: P.S.E. with imperfect knowledge

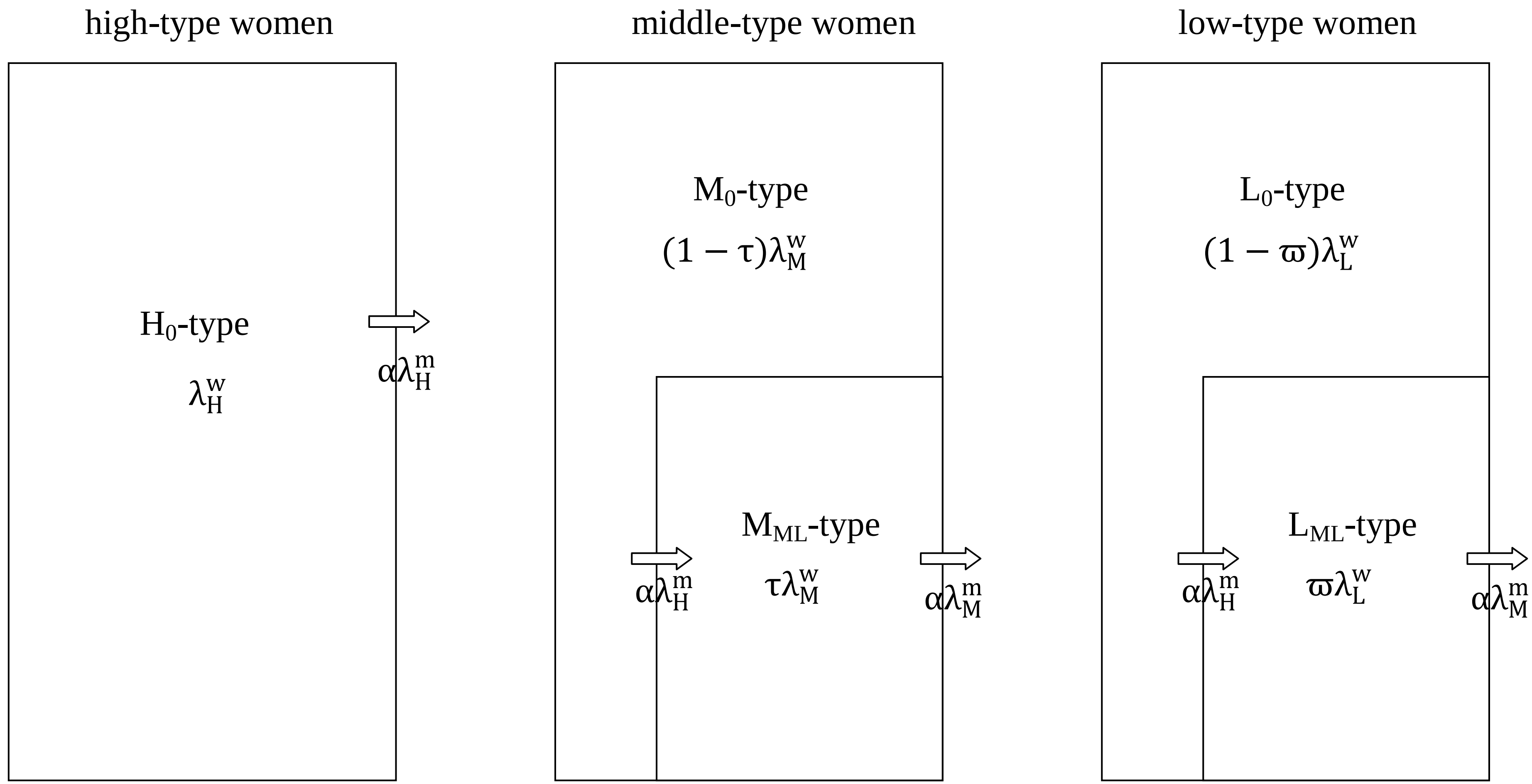

Figure 2: Type 1 equilibrium

