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Abstract

Policy recommendations based on elasticity parameters have assumed greater significance for the firm's financial viability and success in a competitive set up. So there is a need to examine with more prudence the elasticity estimates obtained from various parametric as well nonparametric methods. The main aim of this paper is first to critically examine these methods, then to point out their limitations, and finally to propose an alternative method to be considered for while pursuing further elasticity studies.

Keywords: DEA; cost efficiency; production elasticity; cost elasticity; returns to scale.

1 Introduction

Intensifying pressures in competitive environment have motivated many industries to build larger operating units to achieve the widely advantages of 'scale economies.' This is apparent not only in manufacturing industries but also in regulated/state-owned industries such as electricity, water, telecom, etc., and public sector units such as hospitals and schools. This reflects the

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spread of faith in the underlying benefits of 'scale increases' in the minds of economists, engineers, industrial managers, and governments. From a policy point of view, the estimation of scale elasticity (returns to scale, RTS) parameter is of particular importance concerning whether there is any scope for increased productivity by expanding/contracting, whether minimum efficient scales will allow competitive markets to be established, and if the existing size distribution of firms is consistent with a competitive market outcome. So there is a need that these estimates should be examined with more prudence for the firm's financial viability and success.

We find in the economics literature (Färe *et al.*, 1988 and Førsund, 1996) that there are two approaches to the estimation of scale elasticity or degree of scale economies (DSE): the neoclassical production (cost) function approach (Frisch, 1965) and axiomatic approach (Shephard, 1970). The former approach (whose estimation method is parametric econometric approach) gives us quantitative measure of scale economies whereas the latter approach (whose estimation method is the nonparametric data envelopment analysis (DEA) by Charnes *et al.*, 1978) yields qualitative information on scale economies. Recently, we find in the literature that DEA models also generate quantitative information of scale econmomies (Banker *et al.*, 1996b, Førsund, 1996, Sueyoshi, 1997). Both the methods have become important analytical tools in the empirical evaluation of elasticity. The main purpose of this paper is to empirically examine the nature of scale properties in both the methods, then to point out the limitations, and finally to propose an alternative to get rid of such limitations.

This paper is organized as follows. In Section 2, we first introduce the nonparametric DEA models for the qualitative evalustion of returns to scale, discuss the quantitative evaluation of elasticity in the existing DEA models, then point out their limitations and suggest an alternative method in the same existing DEA framework. Section 3 deals with the discussion of quantitative evaluations of elasticity in the recent parametric models. For empirical comparison, we employ the Nippon Telegraph & Telephone (NTT) data in Sueyoshi (1997) and two other data sets to demonstrate the choice of superior model in Section 4. Section 5 ends with some concluding remarks.

2 Nonparametric DEA Models

Throughout this paper, we deal with n decision making units (DMUs)/firms, each uses m inputs to produce s outputs. For each DMU_o $(o = 1, \ldots, n)$, we denote respectively the input/output vectors by $\boldsymbol{x}_o \in R^m$ and $\boldsymbol{y}_o \in R^s$. The input/output matrices are defined by $X = (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n) \in R^{m \times n}$ and $Y = (\boldsymbol{y}_1, \ldots, \boldsymbol{y}_n) \in R^{s \times n}$. We assume that $X > O$ and $Y > O$.

2.1 Technology and Scale Elasticity

The technology ($\boldsymbol{T}$), which converts inputs into outputs at any given point of time is defined as the set of all feasible input-output combinations,

$$\boldsymbol{T} \equiv \{(\boldsymbol{x}, \boldsymbol{y}) \mid \boldsymbol{x} \text{ can produce } \boldsymbol{y}\}.$$

The standard neoclassical characterization of production function for multiple outputs and multiple inputs is the transformation function $\psi(\boldsymbol{x}, \boldsymbol{y})$ which exhibits the following properties:

$$\psi(\boldsymbol{x}, \boldsymbol{y}) = 0, \ \frac{\partial \psi(\boldsymbol{x}, \boldsymbol{y})}{\partial y_r} < 0 \ (\forall r) \text{ and } \frac{\partial \psi(\boldsymbol{x}, \boldsymbol{y})}{\partial x_i} > 0 \ (\forall i).$$

Alternatively, the technology can be described by its input set

$$L(\boldsymbol{y}) \equiv \{\boldsymbol{x} \mid (\boldsymbol{x}, \boldsymbol{y}) \in \boldsymbol{T}\} \text{ for all } \boldsymbol{y},$$

or by its output set

$$P(\boldsymbol{x}) \equiv \{\boldsymbol{y} \mid (\boldsymbol{x}, \boldsymbol{y}) \in \boldsymbol{T}\} \text{ for all } \boldsymbol{x}.$$

Following Shephard (1970), the output distance function is defined as

$$D_o(\boldsymbol{x}, \boldsymbol{y}) \equiv \inf\{\delta \mid \boldsymbol{y}/\delta \in P(\boldsymbol{x}), \delta > 0\}.$$

For any output vector $\boldsymbol{y}$, $\boldsymbol{y}/D_0(\boldsymbol{x}, \boldsymbol{y})$ is the largest output quantity vector on the ray from the origin through $\boldsymbol{y}$ that can be produced from $\boldsymbol{x}$. Assuming free disposability, the following holds true:

$$\boldsymbol{y} \in P(\boldsymbol{x}) \text{ if and only if } D_0(\boldsymbol{x}, \boldsymbol{y}) \leq 1.$$

Thus, $D_0(\boldsymbol{x}, \boldsymbol{y})$ provides a representation of the technology.

The returns to scale (RTS) or scale elasticity in production (ρ_p) or degree of scale economies (DSE) or *Passus Coefficient*, is defined as the ratio of the maximum proportional (β) expansion of outputs to a given proportional (μ) expansion of inputs. So differentiating the transformation function $\psi(\mu\boldsymbol{x}, \beta\boldsymbol{y}) = 0$ w.r.t. scaling factor μ, and then equating it with zero yields the following local scale elasticity measure:

$$\rho_p(\boldsymbol{x}, \boldsymbol{y}) \equiv -\sum_i^m x_i \frac{\partial\psi}{\partial x_i} \Big/ \sum_r^s y_r \frac{\partial\psi}{\partial y_r}.$$

See Hanoch (1970), Starrett (1977), Panzar and Willig (1977) and Baumol *et al.* (1988) for the detailed discussion.

However, in case of single input and single output technology ρ_p is simply expressed as the ratio of marginal product (MP)[$=dy/dx$] to average product (AP) [$=y/x$], i.e.,

$$\rho_p(x, y) \equiv \frac{\text{MP}}{\text{AP}} = \frac{dy/dx}{y/x}.$$

The scale elasticity also reflects the sensitivity of the output distance function with respect to changes in the input quantity vector where $\psi(\boldsymbol{x}, \boldsymbol{y}) = D_o(\boldsymbol{x}, \boldsymbol{y}) - 1 = 0$ (Färe *et al.*, 1986, and Ray, 1999). Assuming $D_o(\boldsymbol{x}, \boldsymbol{y})$ to be continuously differentiable, ρ_p is then defined by

$$\rho_p(\boldsymbol{x}, \boldsymbol{y}) \equiv -\frac{\sum_{i=1}^m x_i \frac{\partial D_o(\boldsymbol{x}, \boldsymbol{y})}{\partial x_i}}{D_o(\boldsymbol{x}, \boldsymbol{y})}.$$

For a neoclassical 'S-shaped production function' (or *Regular Ultra Passum Law* (RUPL) in the words of Frisch, 1965), $\rho_p(x, y)$ takes on values ranging from 'greater than one' for suboptimal output levels, through 'one' at the optimal scale level, and to values 'less than one' at the superoptimal output levels. So the production function satisfies RUPL if $\partial\rho_p/\partial y < 0$ and $\partial\rho_p/\partial x < 0$ (Førsund and Hjalmarsson, 2002). RTS are increasing, constant and decreasing if $\rho_p > 1$, $\rho_p = 1$, and $\rho_p < 1$ respectively.

Following Baumol *et al.* (1988), the dual measure of production elasticity, called cost elasticity (ρ_c), is defined in multiple input and multiple output environment as

$$\rho_c \equiv C(\boldsymbol{w}, \boldsymbol{y}) \Big/ \sum_{r=1}^s y_r \frac{\partial C(\boldsymbol{w}, \boldsymbol{y})}{\partial y_r}$$

where $C(\boldsymbol{w}, \boldsymbol{y}) \equiv \min_{\boldsymbol{x}}\{\boldsymbol{w}.\boldsymbol{x} \mid \boldsymbol{x} \in L(\boldsymbol{y})\}$ is the minimum cost of producing output vector $\boldsymbol{y}$ when input price vector is $\boldsymbol{w}$. However, ρ_c can be expressed as the ratio of average cost to margianl cost in the case of single output. RTS are increasing, constant or decreasing depending upon whether $\rho_c > 1$, $\rho_c = 1$, or $\rho_c < 1$ respectively.

2.2 Qualitative Information on RTS

The CCR output oriented model (Charnes *et al.*, 1978), which is based on the assumption of constant returns to scale, is used to qualitatively describe local RTS for DMU_o.

$$\begin{array}{ll} \text{[CCR-O]} & \max\ \theta \\ \text{subject to} & \sum_{j=1}^{n} x_{ij}\lambda_j \leq x_o\ (i = 1, \ldots, m) \\ & -\sum_{j=1}^{n} y_{rj}\lambda_j + \theta y_o \leq 0\ (r = 1, \ldots, s) \\ & \lambda_j \geq 0.\ (\forall j) \end{array}$$

If $\sum_{j=1}^{n} \lambda_j = 1$ in any alternate optima, then constant returns to scale (CRS) prevails on DMU_o; if $\sum_{j=1}^{n} \lambda_j < 1$ for all alternate optima, then increasing returns to scale (IRS) prevails; and if $\sum_{j=1}^{n} \lambda_j > 1$ for all alternate optima, then decreasing returns to scale (DRS) prevails.

