GRIPS Discussion Paper 16-16

Little Green Lies: Optimal environmental regulation with partially verifiable information.

Alistair Munro

September 2016

GRIPS
NATIONAL GRADUATE INSTITUTE FOR POLICY STUDIES

National Graduate Institute for Policy Studies
7-22-1 Roppongi, Minato-ku,
Tokyo, Japan 106-8677

Little Green Lies: Optimal environmental regulation with partially verifiable information.

Alistair Munro[☆]

Abstract

When the set of possible messages depends on the actual state of the world, optimal incentive schemes to control environmental problems may not always satisfy the revelation principle. As a result, in equilibrium some agents may send false messages, particularly when the information rents in the truth-telling scheme are high. I characterise optimal pollution regulation schemes and produce some numerical examples to show mechanisms which allow some dishonesty in equilibrium may frequently outperform truth-telling schemes.

Keywords: D82; Q58; H23, optimal incentives, hiding, pollution, adverse selection

1. Introduction.

When the set of possible messages depends on the actual state of the world, optimal incentive schemes may not always satisfy the well-known revelation principle. In a literature that has received intermittent attention, several authors have already shown that the principle need not apply when the feasible message space (e.g. statements about costs) depends on the state of the world or agent type (e.g. actual costs) (Lacker and Weinberg (1989) Green and Laffont (1986) or Postlewaite (1979)). Instead it may be a feature of the optimal scheme that some types of agents lie about their true nature. These concepts have been applied sporadically in the theoretical literature on industrial regulation (e.g. Singh and Wittman (2001) or Celik (2006)), but

[☆]National Graduate Institute for Policy Studies (GRIPS), Roppongi 7-22-1, Minato-ku, Tokyo, Japan, 106-8677. Email: alistair-munro 'at' grips.ac.jp

not so far to environmental issues.[1] In fact though individuals, farmers and businesses often face restrictions on their ability to misrepresent themselves to experienced regulators (Wunder et al. (2008)). A rancher who owns a section of steep and unploughable mountainside may not be able to credibly pretend that all of it could be used for pasture or arable land. A firm with a gleaming chimney and a recently installed processing unit bought from a well-known supplier to the industry may not be able to disguise the fact that abatement costs are lower than for its ageing competitors. In both cases, regulators may still not have full information: the rancher may have neighbours who have managed to turn some of their less severe slopes into grazing land; there may be another firm that installed a slightly older generation of technology only a few years previously. Thus there is neither full information on the part of the regulator or complete freedom for the agent to choose the message it sends. In such a context it seems reasonable to explore the implications for optimal incentive mechanisms when the regulated agents face some restrictions on the set of messages about their types that they are able to send. That is the purpose and contribution of this paper.

Taking its cue from the mechanism design literature (e.g. Lacker and Weinberg (1989)), the general lesson is that dishonesty for at least certain types may be ex-ante Pareto efficient. The main reason is that in order to create incentives for truth-telling, regulators have to allocate information rents to those agents who are blessed with low costs or some other favourable, but hidden feature of their choice environment. When these rents are sufficiently high it may be welfare enhancing to have a mechanism that allows some types to lie while others are induced to report their type truthfully. Of course, theory abounds with special cases which have no empirical relevance, but I also show that the conditions under which dishonesty is optimal are not particularly unusual and match the sort of situation in which the ideas of optimal regulation are typically applied.

The plan of the paper is as follows: in the next section I provide some motivating background. In sections 3 and 4 a simple model is introduced in which there is a benign principal and a set of possible types of agents who have private information about their type. To keep the paper straightforward

[1] Bontems et al. (2005) note in passing that the assumption of independence of the feasible message set and the agent's type and state 'this assumption can be relaxed but at a cost of much greater complexity for the analysis' (footnote 3).

I use a simple model with three types that is closely related to standard models used in both the pollution control and conservation literatures. Section 5 provides some numerical examples and section 6 concludes.

2. The Feasibility of Messages.

In pollution control, the regulatory authority is usually faced with a problem of asymmetric information. Firms may know more about their productivity, production costs and the costs of pollution control compared to the regulator Spulber (1988), Moxey et al. (1999), Jebjerg and Lando (1997). At the same time, consumers may be more informed about the benefits of pollution control. In a similar manner, in conservation policies, suppliers of conservation may be more informed about their costs of supply, Ferraro (2008), Chambers (2002). In both these situation, in the standard approach, a benign regulator sets out an incentive scheme or mechanism. In the scheme firms announce their cost of reducing emissions or supplying conservation services and receive incentives on that basis. In real examples of pollution control, firms may find it hard to overstate or understate the costs of cutting emissions for a number of reasons. Reporting higher costs than the true level may mean that profits do not match reported costs. And while there may be some scope for hiding extra earnings, when the degree of exaggeration is high, the truth may be obvious to the authorities. At the same time some features of the technology may be easily observable by the regulator who may then be able to generate a plausible range of cost functions for the firm. Du and Mao (2015) for example finds that age and installed capacity are highly predictive for the marginal CO_2 abatement cost amongst Chinese coal-burning power stations. Firms may also face limitations on their ability to report costs lower than the truth for similar reasons: profits and technology may send out clues that are informative.

In the context of conservation, misrepresenting cost-relevant information may be difficult if doing so would be associated with large and observable differences in reported income or if there are clues, from soil, landscape type and so on that provide a clear range to the regulator for possible costs of compliance. Ferraro (2008) calls these clues, 'costly to fake' but acknowledges that suppliers of conservation services may still have some ability to hide or manipulate signals about compliance costs to their own advantage. In the context of payments for watershed management in Mexico Muñoz-Piña et al. (2008) notes how econometric models were able to reduce the range of possible opportunity costs of compliance but not eliminate all uncertainty. Meanwhile Dobbs and Pretty (2008) reports that payments for environmentally sensitive management of farms in the U.K. allow some room for negotiation over costs, but benchmarking costs at the regional level means that there are effective limits on how farms may misrepresent their costs.

In both settings therefore, it can be possible for agents to physically send all kinds of messages, but some will have no chance of being believable to regulators.[2] Faced with this and the extreme sanctions that might be received for offering obvious lies, agents may consider that the set of feasible messages in limited and depends in part on their true type. Now, a game in which agents can send any message within a certain set, but cannot send a believable message outside that set represents one simple way of modeling restrictions on messages. However, there are other approaches: for instance agents may be able to send any message but ex-post regulators may be able to, at some cost, verify some aspects of a message. For example if a polluter reports that its costs of abatement are $100 per tonne, a regulator may be able through site inspection to identify some range of possible costs. If $100 lies outside this range and the firm faces a subsequent penalty for lying, it would prefer to send a message within the range. This type of partial verification in an abstract principal agent model is analyzed in Kartik (2009) who has a model in which verification costs are non-zero but it may still be optimal to lie. Alternatively, instead of costly verification for the regulator it may be costly for the agent to lie (see Munro (2014) for a discussion of this in the context of intra-household income hiding). For instance, a firm that mispresents its costs may as a result make higher profits. Hiding these profits, perhaps through unproductive investments or payment of rents to management may be costly. If the costs of lying are increasing and convex in the amount of misrepresentation then it may not be economic for the agent to consider all possible messages. Thus, the model I use here is not the only alternative to the standard case where the feasible message set is the same for all types of agents. Nevertheless it seems a useful starting point to understand why the revelation principal may not always be a part of optimal

[2] As Green and Laffont (1986) put it, 'Whether the variation of the message space with the true observation is purely technological, or whether it is induced by the severity of potential actions of the principal, is largely a matter of interpretation. The important part is that the implementability of any collective decision rule is determined by the interaction of the allowable messages and the associated actions.' p.447-448.

environmental regulation.

