Cost-Benefit Analysis in Monopolistic Competition Models of Urban Agglomeration

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Abstract

Many sources of urban agglomeration involve departures from the first-best world. The benefit evaluation of a transportation project must then take into account agglomeration benefits along with any direct user benefits. Using a monopolistic competition model of differentiated intermediate products, we show that the additional benefits can be expressed as an extended Harberger formula with variety distortion in addition to price distortion. They are positive if variety is procompetitive, but, in the anticompetitive case, we cannot exclude the possibility of negative additional benefits. By introducing the rural sector and multiple cities explicitly, we also show that the agglomeration benefits depend on where the new workers are from.

Keywords: cost-benefit analysis; agglomeration economies; monopolistic competition; new economic geography; second-best economies

JEL Classification: D43; R12; R13

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1. Introduction

Many of the sources of urban agglomeration, such as gains from variety, better matching, and knowledge creation and diffusion, involve departures from the first-best world.[1] The benefit evaluation of a transportation project must then take into account agglomeration benefits along with any direct user benefits. A number of economists have studied this issue, and policy makers in some countries, such as the United Kingdom, have been attempting to include these considerations in their project assessments.[2]

Based on past empirical work, urban agglomeration economies are substantial. For instance, a review by Rosenthal and Strange (2004, p. 2133) summarizes the empirical findings as follows: "In sum, doubling city size seems to increase productivity by an amount that ranges from roughly 3−8%."[3] Agglomeration economies on the consumer side are also substantial, as argued by Glaeser et al. (2001), with estimates by Tabuchi and Yoshida (2000) suggesting economies in the order of 7−12%. The benefit estimates could exceed 10% after combining production and consumption agglomeration economies.

By modeling the microstructure of agglomeration economies, this paper derives second-best benefit evaluation formulae for urban transportation improvements. Venables (2007) investigated the same problem but without modeling explicitly the sources of agglomeration economies. Accordingly, our analysis examines whether the results in this prior work remain valid when monopolistic competition with differentiated products provides the microfoundation of agglomeration economies.

Extending the Henry George Theorem to a second-best setting with distorted prices, Behrens et al. (2010) showed that the optimality condition for the number of cities (or equivalently, the optimal size of a city) must be modified to include Harberger's excess burden, that is, the weighted sum of induced changes in consumption, with the weights being the price distortions. New economic geography (NEG)-type models of

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[1] See Duranton and Puga (2004) for a review of the theoretical analysis of various sources of urban agglomeration, Fujita and Thisse (2002) for the new economic geography approach, and Kanemoto (1990) for the analysis of a nonmonocentric city model.

[2] See, for example, Venables and Gasiorek (1999), Department of Transport (2005), (2008), Graham (2005, 2006), and Vickerman (2008). Arnott (2007) studied second-best congestion tolls in the presence of agglomeration externalities.

[3] See Puga (2010) for a more recent review.

monopolistic competition contain distortions of two forms: a price distortion for each variety of the differentiated good, and a distortion associated with the number of available varieties consumed. Although the former is well known, the latter has largely escaped the attention of the existing literature. Importantly, because these two types of distortions work in opposite directions, the net effect is uncertain. In the constant elasticity of substitution (CES) case, the excess burden is zero, but in general, it can be positive or negative, depending on specific functional forms.[4]

This paper shows that the same technique can be applied to the benefits of transportation improvements, but the result that the two types of distortions work in opposite directions does not in general hold. If an increase in variety is procompetitive in the sense that it makes the price elasticity of demand higher, both distortions work in the same direction to make the additional benefits positive. In the anticompetitive case, however, they work in opposite directions and the additional benefits may become negative.

In yet another departure from Venables (2007), we introduce explicitly the rural sector and multiple cities. We show that the results hinge on whether the new workers are from the rural sector or other cities. If all new workers come from other cities, then the additional benefits are zero.[5]

The remainder of the paper is as follows. In Section 2, we present a model of urban agglomeration economies based on monopolistic competition in differentiated intermediate products. Section 3 derives the properties of the symmetric equilibrium that we consider. Section 4 obtains second-best benefit measures of transportation investment. In Section 5, we examine specific functional forms that have been used in the literature: additively separable and quadratic production functions and a translog cost function. Section 5 concludes.

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[4] Zhelobodko et al. (2011) examined a monopolistic competition model with additively separable preferences and showed that the CES case yields another knife-edge result concerning procompetitive and anticompetitive effects.

[5] This confirms the caution expressed by Glaeser (2010, p. 13):

> For example, advocates of London's Crossrail system emphasized that increasing commuter access to the city would bring in more workers who might generate agglomeration economies. However, those workers would presumably be coming from somewhere else. Any gains to London might be offset by reductions in agglomeration economies elsewhere.

2.   The model

Our model adds three elements to Venables (2007): the microstructure of agglomeration, multiple cities, and an explicit rural sector.[6] We examine agglomeration economies on the production side, using monopolistic competition models with product differentiation in the intermediate goods.[7] The differentiated goods are not transportable to outside a city. The economy contains n cities and a rural area, where all cities are monocentric, i.e., all workers commute to the central business district (CBD). All cities have the same topographical and technological conditions. Workers/consumers are mobile and free to choose where, between the cities and the rural area, to live and work.