The dual of the BCC model (Banker *et al.*, 1984), which is based on the assumption of variable returns to scale (VRS), is also used for obtaining the qualitative information on local RTS for DMU_o.

$$\begin{array}{ll} \text{[BCC-O]} & \min\ \phi = \sum_{i=1}^{m} v_i x_{io} + v_o \\ \text{subject to} & -\sum_{r=1}^{s} u_r y_{rj} + \sum_{i=1}^{m} v_i x_{ij} + v_o \geq 0, (j = 1, \ldots, n) \\ & \sum_{r=1}^{s} u_r y_{ro} = 1 \\ & u_r, v_i \geq 0,\ \text{and}\ v_o:\ \text{free.} \end{array}$$

If $v_o^* = 0$ ($*$ represents optimal value) in any alternate optimal then CRS prevails on DMU_o, if $v_o^* < 0$ in all alternate optimal then IRS prevails, and if $v_o^* > 0$ in all alternate optimal then DRS prevails on DMU_o.

Färe *et al.* (1985) introduced the following 'scale efficiency index' (SEI) method, which is based on non-increasing returns to scale (NIRS), to determine the nature of local RTS for DMU_o as follows:

$$
\begin{aligned}
\text{[SEI-O]} \quad & \max\ f \\
\text{subject to} \quad & \sum_{j=1}^{n} x_{ij}\lambda_j \leq x_o \ (i = 1, \ldots, m) \\
& -\sum_{j=1}^{n} y_{rj}\lambda_j + f y_o \leq 0 \ (r = 1, \ldots, s) \\
& \sum_{j=1}^{n} \lambda_j \leq 1 \\
& \lambda_j \geq 0.\ (\forall j)
\end{aligned}
$$

If $\theta^* = \phi^*$, then DMU_o exhibits CRS; otherwise if $\theta^* < \phi^*$, then DMU_o exhibits DRS iff $\phi^* > f^*$, and DMU_o exhibits IRS iff $\phi^* = f^*$.

These three different RTS methods are equivalent to estimate RTS parameter (Banker *et al.*, 1996b and Färe and Grosskopf, 1994). In empirical applications one, however, finds that the CCR and BCC RTS methods may fail when DEA models have alternate optima. However, the scale efficiency index method does not suffer from the above problem, and hence is found robust. An elaborate discussion on the qualitative evaluation of RTS of differents DEA models is found in Löthgren and Tambour (1996) and Tone and Sahoo (2002a).

In the light of all possible multiple optima problem in the CCR and BCC methods, Banker and Thrall (1992) generalized by introducing new variables v_o^+ and v_o^-, which represent optimal solutions obtained by solving the dual of the output-oriented BCC model, with one more constraint $\sum_{i=1}^{m} v_i x_{io} + v_o = 1$ and replacing the objective function in this model by either $v_o^+ = \max v_o$ or or $v_o^- = \min v_o$. They show here that IRS operates iff $v_o^+ \geq v_o^- > 0$, DRS operates iff $0 > v_o^+ \geq v_o^-$ and CRS operates iff $v_o^+ \geq 0 \geq v_o^-$.

Banker *et al.* (1996b) point out that the concept of RTS is unambiguous only at point on the efficient facets of production technology. So the RTS for the inefficient units may depend upon whether the efficiency estimation is made through an input-oriented or output-oriented manner. A detailed method of doing so is found in the studies of Banker *et al.* (1996a), Tone (1996) and Cooper *et al.* (1999).

2.3 Quantitative Information on RTS

In this subsection we first discuss the quantitative evaluation of production as well as cost elasticity, then point out their limitations, and finally suggest an alternative measure to get rid of such limitations.

2.3.1 Production Elasticity

If a DMU_o is efficient in [BCC-O], then it holds that

$$-\sum_{r=1}^{s} u_r^* y_{ro} + \sum_{i=1}^{m} v_i^* x_{io} + v_o^* = 0$$

In order to unify multiple outputs and multiple inputs, let us define a scalar output y and scalar input x respectively as

$$y = \sum_{r=1}^{s} u_r^* y_{ro}, \text{ and } x = \sum_{i=1}^{m} v_i^* x_{io}.$$

Then, we have output (y) to input (x) relationship as

$$y = x + v_o^*.$$

From this equation, we define MP as

$$\text{MP} = \frac{dy}{dx} = 1,$$

and AP as

$$\text{AP} = \frac{y}{x} = \frac{1}{x} = \frac{1}{1 - v_o^*}, \text{ since } y = \sum_{r=1}^{s} u_r^* y_{ro} = 1.$$

Now, the production elasticity (ρ_p) is defined as

$$\rho_p = \frac{\text{MP}}{\text{AP}} = 1 - v_o^*.$$

However, if DMU_o is inefficient, then ρ_p equals $(1 - \frac{1}{\phi^*}.v_o^*)$. RTS are increasing, constant and decreasing if $v_o^* < 0$, $v_o^* = 0$ and $v_o^* > 0$ respectively.

To note here that as pointed out by Førsund and Hjalmarsson (2002), the production elasticity, ρ_p does not satisfy fully the requirement of RUPL as

$$\frac{\partial \rho_p(x,y)}{\partial x_{io}} = -v_o^* \frac{\partial(1/\phi)}{\partial x_{io}} = (1/\phi^2)v_o^* v_i, \; i = 1, \ldots, m.$$

IRS ($v_o^* < 0$) implies decreasing production elasticity in accordance with RUPL, while DRS ($v_o^* > 0$) implies an increasing ρ_p, thus violating the law.

2.3.2 Cost Elasticity

Sueyoshi (1997) used the following dual of the VRS cost DEA model

$$\begin{array}{ll} \text{[COST]} & \gamma^* = \max \sum_{r=1}^{s} u_r y_{ro} + \omega_o \\ \text{subject to} & -\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} + \omega_o \leq 0, (\forall j) \\ & v_i \leq w_i, (\forall i) \\ & u_r, v_i \geq 0, (\forall r, i), \;\; \omega_o: \text{ free} \end{array}$$

to compute cost elasticty/DSE for DMU_o (where $*$ represents optimal value). Following Baumol *et al.* (1988), he computed DSE at $(\boldsymbol{w}_o, \boldsymbol{y}_o)$ as

$$\rho_c(= \text{DSE}) = \gamma^* / (\sum_{r=1}^{s} u_r^* y_{ro}),$$

and shows the equivalence of IRS with DSE > 1, CRS with DSE $= 0$ and DRS with DSE < 1.

It is to be noted here that under the assumption of unique optimal solution, the production elasticity (ρ_p) in the BCC-O model and cost elasticity (ρ_c) in VRS Cost model are same when $\phi^* = 1$ and $v_o^* = \omega_o^*/(\omega_o^* - \gamma^*)$. Otherwise,

$$\frac{\rho_c}{\rho_p} = \frac{1 - \frac{\omega_o^*}{\omega_o^* - \gamma^*}}{1 - \frac{1}{\phi^*} v_o^*}.$$

However, the the details of the duality relationship between ρ_p and ρ_c can be found in Cooper *et al.* (1996) and Sueyoshi (1999, pp.1603-1604).

2.3.3 An Alternative Measure of Scale Elasticity

This cost model however suffers from two problems: 1) cost elasticity ρ_c is no different from its dual counterpart, i.e., production elasticity ρ_p, thus giving the illusion that RTS and economies of scale are the one and same, and 2) this cost model declares a cost inefficient DMU as efficient one.

Concerning the first problem, it is to be noted here that in the above production-cost relationship it has been implicitly maintained that in the special case of given input factor prices, the cost structure is entirely determined from the underlying production technology where IRS implies economies of scale. However, as the input market is typically imperfect in the real world, these two concepts can no longer be the same. A description concerning the conceptual differences between these two concepts lies beyond the scope of this study. However, the interested readers can refer to our earlier studies, e.g., Sahoo *et al.* (1999) and Tone and Sahoo (2002a) in which both the concepts are critically analyzed and distinguished in the light of classical and neoclassical perspectives, and it is further shown there that they have distinctive causative factors that do not permit them to be used interchangeably.

As regards the second problem, Tone (2001) has recently shown that if any two DMUs (A and B, say) have same amount of inputs and outputs, i.e., $\boldsymbol{x}_A = \boldsymbol{x}_B$ and $\boldsymbol{y}_A = \boldsymbol{y}_B$, and the unit cost of DMU A is twice that of DMU B for each input, i.e., $\boldsymbol{w}_A = 2\boldsymbol{w}_B$, then both the DMUs exhibit the same cost and allocative efficiencies. This finding is very 'strange' because they have achieved the same cost efficiency irrespective of their cost differential[1] . These problems are due to the structure of the supposed production possibility set P as defined by

$$P = \{(\boldsymbol{x}, \boldsymbol{y}) \mid \boldsymbol{x} \geq X\boldsymbol{\lambda}, \boldsymbol{y} \leq Y\boldsymbol{\lambda}, \boldsymbol{e\lambda} = 1, \boldsymbol{\lambda} \geq \boldsymbol{0}\}$$

P is defined only by using technical factors $X = (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n) \in R^{m \times n}$ and $Y = (\boldsymbol{y}_1, \ldots, \boldsymbol{y}_n) \in R^{s \times n}$, but has no concern with the unit input cost $\boldsymbol{w} = (\boldsymbol{w}_1, \ldots, \boldsymbol{w}_n)$.

Let us define an another cost-based production possibility set P_c as

$$P_c = \{(\bar{\boldsymbol{x}}, \boldsymbol{y}) | \bar{\boldsymbol{x}} \geq \bar{X}\boldsymbol{\lambda}, \boldsymbol{y} \leq Y\boldsymbol{\lambda}, \boldsymbol{e\lambda} = 1, \boldsymbol{\lambda} \geq 0\},$$

[1] See Tone (2001), Tone and Sahoo (2002b,c) for the detailed explanation.

where $\bar{X} = (\bar{\boldsymbol{x}}_1, \ldots, \bar{\boldsymbol{x}}_n)$ with $\bar{\boldsymbol{x}}_j = (w_{1j}x_{1j}, \ldots, w_{mj}x_{mj})^T$.

Here, we assume that the matrices X, C and hence $\bar{X}$ are all positive. Also we assume that the elements of $\bar{x}_{ij} = (w_{ij}x_{ij})$ $(\forall(i,j))$ are denominated in homogeneous units, e.g., dollar, cent or pound so that adding up the elements of $\bar{x}_{ij}$ has a meaning.