3. A Basic Model of Efficient Hiding.

In this section, I propose a simple model in which lying about costs can be part of an efficient contract for environmental regulation. I present this as a model of abatement of a polluting activity. In particular it is a discrete version of the model in Jebjerg and Lando (1997) (see also Spulber (1988), Moxey et al. (1999)), but following Mason and Plantinga (2013) and Ferraro (2008) or Sipiläinen and Huhtala (2012), we could also frame this as a problem in conservation in a way that is mathematically equivalent.

There is a constant social marginal value of abatement, p. The profit-maximizing suppliers of abatement, x, (which is observable) supply it at a cost $c(x, \theta)$ where c is strictly increasing, twice differentiable and strictly convex with $c(0, \theta) = 0$.[3] The parameter $\theta \in \{\theta_1, ..., \theta_n\} \equiv \Theta$ represents a factor that affects the cost of abatement and which is private knowledge, with $\theta_i > \theta_{i+1}$ $i = 1, ..., n-1$. In other words, lower values of i are associated with higher costs and marginal costs for supplying abatement.[4] Suppliers differ only in this parameter and the probability distribution of types, π_i is common knowledge.

Let Θ_i be the set of feasible messages for type i. I shall always suppose that $\theta_i \in \Theta_i$ - in other words it is always feasible for any type to report truthfully. In the *standard* case, $\Theta_i = \Theta$ $i = 1, ..., n$. That is the message space is independent of type and equal to the set of possible types. If $\Theta_j \subseteq \Theta_i$ whenever $\theta_j \in \Theta_i$ then the message sets are said to be *nested*.[5] Nesting might mean that a higher type can always send the message sent by a lower type or in the reverse version, the lower type can always send a message that it is feasible for the higher type to send. For instance, it may be possible for firms to always inflate their costs (e.g. by wasteful production).

[3]Although I don't explicitly model it, c(.) can be thought of as the difference between profits with and without an abatement requirement of x.

[4]For simplicity and to match the numerical examples later in the paper, I use a discrete rather than continuous form model of θ.

[5]One feature of this condition worth noting is that it says nothing about whether all message sets are nested. So for instance if firms in one province cannot signal a type from a neighbouring province, the nested condition can still be satisfied if all message sets are nested within each region.

The regulator offers a menu of contracts, $h : \Theta \rightarrow \mathbb{R}^2$ of the form, $h(\theta_i) = (T(\theta_i), x(\theta_i))$ where $T(\theta_i)$ is the transfer that will made to a firm that must supply $x(\theta_i)$ units of abatement after it sends a message that its type is θ_i. Transfers must be funded from public accounts and there is a marginal efficiency cost of $\mu - 1$ where $\mu > 1$, meaning that there is some deadweight cost from raising public funds.

The game form is as follows: First the regulator announces the menu of contracts, the supplier observes its type, then sends a message about type and then the contract is implemented according to the message.

Given this game form and the goal of profit-maximization, firms choose a message $m(\theta_i, h) \in \Theta_i$ such that, $-c(x(m(\theta_i, h)), \theta_i) + T(m(\theta_i, h)) \geq -c(x(\theta), \theta_i) + T(\theta)\ \forall \theta \in \Theta_i$.

An *outcome function*, $f(.)$, $f : \Theta \rightarrow \mathbb{R}^2$, is said to be implementable if and only if there exists a menu $h(.)$ such that for all $\theta \in \Theta$, $f(m(\theta_i, h)) = f(\theta)$. It is said to be *truthfully implementable* if it is implementable and for all i, $m(\theta_i, h) = \theta_i$. It is said to be *dishonestly implementable* if it is implementable but for any menu that implements the outcome function, there is at least one i such that $m(\theta_i, h) \neq \theta_i$.

Denote the value of x and T assigned to type in the outcome function $f(.)$ using subscripts - that is by x_i and T_i. A benign regulator chooses the menu of contracts, h, to maximize expected surplus or

$$W = \sum_{i=1}^{i=n} \pi_i \left[p x_i - c(x_i, \theta_i) - (\mu - 1) T_i \right] \tag{1}$$

subject to the constraints posed by the profit-maximizing behaviour of the suppliers. The result is the optimum. A *truthful equilibrium* is the menu and consequent outcome function that maximizes (1) subject to the constraints posed by profit maximizing behaviour of the suppliers and with the additional requirement that the outcome function be truthfully implementable. Conversely a *dishonest equilibrium* is one in which for the menu that implements the outcome function, there is at least one i such that $m(\theta_i, h) \neq \theta_i$.

3.1. Nested messages sets.

Where there is one sided asymmetric information a sufficient condition for truth-telling to be a feature of the optimum is nesting. In fact the result is more general: any outcome that can be feasibly implemented can be truthfully implemented. (Green and Laffont (1986)).

Proposition 1. *(Green and Laffont (1986)). Suppose message sets are nested. Then the set of implementable outcome functions is equivalent to the set of truthfully-implementable outcome functions.*

Proof. Let $h(.)$ implement $f(.)$ and suppose that $h^*(.)$ defined by $h^*(.) = f(.)$ is not truthfully implementable. Since h^* is not truthfully implementable, there must be some θ_i such that $h^*(\theta_i) = f(\theta_j)\, j \neq i$ that is $-c(x_i, \theta_i) + T_i < -c(x_j, \theta_i) + T_j$ for some $\theta_j \in \Theta_i$. Since message sets are nested then if $\theta_j \in \Theta_i$, $\Theta_j \in \Theta_i$. Since h implements f, then there can be no $\theta \in \Theta_i$ such that $h(\theta) = (T_j, x_j)$ for if there were, then the θ_i type would prefer to send the message θ since this produces higher profits. But since $\Theta_j \in \Theta_i$ then this also means that there is no feasible message the θ_j type can send such that $h(\theta_j) = (T_j, x_j)$. Hence h() does not implement f() - a contradiction. □

3.2. Standard case.

For this case, the revelation principle applies and there exists an optimal mechanism in which each type sends a message that is truthful. For this problem the constraints posed by rational behaviour by the suppliers take two forms: first suppliers can always opt out of the programme, for instance by closing down. This individual rationality (IR) constraint requires that profits are non-negative for all participating suppliers. Secondly, each supplier must prefer its own chosen contract over all other contracts. These are the incentive compatibility (IC) constraints and there are $(n-1)$ of them for each supplier type. Formally the two types of constraint are as follows:

$$T_i - c(x_i, \theta_i) \geq 0 \; i = 1, ..., n \qquad (IR) \tag{2}$$

$$T_i - c(x_i, \theta_i) \geq T_j - c(x_j, \theta_i) \; i = 1, ..., n; \; j = 1, ...n \qquad (IC) \tag{3}$$

As is well known, these n^2 constraints can be reduced to n binding restrictions given the form of the cost function and the regulator's maximand. First, we note that if the IR constraint is satisfied by the supplier with the lowest value of θ, then by the IC constraints it is also satisfied by the other supplier types. That is, $T_1 - c(x_1, \theta_1) \geq 0$ implies $T_1 - c(x_1, \theta_i) > 0$ for $i > 1$ and since $T_i - c(x_i, \theta_i) \geq T_1 - c(x_1, \theta_i)$ then $T_i - c(x_i, \theta_i) > 0 \; i > 1$. In fact, since the maximand is strictly decreasing in T, then the IR constraint will be binding for the $i = 1$ type.

