Dynamic network *range-adjusted* measure vs. dynamic network *slacks-based* measure

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We dedicate this paper to the memory of Professor William W Cooper, 1914 – 2012, whose generous demeanor touched and inspired at least three generations of DEA researchers.

It is up to the DEA community to make sure that his vision and legacy live on.

DEA 研究者 3 世代以上にわたり感動とインスパイア―を与えてきたウィリアム•W•クーパー教授

（1914 - 2012）に、この論文を捧ぐ。
教授から受け継いだ知識とビジョンは、確実にＤＥＡ利用者達に生き続けています。

Abstract: We formulate weighted, dynamic network range-adjusted measure (D-NRAM) and dynamic network slacks-based measure (D-NSBM), run robustness tests and compare results. To the best of our knowledge, the current paper is the first to compare two weighted dynamic network DEA models and it also represents the first attempt at formulating D-NRAM. We illustrate our models using simulated data on *residential aged care*. Insight gained by running D-NRAM in parallel with D-NSBM includes (a) identical benchmark groups, (b) a substantially wider range of efficiency estimates under D-NRAM, and (c) evidence of inefficient DMU size bias. D-NRAM is also shown to have the additional desirable technical efficiency properties of translation-invariance and acceptance of data. Managerial implications are also briefly discussed.

Keywords: Weighted dynamic network DEA, robustness, residential aged care

1. INTRODUCTION

A few years ago Avkiran and Parker ([4], p.1) reported, “Emerging evidence of a *declining* number of influential methodological (theory)-based publications, and a *flattening* diffusion of applications imply an *unfolding maturity of the field*.” Since then data envelopment analysis (DEA) researchers keen to exploit some of the few remaining main avenues for methodological-based studies have shifted their focus to *network* and *dynamic* data envelopment analysis. While both concepts have been around for some time in various forms, consolidating the two in a unified model is a more recent attempt as evidenced in the GRIPS workshop of January 2013 held in Tokyo. Our study contributes to the field by bringing together the core concepts advanced in the network range-adjusted measure by Avkiran and McCrystal [2] and dynamic slacks-based measure by Tone and Tsutsui [11]. In the process, we formulate weighted, dynamic network range-adjusted measure (D-NRAM) and dynamic network slacks-based measure (D-NSBM).

We then proceed to compare and contrast D-NRAM versus D-NSBM and run robustness tests. Thus, to the best of our knowledge, the current paper is the first attempt that compares two weighted dynamic network DEA models and tests the robustness of estimates generated to various data perturbations. It also represents the first attempt at formulating *dynamic network range-adjusted measure*. We illustrate our models using simulated data on residential aged care

(RAC). Our motivation remains that of rising to the challenge laid down by Avkiran and Parker [4] in pushing the DEA research envelope both in methodology and application.

2. DYNAMIC NETWORK RAM AND DYNAMIC NETWORK SBM

We compare robustness test results across weighted, variable returns-to-scale D-NRAM and D-NSBM, both of which are non-radial measures. Part of our motivation is to encourage others to write other comparative studies that apply each approach in various settings of organizational performance. For example, translation invariance of RAM can be of particular significance in a business environment where negative numbers are part of performance measurement (e.g., negative return on equity, negative growth rates, budget deficits, etc.) and data transformation is used. According to Cooper, Park and Pastor [7], RAM is one of those measures that allow easy interpretation in a variety of contexts because it captures the average proportion of inefficiencies that input/output ranges indicate as feasible.

However, RAM's ability to accept free data does not resolve a potential conflict with economic theory sometimes overlooked in applications of DEA. That is, the production process captured as part of *technical efficiency* estimates (rather than cost or price efficiency) may have been represented by negative values in violation of the quantity (volume) measures that should consist of semi-positive numbers, e.g. non-interest income as a measure of bank output can sometimes be negative due to losses being larger than gains.

We emphasize that use of SBM and RAM as the core models in our equations instead of the more traditional CCR (Charnes, Cooper and Rhodes, [6]) or BCC (Banker, Charnes, and Cooper, [5]) radial models allows the analysis to capture the potential non-radial changes in inputs and outputs we would expect in practice. Thus, in a similar manner to Avkiran and Morita [3] and Tone and Tsutsui [11], we also argue that the radial input contractions or output expansions assumed in the CCR and BCC models are inappropriate unless proportionality is established as part of the production process. That is, estimating non-proportional projections through non-radial models is a more realistic representation of a complex business world. We now present the equations behind the dynamic network RAM and SBM models developed in the current study.