The new cost efficiency $\bar{\gamma}^*$ is defined as

$$\bar{\gamma}^* = \boldsymbol{e}\bar{\boldsymbol{x}}_o^*/\boldsymbol{e}\bar{\boldsymbol{x}}_o,$$

where $\bar{\boldsymbol{x}}_o^*$ is the optimal solution of the LP given below.

$$\begin{array}{ll} \text{[NCOST]} & \boldsymbol{e}\bar{\boldsymbol{x}}^* = \min\ \boldsymbol{e}\bar{\boldsymbol{x}} \\ \text{subject to} & \bar{\boldsymbol{x}} \geq \bar{X}\boldsymbol{\lambda} \\ & \boldsymbol{y}_o \leq Y\boldsymbol{\lambda} \\ & \boldsymbol{e}\boldsymbol{\lambda} = 1 \\ & \boldsymbol{\lambda} \geq \mathbf{0}. \end{array}$$

The new cost efficiency is evaluated by the program [NCOST]. The constraint includes m inequalities, since $\bar{\boldsymbol{x}}$ is an m-vector. Considering the objective function form $\boldsymbol{e}\bar{\boldsymbol{x}}$ and the intput constraints in [NCost], the aggregation of these m constraints into one yields the following new program [NCOST-1]:

$$\begin{array}{ll} \text{[NCOST-1]} & \min\ \boldsymbol{e}\bar{\boldsymbol{x}} \\ \text{subject to} & \boldsymbol{e}\bar{\boldsymbol{x}} \geq \boldsymbol{e}\bar{X}\boldsymbol{\lambda} \\ & \boldsymbol{y}_o \leq Y\boldsymbol{\lambda} \\ & \boldsymbol{e}\boldsymbol{\lambda} = 1 \\ & \boldsymbol{\lambda} \geq \mathbf{0}. \end{array}$$

This program is simpler than the former in that it has only one aggregated constraint on the input part.

This aggregated model presents a correspondence between cost (input) and production (outputs). Let us denote $\boldsymbol{e}\bar{\boldsymbol{x}}_j$ by $\bar{w}_j$, i.e.,

$$\bar{w}_j = \sum_{i=1}^{m} x_{ij}w_{ij}.\ (j = 1, \ldots, n)$$

$\bar{w}_j$ is the input cost for the DMU$_j$ for producing the output vector $\boldsymbol{y}_j$. Using this notation and notifying the expressions in [NCOST-1], the new aggregated scheme reduces to the following LP:

$$\begin{aligned}
\text{[NCOST-2]} \quad & \min \sum_{j=1}^{n} \bar{w}_j \lambda_j \\
\text{subject to} \quad & \boldsymbol{y}_o \leq Y\boldsymbol{\lambda} \\
& \boldsymbol{e\lambda} = 1 \\
& \boldsymbol{\lambda} \geq \mathbf{0}.
\end{aligned}$$

In order to compute the cost elasticity, we consider the dual LP for [NCOST-2] to serve the purpose.

$$\begin{aligned}
\text{[NCOST(Dual)]} \quad & \max \sum_{r=1}^{s} u_r y_{ro} + \delta_o \\
\text{subject to} \quad & \sum_{r=1}^{s} u_r y_{rj} + \delta \leq \bar{w}_j \quad (j = 1, \ldots, n) \\
& u_r \geq 0 \ (\forall j), \ \ \delta : \text{free}.
\end{aligned}$$

We have the cost elasticity at $(\bar{w}_o, \boldsymbol{y}_o)$ as

$$\rho_c = \frac{1}{1 - \delta^* / \bar{w}_o}.$$

RTS are *increasing* if $\delta^* > 0$ $(\rho_c < 1)$, *constant* if $\delta^* = 0$ $(\rho_c = 1)$, and *decreasing* if $\delta^* < 0$ $(\rho_c > 1)$.

If there are multiple optima in δ^*, then let its sup (inf) be $\bar{\delta}^*$ $(\underline{\delta}^*)$. Then RTS are characterized as *increasing* if $\underline{\delta}^* > 0$ $(\underline{\rho_c} > 1)$, *constant* if $\bar{\delta}^* \geq 0 \geq \underline{\delta}^*$ $(\underline{\rho_c} \leq 1 \leq \bar{\rho}_c)$, and *decreasing* if $\bar{\delta}^* < 0$ $(\bar{\rho}_c < 1)$.

To note that the method discussed above for characterizing RTS holds true for efficient DMU. However, if a DMU is found inefficient, then project it onto the efficient frontier, and then solve the above LP to compute ρ_c. A detailed method of doing so is extensively discussed in Tone and Sahoo (2000c). The applications of this new method in the area of life insurance and telecommunication are found in Tone and Sahoo (2002b,c).

3 Parametric Models

Following Griliches and Ringstad (1971) and Christensen *et al.* (1973), the technology characterized by translog production function is represented by

$$\ln y = \alpha_o + \sum_{i=1}^{m} \alpha_i \ln x_i + \frac{1}{2} \sum_{i=1}^{m} \sum_{i'=1}^{m} \alpha_{ii'} \ln x_i \ln x_{i'}$$

with the following restrictions to ensure linear homogeneity in input quantities:

$$\sum_{i=1}^{m} \alpha_i = 1, \ \sum_{i'=1}^{m} \alpha_{ii'} = 0 \ (i = 1, \ldots, m), \ \sum_{i=1}^{s} \alpha_{ii'} = 0 \ (i' = 1, \ldots, m).$$

The production elasticity ρ_p is computed by

$$\rho_p = \sum_{i=1}^{m} \frac{\partial \ln y}{\partial \ln x_i} = \sum_{i=1}^{m} \left[\alpha_i + \frac{1}{2} \sum_{i'=1}^{m} \alpha_{ii'} \ln x_{i'} + \frac{1}{2} \sum_{i'=1}^{m} \alpha_{i'i} \ln x_{i'} \right].$$

Following Ray (1999), the technology characterized by a translog output distance function is represented by

$$\begin{aligned} \ln D_o(\boldsymbol{x}, \boldsymbol{y}) \ &= \ \alpha_o + \sum_{i=1}^{m} \alpha_i \ln x_i + \sum_{r=1}^{s} \beta_r \ln y_r \\ &+ \frac{1}{2} \sum_{i=1}^{m} \sum_{i'=1}^{m} \alpha_{ii'} \ln x_i \ln x_{i'} + \frac{1}{2} \sum_{r=1}^{s} \sum_{r'=1}^{s} \beta_{rr'} \ln y_r \ln y_{r'} \\ &+ \sum_{i=1}^{m} \sum_{r=1}^{s} \gamma_{ir} \ln x_i \ln y_r \end{aligned}$$

with the following restrictions to ensure linear homogeneity in output quantities:

$$\sum_{r=1}^{s} \beta_r = 1, \ \sum_{r'=1}^{s} \beta_{rr'} = 0 \ (r = 1, \ldots, s)$$

$$\sum_{r=1}^{s} \beta_{rr'} = 0 \ (r' = 1, \ldots, s), \ \sum_{r=1}^{s} \gamma_{ir} = 0 \ (i = 1, \ldots, m).$$

Then, ρ_p is computed by

$$\begin{aligned}\rho_p &= -\sum_{i=1}^{m}\frac{\partial \ln D_o(\boldsymbol{x},\boldsymbol{y})}{\partial \ln x_i}\\ &= -\sum_{i=1}^{m}\Big[\alpha_i + \frac{1}{2}\sum_{i'=1}^{m}\alpha_{ii'}\ln x_{i'} + \frac{1}{2}\sum_{i'=1}^{m}\alpha_{i'i}\ln x_{i'} + \sum_{r=1}^{s}\gamma_{ir}\ln y_r\Big].\end{aligned}$$

The following translog cost function

$$\begin{aligned}\ln C(\boldsymbol{w},\boldsymbol{y}) &= \alpha_o + \sum_{i=1}^{m}\alpha_i \ln w_i + \sum_{r=1}^{s}\beta_r \ln y_r\\ &\quad + \frac{1}{2}\sum_{i=1}^{m}\sum_{i'=1}^{m}\alpha_{ii'}\ln w_i \ln w_{i'} + \frac{1}{2}\sum_{r=1}^{s}\sum_{r'=1}^{s}\beta_{rr'}\ln y_r \ln r_{r'}\\ &\quad + \sum_{i=1}^{m}\sum_{r=1}^{s}\gamma_{ir}\ln w_i \ln y_r\end{aligned}$$

can be employed to estimate the cost-production relationship with the following restrictions:

$$\sum_{i=1}^{m}\alpha_i = 1,\ \sum_{i'=1}^{m}\alpha_{ii'} = 0\ (i = 1,\ldots,m)$$

$$\sum_{i=1}^{m}\alpha_{ii'} = 0\ (i' = 1,\ldots,m),\ \sum_{i=1}^{m}\gamma_{ir} = 0\ (r = 1,\ldots,s)$$

to ensure linear homogeneity in input prices. The cost elasticity ρ_c is then computed by

$$\begin{aligned}\rho_c &= 1\Big/\sum_{r=1}^{s}\frac{\partial \ln C(\boldsymbol{w}, vy)}{\partial \ln y_r}\\ &= 1\Big/\sum_{r=1}^{s}\Big[\beta_r + \frac{1}{2}\sum_{r'=1}^{s}\beta_{rr'}\ln y_{r'} + \frac{1}{2}\sum_{r'=1}^{s}\beta_{r'r}\ln y_{r'} + \sum_{i=1}^{m}\gamma_{ir}\ln w_i\Big]\end{aligned}$$

To note here that in the multiple input and multiple output environment, at the cost minimizing input vector $\boldsymbol{x}^*(\boldsymbol{w},\boldsymbol{y})$, the production elasticity (ρ_p) and cost elasticity (ρ_c) are same, i.e.,

$$\rho_p \equiv -\sum_{i}^{m} x_i^* \frac{\partial\psi}{\partial x_i}\Big/\sum_{r}^{s} y_r \frac{\partial\psi}{\partial y_r} = C(\boldsymbol{w},\boldsymbol{y})\Big/\sum_{r=1}^{s} y_r \frac{\partial C(\boldsymbol{w},\boldsymbol{y})}{\partial y_r} \equiv \rho_c.$$

See Baumol *et al.* (1988, p.55) for its proof. Further, it is shown there that any differentiable cost function, whatever the number of outputs involved, and whether or not it is derived from a homogeneous production process, has a local degree of homogeneity, which is reciprocal of the homegeneity parameter of the production process.