Meanwhile, the relevant IC constraints are:

$$T_i - c(x_i, \theta_i) \geq T_{i-1} - c(x_{i-1}, \theta_i) \; i = 2, ..., n; \qquad (IC') \tag{4}$$

To see this, consider a situation where type i is indifferent between its own contract and that taken by type $i-1$. In other words, $T_i - c(x_i, \theta_i) = T_{i-1} - c(x_{i-1}, \theta_i)$. By strict convexity, $c(x_i, \theta_{i-1}) - c(x_i, \theta_i) > c(x_{i-1}, \theta_{i-1}) - c(x_{i-1}, \theta_i)$ for $x_i > x_{i-1}$.[6] Thus, $T_i - c(x_i, \theta_{i-1}) > T_{i-1} - c(x_{i-1}, \theta_{i-1})$. In short, type (i-1) strictly prefers its own contract to that chosen by type i, so that the IC constraint is only binding for the 'downward' constraint and not for the 'upward' constraint for neighbouring types. The same convexity-based argument can then be extended to types $(i+1)$ and $(i-2)$ and by induction to all supplier types. Moreover, all suppliers except the first type must have at least one binding constraint because if a constraint was not binding, T_i could be lowered in a way that did not break any incentives but at the same time increased the expected payoff to the regulator. For the $i = 1$ type, it is the IR constraint that binds.

The regulator therefore maximizes:

$$\sum_{i=1}^{i=n} \pi_i \left[p x_i - c(x_i, \theta_i) - (\mu - 1) T_i \right]$$
$$+ \sum_{i=2}^{i=n} \lambda_i \left[T_i - c(x_i, \theta_i) - T_{i-1} + c(x_{i-1}, \theta_i) \right] + \lambda_1 \left[T_1 - c(x_1, \theta_1) \right] \tag{5}$$

From this we get the first order conditions:

$$\pi_i \left[p - c'(x_i, \theta_i) \right] - \lambda_i c'(x_i, \theta_i) + \lambda_{i+1} c'(x_i, \theta_{i+1}) = 0 \; i = 1, ...n-1 \tag{6}$$

$$\pi_i \left[p - c'(x_i, \theta_i) \right] - \lambda_i c'(x_i, \theta_i) = 0 \; i = n \tag{7}$$

$$-\pi_i (\mu - 1) + \lambda_i - \lambda_{i+1} = 0 \; i = 1, ..., n-1 \tag{8}$$

[6] That x is increasing in i comes from the cost function. A contract that offered a lower value of x to a higher value of i could always be removed from the set of options to give the regulator higher expected profits.

$$-\pi_i(\mu - 1) + \lambda_i = 0 \; i = n \tag{9}$$

From this we get that,

$$p - c'(x_i, \theta_i) = \frac{(\mu - 1)}{\pi_i}\left[c'(x_i, \theta_i)\sum_{j=i}^{j=n}\pi_j - c'(x_i, \theta_{i+1})\sum_{j=i+1}^{j=n}\pi_j\right] \; i = 1, ...n - 1 \tag{10}$$

$$p - c'(x_i, \theta_i) = (\mu - 1)c'(x_i, \theta_i) \; i = n \tag{11}$$

For the highest type therefore, the distortion between price and marginal cost only reflects the marginal cost of public funds. For other types, the distortion includes an added cost from asymmetric information.

3.3. Diagrammatic Illustration.

The equilibrium is illustrated for a three type case in Figure 1. Three iso-profit curves, A, B and C are shown with A representing type1, B representing type 2 and C representing type 3. The optimal allocations for the three types are indicated by the lower case a, b and c. Through c there is a concave curve that represents an iso-welfare curve. At the optimum type 3 is indifferent between its own allocation at c and the allocation that is chosen by type 2, b. Similarly type 2 is indifferent between b and a. The iso-profit line A represents zero profit for type 1 firms. Meanwhile at c, the absence of a distortion at the equilibrium for the highest type is represented by the fact that the iso-profit and iso-welfare curves are tangents.

3.4. Non-nested message sets.

Suppose there is at least one triple of agent types, i, j, k with $i \leq j \leq k$ such that not $\Theta_i \subseteq \Theta_j$ and $\Theta_j \subseteq \Theta_k$ then message sets cannot be nested. To be specific, for the jth type let the set of feasible messages be $\Theta_j = \{\theta : \theta_j + \delta \geq \theta \geq \theta_j\} \cap \Theta$ with $\delta \geq 0$[7]. In other words each type j can send a message that it is type j or it can mimic any type that lies within the range θ_j to $\theta_j + \delta$. If there are no such agents then the firm can only send a truthful

[7]For simplicity I set δ as independent of j, but in theory it could vary according to type.

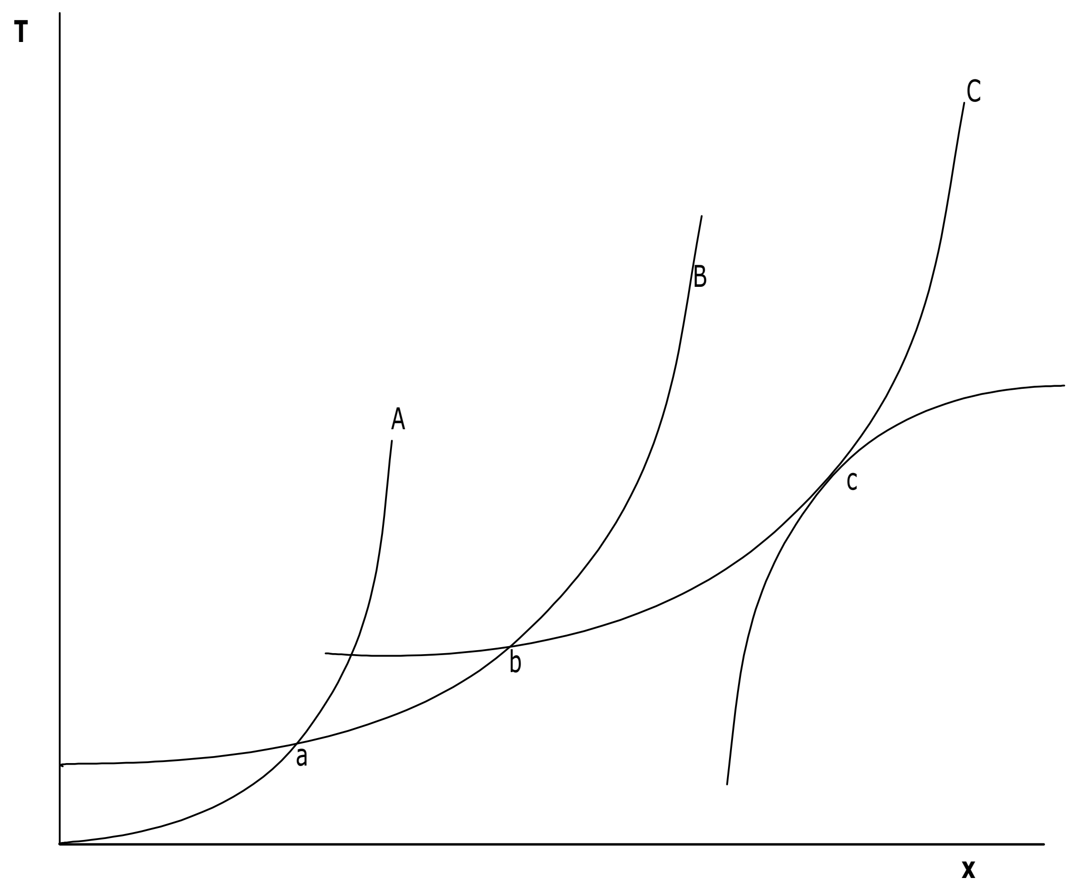

Figure 1: A Truth-telling equilibrium.

message, but if δ is sufficiently large then message sets are nested and the revelation principle applies. Note that an agent with the highest possible θ cannot therefore lie. With this formulation it is possible that the cardinality of the message set will vary amongst types. As a empirical feature this seems reasonable, but the sake of the model that follows I will make the further assumption that δ and Θ are such that for all but highest possible θ, each type can feasibly send two possible messages: one truthful and one in which it imitates the next lowest type. This simplifies the analysis that follows but is not essential to the argument.