The total population in the economy is $\overline{N}$, which is divided into n cities with population N^j, $j = 1, \cdots, n$, and the rural area with population N^A:

$$(1) \qquad \overline{N} = \sum_{j=1}^{n} N^j + N^A = N^U + N^A ,$$

where N^U is the total urban population. The number of cities n is fixed.

The production of an urban final good requires differentiated intermediate inputs.[8] We assume the final good is homogeneous. The final good can be transported costlessly between cities and the rural area, but, as stated above, intermediate goods can be used only within a city. Final-good producers are competitive within a city, taking both output and input prices as given. We assume free entry for final-good producers. For simplicity, we assume the rural area produces the same final product, albeit with a different technology. While the final good is consumed directly by consumers, it is also used in the production of transportation services.

The intermediate good producers are monopolistically competitive. We assume

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[6] We ignore income tax distortions because Venables' analysis is applicable to our model without modification.

[7] See Kanemoto (2012) for the analysis of differentiated consumer goods. Although there are minor differences, most of the qualitative results are the same.

[8] Duranton and Puga (2004) distinguished three types of micro-foundations of urban agglomeration: sharing, matching and learning mechanisms. Our framework is an example of the sharing models in this classification. Specifically, it generalizes the differentiated intermediate good model of Abdel-Rahman and Fujita (1990) from its CES production function to a general functional form.

free entry for intermediate good production as well as for final-good production. Following Venables (2007), we use a monocentric city model with commuting transportation and assume absentee landlords own land in both urban and rural areas.

Production of the final good

The production of the urban final good requires differentiated intermediate inputs only, and the production function is $y_0 = F(\{y_i\}_{i \in M})$, where y_0 and y_i, respectively, denote the homogeneous final good and differentiated intermediate input i, and M is the set of available intermediate goods. Unlike in typical NEG models, we do not assume specific functional forms. We only assume the production function is symmetric in the y_is, and that it is well behaved, so profit maximization yields a unique interior solution. The mass of the set of intermediate goods that are actually used for production (i.e., $y_i > 0$) is denoted by m and called the variety. An example of production functions satisfying these conditions is a separable function, $y_0 = \left(\int_0^m f(y_i) di \right)^{1+\rho}$, which includes the CES form commonly used in NEG models: $f(y_i) = \alpha(y_i)^{(\sigma-1)/\sigma}$. Other functional forms examined later are quadratic functions (Ottaviano et al. (2002) and Peng et al. (2006)) and a translog cost (expenditure) function (Feenstra (2003)). The final good, y_0, is homogeneous and its transportation cost is zero.

The final-good industry is competitive within a city and we assume free entry. The profit of a producer is $\pi = y_0 - \int_0^m p_i y_i di$, where p_i is the price of intermediate good i and we normalize the price of the final good to one (1). A producer takes the prices of intermediate goods, as well as that of the final good, as fixed. For the choice of y_i, profit maximization yields the usual first-order condition: $\partial F / \partial y_i = p_i$. The choice of variety m, however, is constrained by the entry decisions of intermediate good producers. Even if adding another variety increases profit, it may not be available in the market. The first-order condition is therefore in an inequality form: $\partial F / \partial m \geq p_m y_m$. In fact, the inequality is strict in most cases. The zero profit condition from free entry is: $\pi = F(\{y_i\}_{i \in M}) - \int_0^m p_i y_i di = 0$.

In the rural area, the production of the final good requires only the labor input. The production function of the rural sector is $Y_0^A = G(N^A)$.

Production of differentiated intermediate goods

Next, let us turn to the producers of the intermediate goods. An intermediate good producer has monopoly power because of product differentiation. Under the standard monopolistic competition assumption, however, a producer is small enough to ignore impacts on other producers. Profit maximization of a final-good producer yields the demand function for each intermediate product. Omitting the variables that are taken as fixed by a producer, we can write the demand function of input i as $y_i = d(p_i)$. Aggregating over all final-good producers, we obtain the market demand for input i: $Y_i = D(p_i)$. The (perceived) price elasticity of demand is $\eta_i = -D'(p_i)p_i / Y_i$.

Production of an intermediate good requires only labor as an input. The labor input required for producing Y_i of variety i is $N_i = cY_i + a$, where the fixed cost and the marginal cost are constant at a and c (measured in terms of labor units), respectively. Given the perceived demand function, an intermediate good producer maximizes the profit $\pi_i = p_i Y_i - w(cY_i + a)$, where w is the wage rate. The first-order condition for profit maximization yields the familiar condition that the price margin equals the inverse of the perceived price elasticity: $(p_i - wc)/p_i = 1/\eta_i$. From free entry, the maximized profit is zero: $\pi_i = 0$.

Commuting costs and migrational equilibrium

Following Venables (2007), we assume a simple monocentric city, where all urban workers commute to the CBD and the lot sizes of all houses are fixed and equal. We ignore the structural part of a house and assume the alternative cost of urban land is zero. We also ignore the allocation within the CBD. In this simple framework, the total transportation cost in a monocentric city can be expressed as a function of the population of a city N and a transportation cost parameter t: $TC(N,t)$. The total transportation cost is related to the commuting cost for a resident living at the edge of the city, $T(N,t)$, by:

(2) $\qquad \dfrac{\partial TC(N,t)}{\partial N} = T(N,t)$.