4 Empirical Results

4.1 The Data

For the empirical application, we have used the Japanese telecommunication industry data (NTT), which have been used in the earlier study[2] in which three outputs (Toll revenues, y_1, Local revenues, y_2 and Other, y_3) and three inputs (Total assets, x_1, Employess, x_2 and Access lines, x_3) were considered for 39 financial years (from 1953-54 to 1991-92). Each year's NTT annual performance is considered here a distinct DMU. We have made some changes in the data concerning the aggregation of outputs and scaling of data. First, for the estimation of parameters of both translog production and cost functions with greater precision, since the number of observations is less, we have considered only one output, i.e., total revenue, which is the sum of toll revenues, local revenues and other. Second, since price data are in decimal terms, taking logarithm of these data points yields negative scores. We have therefore scaled up these scores by multiplying each with 100 ensuring that the logarithmic transformation does not yield negative values. The units of original financial data are in one billion yen, but now they are in 10 million yen. Since the units of revenue and total assets data are also in one billion yen, we have multiplied them by 100 in order to maintain parity at least in terms of expressing them all in the same unit, i.e., 10 million yen. The data are reported in Appendix A.

A closer look at data set reveals that there is a trend associated with output and cost over years. And as regards the curvature of the cost frontier, the scatter plot of output *vis-a-vis* cost is shown below in Figure 1, and the frontier is then drawn keeping in mind the fact that the cost-based technology set is convex.

[2] See Sueyoshi (1997, pp.788-789).

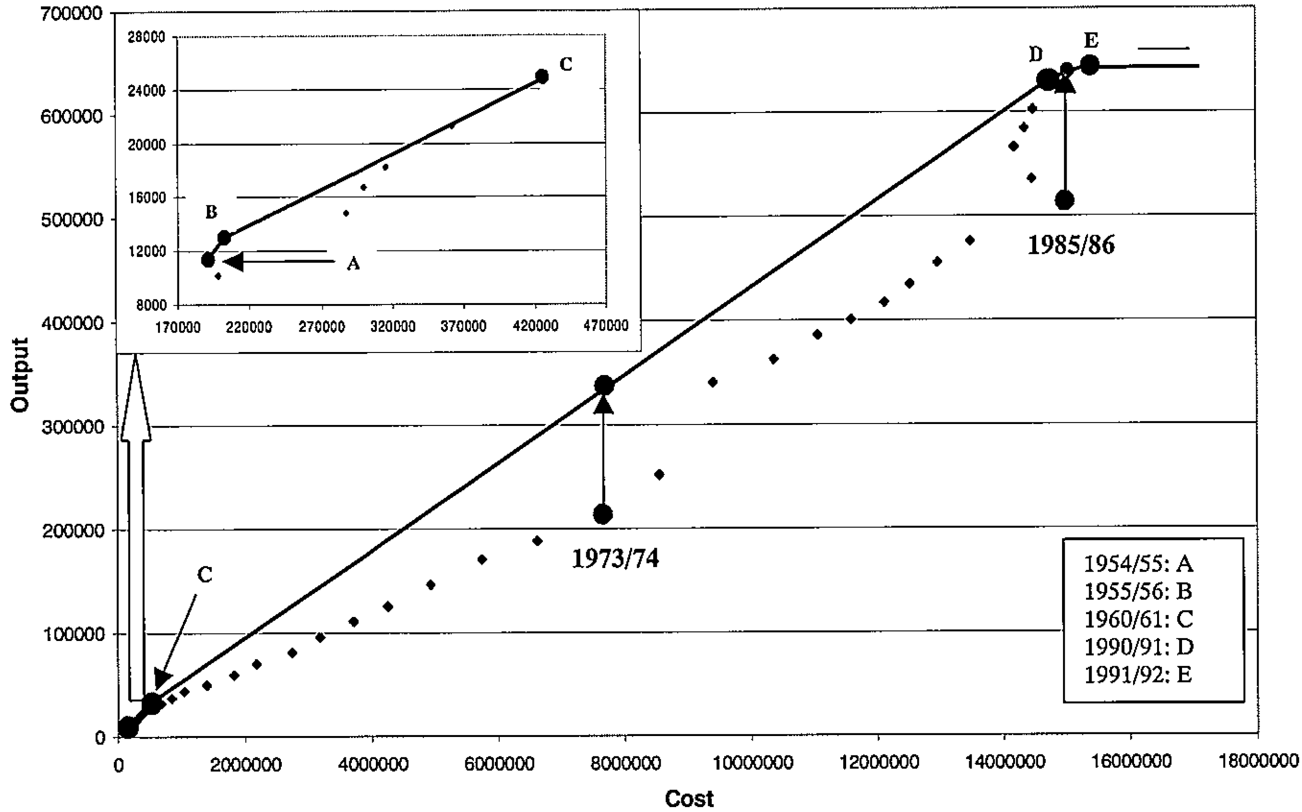

Figure 1: Cost Frontier in NTT

4.2 RTS Results from Nonparametric Models

4.2.1 RTS: Qualitative Information

We have employed CCR-O, BCC-O and SEI-O DEA models to obtain the qualitative information on returns to scale in production for 39 years of operation of NTT. The results are reported in Table 1. We find here that all the three methods are in agrement that NTT has been operating under IRS for the first 34 years, CRS for the last five years, and no DRS for any year of our sample period. However, the trend of degree of scale economies is not clear here.

Table 1: Qualitative Information on RTS in DEA

	CCR Model		BCC Model		SEI Model
DMU	$\sum_j \lambda_j$	RTS	v_o	RTS	RTS
1953/54	0.023	IRS	-0.428	IRS	IRS
54/55	0.025	IRS	-1.121	IRS	IRS
55/56	0.029	IRS	-1.009	IRS	IRS
56/57	0.032	IRS	-0.295	IRS	IRS
57/58	0.036	IRS	-0.262	IRS	IRS
58/59	0.041	IRS	-0.241	IRS	IRS
59/60	0.047	IRS	-0.206	IRS	IRS
60/61	0.059	IRS	-0.363	IRS	IRS
61/62	0.072	IRS	-0.377	IRS	IRS
62/63	0.085	IRS	-0.344	IRS	IRS
63/64	0.108	IRS	-0.297	IRS	IRS
64/65	0.128	IRS	-0.254	IRS	IRS
65/66	0.150	IRS	-0.222	IRS	IRS
66/67	0.174	IRS	-0.186	IRS	IRS
67/68	0.199	IRS	-0.158	IRS	IRS
68/69	0.215	IRS	-0.111	IRS	IRS
69/70	0.236	IRS	-0.094	IRS	IRS
70/71	0.266	IRS	-0.060	IRS	IRS
71/72	0.305	IRS	-0.053	IRS	IRS
72/73	0.359	IRS	-0.030	IRS	IRS
73/74	0.411	IRS	-0.026	IRS	IRS
74/75	0.465	IRS	-0.023	IRS	IRS
75/76	0.516	IRS	-0.021	IRS	IRS
76/77	0.564	IRS	-0.017	IRS	IRS
77/78	0.641	IRS	-0.021	IRS	IRS
78/79	0.722	IRS	-0.025	IRS	IRS
79/80	0.772	IRS	-0.029	IRS	IRS
80/81	0.802	IRS	-0.028	IRS	IRS
81/82	0.820	IRS	-0.027	IRS	IRS
82/83	0.857	IRS	-0.025	IRS	IRS
83/84	0.885	IRS	-0.024	IRS	IRS
84/85	0.918	IRS	-0.023	IRS	IRS
85/86	0.935	IRS	-0.022	IRS	IRS
86/87	0.974	IRS	-0.021	IRS	IRS
87/88	1.000	CRS	0	CRS	CRS
88/89	1.000	CRS	0	CRS	CRS
89/90	1.000	CRS	0	CRS	CRS
90/91	1.000	CRS	0	CRS	CRS
91/92	1.000	CRS	0	CRS	CRS

4.2.2 RTS: Quantitative Information

Let us now turn to see the quantitative information on scale economies, i.e., production elasticity and cost elasticity. The results are reported in Table 2. We observe here that the production elasticity has exhibited a declining trend since the begning period of our study, and the trend has become flat for the last five years of operation of NTT characterized by constant returns to scale. The economic interpretation here is that NTT has exploited all the productivity gains that were available up to year 1987-88, a year of minimum efficient scale (MES) operation after which no scale economies are found.

The RTS results in our new method, which are reported in the last four columns in Table 2, are quite opposite as compared to those obtained in the

Table 2: Quantitative Information on RTS in DEA

DMU	BCC Model		NCOST Model			
	ρ_p	RTS	$\inf \rho_c$	$\sup \rho_c$	Avg. ρ_c	RTS
1953/54	1.428	IRS	2.444	2.444	2.444	IRS
54/55	2.121	IRS	2.543	1E+15	5E+14	IRS
55/56	1.933	IRS	0.815	2.390	1.602	CRS
56/57	1.268	IRS	0.863	0.863	0.863	DRS
57/58	1.235	IRS	0.868	0.868	0.868	DRS
58/59	1.200	IRS	0.874	0.874	0.874	DRS
59/60	1.169	IRS	0.888	0.888	0.888	DRS
60/61	1.275	IRS	0.721	0.903	0.812	DRS
61/62	1.284	IRS	0.767	0.767	0.767	DRS
62/63	1.238	IRS	0.807	0.807	0.807	DRS
63/64	1.202	IRS	0.839	0.839	0.839	DRS
64/65	1.170	IRS	0.864	0.864	0.864	DRS
65/66	1.144	IRS	0.895	0.895	0.895	DRS
66/67	1.122	IRS	0.918	0.918	0.918	DRS
67/68	1.103	IRS	0.930	0.930	0.930	DRS
68/69	1.072	IRS	0.943	0.943	0.943	DRS
69/70	1.062	IRS	0.951	0.951	0.951	DRS
70/71	1.040	IRS	0.958	0.958	0.958	DRS
71/72	1.035	IRS	0.963	0.963	0.963	DRS
72/73	1.019	IRS	0.968	0.968	0.968	DRS
73/74	1.017	IRS	0.972	0.972	0.972	DRS
74/75	1.015	IRS	0.976	0.976	0.976	DRS
75/76	1.013	IRS	0.979	0.979	0.979	DRS
76/77	1.012	IRS	0.981	0.981	0.981	DRS
77/78	1.018	IRS	0.983	0.983	0.983	DRS
78/79	1.022	IRS	0.984	0.984	0.984	DRS
79/80	1.025	IRS	0.985	0.985	0.985	DRS
80/81	1.024	IRS	0.986	0.986	0.986	DRS
81/82	1.024	IRS	0.987	0.987	0.987	DRS
82/83	1.023	IRS	0.987	0.987	0.987	DRS
83/84	1.022	IRS	0.987	0.987	0.987	DRS
84/85	1.021	IRS	0.988	0.988	0.988	DRS
85/86	1.021	IRS	0.550	0.550	0.550	DRS
86/87	1.020	IRS	0.989	0.989	0.989	DRS
87/88	1	CRS	0.989	0.989	0.989	DRS
88/89	1	CRS	0.989	0.989	0.989	DRS
89/90	1	CRS	0.989	0.989	0.989	DRS
90/91	1	CRS	0.546	0.989	0.767	DRS
91/92	1	CRS	1E-12	0.556	0.278	DRS

previous three DEA models. Here $\inf \rho_c$ and $\sup \rho_c$ represent respectively the lower and upper bounds of cost elasticity, and Avg. ρ_c represnts average of these two lower and upper bounds. We observe here that NTT enjoys increasing returns to scale only for the first two years, then reaches the minimum efficient scale after which diminishing returns to scale completely set in for the remaining years. A comparison between elasticity estimates of [BCC] and [NCOST] is also exhibited in Figure 2.