The transition from NRAM to input-oriented D-NRAM proceeds by introduction of the extra multiplicative weighting by the time periods in the objective function (see equation 1 in Appendix A). We follow the approach of Tone and Tsutsui [12] in incorporating bad carry-overs as inputs in dynamic network modeling. In earlier published research, bad carry-overs and undesirable intermediate outputs were treated as inputs in the constraints. For example, Tone and Tsutsui [10] incorporate link flows into efficiency measurements in the input-oriented case. Vaz et al. [13] and Fukuyama and Weber [8] also treat undesirable outputs as inputs. Equation (2) in Appendix A represents input-oriented D-NSBM.

3. RESEARCH DESIGN

3.1. Network structure and data simulation

In the simulated network structure of residential aged care, there are five input variables and one final output variable per division, and one intermediate product linking the two divisions of low-level care (LLC) and high-level care (HLC) (see Figure 1 in Appendix B). We also incorporate in our conceptual model three undesirable outputs that become part of the dynamic modeling. Negative

outcomes such as hospitalizations or mortality are designated as *undesirable outputs* or *carry-overs* from one period to the next. This approach acknowledges that some divisions may enter a period at a relative disadvantage if they have higher undesirable carry-overs than others. Finally, the number of residents being transferred from one level of care to the next represents divisional links. For example, people being transferred from an LLC to an HLC division become an *undesirable output* for an LLC division and an *input* for an HLC division.

According to the Australian Institute of Health and Welfare ([1], p.83), about three-quarters of permanent residents were appraised as high-care as at 30 June 2009. We allow this ratio to guide our initial data simulation for number of beds. In recognition of the current Australian federal government plans to shift low-care to residents' homes, we then build into data simulation a scenario where RAC networks undertake growth in their number of high-care beds across a three year period randomly selected in the range of 10-25% per annum. This growth scenario targets the bottom 20% of the RAC networks in the initial sample sorted in descending order on the ratio of high-care to low-care beds, i.e. those networks that have a relatively low number of high-care beds at the start of the growth period. We generate data for 526 RAC networks for 4 years (2012-2015), i.e. the total number of observations equals 2104 (526x4). Further details of the data simulation are available from the corresponding author.

3.2. Discrimination across various sample sizes

The population of 526 RAC networks or DMUs with the full complement of variables and divisional weights (LLC 0.6, HLC 0.4) is hereafter referred to as the *core model*, and results from the core model in the absence of perturbations are referred to as *baseline results*. Initially, the research design calls for comparisons *across different sample sizes* by monitoring discrimination. Different samples are created via nested sampling. That is, starting with N=526, we remove the top 100 DMUs four times; thus, the first 100 DMUs removed are those numbered 526-427. Monitoring discrimination involves observing descriptive statistics of D-NDEA efficiency estimates, efficient versus inefficient DMUs, membership of the benchmark group, repositioning of the benchmark DMUs as sample size grows, and so on.

3.3. Perturbations

Here, we focus our attention on the core model and expose it to a series of data perturbations. We start by removing network variables, followed by removal of efficient DMUs (i.e. layering), and finally change divisional weights. Following each perturbation, we examine the distribution of emerging efficiency estimates and composition of the benchmark group that defines the efficient frontier.

4. ROBUSTNESS TESTS AND AN ILLUSTRATIVE APPLICATION

4.1. Observations related to sample size

As sample size grows, discrimination improves; improvement in discrimination stops after N=326. The negative but falling skewness suggests that the majority of estimates are closer to 1. The low rate of survival of benchmark groups from one sample to the next underscores the relative nature of DEA where new DMUs outperform DMUs in the previous sample's benchmark group. Benchmark groups across D-NRAM and D-NSBM are *identical*. The main difference between the two models is a *substantially wider range* of efficiency estimates under D-NRAM.

4.2. Observations related to data perturbations

In three separate perturbations, we remove the inputs of *registered nurses average length of service* and *other caregivers* first, the undesirable output of *average severity of hospitalizations* as the second

perturbation, and finally we remove all three variables simultaneously. Compared to the baseline results, new D-NRAM and D-NSBM efficiency estimates are over a wider range as degrees of freedom rises – similar to what we would expect to find with traditional DEA. We also note that the distribution of efficiency estimates remains negatively skewed, with one exception where simultaneous removal of all three variables results in positively skewed D-NSBM estimates. All the absolute skewness values are under 0.3 for perturbed data results. Membership of the efficient frontier drops from 31 to 12 when the two inputs are removed. Similarly, this number becomes 6 when only the undesirable output is removed, suggesting greater sensitivity of the frontier to this kind of variable. Removing all three variables lowers the number of benchmark DMUs to 3 as more degrees of freedom are released.