If a worker is added to a city, this person must be located at the edge of the city and the total transportation cost increases by the commuting cost at the edge.

The budget constraint for a resident living at the edge of the city is:

(3) $\qquad w = x_0 + T(N,t)$.

In the rural area, the wage rate equals the value of the marginal product of labor in the final-good sector: $w^A = G'(N^A)$. The budget constraint for a rural worker is then $w^A = x_0^A$. Free migration equalizes the consumption levels in all cities and the rural area, i.e., $x_0^A = x_0^j, j = 1, \cdots, n$. This implies net income equalization:

(4) $\qquad w^j - T(N^j, t^j) = w^A, \; j = 1, \cdots, n$.

3. Symmetric equilibrium in urban production

The conditions outlined in the preceding section determine a market equilibrium given a set of transportation cost parameters t^j in cities. Our task is to evaluate the welfare changes caused by a decrease in transport costs in one of the cities. Toward this goal, we first derive the properties of a symmetric equilibrium that will be used in the welfare analysis. Even though we do not assume a specific functional form for the production function, the symmetry assumption results in strong restrictions as shown in this section.

First, in a competitive homogeneous good industry where all firms have the same technology, free entry ensures that they choose the production scale at which constant returns to scale prevail; i.e., the average cost equals the marginal cost. The aggregate production function then exhibits constant returns to scale. This can be seen as follows. In a symmetric equilibrium, the production function of the final good can be written as a function of the quantity of each input, y, and variety, m:

(5) $\qquad y_0 = F\left(\{y_i\}_{i \in M}\right)\Big|_{y_i = y} \equiv \hat{F}(y, m)$,

where the first-order conditions for profit maximization become

$$(6) \qquad \frac{\partial \hat{F}}{\partial y} = m \frac{\partial F}{\partial y_i} = mp ,$$

$$(7) \qquad \frac{\partial \hat{F}}{\partial m} = \frac{\partial F}{\partial m} \geq py .$$

The zero profit condition of free entry is:

$$(8) \qquad \hat{F}(y,m) = mpy .$$

Combining (6) and (8) yields:

$$(9) \qquad \frac{\partial \hat{F}(y,m)}{\partial y} = \frac{\hat{F}(y,m)}{y} ,$$

which is the standard result that the marginal product equals the average product at a free-entry equilibrium. This condition determines the scale of production y as a function of variety m : $y = y(m)$. The price of an intermediate input, p, and the output–input ratio, $\varphi = y_0 / y$, can also be expressed as functions of variety, m: $p(m) \equiv \hat{F}(y(m),m) / my(m)$ and $\varphi(m) \equiv \hat{F}(y(m),m) / y(m)$. The aggregate production function of a city then becomes:

$$(10) \qquad Y_0 = Y\varphi(m) \equiv \widetilde{F}(Y,m) .$$

Next, let us turn to the producers of the intermediate good. Here, we assume Cournot-Nash behavior where each producer takes the quantities supplied by other producers as given.[9] In a symmetric equilibrium, the first-order condition for profit maximization of a final-good producer can be rewritten as:

$$(11) \qquad \frac{\partial F(y_i, y, m)}{\partial y_i} = p_i ,$$

where y denotes the common quantity of intermediate inputs other than i. This yields demand for input i by a final-good producer: $y_i = d(p_i, y, m)$. Denoting the number of

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[9] Alternatively, we can assume Bertrand–Nash behavior. The qualitative results are the same although equilibrium prices and quantities are in general different. In the translog cost function example in Section 5, we use the Bertrand–Nash assumption.

final-good producers by k, we can write the market demand for input i as:

$$Y_i = D(p_i; y, m, k) \equiv kd(p_i; y, m), \quad i \in M .$$

The price elasticity of demand that the producer of intermediate good i is faced with is then:

$$\eta_i = -\frac{\partial D_i}{\partial p_i} \frac{p_i}{Y_i} = -\frac{\partial d_i}{\partial p_i} \frac{p_i}{y_i} = \hat{\eta}(p_i; y, m) .$$

In a symmetric equilibrium where $p_i = p(m)$ and $y = y(m)$, this becomes a function of variety only: $\eta = \hat{\eta}(p(m), y(m), m) \equiv \eta(m)$. From the first-order condition for profit maximization (11), this elasticity satisfies:

$$\eta(m) = -\frac{1}{\left. \dfrac{\partial F^2(y_i, y(m), m)}{\partial y_i^{\,2}} \right|_{y_i = y(m)}} \frac{p(m)}{y(m)} .$$

Define the elasticity of the price elasticity with respect to variety as:

$$\xi(m) = \frac{m \eta'(m)}{\eta(m)} .$$

If this is positive, variety m is procompetitive in the sense that an increase in variety makes demand more elastic, leading to a lower price. We will see later that this is the case for functional forms that have been used in the literature: additively separable and quadratic production functions and a translog cost function. Because $\eta'(m)$ depends on the third order derivatives of the production function, however, we cannot exclude the possibility that variety is anticompetitive.