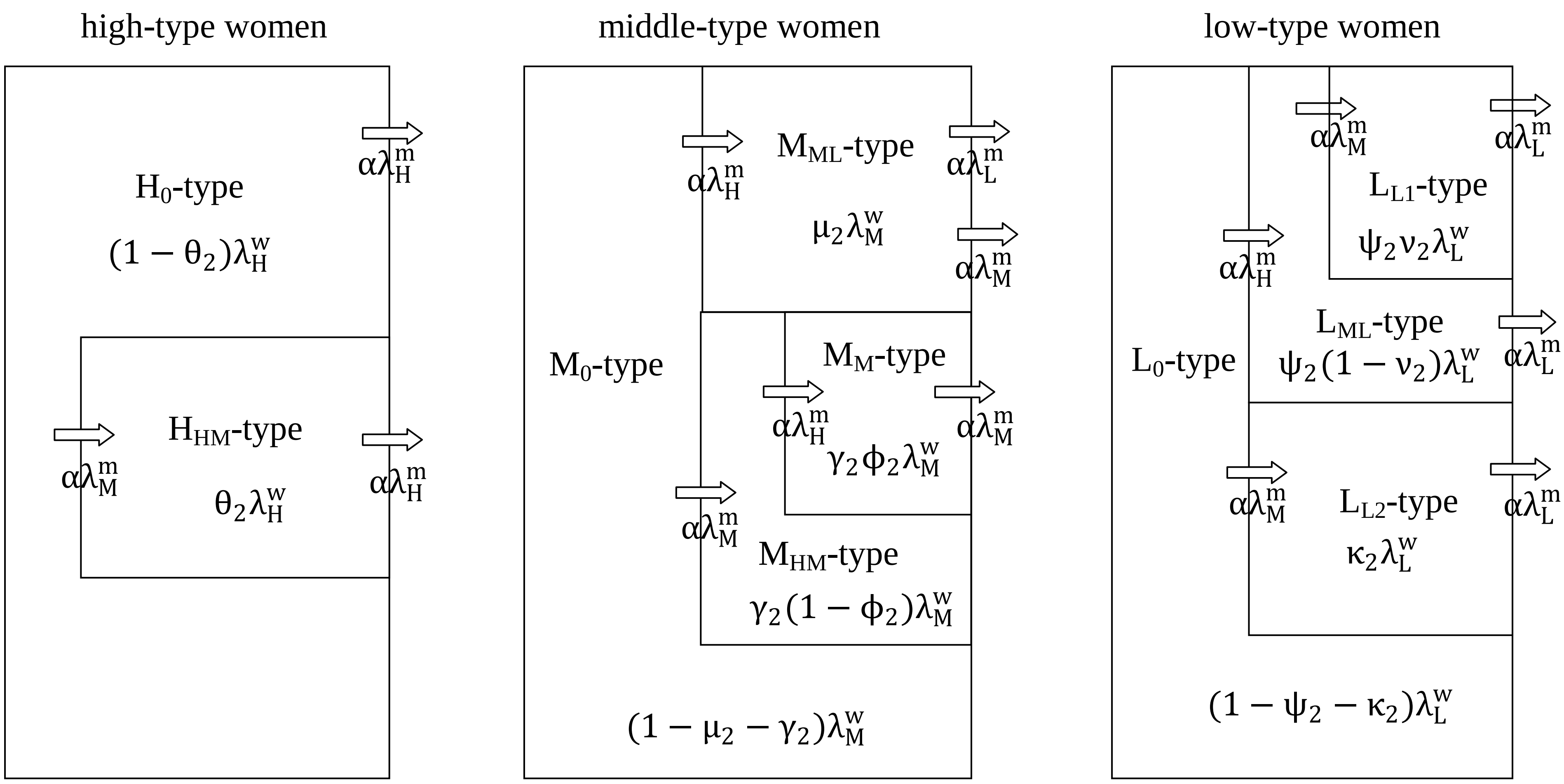

Figure 3:　Type 2 equilibrium