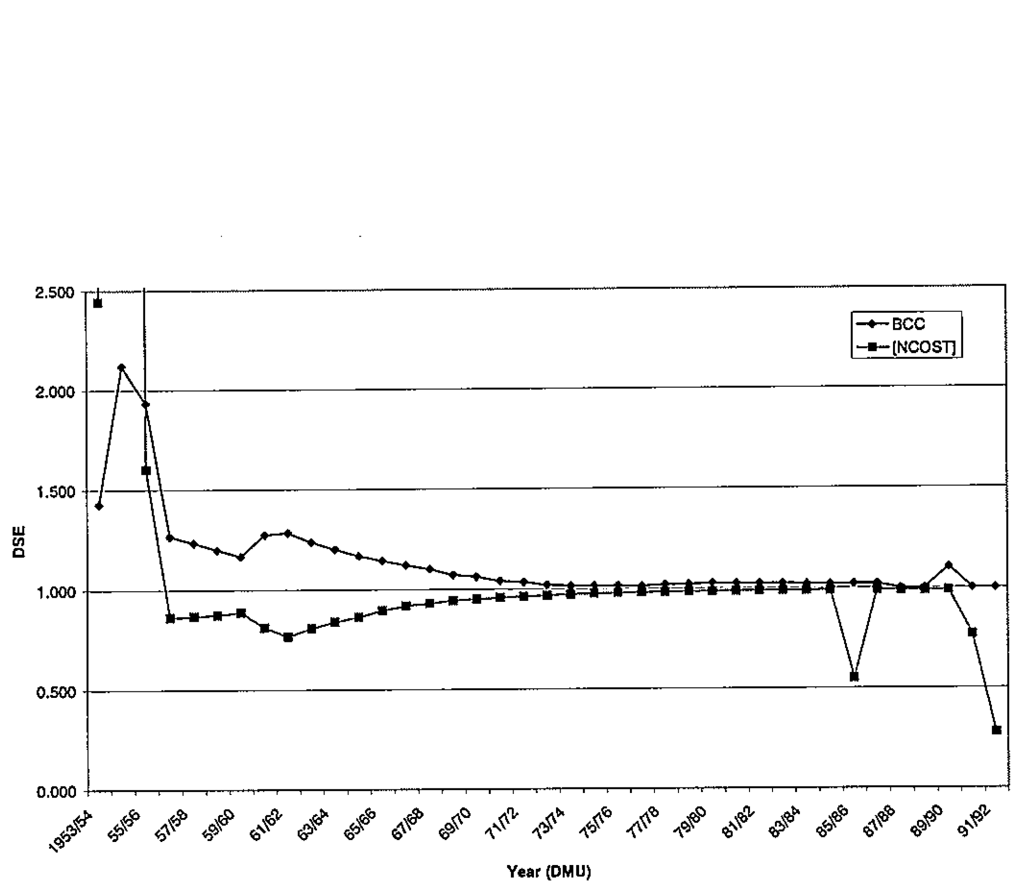

Figure 2: A Comparison Between [BCC] and [NCOST] RTS Estimates

4.3 RTS Results from Parametric Models

For the computation of production elasticity we have employed translog output distance function as well as translog production function, each being separately estimated with and without time trend. And similarly, the translog cost function (with and without time trend) is employed for the estimation of cost elasticity. The results are reported in Table 3.

As is seen in this table, the consideration of time trend has a noticeable impact on the elasticity estimates of production and cost. When no time trend is considered, there is a significant irregularity in the NTT's RTS behavior found in the production elasticity estimates that were obtained from both translog output distance function and translog production function. These results can not be trusted on the ground that there is a trend visible in the data. However, there is no such irregularity found in the cost elasticity estimates. We observe consistency in the production as well as cost elasticity estimates when time trend is introduced. On the comparison between production and cost based elasticity estimates (with trend), the difference found in these estimates can be well attributed to the fact that the translog cost function is not the self-dual of translog production function, otherwise both sets of estimates will be the same as is in the case of homegeneous technology.

Table 3: Information on RTS in Parametric Models

	With no trend		With trend		With no trend		With trend		With no trend		With trend	
DMU	ρ_p^1	RTS	ρ_p^2	RTS	ρ_p^3	RTS	ρ_p^4	RTS	ρ_c^5	RTS	ρ_c^6	RTS
1953/54	0.84	D	1.00	I	-0.61	N	-0.21	N	0.80	D	0.63	D
54/55	0.91	D	1.00	D	-0.50	N	-0.27	N	0.82	D	0.64	D
55/56	0.99	D	1.03	I	-0.23	N	-0.14	N	0.83	D	0.65	D
56/57	0.96	D	1.03	I	-0.21	N	-0.04	N	0.85	D	0.67	D
57/58	0.94	D	1.03	I	-0.21	N	0.03	D	0.86	D	0.69	D
58/59	1.03	I	1.07	I	0.08	D	0.19	D	0.88	D	0.69	D
59/60	1.09	I	1.11	I	0.33	D	0.39	D	0.90	D	0.72	D
60/61	1.30	I	1.22	I	1.04	I	0.84	D	0.93	D	0.74	D
61/62	1.44	I	1.29	I	1.55	I	1.18	I	0.96	D	0.77	D
62/63	1.43	I	1.31	I	1.65	I	1.34	I	0.97	D	0.78	D
63/64	1.45	I	1.34	I	1.85	I	1.58	I	1.00	I	0.82	D
64/65	1.40	I	1.33	I	1.84	I	1.68	I	1.03	I	0.85	D
65/66	1.35	I	1.33	I	1.82	I	1.77	I	1.06	I	0.89	D
66/67	1.29	I	1.31	I	1.74	I	1.78	I	1.10	I	0.95	D
67/68	1.19	I	1.27	I	1.54	I	1.73	I	1.14	I	0.99	D
68/69	1.08	I	1.22	I	1.31	I	1.67	I	1.18	I	1.04	I
69/70	0.98	D	1.19	I	1.12	I	1.63	I	1.22	I	1.09	I
70/71	0.85	D	1.13	I	0.82	D	1.52	I	1.27	I	1.14	I
71/72	0.82	D	1.12	I	0.81	D	1.57	I	1.31	I	1.18	I
72/73	0.72	D	1.07	I	0.56	D	1.42	I	1.36	I	1.23	I
73/74	0.65	D	1.04	I	0.45	D	1.42	I	1.41	I	1.28	I
74/75	0.61	D	1.02	I	0.38	D	1.40	I	1.45	I	1.30	I
75/76	0.58	D	1.01	I	0.37	D	1.44	I	1.50	I	1.35	I
76/77	0.60	D	1.03	I	0.48	D	1.55	I	1.58	I	1.48	I
77/78	0.71	D	1.09	I	0.85	D	1.79	I	1.75	I	1.75	I
78/79	0.77	D	1.12	I	1.06	I	1.93	I	1.79	I	1.79	I
79/80	0.82	D	1.15	I	1.23	I	2.03	I	1.83	I	1.84	I
80/81	0.91	D	1.18	I	1.46	I	2.13	I	1.85	I	1.84	I
81/82	0.98	D	1.21	I	1.66	I	2.23	I	1.88	I	1.87	I
82/83	0.99	D	1.19	I	1.64	I	2.15	I	1.91	I	1.88	I
83/84	1.01	I	1.19	I	1.66	I	2.09	I	1.94	I	1.90	I
84/85	1.02	I	1.17	I	1.63	I	2.01	I	1.96	I	1.91	I
85/86	1.18	I	1.22	I	2.01	I	2.10	I	2.00	I	1.91	I
86/87	1.13	I	1.17	I	1.77	I	1.87	I	2.05	I	1.96	I
87/88	1.13	I	1.15	I	1.70	I	1.74	I	2.10	I	2.05	I
88/89	1.12	I	1.11	I	1.57	I	1.54	I	2.13	I	2.04	I
89/90	1.24	I	1.13	I	1.78	I	1.51	I	2.16	I	2.04	I
90/91	1.22	I	1.08	I	1.62	I	1.29	I	2.20	I	2.08	I
91/92	1.25	I	1.07	I	1.62	I	1.18	I	2.22	I	2.11	I

Note: I: IRS, C: CRS, D: DRS, N: Negative returns

1: ρ_p is computed from translog distance function
2: ρ_p is computed from translog distance function
3: ρ_p is computed from translog production function
4: ρ_p is computed from translog production function
5: ρ_c is computed from translog cost function
6: ρ_c is computed from translog cost function

A graphical illustration for the comparison of scale elasticity estimates among parametric models is shown below in Figure 3.