Using the price elasticity thus obtained, the first-order condition for profit maximization becomes:

(12) $$\frac{p(m) - wc}{wc} = \frac{1}{\eta(m) - 1} .$$

Combining this with the free entry condition, $pY - w(cY + a) = 0$, yields:

(13) $$Y = \frac{a}{c}(\eta(m) - 1).$$

Thus, the supply of an intermediate good is a linear function of the price elasticity. An important implication is that if the price elasticity is constant, the production level of an intermediate good, *Y*, is fixed and, in particular, is not affected by a transportation improvement.

The population of a city, $N = \int_0^m N_i di$, satisfies $N = m(cY + a)$. Substituting (13) into this equation yields:

(14) $N = am\eta(m)$.

If the price elasticity is constant, the total labor force is proportional to variety. If variety is procompetitive, an increase in variety raises the production level of each variety, which results in a more than proportionate increase in employment.

Inverting (14) yields variety *m* as a function of city size, $m = \tilde{m}(N)$, which satisfies:

(15) $\tilde{m}'(N) = \dfrac{m}{N} \dfrac{1}{1 + \xi(m)}$.

If the elasticity of the price elasticity is not less than minus one (i.e., $\xi(m) \geq -1$), then an increase in city size increases the variety. If it is less than minus one ($\xi(m) < -1$), the variety is reduced because a reduction in variety is accompanied by a more than proportionate increase in the production of each variety. Substituting $m = \tilde{m}(N)$ into (13) yields the production of each variety as a function of city size *N*:

(16) $\tilde{Y}(N) = \dfrac{a}{c}\big(\eta(\tilde{m}(N)) - 1\big)$,

where

(17) $\tilde{Y}'(N) = \dfrac{1}{cm} \dfrac{\xi(m)}{1 + \xi(m)}$.

If the price elasticity is constant (or $\xi(m) = 0$), then the city size does not affect the production level of a variety, *Y*. If $\xi(m) > 0$ or $\xi(m) < -1$, an increase in city size increases the production level, but the opposite result holds in the intermediate case of

$-1 < \xi(m) < 0$.

4. Benefits of transportation investment

We now examine the general-equilibrium impacts of small transportation improvements in a city. Our goal is to estimate the benefits of a marginal reduction in transportation costs in city 1, taking into account the effects on urban agglomeration.

Price distortions

Before examining the general-equilibrium impacts of a transportation project, we define price distortions. First, the marginal social benefit of an intermediate good can be measured by an increase in the final-good production caused by a marginal increase in an intermediate input: $MB_Y = \partial F / \partial y_i$.   From the first-order condition of profit maximization, this equals the price of an intermediate input: $MB_Y = p$. Because the marginal social cost is its production cost, $MC_Y = wc$, the price distortion of an intermediate good is:

(18) $\tau_Y = MB_Y - MC_Y = p - wc = \dfrac{wc}{\eta(m)-1} \geq 0$.

The marginal social benefit of increasing the variety of differentiated goods is the resulting increase in the production of the final good, $MB_m \equiv \partial \widetilde{F}(Y,m)/\partial m$, and the marginal cost is the cost of producing the additional variety, $MC_m \equiv w(cY + a)$. The price distortion of variety is then:

(19) $\tau_m = MB_m - MC_m = \dfrac{\partial \widetilde{F}(Y,m)}{\partial m} - w(cY+a) \geq 0$,

where the inequality follows from the first-order condition (7) for profit maximization by a final-good producer.

Harberger formula

Now, we turn to the impacts of a transportation project. We first derive a general formula that can be interpreted as an extension of the Harberger triangles to urban agglomeration. Given that there is only one consumption good in our model, we can define the social surplus as the total amount of the good available for consumption by

urban and rural workers and absentee landlords:

$$S = \sum_j \left(Y_0^j - TC^j \right) + G(N^A).$$

Substituting the aggregate production function (10), the total transport cost function (2), and the population constraint (1) into this yields:

$$S = \sum_j \left(\widetilde{F}(Y^j, m^j) - TC(N^j, t^j) \right) + G\left(\overline{N} - \sum_j N^j \right).$$

Our task is to evaluate a change in the social surplus caused by a marginal change in transportation costs t^j. Totally differentiating the social surplus equation, we obtain:

$$dS = \sum_j \left(\frac{\partial \widetilde{F}}{\partial Y^j} dY^j + \frac{\partial \widetilde{F}}{\partial m^j} dm^j - \frac{\partial TC}{\partial N^j} dN^j - \frac{\partial TC}{\partial t^j} dt^j \right) - G' \sum_j dN^j.$$

Applying equilibrium conditions obtained in the preceding section to this equation yields the Harberger formula. First, using $G' = w^A$, the equal-income condition (4), and the definitions of marginal social benefits, we obtain:

(20) $$dS = \sum_j \left(m^j MB_Y^j dY^j + MB_m^j dm^j - MB_t^j dt^j \right) - \sum_j w^j dN^j.$$

Next, the total differentiation of the labor force requirement in the differentiated good industry, $N^j = m^j(cY^j + a)$, yields:

$$dN^j = (cY^j + a)dm^j + m^j c dY^j.$$

Substituting this into the equation above and using the definitions of price distortions, (18) and (19), we can further rewrite (20) as:

(21) $$dS = -\sum_j MB_t^j dt^j + \sum_j \left(m^j \tau_Y^j dY^j + \tau_m^j dm^j \right),$$

where $MB_t^j = -\partial TC / \partial t^j$ is the marginal direct benefit of a reduction in t^j. This is an extension of Harberger's measure of welfare change (Harberger, 1964); i.e., a change in surplus can be decomposed into the direct benefit and the changes in the excess burden, where the excess burden is given by the weighted sum of induced changes, with the weights being the price distortions. As noted by Behrens et al. (2010), the Harberger formula must be extended to include the variety distortion when the variety is endogenous.