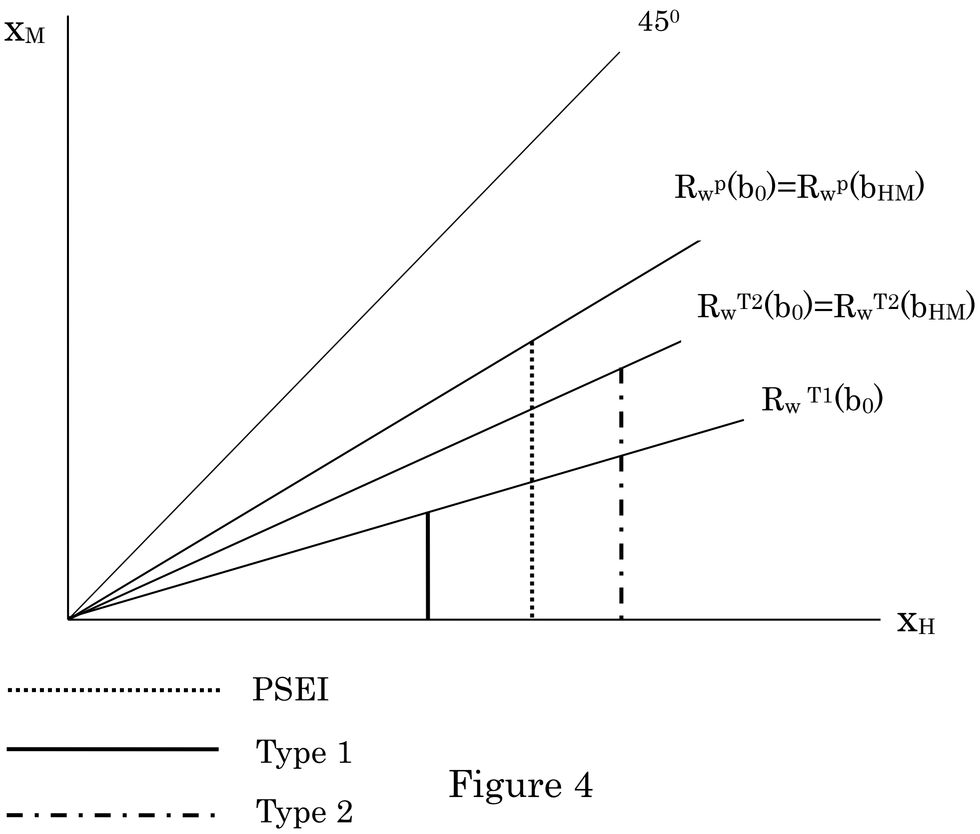

Figure 4

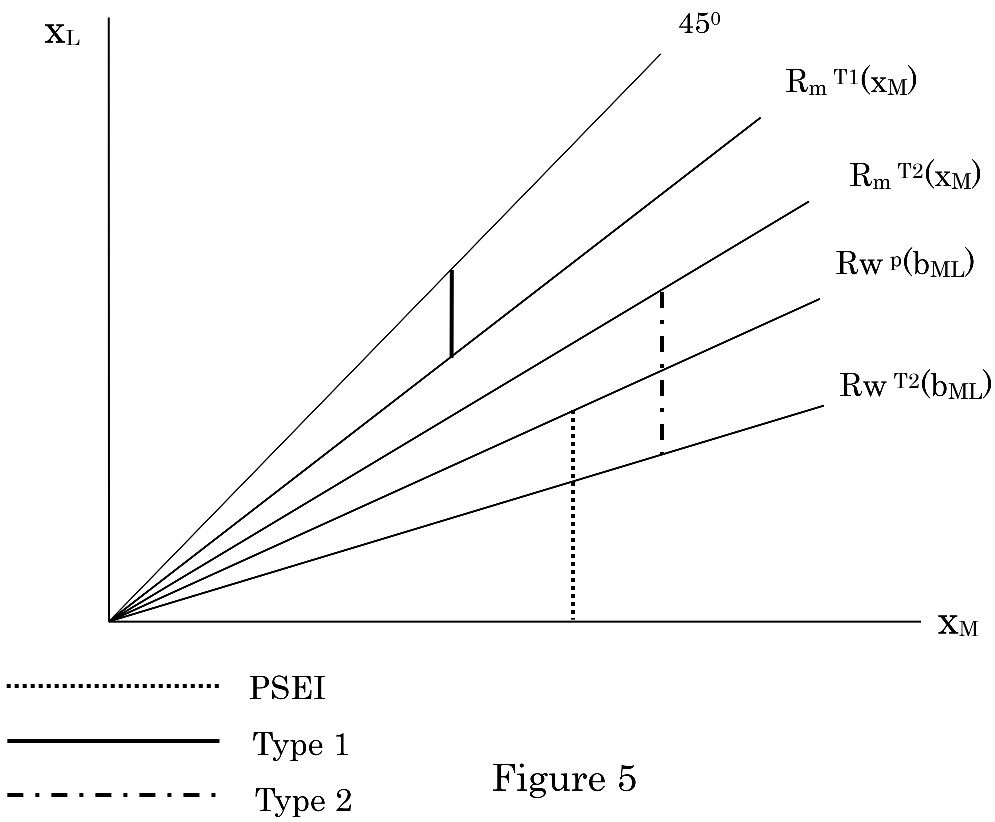

Figure 5