As layering creates a smaller sample at each step, there is evidence of some loss of discrimination in the steadily but slowly rising mean and median of estimates corresponding to the *core inefficient cohort* (this trend is less discernible with D-NRAM). Significant rank correlations range between 0.973-1.000 for the core efficient cohort when compared across two consecutive ~~periods~~ layers. Thus, there is a core group of inefficient DMUs whose measure of relative performance is not substantially affected by omission of benchmark DMUs in the sample. Knowing that relative ranking of those comprising the core inefficient cohort is not severely impacted by any particular group of benchmark DMUs helps management better target activities geared towards performance improvement.

The divisional weights of the core model (i.e., LLC 0.6, HLC 0.4) are first swapped (i.e., LLC 0.4, HLC 0.6), and then changed again to LLC 0.2 and HLC 0.8. As we change divisional weights, composition of the benchmark group remains unchanged, and rank correlations are positive and 0.9 or above. The range of estimates is almost identical with D-NRAM and highly comparable with D-NSBM, when compared against baseline results. These observations suggest that both D-NRAM and D-NSBM can accommodate a range of divisional weights determined by management without necessarily compromising the baseline benchmark group or the emerging overall rankings. Such flexibility would make it easier to promote DEA in the workplace.

4.3. Additional notes on D-NRAM vs. D-NSBM

Steinmann and Zweifel [9], in their critique of RAM, report that large inefficient DMUs appear less efficient than small inefficient DMUs, thus claiming RAM to be biased against large DMUs (also see rebuttal by Cooper, Park and Pastor [7] in the same journal and issue). We test for this potential size bias in the samples N=526 and N=126 by focusing on the inefficient DMUs. Rank correlations between yearly D-NRAM efficiency estimates at the DMU level and the size proxy of number of beds are significant at the 1% level and range between -0.352 and -0.491, with no substantial difference between samples. D-NSBM results are similar where the rank correlations range between -0.294 and -0.426. In conclusion, we observe *moderate size bias* across both D-NDEA models regarding inefficient DMUs.

Finally, we test whether D-NRAM retains the properties of *translation-invariance* and *acceptance of free data* normally associated with RAM. We force the following simultaneous changes on the data based on arbitrarily selected numbers and compare the emerging efficiency estimates to those from our core model:

- Add 73 to the input of registered nurses average length of service in HLC (testing translation invariance);
- Add 25 to the undesirable output of number of hospitalizations in LLC (testing translation invariance); and

- Subtract 101 from the input of average resident classification score in LLC, thus converting all numbers to negative (testing acceptance of free data and translation invariance).

Results available from the corresponding author show precisely the same efficiency estimates as those from the core model, thus confirming the presence of translation invariance and acceptance of free data for D-NRAM.

4.4. Observations related to the growth scenario

We now report the findings from our illustrative application of D-NRAM and D-NSBM to residential aged care. The resulting average annual growth rate in high-care beds selected for growth is 17.25%, where the actual minimum and maximum growth rates are 10.02% and 24.7%, respectively. Chart A in Figure 2 (see Appendix B) plots mean annual D-NRAM efficiency estimates at the DMU and divisional levels using baseline results, as well as the corresponding total number of beds. The mean DMU efficiency appears to closely follow the efficiency of LLC divisions where the number of beds is kept constant across the study period. On the other hand, the mean HLC estimates start falling after the first year of growth in number of beds. Overall, an *optimal* total number of beds is reached after two years of growth based on organizational efficiency as conceptualized in Figure 1 (see Appendix B).

Chart B in Figure 2 plots the case for D-NSBM where all three efficiency lines follow a similar path as the number of beds grows. Once again, growth within the assumed range beyond the second year appears to be sub-optimal in terms of efficiency. The patterns of changes in mean efficiency estimates plotted in Figure 2 hold when we swap the divisional weights.