Proposition 2. *For any optimum allocation there is a mechanism in which agents with the highest and lowest types report honestly.*

Proof. By construction it is not possible for a player in the lowest type to be dishonest. Suppose a highest type agent is dishonest about its type. Since no other type can send a message that it is type n, then the message(s) associated with the n-type are redundant. Replacing (T_n, x_n, θ_n) with $(T_{n-1}, x_{n-1}, \theta_n)$ produces the same outcomes but without any misrepresentation. □

Corollary 3. *When there are at most two types of the world, then the optimum is a truthful equilibrium.*

4. An example with dishonesty.

Figure 2 provides some intuition about the value of allowing dishonesty to the regulator. It shows the same payoff structure as in the previous figure but it also shows a dishonest allocation which may ex ante dominate the honest equilibrium. In this diagram A, B, C and a, b, c are as before. The dishonest allocation is represented by a, b' and c'. That is the allocation is unchanged for the lowest type. (This allocation may therefore not represent the optimal allocation with dishonesty. Nevertheless it is useful to fix a in order to focus on the changes to the allocation to types 2 and 3.) The allocation b' is such that type 2 prefers to lie and pretend to be type 1 rather than take b'. Conversely b' is such that type 3 prefers to take c' over b'. Notice that in this diagram, type 3 would actually prefer to state that it is type 1, but because this message is not feasible it takes c'. For the regulator the net benefits of dishonest allocation depend on a comparison of a gain in welfare from moving type 3 from c to c' against the loss of welfare from moving type 2 from b to b'. In fact in the diagram it appears that possibly the loss outweighs the gains,

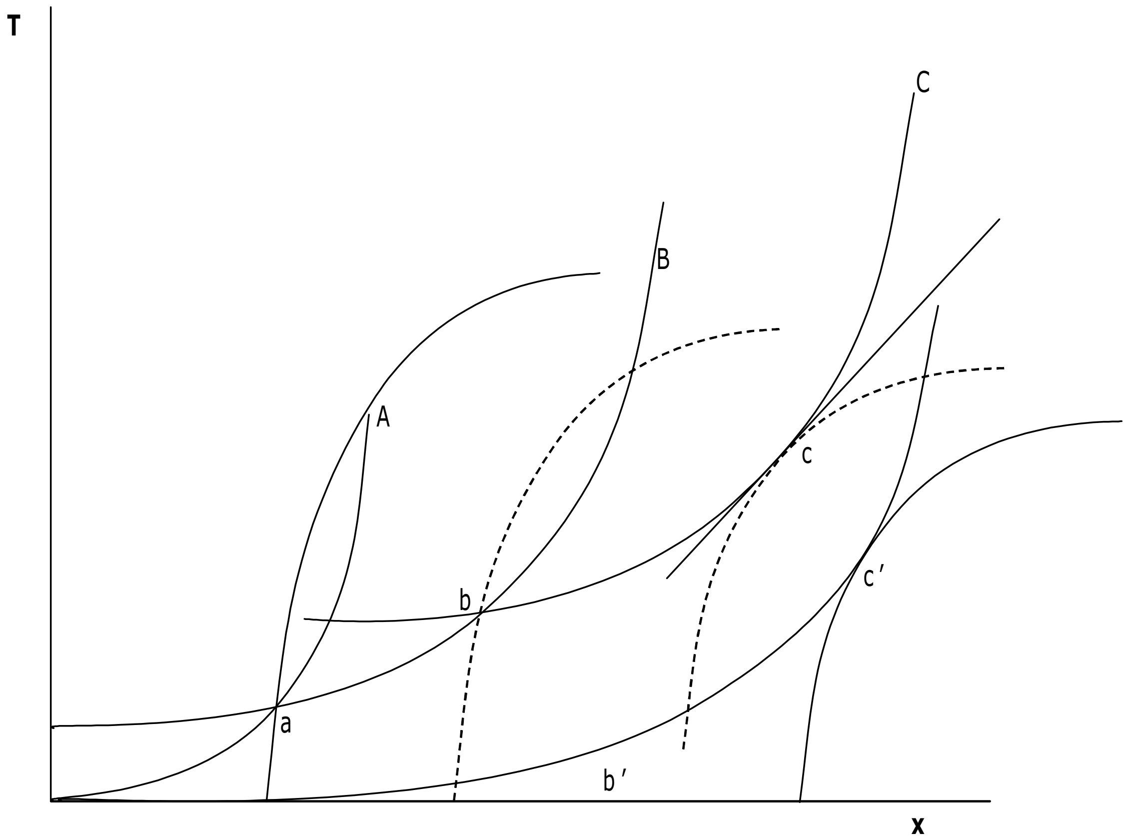

Figure 2: Equilibrium with Dishonesty

but if say, a was closer to b, then the net benefit would be possible. Could the same outcome be achieved by offering an allocation in which players tell the truth? Fixing a and c', any b' that is preferred by type 2 to a must also be preferred by type 3 to c'. In other words it is not possible to support the allocation of a to types 1 and 2 and c' to type 3 through truth-telling.

The diagram is suggestive, but not conclusive. I now present an extended example to show that dishonesty may be optimal. Specifically, I use the cost function, $c = \frac{\theta_i(x_i+1)^2}{2} - \frac{\theta_i}{2}$ and also to make the example clearer, I select p

is such that $x_1 > 0$ so that abatement is strictly positive for all types.[8]. In other words, $\theta_1 < p/\mu$. The planner aims to maximize,

$$W = \sum_{i=1}^{i=3} \pi_i \left(px_i - \frac{\theta_i}{2}(x_i+1)^2 + \frac{\theta_i}{2} - (\mu-1)T_i \right) \tag{12}$$

The relevant cases are as follows:

4.1. Full information case.

For this case, the Lagrangean is,

$$W + \lambda_1 \left(\frac{\theta_1}{2} \left(1-(x_1+1)^2\right) + T_1 \right) + \lambda_2 \left(\frac{\theta_2}{2} \left(1-(x_2+1)^2\right) + T_2 \right) + \lambda_3 \left(\frac{\theta_3}{2} \left(1-(x_3+1)^2\right) + T_3 \right) \tag{13}$$

The constraints here are the participation constraints for the three types.We get the following solution for abatement:

$$\begin{aligned} x_i &= \frac{p}{\mu\theta_i} - 1 \\ T_i &= \frac{\theta_i}{2}\left(\frac{p^2}{\mu^2\theta_i^2} - 1 \right) \end{aligned} \tag{14}$$

4.2. The truth-telling equilibrium

In this case the Lagrangean is

$$\begin{aligned} W &+ \lambda \left(\frac{\theta_1}{2} \left(1-(x_1+1)^2\right) + T_1 \right) \\ &+ \gamma_2 \left(-\frac{\theta_2}{2}(x_2+1)^2 + \frac{\theta_2}{2}(x_1+1)^2 + T_2 - T_1 \right) \\ &+ \gamma_3 \left(-\frac{\theta_3}{2}(x_3+1)^2 + \frac{\theta_3}{2}(x_2+1)^2 + T_3 - T_2 \right) \end{aligned} \tag{15}$$

[8] In addition to make the problem well-behaved I impose: $\sum_{j=i}^{j=3} \theta_i \pi_j > \sum_{j=i+1}^{j=3} \theta_{i+1} \pi_j$