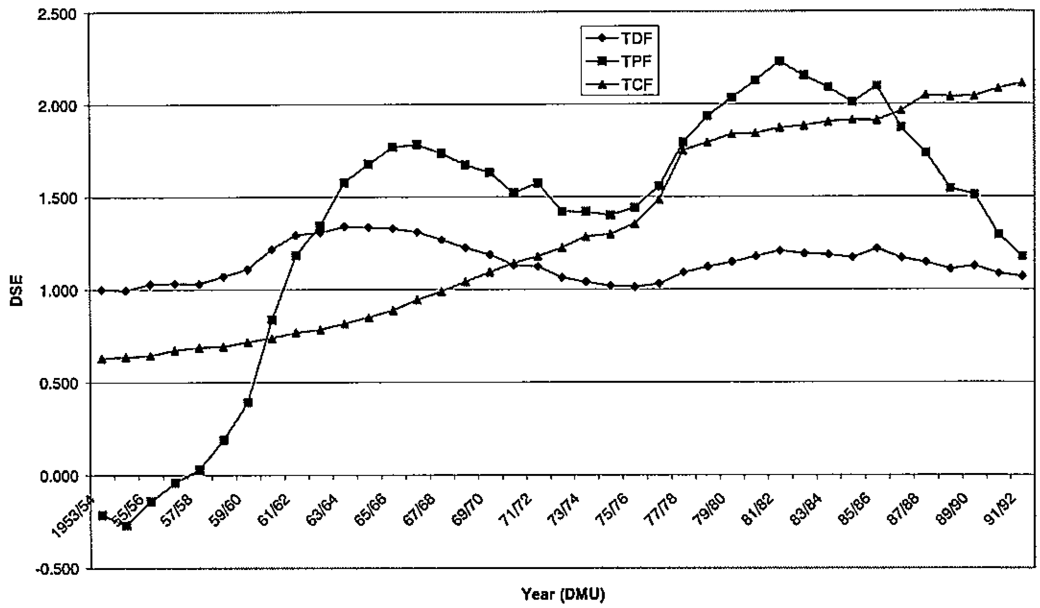

Figure 3: A Comparison of RTS Estimates Among Parametric Models

4.4 RTS Results: A Comparison

Let us now turn to compare the cost elasticity estimates obtained from the translog cost function with those in our new method. We find here that in the former method, NTT operates under DRS for the first 15 years of our sample period followed by IRS for the remaining years, which in turn implies that average cost curve is inverted U-shaped (concave from below). This finding calls into question the widely preferred translog cost method for the computation of cost elasticity, which goes against the principle of 'S-shape production function' that has long been maintained in the neoclassical microeconomic theory. However, in our new method NTT operates under IRS for the first two years, then under CRS for one year after which DRS completely sets in. This finding is quite natural in the sense of being free from above mentioned trouble. And concerning the investigation of optimal scale of operations of NTT, our new model [NCOST] offers different policy prescription in that it suggests the lower level of operation (output corresponding to DMU 1955-56's operation) as against the very very high level of such operation which is yet to come in the translog cost model.

The most pertinent question now is: Does the translog cost model always violate the principle of S-shaped neoclassical production function when the cost-based technology set is maintained to be conex (as per theory), but the observed set is not?. The answer to this question is generally a positive one. We have shown this here by considering two artificial time-series data sets (both at which cost-based technology set is maintained (assumed) to be convex, but the observed cost-based set satisfies this requirement in Data Set 2, and violates this requirement in Data Set 1) that in the former both the models are in broad agrement in exhibiting information relating to RTS whereas in the latter they are not.

4.4.1 RTS in [NCOST]: Is It Rational?

Before proceeding further, let us first intutively demonstarete the rationality of the empirical evaluation of cost elasticity in our new method. We see in Figure 1 that the cost-based technology frontier is piecewise-linear comprising only five efficient DMUs (A: 1954/55, B: 1955/56, C: 1960/61, D: 1990/91 and E: 1991/92). To note here that since the first two facets ($\overline{AB}$ and $\overline{BC}$) of the frontier are not clearly visible, we have shown their expanded view in the top left corner of this figure. We clearly see here that the first facet ($\overline{AB}$) is characterized by IRS, and DRS prevails on the remaining facets ($\overline{BC}$, $\overline{CD}$ and $\overline{DE}$). The cost, output and the computational procedure for the calculation of scale elasticity are all shown in Table 4.

Table 4: Empirical Evaluation of Cost Elasticity

DMUs	Cost (C)	Output (Y)	AC (=C/Y)	MC ($= \frac{dC}{dY}$)		$\rho_c = \frac{AC}{MC}$
1954/55	192773.7	11690	16.490	$\frac{(201139.4-192773.7)}{(12980-11690)} =$	6.485	2.543
1955/56	201139.4	12980	15.496	$\frac{(201139.4-192773.7)}{(12980-11690)} =$	6.485	2.390
1960/61	424806.9	24750	17.164	$\frac{(424806.9-201139.4)}{(24750-12980)} =$	19.003	0.903
1990/91	14714659.4	625160	23.537	$\frac{(14714659.4-424806.9)}{(625160-24750)} =$	23.8	0.989
1991/92	15347520.7	639840	23.986	$\frac{(15347520.7-14714659.4)}{(639840-625160)} =$	43.110	0.556

Now let us compute the scale elasticity of an inefficient DMU such as 1973/74 (also indicated in Figure 1). This DMU's efficiency score in our [NCOST] model is 0.686987 (not reported anywhere), and its peer DMUs are C (1960/61) and D (1990/91). So the projected cost and output values of this DMU are respectively 5742599 and 248185.2. The average cost, marginal cost and scale elasticity are computed as follows:

$$\begin{aligned} \text{AC} &= C/Y = 5742599/248185.2 = 23.138, \\ \text{MC} &= \frac{1}{\text{slope of facet } \overline{CD}} = dC/dY = 23.8, \\ \text{Cost Elasticity} &= \text{AC/MC} = 23.138/23.8 = 0.972. \end{aligned}$$

Now turning to Table 2 we verify that the the cost elasticity values of these DMUs are no different from those reported in Table 4. The fundamental difference between our new [NCOST] method and translog cost method for the estimation of cost frontier is that in the former the structure of the frontier is assumed to be convex whereas in the latter the structure, depending upon data, may either be convex or non-convex.

4.4.2 Verifying RTS in New Data Sets

We now examine this relationship in two new data sets. The scatter plots of output *vis-a-vis* cost corresponding to data sets reported in Appendix B and Appendix C are respectively exhibited in Figure 4 and Figure 5. We find here that the observed cost frontier in Figure 5, corresponding to Data Set 2, is convex whereas corresponding to Data Set 1 in Figure 4, it is not. However, in both the cases the maintained hypothesis of convex structure for the technology is maintained as can be seen from both the figures where the cost frontier is made of thick piecewise linear lines.