When we probe mean slacks at variable level, we notice that the three undesirable outputs harbor the *largest growth in slacks* as of year 1. As growth in number of beds sets in, we also notice rising inefficiencies in the discretionary input variables such as number of beds and staff numbers. Focusing on the four discretionary inputs in Figure 1 reveals that, under D-NRAM, on average, the number of beds (32.35%), followed by registered nurses employed (28.12%), contribute the largest proportion of the inefficiencies in variables under managerial control. This order is somewhat different under D-NSBM where the greatest contributor to slacks is other caregivers (31.27%), followed by registered nurses (27.71%). Equally revealing, the two models share *registered nurses average length of service* as the lowest contributor to slacks embedded in discretionary input variables. This insight suggests that management ought to focus more attention on the three discretionary input variables, whereas the average length of service of registered nurses is less critical in running efficient operations.

5. CONCLUDING REMARKS

This paper develops two non-radial, weighted, dynamic network DEA models, reports a number of robustness tests, and applies the models in the context of residential aged care using simulated data.

Key findings regarding different sample sizes and various data perturbations can be summarized as, (a) up-to a point, increasing sample size improves discrimination, (b) removing a relevant input variable improves discrimination and changes the composition of the benchmark group where the frontier is more sensitive to removal of an undesirable output, (c) layering results suggest that the core inefficient cohort is resilient against omission of benchmark DMUs, and (d) changing divisional weights produces efficiency estimates with a similar range to baseline results and the benchmark group remains the same.

Additional insight gained by running D-NRAM in parallel with D-NSBM includes

- identical benchmark groups across D-NRAM and D-NSBM;
- a substantially wider range of efficiency estimates across different sample sizes under D-NRAM; and
- evidence of inefficient DMU size bias among D-NRAM and D-NSBM estimates.

Furthermore, D-NRAM is shown to have the additional desirable technical efficiency properties of translation-invariance and acceptance of free data.

We conclude the paper by highlighting some managerial implications. For example, identification of a *core inefficient cohort* enables designing performance improvement for those networks that are most likely to benefit. Similarly, the resilience of results from both mathematical models to a range of divisional weights suggests that management would be able to make DEA more palatable to those whose performance is being measured. Finally, results on the growth scenario we have demonstrated highlight a potentially powerful planning tool in dynamic network DEA where optimal capacity can be guided by technical efficiency of operations.

Acknowledgements: We thank Professor Tone for organizing the workshop at GRIPS, Tokyo, and sharing his paper prior to the workshop. We are also grateful to Ms Satomi Hiraoki for translating the dedication.

REFERENCES

[1] Australian Institute of Health and Welfare (2010) Residential aged care in Australia 2008–09: a statistical overview. Aged care statistics series no. 31. Cat. no. AGE 62. Canberra: AIHW.

[2] Avkiran NK, McCrystal A, (2012), Sensitivity analysis of network DEA: NSBM versus NRAM, Applied Mathematics and Computation, 218(22), 11226-11239.

[3] Avkiran NK, Morita H 2010. Predicting Japanese bank stock performance with a composite relative efficiency metric: a new investment tool, Pacific-Basin Finance Journal, 18(3), 254-271.

[4] Avkiran NK, Parker BR, 2010, Pushing the DEA research envelope, Socio-Economic Planning Sciences, 44(1), 1-7.

[5] Banker RD, Charnes A, Cooper WW 1984. Some models for estimating technical and scale inefficiencies in data envelopment analysis. Management Science 30, 1078-1092.

[6] Charnes A, Cooper WW, Rhodes E. 1978. Measuring the efficiency of decision making units. European Journal of Operational Research 2, 429-444.

[7] Cooper WW, Park KS, Pastor JT 2001. The Range Adjusted Measure (RAM) in DEA: A response to the comment by Steinmann and Zweifel. Journal of Productivity Analysis 15, 145-152.

[8] Fukuyama H, Weber WL. 2010. A slacks-based inefficiency measure for a two-stage system with bad outputs. OMEGA – International Journal of Management Science 38, 398-409.

[9] Steinmann L, Zweifel P 2001. The range adjusted measure (RAM) in DEA: Comment. Journal of Productivity Analysis 15, 139-144.

[10] Tone K, Tsutsui M. 2009. Network DEA: A slacks-based measure approach. European Journal of Operational Research 197, 243-252.

[11] Tone K, Tsutsui M. 2010. Dynamic DEA: A slacks-based measure approach. OMEGA – International Journal of Management Science 38, 145-156.