The constraints represent a participation constraint for type 1 firm and incentive compatibility constraints for the other two types. The first order conditions for an interior solution are,

$$\begin{aligned} \pi_1 \left(p - \theta_1(x_1+1)\right) - \lambda\theta_1(x_1+1) + \gamma_2\theta_2(x_1+1) &= 0 \\ -\pi_1(\mu-1) + \lambda - \gamma_2 &= 0 \\ \pi_2 \left(p - \theta_2(x_2+1)\right) - \gamma_2\theta_2(x_2+1) + \gamma_3\theta_3(x_2+1) &= 0 \\ -\pi_2(\mu-1) + \gamma_2 - \gamma_3 &= 0 \\ \pi_3 \left(p - \theta_3(x_3+1)\right) - \gamma_3\theta_3(x_3+1) &= 0 \\ -\pi_3(\mu-1) + \gamma_3 &= 0 \end{aligned} \tag{16a}$$

The solution for abatement (but not the associated payment) by the third type is unchanged from the full information case. For the other types we get:

$$x_2 = \frac{\pi_2 p}{\pi_2\mu\theta_2 + \pi_3\left(\mu-1\right)\left[\theta_2-\theta_3\right]} - 1 \tag{17}$$

$$x_1 = \frac{\pi_1 p}{\pi_1\theta_1 + \left(\mu-1\right)\left[\theta_1-(1-\pi_1)\theta_2\right]} - 1 \tag{18}$$

Meanwhile,

$$T_1 = \frac{\theta_1}{2}\left[(x_1+1)^2-1\right]$$

$$T_2 = \frac{\theta_2}{2}\left[(x_2+1)^2-1\right] + T_1\left[1-\frac{\theta_2}{\theta_1}\right]$$

$$T_3 = \frac{\theta_3}{2}\left[(x_3+1)^2-1\right] + T_2\left[1-\frac{\theta_3}{\theta_2}\right] - \frac{\theta_3}{\theta_2}T_1\left[1-\frac{\theta_2}{\theta_1}\right]$$

From these we get,

Proposition 4. *In the truth-telling equilibrium, T_i and x_i are increasing in p, decreasing in μ and θ_i and increasing in π_i for $i = 1,2$. x_3 is increasing in p, decreasing in μ and θ_3 and T_i is increasing in p and decreasing in μ.*

Proof. By partial differentiation of the preceding equations and equation (14) for x_3. □

4.3. The dishonest equilibrium.

Since type 2 will not choose their own bundle, it can be designed in a way that makes type 3 indifferent between participation and opting out. Similarly the bundle aimed at types 1 and 2 (labeled here as '1') is chosen subject to the condition that type 1 participates. As a result the problem can be represented by the Lagrangean

$$W + \lambda_1 \left(\frac{\theta_1}{2} \left(1 - (x_1 + 1)^2\right) + T_1 \right) + \lambda_3 \left(\frac{\theta_3}{2} \left(1 - (x_3 + 1)^2\right) + T_3 \right) \tag{19}$$

The first order conditions for an interior solution are,

$$\begin{aligned} \pi_1 \left(p - \theta_1(x_1 + 1)\right) + \pi_2 \left(p - \theta_2(x_1 + 1)\right) - \lambda_1\theta_1(x_1 + 1) = 0 \\ -\left(\pi_1 + \pi_2\right)\left(\mu - 1\right) + \lambda_1 = 0 \\ \pi_3 \left(p - \theta_3(x_3 + 1)\right) - \lambda_3\theta(x_3 + 1) = 0 \\ -\pi_3(\mu - 1) + \lambda_3 = 0 \end{aligned} \tag{20a}$$

The solution for abatement by the third type is again unchanged from the full information case. Using 'd' to indicate the dishonest equilibrium, for the other two types, the solution is,

$$x_1^d = x_2^d = \frac{p(\pi_1 + \pi_2)}{\mu\pi_1\theta_1 + \pi_2 \left[\theta_2 + (\mu - 1)\,\theta_1\right]} - 1 \tag{21}$$

In this situation, transfers are:

$$T_2^d = T_1^d = \frac{\theta_1}{2}\left[\left(x_1^d + 1\right)^2 - 1\right]$$

$$T_3^d = \frac{\theta_3}{2}\left[\left(\frac{p}{\mu\theta_3}\right)^2 - 1\right]$$

From these we get,

Proposition 5. *In the dishonest equilibrium, T_i^d and x_i^d are increasing in p, decreasing in μ and increasing in π_i for $i = 1, 2$. T_i^d and x_i^d are increasing in p, decreasing in μ and θ_3.*

Proof. By partial differentiation of the preceding equations and equation (14) for x_3. □

The net gain from dishonesty rather than truth-telling is,

$$\Delta W = W^d - W = \pi_1 \left(p\left(x_1^d - x_1\right) + \mu\left(T_1 - T_1^d\right)\right) + \pi_2 \left(p\left(x_1^d - x_2\right) + \mu\left(T_2 - T_1^d\right) - T_1\left[1 - \frac{\theta_2}{\theta_1}\right]\right) + \pi_3\left(\left(\mu - 1\right)\left(T_3 - T_3^d\right)\right) \quad (22)$$

Proposition 6. *The net gain to dishonesty is increasing in π_3 and decreasing in θ_3.*

Proof. By partial differentiation of equation (22) (see Appendix for details). □

The key point here is that the second type has to be relatively rare compared to type 3 in order for dishonesty to be optimal. The reason is that the ability to mimic type 2 gives type 3 informational rents in the truth-telling equilibrium. Removing this ability by encouraging type 2 to mimic type 1's bundle and offering a bundle to type 2s that is unappealing removes these informational rents but comes at a cost of also lowering abatement by type 2s. When type 2 is more prevalent this cost outweighs the value of the financial savings from moving to the dishonest equilibrium. On the other hand, if type 2 is rare, the cost of the information rents paid out to type 3 can be become large compared to the value of the abatement that is obtained from type 2s at the truth-telling equilibrium.

4.4. n types.

A question is what happens if there are more than 3 types. Is it possible for more than one type to be dishonest at an optimum for instance if $n > 3$? It is clear from the sections above that it is not possible to show that in general a dishonest equilibrium yields higher welfare than the truthful equilibrium, however it is possible to show that under certain conditions that there exist implementable menus in which nearly all types send dishonest messages and as the number of types increases, the difference in welfare between the dishonest equilibrium and the full information outcome tends to zero.

To see this, I revert back to the general cost function, $c(x,\theta)$. Fix $\theta_1 = \underline{\theta}$ and fix $\theta_n = \bar{\theta}$. Let $\theta_i = \left(\frac{i-1}{n-1}\right)\theta_n + \left(\frac{n-i+1}{n-1}\right)\theta_1$ and suppose that $\delta = (\theta_n - \theta_1)/(n-1)$ thus $\Theta_j = \{\theta_j, \theta_{j-1}\}$ for $j > 1$ and $\Theta_1 = \{\theta_1\}$. Meanwhile let $\pi_i = 1/n$. Let $W* = \sum_i \pi_i w_i^*$ where w_i^* is the value of $px_i - c(x_i,\theta_i) - (\mu-1)T_i$ in the full information case. Let W^d be the equivalent function for the dishonest equilibrium.