Wage distortion

Now, we convert Harberger's measure into a form that involves city size. First, we show that the price and variety distortions can be combined to obtain the wage distortion. Substituting $m = \tilde{m}(N)$ and $Y = \tilde{Y}(N)$ obtained in (14) and (16) into the aggregate production function (10), we obtain a reduced-form aggregate production function linking aggregate production to the total labor force in a city:

$$Y_0 = \tilde{F}(\tilde{Y}(N), \tilde{m}(N)) \equiv \tilde{Y}_0(N) .$$

The marginal social benefit of a worker in the differentiated good industry is then:

$$MB_N = \tilde{Y}_0{}'(N) = \frac{\partial \tilde{F}}{\partial Y}\tilde{Y}'(N) + \frac{\partial \tilde{F}}{\partial m}\tilde{m}'(N) = mMB_Y\tilde{Y}'(N) + MB_m\tilde{m}'(N) .$$

The marginal social cost of a worker equals the wage rate w, $MC_N = w$, and the wage distortion is the difference between these two:

$$\tau_N = MB_m\tilde{m}'(N) + mMB_Y\tilde{Y}'(N) - w .$$

Using the definitions of price and variety distortions in (18) and (19), we can rewrite this equation as:

(22) $$\tau_N = \tau_m\tilde{m}'(N) + m\tau_Y\tilde{Y}'(N) .$$

Thus, the wage distortion captures both the price and variety distortions of differentiated intermediate goods. Using (15) and (17), we can rewrite this equation as:

(23) $$\tau_N = \left(\frac{m}{N}\tau_m + \frac{\xi(m)}{c}\tau_Y\right)\frac{1}{1+\xi(m)} .$$

Note that we cannot in general sign the elasticity of the price elasticity $\xi(m)$. Therefore, we cannot exclude the possibility that the wage distortion is negative. As will be shown later, however, the wage distortion is positive in all the functional forms that we have examined, including the additively separable and quadratic production functions and the translog cost function.

Using the price distortion of labor (22), we can simplify (21) as:

(24) $$dS = -\sum_j MB_t^j dt^j + \sum_j \tau_N^j dN^j .$$

Thus, the excess burden can be measured by the wage distortion only. This result shows

that the agglomeration externality measure in Venables (2007) is valid if it is obtained from a reduced-form aggregate production function with differentiated intermediate inputs.

Benefits of transportation investment in a city

Next, we consider a change in transportation costs in city 1, starting from a symmetric equilibrium where all cities are identical replicas of each other. Its direct benefit is the change in the total transportation cost in city 1: $MB_t^1 = \partial TC(N^1, t^1)/\partial t^1$. From the Harberger formula (21), the change in the social surplus is:

$$(25) \qquad -\frac{dS}{dt^1} = MB_t^1 - \left(m\tau_Y \frac{dY^1}{dt^1} + \tau_m \frac{dm^1}{dt^1} \right) - (n-1)\left(m\tau_Y \frac{dY}{dt^1} + \tau_m \frac{dm}{dt^1} \right),$$

where superscript 1 denotes city 1 and variables without a superscript refer to other cities, and we have used the fact that all the variables are equal at the initial symmetric equilibrium: $\tau_Y^1 = \tau_Y$, $\tau_m^1 = \tau_m$, and $m^1 = m$. Note that we attach a minus sign to dS/dt^1 to indicate the impact of a marginal *decrease* in transportation costs (i.e., $-dt^1$).

From (24), we can express the change in the social surplus using the wage distortion as:

$$(26) \qquad -\frac{dS}{dt^1} = MB_t^1 + \tau_N\left(-\frac{dN^1}{dt^1} - (n-1)\frac{dN}{dt^1} \right) = MB_t^1 - \tau_N \frac{dN^U}{dt^1}.$$

Thus, if a transportation improvement in a city increases the total urban population, $dN^U/dt^1 > 0$, and if the wage distortion is positive, then there will be positive additional benefits. We show that the stability condition for population migration ensures the first condition, but we cannot exclude the possibility that the wage distortion becomes negative.