[12] Tone K, Tsutsui M. 2012. Dynamic DEA with network structure; A slacks-based measure approach. Working paper, National Graduate Institute for Policy Studies, Tokyo, Japan.

[13] Vaz CB, Camanho AS, Guimarães RC, 2010. The assessment of retailing efficiency using Network Data Envelopment Analysis, Annals of Operations Research 173, 5-24.

APPENDIX A: Equations

$$\Gamma_o^* = \min_{\lambda_k^t, s_{ok}^{t-}, s_{ok}^{t+}, s_{o(k,h)}^t, s_{ok}^{(t,t+1)}} 1 - \left\{ \sum_{t=1}^{T} W_t \sum_{k=1}^{K} \frac{w_k}{m_k + i_k + b_k} \left[\sum_{m=1}^{m_k} \frac{s_{mok}^{t-}}{R_{mk}^{t-}} + \sum_{l=1}^{i_k} \frac{s_{lo(k,h)}^{t}}{R_{l(k,h)}^{t}} + \sum_{n=1}^{b_k} \frac{s_{nok}^{(t,t+1)}}{R_{nk}^{(t,t+1)}} \right] \right\} \quad (1)$$

subject to

$$x_{ok}^t = X_k^t \lambda_k^t + s_{ok}^{t-} \qquad (k = 1,...,K; t = 1,...,T) \quad (1a)$$

$$y_{ok}^t = Y_k^t \lambda_k^t - s_{ok}^{t+} \qquad (k = 1,...,K; t = 1,...,T) \quad (1b)$$

$$z_{o(k,h)}^t = Z_{(k,h)}^t \lambda_k^t + s_{o(k,h)}^t \qquad (\forall (k,h)\ input; \forall t) \quad (1c)$$

$$z_{ok}^{(t,t+1)} = Z_k^{(t,t+1)} \lambda_k^t + s_{ok}^{(t,t+1)} \qquad (\forall k; t = 1,...,T-1) \quad (1d)$$

$$\sum_{n=1}^{N} \lambda_{nk}^t = 1\ (\forall k,t) \qquad \lambda_{nk}^t \geq 0\ (\forall n,k,t) \quad (1e)$$

$$Z_{(k,h)}^t \lambda_h^t = Z_{(k,h)}^t \lambda_k^t \qquad (\forall (k,h)\ input; \forall t) \quad (1f)$$

$$Z_k^{(t,t+1)} \lambda_k^t = Z_k^{(t,t+1)} \lambda_k^{t+1} \qquad (\forall k; t = 1,...,T-1) \quad (1g)$$

$$\lambda_k^t \geq 0, s_{ok}^{t-} \geq 0, s_{ok}^{t+} \geq 0, s_{o(k,h)}^t \geq 0\ (\forall k,h,t),\ s_{ok}^{(t,t+1)} \geq 0\ (\forall k; t = 1,...,T-1) \quad (1h)$$

where,

o = the observed DMU, $o = 1,...,N$

N = the number of DMU's

k, h = a division (K = number of divisions)

m_k = number of inputs for division k

r_k = number of outputs for division k

i_k = number of input links for division k

b_k = number of bad carry overs for division k

s_{ok}^{t-} = input slack for division k, time t

s_{ok}^{t+} = output slack for division k, time t

$s_{o(k,h)}^t$ = slack for intermediate product link between division k and division h, time t

$s_{ok}^{(t,t+1)}$ = slack for bad carry over for division k from time t to time $(t+1)$

λ_k^t = intensity vector for division k, time t

X_k^t is the input matrix for division k, time t where $X_k^t = (x_1^t,...,x_K^t) \in R^{m_k \times T}$

Y_k^t is the output matrix for division k, time t where $Y_k^t = (y_1^t,...,y_K^t) \in R^{r_k \times T}$

$Z_{(k,h)}^t$ = intermediate product input link between division k and division h at time t

$Z_k^{(t,t+1)}$ = bad carry over for division k from time t to time $(t+1)$

R_{mk}^{t-} = range of input m, division k, time t = $\max(x_{mk}^{t-}) - \min(x_{mk}^{t-})$

$R_{l(k,h)}^t$ = range of l^{th} input link variable between division k and h, time t = $\max(z_{l(k,h)}^t) - \min(z_{l(k,h)}^t)$

$R_{nk}^{(t,t+1)}$ = range of n^{th} bad carry over variable, division k, time t = $\max(z_{nk}^{(t,t+1)}) - \min(z_{nk}^{(t,t+1)})$