Proposition 7. *$W^* - W^d \to 0$ as $n \to \infty$.*

Proof. Suppose that $n > 3$ and consider the following menu: $h(\theta_j) = (x_{j+1}^*, T_{j+1}^*) j = 3,..,n-2$; $h(\theta_n) = (x_n^*, T_n^*)$, $h(\theta_j) = (0,0)$ $j = n-1$, $h(\theta_1) = (x_1^d, T_1^d)$ $j = 1$ where the superscript * refers to the full-information case and ^{d} refers to the dishonest equilibrium. Under this menu, the n-th type sends a truthful message. Each of types j = 3 to n-1 sends a message that they are type $j-1$ and in return gets the outcome for the jth type in the full-information case. The $j = 1$ and $j = 2$ types both send the message that they are j=1 and in return get (x_1^d, T_1^d). Type 1 has no choice about its message. For type $j > 1$ the alternative feasible message either yields negative payoff (for $j = 2$ to n-1) or 0 (for $j = n$). It follows that h is implementable and it involves dishonesty, but note that this is not necessarily the optimal dishonest menu. Thus, the resulting value of W is such that $W \leq W^d$. $W^* - W^d \leq \frac{1}{n}\left(w_1^* + w_2^* - 2w_1^d\right)$. Since $w_j^* \leq w_{j+1}^*$ $j = 1,..,n$ (because if it was not then it would be feasible to replace $h(\theta_{j+1}) = (x_{j+1}^*, T_{j+1}^*)$ with $h(\theta_{j+1}) = (x_j^*, T_j^*)$ and the result would yield higher W) then $W^* - W^d \leq \frac{1}{n}\left(2w_n^* - 2w_1^d\right) \leq \frac{2}{n}(w_n^*)$. But since $w^*(\theta_n) = w^*(\bar{\theta})$ then $\frac{2}{n}(w_n^*) \to 0$ as $n \to \infty$ and the result follows. □

This result shows that when the number of types is sufficiently large, a menu in which all but 2 types are dishonest can yield welfare that approximates the full-information outcome. The 'trick' is that the information rent of the highest type is eliminated by making it unattractive to send an untruthful message. For the other types (excluding the lowest), the full information outcome is also implementable because the only alternative message forces them to supply the abatement level of a higher type and since that abatement level produced zero profits for the higher type, copying it would produce losses. It's clear therefore that this is a special case but if each type's message set included m other sequentially lower types then a menu in which all but the bottom m+1 types abated at the full-information level would still be possible.[9]

5. Numerical Example.

The diagrams are suggestive but are not numerically exact. I now give some specific numerical examples using the cost function of the previous section. Of course, any numbers can be used to produce an existence result, but I wish also to show that the examples are reasonable, in the sense that the parameters are not out of line with those used in some of the literature on abatement or conservation problems.

The key parameters are p, μ, π and θ. Estimates for the marginal costs of public funds (μ) vary somewhat but typically lie in the range $1-2.0$ (Dahlby (2008); Hansson and Stuart (1985); Snow and Warren (1996)). For marginal benefits, p and marginal costs, θ, I draw on a variety of sources (e.g. Gray and Shadbegian (2004)) but set p large relative to marginal costs for the purpose of having examples in which abatement is optimal. For example, for the case of ecosystem service payments, Holmes et al. (2004) for instance find a range of 4.03 to 15.65 for benefit to cost ratios for river restoration, figures which are similar to earlier estimates from Loomis et al. (2000). The ratio p/θ that I use in the examples is compatible with these estimates. In fact the shapes of the curves in the figures are not sensitive to values of p above 2. In terms of plausibility, the key difficulty is finding values for π, so I allow these parameters to vary in the first example below. In the second example I use a specific paper to guide the estimates.

Figure 3 shows, ΔW, the net gain to dishonesty (vertical axis) against the probability of type 2 (horizontal axis) for different values of the probability of type 1. In this chart, $p = 4$, $\mu = 1.5$ and $\theta = [0.5, 0.3, 0.2]$. For these values the net gain is decreasing in π_2 and π_1, but when the probability of type 2 is sufficiently rare and the probability of type 3 is sufficiently high, then the gains to dishonesty are positive. For the highest positive number in the figure, the gain in welfare from switching to the dishonesty equilibrium represents about 19% of the welfare from the full-information case.

Figure 4 shows how the advantage of the dishonest equilibrium changes in response to variation in μ and θ_2. For this example $p = 4$, $\theta_1 = 0.5$, $\theta_3 = 0.2$, $p_1 = 0.3$, $p_2 = 0.2$. As θ_2 tends to θ_1 the advantage becomes positive

[9] If δ did not fall with n, then as n increases the cardinality of the message sets would increase too and there is therefore no similar convergence result.

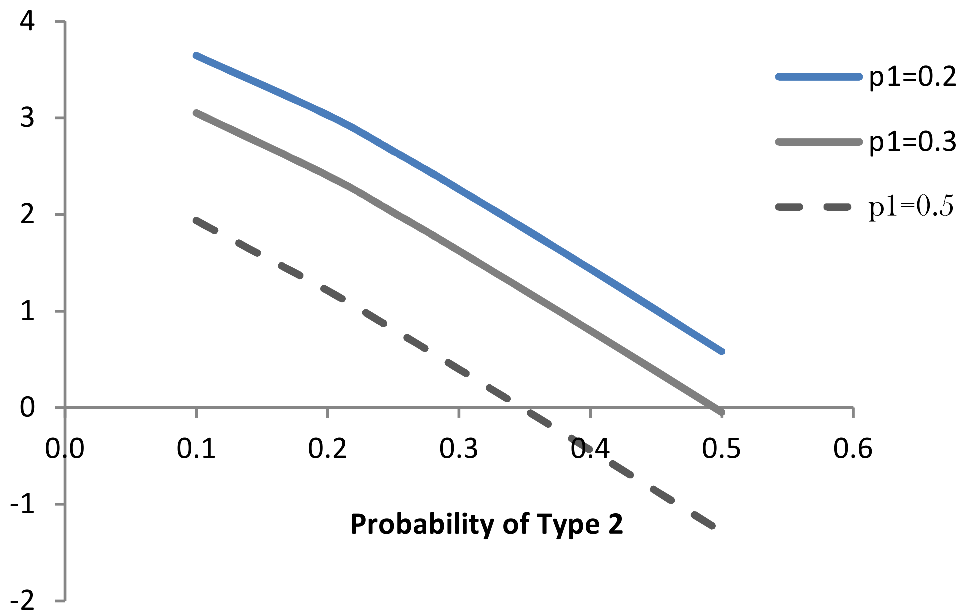

Figure 3: ΔW for changes in π_2

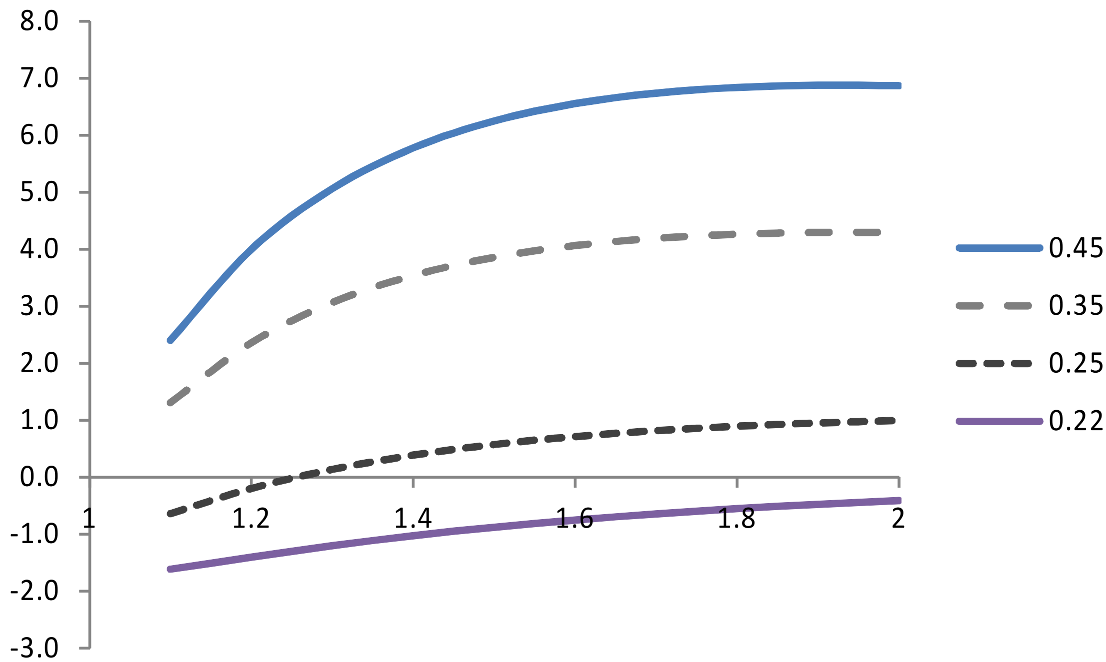

Figure 4: ΔW against changes in μ (horizontal axis) for different values of θ_2.