Equilibrium within a city determines the wage rate as a function of its population, $w^j = \widetilde{w}(N^j)$. The equilibrium condition for population movement (25) can then be rewritten as:

$$\widetilde{w}(N^1) - T(N^1, t^1) = \widetilde{w}(N) - T(N, t) = G'(N^A).$$

The effect of a marginal change in t^1 is then:

$$\left(\widetilde{w}' - T_N\right)\frac{dN^1}{dt^1} - T_t = \left(\widetilde{w}' - T_N\right)\frac{dN}{dt^1} = G''(N^A)\left(-\frac{dN^1}{dt^1} - (n-1)\frac{dN}{dt^1}\right).$$

From the first equality, we obtain:

$$\frac{dN^1}{dt^1} = \frac{dN}{dt^1} + \frac{T_t}{\widetilde{w}' - T_N}.$$

Substituting this into the second equality yields:

$$\frac{dN}{dt^1} = -\frac{G''(N^A)T_t}{\left(\widetilde{w}' - T_N + nG''(N^A)\right)\left(\widetilde{w}' - T_N\right)}.$$

Combining these two relationships with the population constraint (1), we obtain:

$$\frac{dN^U}{dt^1} = \frac{dN^1}{dt^1} + (n-1)\frac{dN}{dt^1} = \frac{T_t}{\widetilde{w}' - T_N + nG''(N^A)}.$$

Now, one of the necessary conditions for stability is that if a random perturbation increases the population in all cities equally and decreases that in the rural area accordingly, the utility in cities becomes lower than that in the rural area, inducing counteractive population movement from cities to the rural area:

$$\frac{d}{dN}\left(\widetilde{w}(N) - T(t,N) - G'\right) = \widetilde{w}' - T_N + nG''(N^A) \le 0.$$

This implies:

$$\frac{dN^U}{dt^1} = \frac{T_t}{\widetilde{w}' - T_N + nG''(N^A)} \le 0.$$

Thus, a transportation improvement in city 1 tends to increase the total population. The additional benefits are then positive or negative depending on the sign of the wage distortion:

$$(27) \qquad -\frac{dS}{dt^1} = MB_t^1 - \tau_N \frac{dN^U}{dt^1} \underset{<}{\overset{>}{}} MB_t^1 \quad as \quad \tau_N \underset{<}{\overset{>}{}} 0,$$

where the wage distortion satisfies (23). As noted above, we cannot exclude the possibility that the wage distortion is negative. If the elasticity of the price elasticity with respect to variety, $\xi(m)$, is nonnegative, then the wage distortion and hence the additional benefits are positive. They may be negative, however, if $\xi(m)$ is negative. For example, if $-1 < \xi < 0$, then the wage distortion is negative when the term involving

the price distortion is large compared with the variety distortion term. If $\xi < -1$, then it is negative in the opposite case where the variety distortion term is large relative to the price distortion.

Another important implication of our result is that additional agglomeration benefits are positive only when the total urban population increases. A transportation improvement increases the size of the city where it occurred, but it reduces the size of other cities. The adverse effects on other cities (at least partially) offset the benefits in city 1. If the total urban population (or equivalently, the total population of the rural area) is fixed, then these effects cancel each other out and there will be no extra benefits besides the direct benefit: $-dS/dt^1 = MB_t^1$.

5.   Examples: Additively separable, quadratic, and translog functions

We now examine three functional forms that have been used in the literature, additively separable and quadratic production functions, and a translog cost function.

Additively separable production function

If the production function of the final good is additively separable, $y_0 = \left(\int_0^m f(y_i)di \right)^{1+\rho}$, demand for an intermediate good by a final-good producer, y, and its total supply by an intermediate good producer, *Y*, do not depend on variety *m*. This can be seen as follows.

First, in a symmetric equilibrium we have $\hat{F}(y,m) = (mf(y))^{1+\rho}$, and condition (9) obtained from combining the first-order condition for profit maximization (6) and the free-entry condition (8) becomes:

(28) $(1+\rho)f'(y) = f(y)/y$.

This equation fully determines *y* independent of variety *m*: $y'(m) = 0$.

Second, the first-order condition for profit maximization of a final-good producer, (11), becomes:

$$(1+\rho)(mf(y))^{\frac{\rho}{1+\rho}} f'(y_i) = p_i,$$

in a symmetric equilibrium. The demand for input *i* by a final-good producer then

satisfies:

$$\frac{\partial d(p_i; y, m)}{\partial p_i} = \frac{1}{f''(y_i)(1+\rho)(y_0)^{\frac{\rho}{1+\rho}}},$$

and the price elasticity of market demand is

$$\eta = -\frac{f'(y)}{yf''(y)} = \frac{1}{R_R(y)},$$

where $R_R(y) \equiv -yf''/f'$ is equivalent to the measure of relative risk aversion in expected utility theory. As y does not depend on variety, the price elasticity does not depend on it either: $\eta'(m) = 0$. Equation (12) determines the production level of an intermediate good as $Y = (\eta - 1)a/c$ so that $Y'(m) = 0$. Thus, transportation improvements do not affect the production level Y, and any change in production occurs only through variety m.

The price distortion (18) and the variety distortion (19) satisfy: $\tau_Y = wc/(\eta - 1) > 0$ and $\tau_m = \rho wa/R_R > 0$. Although the price distortion exists in the additively separable case, it does not cause any excess burden because the output level Y does not change. The wage distortion is proportional to the variety distortion and satisfies: $\tau_N = \tau_m m/N = \rho w$. Note that the measure of relative risk aversion R_R is the key parameter for the price distortion, whereas the returns-to-scale parameter ρ determines the variety and wage distortions.