$\sum_{k=1}^{K} w_k = 1,\ w_k \geq 0\ (\forall k)$ where w_k is the relative weight of division k determined exogenously

$\sum_{t=1}^{T} W_t = 1,\ W_t \geq 0 (\forall t)$ where W_t is the relative weight of time t determined exogenously

$$\rho_o^* = \min_{\lambda_k^t, s_{ok}^{t-}, s_{ok}^{t+}, s_{o(k,h)}^t, s_{ok}^{(t,t+1)}} \sum_{t=1}^{T} W_t \sum_{k=1}^{K} w_k \left\{ 1 - \left[\frac{1}{m_k + i_k + b_k} \left[\sum_{m=1}^{m_k} \frac{s_{mok}^{t-}}{x_{mok}^j} + \sum_{l=1}^{i_k} \frac{s_{lo(k,h)}^t}{z_{lo(k,h)}^t} + \sum_{n=1}^{b_k} \frac{s_{nok}^{(t,t+1)}}{z_{nok}^{(t,t+1)}} \right] \right] \right\} \quad (2)$$

subject to

$$x_{ok}^t = X_k^t \lambda_k^t + s_{ok}^{t-} \qquad (k = 1,...,K; t = 1,...,T) \qquad (2a)$$

$$y_{ok}^t = Y_k^t \lambda_k^t - s_{ok}^{t+} \qquad (k = 1,...,K; t = 1,....,T) \qquad (2b)$$

$$z_{o(k,h)}^t = Z_{(k,h)}^t \lambda_k^t + s_{o(k,h)}^t \qquad (\forall (k,h)\ input; \forall t) \qquad (2c)$$

$$z_{ok}^{(t,t+1)} = Z_k^{(t,t+1)} \lambda_k^t + s_{ok}^{(t,t+1)} \qquad (\forall k; t = 1,...,T-1) \qquad (2d)$$

$$\sum_{n=1}^{N} \lambda_{nk}^t = 1\ (\forall k,t) \qquad \lambda_{nk}^t \geq 0\ (\forall n,k,t) \qquad (2e)$$

$$Z_{(k,h)}^t \lambda_h^t = Z_{(k,h)}^t \lambda_k^t, \qquad (\forall (k,h)\ input; \forall t) \qquad (2f)$$

$$Z_k^{(t,t+1)} \lambda_k^t = Z_k^{(t,t+1)} \lambda_k^{t+1} \qquad (\forall k; t = 1,...,T-1) \qquad (2g)$$

$$\lambda_k^t \geq 0, s_{ok}^{t-} \geq 0, s_{ok}^{t+} \geq 0, s_{o(k,h)}^t \geq 0\ (\forall k,h,t),\ \ s_{ok}^{(t,t+1)} \geq 0\ (\forall k; t = 1,...,T-1) \qquad (2h)$$

where,

o = the observed DMU, $o = 1,...,N$

N = the number of DMU's

k, h = a division (K = number of divisions)

m_k = number of inputs for division k

r_k = number of outputs for division k

i_k = number of input links for division k

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s_{ok}^{t-} = input slack for division k, time t

s_{ok}^{t+} = output slack for division k, time t

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$R_{l(k,h)}^t$ = range of l^{th} input link variable between division k and h, time t = $\max(z_{l(k,h)}^t) - \min(z_{l(k,h)}^t)$

$R_{nk}^{(t,t+1)}$ = range of n^{th} bad carry over variable, division k, time t = $\max(z_{nk}^{(t,t+1)}) - \min(z_{nk}^{(t,t+1)})$

$\sum_{k=1}^{K} w_k = 1,\ w_k \geq 0\ (\forall k)$ where w_k is the relative weight of division k determined exogenously

$\sum_{t=1}^{T} W_t = 1,\ W_t \geq 0 (\forall t)$ where W_t is the relative weight of time t determined exogenously