and larger in absolute size since the rent that must be paid to type 3 in the truth-telling equilibrium becomes large compared to the rent paid to type 2. At the same time, as the cost of public funds rises then the advantage of the dishonest equilibrium rises, since this involves lower transfers to type 3. In the most extreme case shown, the gain in welfare from switching to the dishonest equilibrium represents 65% of the welfare obtainable from the full-information case. In other words, the standard, truth-telling incentive mechanism is very inefficient.

5.1. Varying the size of the message set.

In this example, I consider a case where the spread of possible costs varies and also the size of the message set varies according to the parameter δ. Suppose for instance that θ follows a half-normal distribution where σ is the standard deviation of the associated normal distribution. This could arise when for example the distribution has been estimated using a stochastic frontier approach as in Sheriff (2009) who derives the distribution of cost-related types for a land set aside program in the mid-west of the USA. He estimates a half-normal distribution of cost parameters that cannot be related to observable characteristics. Specifically, consider a case where θ_3 is fixed at 0.2. Suppose θ_2 and θ_1 are defined such $\theta_2 = 0.2 + 0.5\sigma$ and $\theta_1 = 0.2 + \sigma$. If θ follows a half-normal distribution where σ is the standard deviation of the associated normal distribution, then 0.38 of the distribution lies between θ_1 and θ_2 0.30 between θ_2 and θ_3 and 0.32 has a value more than θ_1. Suppose for simplicity that exactly all types have $\theta = \theta_i$. Table 1 shows the achievable welfare as a fraction of the full information value, for different values of σ and δ in these circumstances,[10] with $p = 4$, $\mu = 1.5$. If δ is sufficiently small, firms cannot falsify their own type. In this case, the full information allocation is acheivable. Such cases are indicated by the green italicised font in the table. For intermediate values of δ , $\Theta_i = \{\theta_i, \theta_{i-1}\}$ $i > 1$. In this case the optimum is the dishonest allocation (font in bold black) and when δ is sufficiently large, message sets are nested and the truth-telling equilibrium is the optimum (font in blue, normal). As the message set expands so welfare falls, as more information rents must be granted to lower cost abating firms in order to induce their compliance. It can be seen that the welfare cost of using a truthful mechanism can be substantial. In the case of $\sigma = 0.3$ for example, welfare is reduced by 24.2% from the full information level if the optimal truthful mechanism is used, but by only 4.5% if the best dishonest scheme is used.

6. Conclusions.

In a world where regulators have some insight into the cost structures of potential suppliers of environmental services, it may not be possible for all types of supplier to send credible messages about all other possible types. The possibility that feasible messages may be contingent on the true state of the world implies that the revelation principle need not be a feature of

[10]In Sheriff (2009) the lowest cost type is normalized with $\theta = 1$. His model is actually for the inverse of my θ and he gets 0.861 for its standard deviation. Maintaining the same coefficient of variation would produce a value for θ_2 of roughly 0.35 - i.e. the same value generated in my example by using $\sigma = 0.3$. However since his parameter is the inverse of mine, θ_1 would be significantly larger (around 1.4 for $\sigma = 0.3$) if I was to match this part of the distribution. For the parameter values used to generate Table 1, this would eliminate the truthful equilibrium as the optimum. While it would expand the range of values for which the dishonest equilibrium was optimal the welfare cost of using the best truthful mechanism relative to the dishonest equilibrium would decrease.

	σ			
δ	0.1	0.2	0.3	0.4
0.05	**0.976**	*1.000*	*1.000*	*1.000*
0.10	0.921	**0.962**	*1.000*	*1.000*
0.15	0.921	**0.962**	**0.955**	*1.000*
0.20	0.921	0.840	**0.955**	**0.951**
0.25	0.921	0.840	**0.955**	**0.951**
0.30	0.921	0.840	0.758	**0.951**

Table 1: Welfare as a Function of Message Set Size

optimal incentives for pollution control and conservation supply. Rather, in some cases, the cost of getting suppliers to reveal their information may be too high to justify truth-telling contracts. Instead, lying may be optimal in equilibrium at least for some suppliers. Allowing some types to send inaccurate messages means that the incentive compatibility constraints on other types can be relaxed. I take a straightforward and well-known model and show that for a variety of parameter values this is indeed the case. Although the parameter values for the numerical examples are not closely matched to any particular empirical example, it is notable that the cases in which dishonesty is optimal are not pathological. In fact just the opposite.

The paper also has some implications for an empirical strategy that uses the first-order conditions for a optimal mechanism to estimate a structural model of the cost distribution for an industry supplying conservation or abatement services (e.g. Lavergne and Thomas (2005) for water pollution in France or Ivaldi and Martimort (1994) for energy inputs). One of the identifying assumptions of the literature is that the mechanism is truthful. In the case that the optimal mechanism is actually dishonest then the estimates from the standard approach that assumes the revelation principle may be biased.

A feature of the optimal dishonest contract is that it can be simpler than the contract that induces truthful revelation in the sense that the number of actively reported types is smaller. Thus allowing some dishonesty can be a means for a regulator to raise welfare and simplify the incentive structure at the same time.

References.

Bontems, P., Rotillon, G., Turpin, N., 2005. Self-selecting agri-environmental policieswith an application to the Don watershed. Environmental and resource economics 31 (3), 275–301.

Celik, G., 2006. Mechanism design with weaker incentive compatibility constraints. Games and Economic Behavior 56 (1), 37–44.

Chambers, R. G., 2002. Chapter 34 information, incentives, and the design of agricultural policies. In: Gardner, B. L., Rausser, G. C. (Eds.), Agricultural and Food Policy. Vol. 2, Part B of Handbook of Agricultural Economics. Elsevier, pp. 1751 – 1825.
URL http://www.sciencedirect.com/science/article/pii/S1574007202100211

Dahlby, B., 2008. The marginal cost of public funds: Theory and applications. MIT Press Books 1.

Dobbs, T. L., Pretty, J., 2008. Case study of agri-environmental payments: The united kingdom. Ecological Economics 65 (4), 765 – 775, payments for Environmental Services in Developing and Developed Countries.
URL http://www.sciencedirect.com/science/article/pii/S0921800907004260

Du, L., Mao, J., 2015. Estimating the environmental efficiency and marginal CO2 abatement cost of coal-fired power plants in china. Energy Policy 85, 347 – 356.
URL http://www.sciencedirect.com/science/article/pii/S0301421515002402

Ferraro, P. J., 2008. Asymmetric information and contract design for payments for environmental services. Ecological Economics 65 (4), 810–821.

Gray, W. B., Shadbegian, R. J., 2004. Optimal pollution abatement-whose benefits matter, and how much? Journal of Environmental Economics and Management 47 (3), 510–534.

Green, J. R., Laffont, J.-J., 1986. Partially verifiable information and mechanism design. The Review of Economic Studies 53 (3), 447–456.

Hansson, I., Stuart, C., 1985. Tax revenue and the marginal cost of public funds in sweden. Journal of Public economics 27 (3), 331–353.