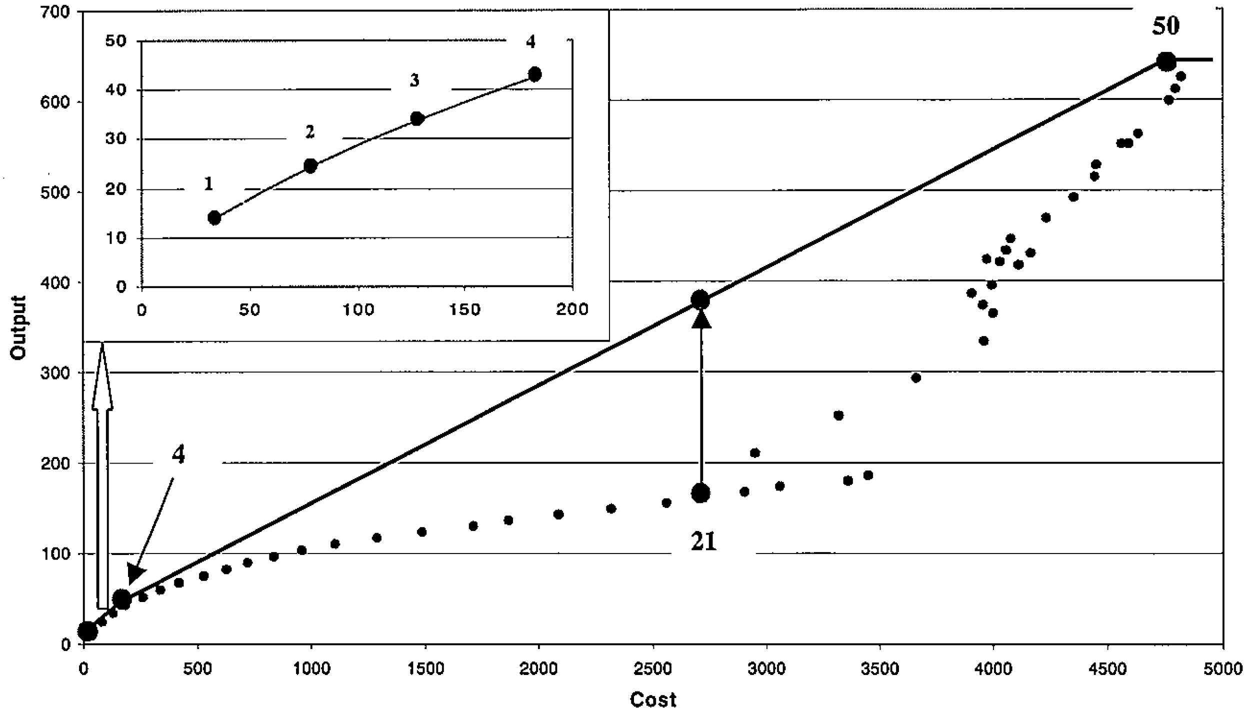

Figure 4: Cost Frontier in Data Set 1

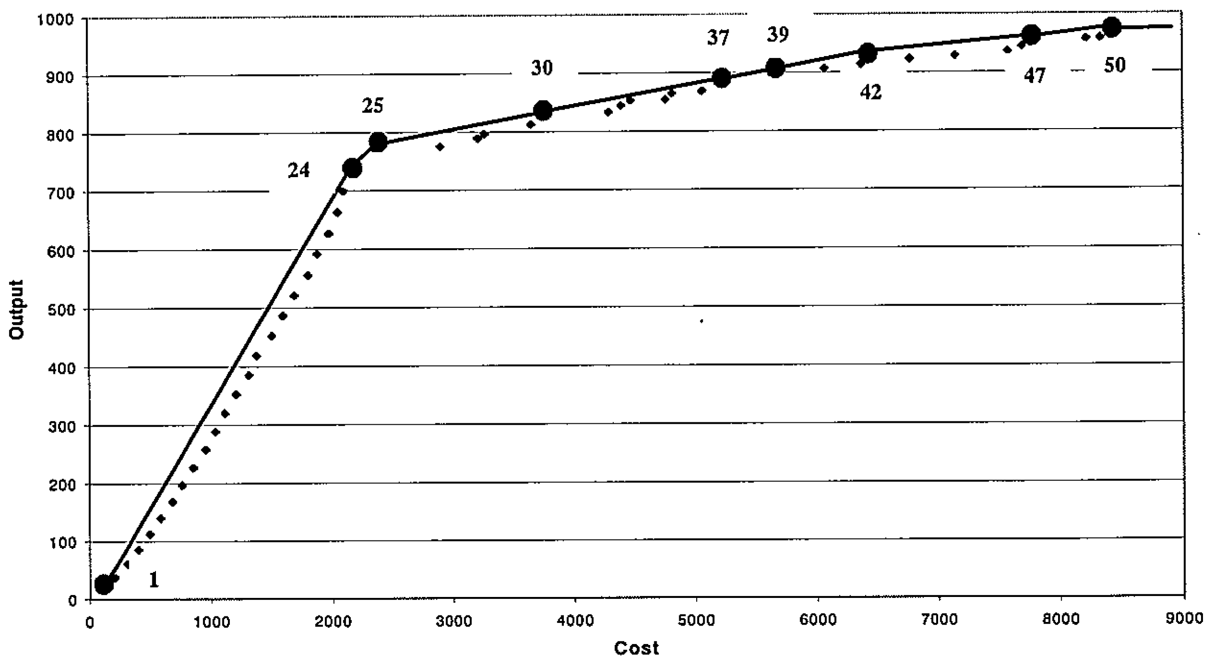

Figure 5: Cost Frontier in Data Set 2

We now report cost elasticity estimates (inf, sup and avg.) and the corresponding RTS status for each DMU in Table 5. The computational procedure is just the same as the one we explained while demonstrating the rationality of empirical evaluation of cost elasticity in Table 4. A simple glance at Figure 4 and Figure 5 reveals that the cost frontier, corresponding to Data Set 1 and Data Set 2, is made of five (1, 2, 3, 4 and 50) and nine (1, 24, 25, 30, 37, 39, 42, 47 and 50) efficient DMUs respectively. We find here that in case of former (where like NTT case, the observed cost frontier is not convex), CRS operates only in the first year after which DRS completely sets in whereas in case of latter (where unlike NTT case, the observed cost frontier is actually convex), the first 23 years of operation are under IRS and the remaining 27 years under DRS.

Now let us re-examine the scale elasticity estimates in the translog cost model, which is estimated first without time trend and then with time trend. The results are reported in Table 6. Since there is a clear trend visible in the data, the parameter estimates of the translog cost function without trend are greatly distorted, which in turn yield misleading information on RTS. Though we have reported both sets of estimates in this table for mere purpose of exposition, our analysis is purely restricted to DSE estimates with trend only. We find here that in case of Data Set 1 the first six years of operation are characterized by DRS followed by the remaining 44 years of operation under IRS. However, in case of Data Set 2 IRS prevails for the first 25 years followed by DRS for the remaining 25 years.

The potential problem arises in the choice of these two methods when the observed output-cost relationship violates the requirement of convex structure of the cost-based technology set, which is very much warranted to ensure the long-run average cost curve to be U-shaped, as has long been maintained in the neoclassical microeconomic theory. The translog cost model has the well reputation of being flexible in approximating arbitrary production technologies. True but certainly at a price. The price here is that in real-life situation when it is likely the case where the observed behavior of cost-output relationship violates the assumption of convex structure, the translog cost model yields undesirable estimates of cost elasticity, leading to erroneous implications concerning the recommendation of policy for restructuring any sector in the economy. So the comparison between these two sets of RTS estimates leads us to further restating our claim that both methods broadly yield same information on RTS in the case of data where the observed cost-based technology structure is convex, and in case of data (like NTT data as well as Data Set 1) where this convex structure is violated, the translog cost model yields misleading information on returns to scale.

5 Concluding Remarks

Investigation of cost elasticity for obtaining optimal scale of operations has significant bearings while recommending policy for restructuring any sector in a competitive economy. So due care is warranted in this regard to ensure that the cost elasticity estimates are free from error. We find in the parametric literature that the translog cost function has been very popular to estimating cost elasticity. We show here that the cost elasticity estimates obtained from this are misleading, and so also are elasticity estimates obtained from traditional nonparametric method (e.g., old cost model by Sueyoshi, 1997). We suggest a new nonparametric method for the estimation of cost elasticty while pursuing further elasticity studies.

Table 5: RTS in [NCOST]: Re-examination in a New Data Set

DMU	Data Set 1				Data Set 2			
	inf ρ_c	sup ρ_c	Avg. ρ_c	RTS	inf ρ_c	sup ρ_c	Avg. ρ_c	RTS
1	0.576	1E+15	5E+14	C	2.171	1E+15	5E+14	I
2	0.626	0.757	0.691	D	1.374	1.374	1.374	I
3	0.607	0.729	0.668	D	1.219	1.219	1.219	I
4	0.555	0.688	0.621	D	1.159	1.159	1.159	I
5	0.638	0.638	0.638	D	1.125	1.125	1.125	I
6	0.695	0.695	0.695	D	1.104	1.104	1.104	I
7	0.740	0.740	0.740	D	1.088	1.088	1.088	I
8	0.782	0.782	0.782	D	1.078	1.078	1.078	I
9	0.810	0.810	0.810	D	1.069	1.069	1.069	I
10	0.830	0.830	0.830	D	1.061	1.061	1.061	I
11	0.850	0.850	0.850	D	1.056	1.056	1.056	I
12	0.867	0.867	0.867	D	1.052	1.052	1.052	I
13	0.882	0.882	0.882	D	1.048	1.048	1.048	I
14	0.897	0.897	0.897	D	1.044	1.044	1.044	I
15	0.910	0.910	0.910	D	1.042	1.042	1.042	I
16	0.921	0.921	0.921	D	1.038	1.038	1.038	I
17	0.927	0.927	0.927	D	1.036	1.036	1.036	I
18	0.934	0.934	0.934	D	1.034	1.034	1.034	I
19	0.940	0.940	0.940	D	1.031	1.031	1.031	I
20	0.946	0.946	0.946	D	1.030	1.030	1.030	I
21	0.948	0.948	0.948	D	1.029	1.029	1.029	I
22	0.952	0.952	0.952	D	1.028	1.028	1.028	I
23	0.954	0.954	0.954	D	1.027	1.027	1.027	I
24	0.958	0.958	0.958	D	0.661	1.026	0.843	C
25	0.959	0.959	0.959	D	0.119	0.677	0.398	D
26	0.952	0.952	0.952	D	0.142	0.142	0.142	D
27	0.957	0.957	0.957	D	0.155	0.155	0.155	D
28	0.961	0.961	0.961	D	0.158	0.158	0.158	D
29	0.964	0.964	0.964	D	0.173	0.173	0.173	D
30	0.964	0.964	0.964	D	0.177	0.178	0.177	D
31	0.964	0.964	0.964	D	0.196	0.196	0.196	D
32	0.964	0.964	0.964	D	0.200	0.200	0.200	D
33	0.964	0.964	0.964	D	0.203	0.203	0.203	D
34	0.965	0.965	0.965	D	0.213	0.213	0.213	D
35	0.966	0.966	0.966	D	0.215	0.215	0.215	D
36	0.965	0.965	0.965	D	0.224	0.224	0.224	D
37	0.964	0.964	0.964	D	0.204	0.229	0.216	D
38	0.965	0.965	0.965	D	0.218	0.218	0.218	D
39	0.965	0.965	0.965	D	0.165	0.219	0.192	D
40	0.966	0.966	0.966	D	0.174	0.174	0.174	D
41	0.967	0.967	0.967	D	0.181	0.181	0.181	D
42	0.968	0.968	0.968	D	0.153	0.182	0.168	D
43	0.968	0.968	0.968	D	0.160	0.160	0.160	D
44	0.969	0.969	0.969	D	0.168	0.168	0.168	D
45	0.969	0.969	0.969	D	0.176	0.176	0.176	D
46	0.969	0.969	0.969	D	0.178	0.178	0.178	D
47	0.970	0.970	0.970	D	0.152	0.180	0.166	D
48	1E-12	0.970	0.485	D	0.159	0.159	0.159	D
49	1E-12	0.970	0.485	D	0.161	0.161	0.161	D
50	1E-12	0.970	0.485	D	1E-12	0.163	0.081	D