APPENDIX B: Figures

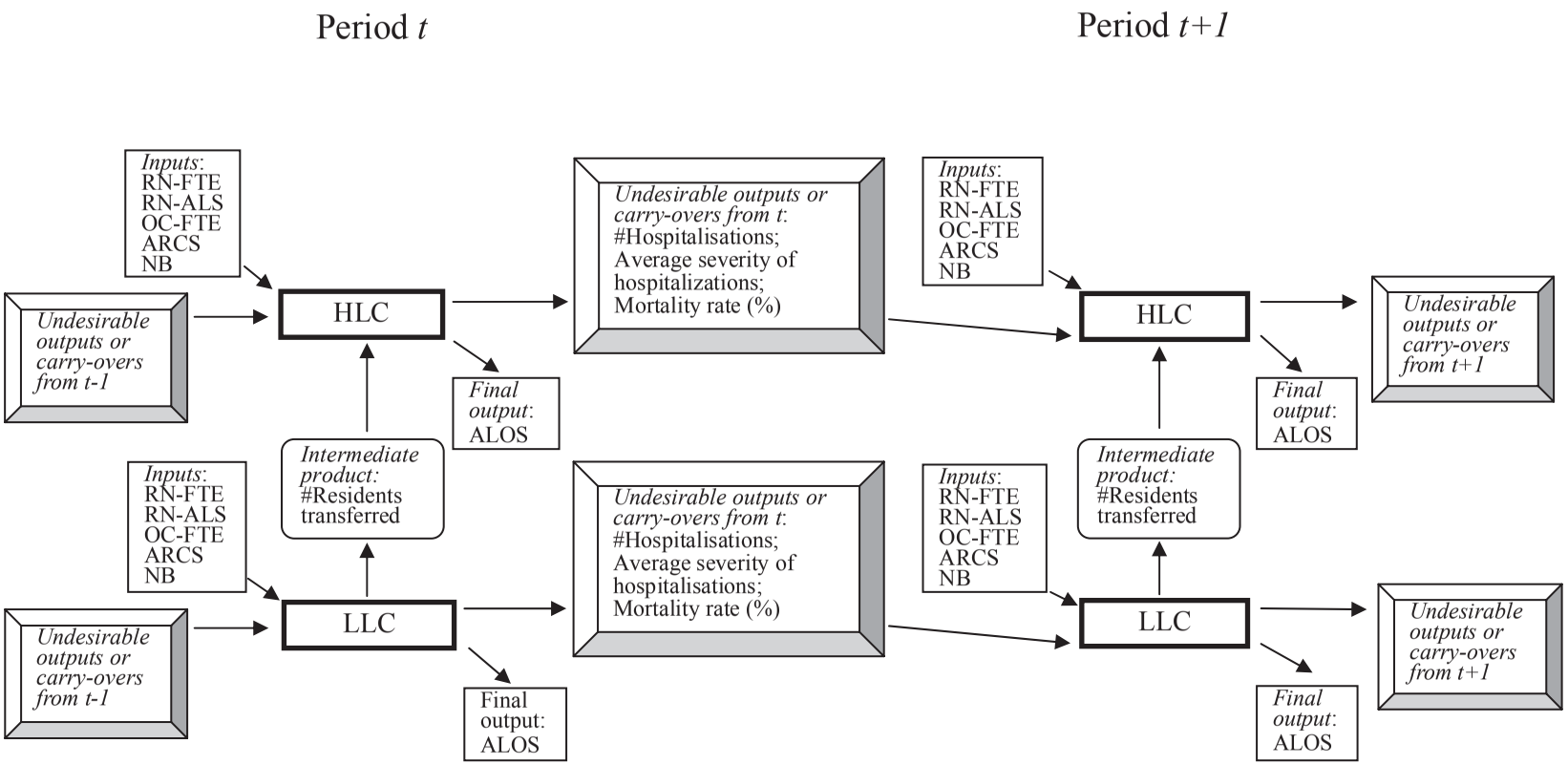
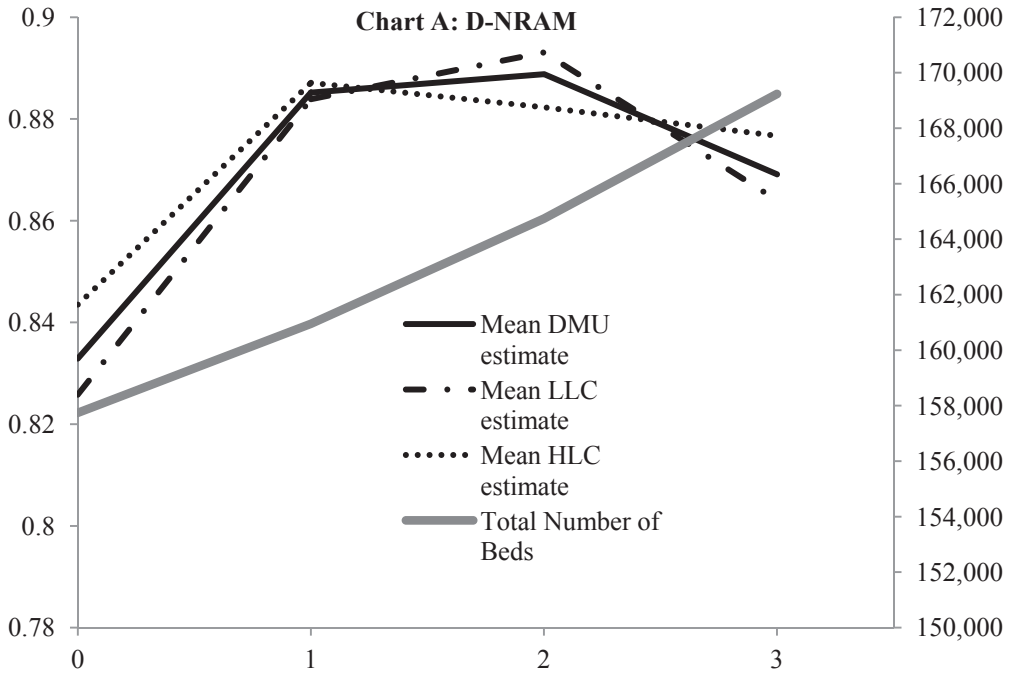