Holmes, T. P., Bergstrom, J. C., Huszar, E., Kask, S. B., Orr, F., 2004. Contingent valuation, net marginal benefits, and the scale of riparian ecosystem restoration. Ecological Economics 49 (1), 19–30.

Ivaldi, M., Martimort, D., 1994. Competition under nonlinear pricing. Annales d'Economie et de Statistique, 71–114.

Jebjerg, L., Lando, H., 1997. Regulating a polluting firm under asymmetric information. Environmental and Resource Economics 10 (3), 267–284.

Kartik, N., 2009. Strategic communication with lying costs. Review of Economic Studies 76 (4), 1359–1395.

Lacker, J. M., Weinberg, J. A., 1989. Optimal contracts under costly state falsification. The Journal of Political Economy, 1345–1363.

Lavergne, P., Thomas, A., 2005. Semiparametric estimation and testing in a model of environmental regulation with adverse selection. Empirical Economics 30 (1), 171–192.

Loomis, J., Kent, P., Strange, L., Fausch, K., Covich, A., 2000. Measuring the total economic value of restoring ecosystem services in an impaired river basin: results from a contingent valuation survey. Ecological economics 33 (1), 103–117.

Mason, C. F., Plantinga, A. J., 2013. The additionality problem with offsets: Optimal contracts for carbon sequestration in forests. Journal of Environmental Economics and Management 66 (1), 1–14.

Moxey, A., White, B., Ozanne, A., 1999. Efficient contract design for agri-environment policy. Journal of Agricultural Economics 50 (2), 187–202. URL http://dx.doi.org/10.1111/j.1477-9552.1999.tb00807.x

Muñoz-Piña, C., Guevara, A., Torres, J. M., Braña, J., 2008. Paying for the hydrological services of Mexico's forests: Analysis, negotiations and results. Ecological economics 65 (4), 725–736.

Munro, A., 2014. Hide and seek: A theory of efficient income hiding within the household. National Graduate Institute for Policy Studies.

Postlewaite, A., 1979. Manipulation via endowments. The Review of Economic Studies 46 (2), 255–262.

Sheriff, G., 2009. Implementing second-best environmental policy under adverse selection. Journal of Environmental Economics and Management 57 (3), 253–268.

Singh, N., Wittman, D., 2001. Implementation with partial verification. Review of Economic Design 6 (1), 63–84.

Sipiläinen, T., Huhtala, A., 2012. Opportunity costs of providing crop diversity in organic and conventional farming: would targeted environmental policies make economic sense? European Review of Agricultural Economics, jbs029.

Snow, A., Warren, R. S., 1996. The marginal welfare cost of public funds: theory and estimates. Journal of Public Economics 61 (2), 289–305.

Spulber, D. F., 1988. Optimal environmental regulation under asymmetric information. Journal of Public Economics 35 (2), 163–181.
URL http://www.sciencedirect.com/science/article/pii/0047272788900527

Wunder, S., Engel, S., Pagiola, S., 2008. Taking stock: A comparative analysis of payments for environmental services programs in developed and developing countries. Ecological economics 65 (4), 834–852.

Appendix.

$$\begin{aligned}
\frac{\partial \Delta W}{\partial \mu} &= \frac{p^2\pi_3(\mu-1)(\theta_2/\theta_1-1)}{\theta_1\mu^3} + \frac{\mu p^2(\pi_1+\pi_2)^2(\theta_1\pi_1+\theta_1\pi_2)^2}{(\pi_2(\theta_2+\theta_1(\mu-1))+\theta_1\mu\pi_1)^3} \\
&- \frac{\pi_1 p^2/(\theta_1\mu^2)}{(\pi_2(\theta_2+\theta_1(\mu-1))+\theta_1\mu\pi_1)^3} - \frac{\theta_1 p^2(\pi_1+\pi_2)^3}{2(\pi_2(\theta_2+\theta_1(\mu-1))+\theta_1\mu\pi_1)^2-1))} \\
&+\pi_1\left[p\left(\frac{p\pi_1(\theta_1+\theta_2(\pi_1-1))}{(\theta_1\pi_1+(\theta_1+\theta_2(\pi_1-1))(\mu-1))^2}\right)\right] - \frac{p^2(\pi_1+\pi_2)^2(\theta_1\pi_1+\theta_1\pi_2)}{(\pi_2(\theta_2+\theta_1(\mu-1))+\theta_1\mu\pi_1)^2} \\
&+ \frac{\pi_1\theta_1}{2}\left(\frac{p^2}{\theta_1^2\mu^2}-1\right) - \left[\frac{\pi_3}{2}\left(\frac{p^2}{\theta_1^2\mu^2}-1\right)(\theta_2-\theta_1)\right] \\
&+ \pi_2 p^2\left[\frac{\pi_2(\theta_2\pi_2+\pi_3(\theta_2-\theta_3))}{(\pi_3(\theta_2-\theta_3)(\mu-1)+\theta_2\mu\pi_2)^2}\right] + \mu\pi_2\left[\frac{p^2(\theta_2/\theta_1-1)}{\theta_1\mu^3} - \frac{p^2}{\theta_2\mu^3}\right] \\
&+ \frac{\pi_2\theta_2}{2}\left(\frac{p^2}{{\theta_2}^2\mu^2}-1\right) - \pi_2\left[\left(\frac{p^2}{\theta_1^2\mu^2}-1\right)\frac{(\theta_2-\theta_1)}{2} + \frac{p^2(\theta_2/\theta_1-1)}{\theta_1\mu^3}\right] \qquad (23)
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial \Delta W}{\partial p} = -&\frac{2p\pi_1^2}{(\theta_1\pi_1 + (\theta_1 + \theta_2(\pi_1 - 1))(\mu - 1))})\\
&+ \mu \left(\frac{p}{(\theta_1\mu^2)} - \frac{\theta_1 p(\pi_1 + \pi_2)^2}{(\pi_2(\theta_2 + \theta_1(\mu - 1)) + \theta_1\mu\pi_1)^2} \right)\\
&+ \frac{2p(\pi_1 + \pi_2)^2}{(\pi_2(\theta_2 + \theta_1(\mu - 1)) + \theta_1\mu\pi_1)} - \frac{2p\pi_2^2}{(\pi_3(\theta_2 - \theta_3)(\mu - 1) + \theta_2\mu\pi_2)}\\
&- \mu \left(\frac{p(\theta_2/\theta_1 - 1)}{\theta_1\mu^2} - \frac{p}{\theta_2\mu^2} + \frac{\theta_1 p(\pi_1 + \pi_2)^2}{(\pi_2(\theta_2 + \theta_1(\mu - 1)) + \theta_1\mu\pi_1)^2} \right)\\
&+ \frac{(p(\theta_2/\theta_1 - 1))}{(\theta_1\mu^2)}) - \frac{p\pi_3(\mu - 1)(\theta_2/\theta_1 - 1)}{\theta_1\mu^2} \qquad (24)
\end{aligned}
$$

$$
\frac{\partial \Delta W}{\partial \theta_3} = -\frac{p^2\pi_2^2\pi_3(\mu - 1)}{(\pi_3(\theta_2 - \theta_3)(\mu - 1) + \theta_2\mu\pi_2)^2} \qquad (25)
$$

$$
\frac{\partial \Delta W}{\partial \pi_3} = \frac{p^2 p_2^2(\theta_2 - \theta_3)(\mu - 1)}{(\pi_3(\theta_2 - \theta_3)(\mu - 1) + \theta_2\mu\pi_2)^2} - \frac{(\theta_2 - \theta_1)(\mu - 1)}{2} \left(\frac{p^2}{\theta_1^2\mu^2} - 1 \right) \qquad (26)
$$