Note: I: IRS, C: CRS and D: DRS

Table 6: RTS in Translog Cost Model: Re-examination in a New Data Set

	Data Set 1				Data Set 2			
DMU	ρ_c^1	RTS	ρ_c^2	RTS	ρ_c^1	RTS	ρ_c^2	RTS
1	0.542	D	0.543	D	2.258	I	1.274	I
2	0.692	D	0.677	D	1.741	I	1.217	I
3	0.808	D	0.771	D	1.446	I	1.180	I
4	0.918	D	0.858	D	1.315	I	1.157	I
5	1.035	I	0.948	D	1.262	I	1.143	I
6	1.114	I	0.998	D	1.212	I	1.131	I
7	1.190	I	1.043	I	1.143	I	1.118	I
8	1.313	I	1.128	I	1.123	I	1.110	I
9	1.391	I	1.170	I	1.076	I	1.100	I
10	1.415	I	1.168	I	1.054	I	1.094	I
11	1.491	I	1.206	I	1.051	I	1.090	I
12	1.568	I	1.244	I	1.006	I	1.081	I
13	1.606	I	1.253	I	0.981	D	1.075	I
14	1.707	I	1.305	I	0.971	D	1.071	I
15	1.814	I	1.358	I	0.979	D	1.069	I
16	1.882	I	1.383	I	0.985	D	1.067	I
17	1.863	I	1.353	I	0.967	D	1.063	I
18	1.873	I	1.343	I	1.005	I	1.065	I
19	1.887	I	1.336	I	1.001	I	1.062	I
20	1.904	I	1.331	I	1.031	I	1.064	I
21	1.958	I	1.349	I	1.014	I	1.060	I
22	2.082	I	1.403	I	1.049	I	1.062	I
23	2.139	I	1.419	I	1.074	I	1.063	I
24	2.118	I	1.395	I	1.058	I	1.059	I
25	2.292	I	1.468	I	1.017	I	1.054	I
26	12.030	I	3.490	I	0.355	D	0.883	D
27	20.152	I	3.543	I	0.346	D	0.877	D
28	-29.225	N	4.700	I	0.346	D	0.877	D
29	-9.641	N	6.417	I	0.338	D	0.871	D
30	-8.759	N	6.210	I	0.338	D	0.870	D
31	-7.616	N	6.872	I	0.365	D	0.887	D
32	-5.903	N	9.343	I	0.365	D	0.887	D
33	-5.680	N	9.636	I	0.364	D	0.886	D
34	-5.023	N	11.457	I	0.358	D	0.882	D
35	-4.484	N	15.369	I	0.357	D	0.882	D
36	-4.953	N	11.666	I	0.384	D	0.897	D
37	-5.050	N	10.916	I	0.383	D	0.896	D
38	-4.603	N	13.624	I	0.376	D	0.892	D
39	-4.576	N	12.894	I	0.376	D	0.892	D
40	-3.941	N	21.921	I	0.369	D	0.888	D
41	-3.759	N	24.733	I	0.391	D	0.900	D
42	-3.545	N	32.788	I	0.391	D	0.899	D
43	-3.578	N	25.970	I	0.384	D	0.896	D
44	-3.216	N	102.787	I	0.378	D	0.893	D
45	-3.311	N	49.171	I	0.373	D	0.889	D
46	-3.286	N	46.836	I	0.373	D	0.889	D
47	-3.037	N	235.103	I	0.372	D	0.889	D
48	-3.005	N	225.302	I	0.367	D	0.886	D
49	-2.980	N	194.044	I	0.376	D	0.891	D
50	-3.000	N	84.288	I	0.376	D	0.890	D

Note: I: IRS, C: CRS, D: DRS and N: Negative Returns
^{1}: Without Time Trend, and ^{2}: With Time Trend

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Appendix A: The NTT Data Set

	Output		Inputs			Input Prices		Cost
DMU	TR	x_1	x_2	x_3	w_1	w_2	w_3	$\sum_{i=1}^{3} w_i x_i$
1953/54	10220	28200	1630	176.9	06.77	02.21	19.62	197987.1
54/55	11690	31380	1600	196.6	05.90	02.52	18.31	192773.7
55/56	12980	35850	1610	217.5	05.38	02.70	18.02	201139.4
56/57	14830	40060	1670	239.7	06.94	02.86	18.40	287203.1
57/58	16700	44520	1720	263.8	06.49	03.05	18.95	299179.8
58/59	18180	50840	1740	290.3	05.98	03.29	18.39	315086.4
59/60	21240	58620	1790	321.6	05.97	03.53	18.31	362168.6
60/61	24750	72870	1840	363.3	05.64	03.87	18.44	424806.9
61/62	29340	89560	1900	415.3	05.85	04.44	19.17	540323.3
62/63	32130	105820	1990	478.1	06.32	04.88	19.33	687735.3
63/64	37240	126270	2090	547.7	06.58	05.30	19.90	852832.8
64/65	43580	147990	2190	633.9	06.87	05.89	20.62	1042661.4
65/66	49840	172470	2290	730.3	07.97	06.56	21.51	1405317.1
66/67	59610	199640	2370	846.6	08.98	07.30	23.33	1829819.4
67/68	70050	229550	2460	988.9	09.32	08.28	24.01	2183518.3
68/69	80940	258600	2550	1136.2	10.40	09.08	24.39	2740305.9
69/70	95680	291750	2640	1300.5	10.69	10.40	24.22	3177761.6
70/71	111080	330760	2730	1517.3	10.98	11.95	24.40	3701390.4
71/72	125290	380050	2820	1731.3	10.98	13.56	24.50	4253605.1
72/73	146250	446650	2880	2098.5	10.84	15.51	23.13	4934893.1
73/74	170500	511740	2980	2416.6	11.01	17.81	22.87	5742598.8
74/75	188200	578440	3040	2744.4	11.21	22.66	24.74	6621095.3
75/76	211030	642770	3120	3034.3	11.67	25.67	26.28	7660957.7
76/77	251820	701330	3190	3242.7	11.95	27.33	27.85	8558385.4
77/78	340360	772280	3230	3394.5	11.93	30.52	29.14	9410795.7
78/79	362240	831460	3270	3549.4	12.22	33.02	29.15	10371881.6
79/80	385560	887410	3290	3704.6	12.22	34.89	28.69	11065223.3
80/81	400630	945910	3270	3849.0	12.01	38.02	29.25	11597287.8
81/82	416710	990070	3270	3933.1	12.00	40.22	30.15	12130942.4
82/83	434430	1024880	3230	4110.4	11.97	43.15	29.57	12528732.6
83/84	455240	1052190	3180	4245.5	12.06	46.51	29.11	12960899.7
84/85	475620	1079170	3140	4401.9	12.23	50.73	28.74	13484051.9
85/86	509150	1136860	3040	4486.1	12.88	57.53	24.98	14929710.8
86/87	535360	1137750	2980	4672.5	12.44	61.66	27.75	14467018.7
87/88	566200	1145590	2911	4797.7	12.09	64.50	30.95	14186431.4
88/89	584190	1155970	2833	4990.4	12.10	71.66	32.01	14349992.5
89/90	602240	1215840	2729	5199.2	11.60	76.57	33.12	14484901.0
90/91	625160	1222520	2649	5408.4	11.70	82.27	35.73	14714659.4
91/92	639840	1244490	2577	5580.0	11.99	85.68	36.79	15347520.7

Note: TR: Total revunue, x_1: Total assets, x_2: Employees, x_3: Access lines.
w_1, w_2, and w_3 are respectively the unit cost of x_1, x_2 and x_3.
The unit of output is 10 million yen, The units of inputs are respectively 10 million yen, 100 employees and 10,000 lines.

Appendix B: Data Set 1

	Output	Inputs		Input Prices		Cost
DMU	y	x_1	x_2	w_1	w_2	$\sum_{i=1}^{2} w_i x_i$
1	14.142	2	1	10	15	35
2	24.623	4	2	12	16	80
3	34.057	6	3	13	17	129
4	42.871	8	4	14	18	184
5	51.250	10	5	16	20	260
6	59.297	12	6	17	22	336
7	67.080	14	7	18	24	420
8	74.643	16	8	20	26	528
9	82.018	18	9	21	28	630
10	89.231	20	10	21	30	720
11	96.301	22	11	22	32	836
12	103.243	24	12	23	34	960
13	110.070	26	13	24	37	1105
14	116.793	28	14	26	40	1288
15	123.421	30	15	28	43	1485
16	129.960	32	16	30	47	1712
17	136.419	34	17	30	50	1870
18	142.802	36	18	31	54	2088
19	149.114	38	19	32	58	2318
20	155.360	40	20	33	62	2560
21	161.544	42	21	33	63	2709
22	167.669	44	22	34	64	2904
23	173.739	46	23	34	65	3059
24	179.756	48	24	35	70	3360
25	185.724	50	25	35	68	3450
26	210.506	15	10	110	130	2950
27	251.713	17	12	100	135	3320
28	292.911	18	15	95	130	3660
29	333.245	19	18	90	125	3960
30	364.113	20	20	80	120	4000
31	373.105	20	21	77	115	3955
32	386.067	21	21	76	110	3906
33	395.153	21	22	75	110	3995
34	417.407	22	23	73	109	4113
35	430.599	23	23	73	108	4163
36	421.135	23	22	72	108	4032
37	423.894	24	21	71	108	3972
38	433.870	24	22	71	107	4058
39	446.446	25	22	69	107	4079
40	469.186	26	23	69	106	4232
41	492.108	27	24	67	106	4353
42	515.206	28	25	65	105	4445
43	528.018	29	25	63	105	4452
44	551.406	30	26	63	104	4594
45	551.406	30	26	62	104	4564
46	561.910	30	27	61	104	4638
47	598.665	32	28	59	103	4772
48	611.700	33	28	58	103	4798
49	624.618	34	28	57	103	4822
50	637.421	35	28	55	102	4781

Appendix C: Data Set 2

	Output	Inputs		Input Prices		Cost
DMU	y	x_1	x_2	w_1	w_2	$\sum_{i=1}^{2} w_i x_i$
1	16.245	2	1	30	42	102
2	37.321	4	2	28	45	202
3	60.711	6	3	28	46	306
4	85.742	8	4	27	46	400
5	112.069	10	5	26	47	495
6	139.477	12	6	25	47	582
7	167.818	14	7	25	47	679
8	196.983	16	8	24	47	760
9	226.888	18	9	24	47	855
10	257.467	20	10	24	48	960
11	288.664	22	11	23	48	1034
12	320.434	24	12	23	47	1116
13	352.739	26	13	23	47	1209
14	385.545	28	14	23	48	1316
15	418.823	30	15	22	48	1380
16	452.548	32	16	22	50	1504
17	486.698	34	17	22	50	1598
18	521.252	36	18	21	52	1692
19	556.193	38	19	21	53	1805
20	591.503	40	20	20	54	1880
21	627.168	42	21	20	54	1974
22	663.175	44	22	19	55	2046
23	699.511	46	23	18	55	2093
24	736.164	48	24	18	55	2184
25	773.124	50	25	19	56	2350
26	774.959	250	200	10	2	2900
27	788.490	255	205	11	2	3215
28	796.183	260	205	11	2	3270
29	812.535	270	206	12	2	3652
30	828.648	280	207	12	2	3774
31	832.805	282	208	13	3	4290
32	844.536	290	208	13	3	4394
33	853.228	296	208	13	3	4472
34	854.234	295	210	14	3	4760
35	864.649	298	215	14	3	4817
36	868.754	300	216	14	4	5064
37	884.339	310	217	14	4	5208
38	898.489	320	217	15	4	5668
39	901.293	322	217	15	4	5698
40	906.731	325	218	16	4	6072
41	913.680	330	218	16	5	6370
42	920.341	333	220	16	5	6428
43	922.843	333	222	17	5	6771
44	928.104	335	224	18	5	7150
45	935.005	340	224	19	5	7580
46	944.480	346	225	19	5	7699
47	949.924	350	225	19	5	7775
48	956.685	355	225	20	5	8225
49	957.959	355	226	20	6	8343
50	962.112	360	224	20	6	8432