Figure 1: Modeling an intertemporal productivity network of residential aged care with two levels of care

Notes: For brevity, undesirable outputs or carry-overs are detailed only once, although the same items are assumed entering and exiting a given period. The intermediate product of 'number of residents transferred' is an output from the LLC division that becomes an input to the HLC division.

Legend: RN-FTE, registered nurses full-time equivalent; RN-ALS, registered nurses average length of service; OC-FTE, other caregivers full-time equivalent; ARCS, average resident classification score; NB, number of beds; ALOS, average length of stay; HLC, high-level care; LLC, low-level care.



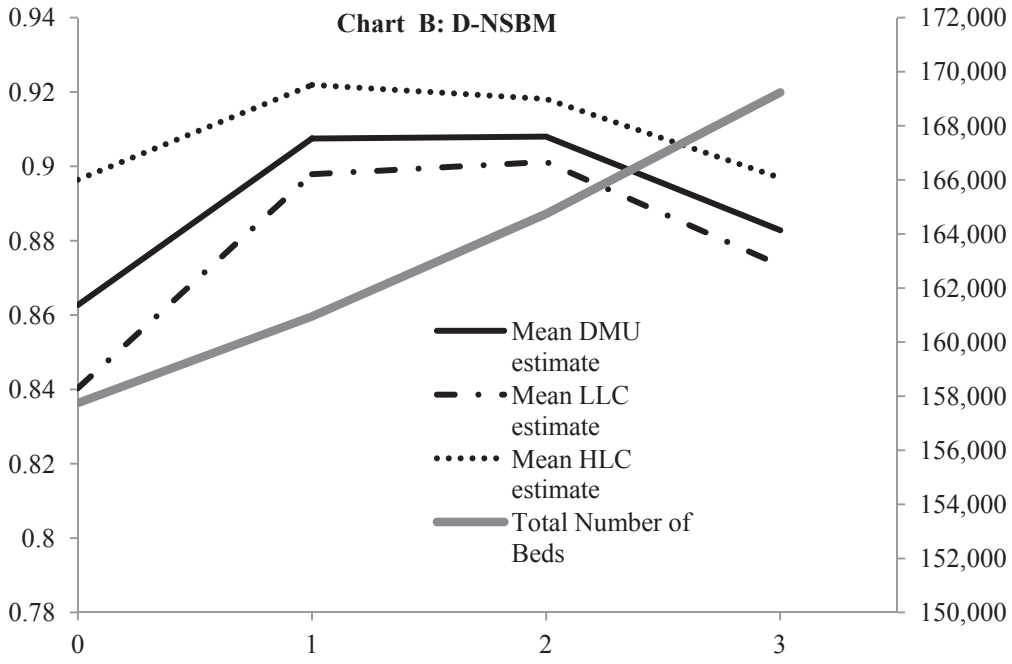


Figure 2: Plotting mean DMU and divisional efficiency estimates against number of beds