If we restrict the functional form to CES, condition (28) holds only when $\rho = 1/(\sigma - 1)$. In this case, the scale of a final-good producer is indeterminate, but the aggregate production function exists and satisfies $Y_0 = \alpha^{\sigma/\sigma-1} m^{\sigma/\sigma-1} Y$ in a symmetric equilibrium. The price elasticity is constant at $\eta = \sigma$, and the production level can be solved explicitly as $Y = (\sigma - 1)a/c$. Because in the CES case the returns-to-scale parameter ρ and the elasticity of substitution parameter σ are linked perfectly, one parameter (either σ or ρ) determines all the distortions: $\tau_Y = wc/(\sigma - 1)$, $\tau_m = wa(\sigma/(\sigma - 1))$, and $\tau_N = w/(\sigma - 1)$.

Zhelobodko et al. (2011) examined an additively separable utility function in a

model of differentiated consumer goods and obtained results that are different from ours. In their model, the price elasticity and the production level of a differentiated good in general depend on city size, and the CES is the knife-edge case where these are constant. The separability assumption is more restrictive for differentiated intermediate goods: the elasticity and the production level are constant for any additively separable production function.

Quadratic production function

The next example is a quadratic production function. The quadratic form has been used extensively for utility functions, e.g., in Vives (1985) and Ottaviano et al. (2002), but not often for production functions. The exception is Peng et al. (2006). We use a slightly modified version of their production function:

$$y_0 = \alpha \int_0^m y_i \, di - \frac{1}{2}(\beta - \gamma) \int_0^m (y_i)^2 \, di - \frac{1}{2}\gamma \left(\int_0^m y_i \, di \right)^2 - 1, \quad \alpha > 0, \ \beta > \gamma.$$

They introduced scale economies by assuming that production is zero unless one unit of labor is used. Our approach is to add minus one to the production function so that positive production is not possible with low levels of inputs.

With this quadratic production function, the first-order condition for profit maximization (11) is:

$$\frac{\partial F}{\partial y_i} = \alpha - (\beta - \gamma)y_i - \gamma m y = p_i$$

in a symmetric equilibrium. The perceived demand function is then:

$$d(p_i, y, m) = \frac{1}{\beta - \gamma}(\alpha - \gamma m y - p_i),$$

and the price elasticity satisfies

$$\eta = \frac{1}{\beta - \gamma}\left(\frac{\alpha}{y} - \gamma(m + \frac{\beta - \gamma}{\gamma}) \right).$$

Next, condition (9) yields

$$y(m) = \left(\frac{2}{\gamma}\right)^{1/2}\left(m(m + \frac{\beta - \gamma}{\gamma})\right)^{-1/2}.$$

Substituting this into the price elasticity, we obtain:

$$\eta(m) = \frac{1}{\beta - \gamma}\left(\frac{\gamma}{2}\right)^{1/2}\left(m + \frac{\beta - \gamma}{\gamma}\right)^{1/2}\left(\alpha m^{1/2} - (2\gamma)^{1/2}\left(m + \frac{\beta - \gamma}{\gamma}\right)^{1/2}\right).$$

The derivative of the price elasticity is:

$$\eta'(m) = \frac{1}{\beta - \gamma}\frac{\alpha\left(m + \frac{\beta - \gamma}{2\gamma}\right) - \left(2\gamma m(m + \frac{\beta - \gamma}{\gamma})\right)^{1/2}}{\left(\frac{2}{\gamma}\right)^{-1/2}\left(m(m + \frac{\beta - \gamma}{\gamma})\right)^{-1/2}} > 0,$$

where the inequality follows from the condition that the price elasticity must be positive.[10] Thus, in the quadratic case, an increase in variety is procompetitive. From

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[10] The inequality can be proved as follows. First, for $\eta(m)$ to be positive, the last bracket must be positive. Hence,

$$\alpha m^{1/2} > (2\gamma)^{1/2}\left(m + \frac{\beta - \gamma}{\gamma}\right)^{1/2}.$$

Because both sides of the inequality are positive, the inequality is preserved when we take squares of both sides of the inequality:

$$\alpha^2 m > 2\gamma(m + \frac{\beta - \gamma}{\gamma}).$$

This requires that

$$\alpha^2 > 2\gamma \text{ and } m > \frac{2(\beta - \gamma)}{\alpha^2 - 2\gamma}.$$

Next, we show that $\eta'(m)$ is positive by proving

$$\alpha\left(m + \frac{\beta - \gamma}{2\gamma}\right) > \left(2\gamma m(m + \frac{\beta - \gamma}{\gamma})\right)^{1/2}.$$

Taking the squares of both sides and moving the right hand side to the left hand side, we obtain

$$\alpha^2\left(m + \frac{\beta - \gamma}{2\gamma}\right)^2 - 2\gamma m(m + \frac{\beta - \gamma}{\gamma})$$

$$= (\alpha^2 - 2\gamma)m(m + \frac{\beta - \gamma}{\gamma}) + \alpha^2\left(\frac{\beta - \gamma}{2\gamma}\right)^2 > \alpha^2\left(\frac{\beta - \gamma}{2\gamma}\right)^2 > 0$$

(15) and (17), both variety and the output level are increasing in city size: $\tilde{m}'(N) > 0$ and $\tilde{Y}'(N)$.

Translog cost function

Feenstra (2003) developed a method of handling a variable number of goods in a symmetric translog expenditure function. Applying his methodology to a cost function with variable returns to scale, we obtain a translog cost function:

$$\ln C = \delta_0 \ln y_0 + \frac{1}{2}\varepsilon_0(\ln y_0)^2 + a_0 + \sum_{i=1}^{m} a_i \ln p_i + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m} b_{ij} \ln p_i \ln p_j ,$$

where $a_0 = \alpha_0 + (\overline{m} - m)/2\gamma m\overline{m}$, $a_i = 1/m, b_{ii} = -\gamma(m-1)/m$ and $b_{ij} = \gamma/m$ for $i \neq j$, with $i, j = 1, \cdots, m$, and $\overline{m}$ is the total number of goods conceivably available. If $\overline{m}$ is large, then we can approximate a_0 by $a_0 = \alpha_0 + 1/(2\gamma m)$. The cost share s_i of i satisfies:

$$s_i = \frac{p_i y_i}{m\overline{py}} = (1/m) + \gamma(\overline{\ln p} - \ln p_i) ,$$

where upper bars denote averages: $\overline{py} = \sum_{i=1}^{m} p_i y_i / m$ and $\overline{\ln p} = \sum_{i=1}^{m} \ln p_i / m$. The demand function is then:

$$y_i = \frac{\overline{py}}{p_i}\left[1 + \gamma m(\overline{\ln p} - \ln p_i)\right].$$

If producer i takes the average price and the variety as given,[11] the price elasticity of the perceived demand is:

$$\eta_i = -\frac{\partial y_i}{\partial p_i}\frac{p_i}{y_i} = \left(\frac{y_i}{p_i} + \frac{m\overline{py}}{p_i}\gamma\frac{1}{p_i}\right)\frac{p_i}{y_i} = 1 + \gamma\frac{m\overline{py}}{p_i y_i} .$$

In a symmetric equilibrium, this becomes $\eta_i = \eta(m) = 1 + \gamma m$. Hence, an increase in variety makes the price elasticity higher, i.e., it is procompetitive: $\eta'(m) = \gamma > 0$. This

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where we used inequality $\alpha^2 > 2\gamma$ obtained above.

[11] Note that this is a Bertran–Nash assumption as opposed to the Cournot–Nash assumption made in the additively separable and quadratic cases.

implies that both variety and the output level are increasing in city size: $\widetilde{m}'(N) > 0$ and $\widetilde{Y}'(N) > 0$.

6.   Concluding remarks

This paper developed cost-benefit measures for the case where monopolistic competition with differentiated products provides a microfoundation of agglomeration economies. Our major results can be summarized as follows.

First, the Harberger formula for excess burden represents the extra benefits of transportation investment additional to the direct user benefits if we extend it to include variety distortion. This measure of excess burden can also be expressed by using wage distortion that captures both variety and price distortions. The agglomeration externality measure in Venables (2007) obtained from a reduced-form aggregate production function is equivalent to this measure. If the production function of the final good is additively separable with respect to intermediate inputs, then the production level of each intermediate input does not depend on the city size. This implies that the excess burden from the price distortion is zero. The excess burden comes only from the variety distortion and the additional benefits in this case are always positive as in Venables (2007). For a general functional form, however, the production of an intermediate input may change in either direction and we cannot exclude the possibility that the additional benefits are negative.

Second, an improvement in urban transportation in one city increases the population in that city but reduces the populations in other cities. If the population of the rural area (or equivalently, the total population of the urban areas) is fixed, then the changes in the excess burden cancel each other out and only the direct benefit remains. If migration between the rural area and cities is possible, then a transportation improvement increases the total urban population and there will be positive additional benefits.

There are two practical implications of our findings. First, at least in a model of differentiated intermediate products, one can use a reduced-form aggregate production function, as in Venables (2007), to estimate the "wider" benefits of transportation improvements. Second, whether or not substantial agglomeration benefits exist depends on where the new workers are from. If they are from another city with a similar

agglomeration economy, there will be little additional benefit. Conversely, if they are from rural areas with no agglomeration economies, or from small cities with only small agglomeration economies, the additional benefits may be substantial.[12]

Graham (2005, 2006, 2007a, 2007b) and the Department of Transport (2005, 2008) employ a framework unlike that of Venables (2007) in modeling urban agglomeration. These particular studies use the concept of "effective density" to measure relative proximity to urban activities, as defined for each location using a gravity-model-type equation: for example, the weighted sum of the number of workers, with weights determined as a decreasing function of distance. However, even in a model of this type, we need to consider the adverse effects on areas that lose workers. We defer to future work the analysis of a second-best benefit measure based on the microfoundations of effective density.

If transportation improvements cause a merger of two cities, agglomeration might be increased without reducing agglomerations in other cities. In order to analyze a merger in our model, transportation improvements have to open up the possibility of transporting differentiated goods to another city. Using simulation models of this type, Venables and Gasiorek (1999) showed that the additional benefits are substantial amounting to around 30% to 40% of the direct benefits. Another direction for future work is to apply the technique developed in this paper to examine the generality of their results.

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[12] Agglomeration economies tend to be larger in larger metropolitan areas. See Kanemoto et al. (2005) for an example of such a finding.

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