THREE ESSAYS ON TIME CONSISTENCY OF MONETARY POLICY IN OPEN ECONOMY

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I would like to dedicate this thesis to my wife.

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Abstract

This dissertation aims to study the time consistency problem of monetary policy in two-country model. We raise two research questions: 1) Is the Friedman rule less likely to be sustained under cooperation than under non-cooperation when the governments lack commitment technology? 2) Is delegation more effective to solve the time consistency when the economy becomes more open?

Chapter 1 describes the motivation, research objectives, methodology, and organization.

Chapter 2 investigates the time-consistency problem of monetary policy under open economy. We first consider the consequences and the solutions in the general macroeconomic framework, and then examine the role of open economy in solving the time-inconsistent monetary policies. Three main consequences have been pointed out in the literature: inflationary bias, expectation trap, and free rider. To eliminate the problem, different types of commitment technologies have been suggested: rules, reputation, maturity structures of public debt, and monetary delegation method. An open economy sheds more light on resolving the problem.

Chapter 3 uses a microfounded New Keynesian two-country model to revisit the counterproductive cooperation when governments cannot commit to their future monetary policy. When monopolistic distortions are large, the optimal policy under commitment is characterized by the global Friedman rule irrespective of policy regime. The counterproductive cooperation between governments displays in two aspects: under cooperation, the discretionary outcome may be worse, and the global Friedman rule is less likely to be sustained than under non-cooperation.

Chapter 4 assesses the effects of openness on the effectiveness of delegation in solving the time-inconsistency problem of monetary policy. We built a micro-founded New Keynesian two-country model with a government structure including a government and a central bank. We explicitly formulate delegation process which allows the government to reappoint the current central bank with a reappointment cost. Our findings are two-fold: when the economy becomes more open, a lower threshold of reappointment cost parameter is required for central bank appointment being sustained and when the reappointment cost is less than such a threshold, the current central bank is less likely to be reappointed.

Chapter 5 provides conclusions and policy implications.

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Chapter 1

Introduction

1.1 Motivation

In recent years, there have been two sources which call for international monetary policy cooperation. First, the 2008-2009 financial crisis gives rise to a need for coordination to recover the world economy. Second, the recent behaviors of major currencies generate growing concern about adverse spillovers from developed countries into developing countries. The finance minister of Brazil alarms a currency war in which the major currencies depreciate against emerging ones. The central banker of India criticizes the unconventional policy in the U.S. and near-zero interest rate policy in some developed nations.

The economists capture the increasing concern in different dimensions. In terms of shocks, Coenen et al. (2010) use the New Open Economy New Keynesian to compare the welfare under cooperation and non-cooperation for a range of typical shocks. Corsetti et al. (2011) treat news as a type of unconventional shocks and investigate the effects of news on the optimal cooperation. They find that with the presence of private information, news could aggravate the informational distortion. Regarding distortions, in addition to canonical sticky price and monopolistic competition, recent papers have focused on other sources. Engel (2011) and Rodriguez-Lopez (2011) model the pricing to market distortion in the open economy; Benigno (2009) and Corsetti et al. (2010) consider financial market distortions; Bianchi (2011) examines the one caused by current account imbalance. For the zero lower bound, Fujiwara et al. (2010) study optimal monetary policy under cooperation.

The above New Keynesian literature, however, assumes that policymakers own commitment technology to their announced policies. In reality, even in developed countries, the governments only have a limited commitment ability. The lack of such ability causes a time consistency problem pioneered by a seminal contribution of Kydland and Prescott (1977). In the context of monetary policy, Barro and Gordon (1983a) show that this problem leads to an inflationary bias which then becomes a workhorse of monetary policy analysis as well as a major concern of modern central bank whose ultimate goal is to fight inflation. Putting this problem under the open economy, it is natural to ask whether cooperation helps to reduce the bias. Rogoff (1985a) extends the model of Barro and Gordon to a two-country model and points out that cooperation might be counterproductive. Although this result is striking, his theory based on ad hoc objectives for policymakers. It is more reasonable to reflect the actual goals of monetary authorities by using a more descriptive objective function which is derived from a micro-founded framework. In response to this requirement, several remedies are proposed to eliminate or mitigate the inflationary bias by dealing with the ability to commit. Rogoff (1985b) proposes that delegating monetary policy to a conservative and independent central banker could remove the inflation bias. Subsequent papers develop this idea and establish a research line of monetary delegation method (Walsh, 1995; Jensen, 1997). These papers rely on ad hoc utility function and focus on the closed economy.

Given the growing concern about the international monetary policy cooperation and the development of literature on the New Keynesian open economy model, this dissertation aims to revisit the counterproductive cooperation and to access the effectiveness of monetary delegation method in solving the time consistency problem under a New Keynesian two-country framework.

1.2 Research objectives

This dissertation consists of three main chapters which have different research objectives. The achievements of these research objectives represent our contributions. Chapter 2 investigates the literature on the time consistency problem in open economy. It shows the consequences and the solutions to the problem in a closed economy and then compares the effectiveness of the solutions with those in open economy. This chapter also figures out what has not yet been done in studying the time consistency in the open economy.

Chapter 3 revisits the counterproductive cooperation. It compares the sustainability constraints under various policy regimes to see whether the Friedman rule is less likely to be sustained under cooperation than under non-cooperation when the governments lack commitment technology.

Chapter 4 assesses the effectiveness of monetary delegation method to solve time consistency problem in open economy in two aspects. First, it examines that effect of openness of an economy on the lower threshold of reappointment cost parameter to keep the Friedman rule sustained. Second, it compares the conditions for the Friedman rule being sustainable when the economy becomes more open.

1.3 Methodology

Chapter 2 of this dissertation surveys a literature on the time consistency problem in the open economy framework. The last two essays use a Dynamic Stochastic General Equilibrium method (DSGE). We develop a New Keynesian open economy model with utility optimizing households, imperfect competition in the goods market, and one-period sticky price. These models are elaborate in their microfoundation but simple enough to derive analytical solutions.

In Chapter 3, we use Arseneau (2007, 2012)'s model to derive the conditions for the Friedman rule being sustained when the governments cannot commit to their future policies under various policy regimes. These sustainability conditions correspond with some sets of discount factor. The bigger the set is, the more sustainable the Friedman rule is. By comparing the sets under different policy regimes, we can show under which regime the Friedman rule is more likely to be sustained.

In Chapter 4, we extend Basso (2009)'s model to a two-country model. A governmental structure includes a government and a central bank. Instead of directly setting a monetary policy, the government initially appoints a central bank and then can replace to a new central bank with some reappointment cost after prices are set by firms. With an absolute value reappointment cost function, there exists a lower threshold of reappointment cost parameter to restore the Friedman rule as a solution under discretion. This threshold is a decreasing function of openness. When the reappointment cost parameter is lower than such a threshold, we derive the sustainability constraints of the Friedman rule under the open and show that these constraints are more likely to hold when the economy becomes more open.

1.4 Organization

Chapter 2, entitled "Time Consistency Problem of Monetary Policy under Open Economy: A Survey", investigates the time-consistency problem of monetary policy under open economy. Chapter 3, entitled "Time Consistency and Counterproductive Monetary Policy Cooperation in a New Keynesian Two-country Model", revisits the counterproductive cooperation when governments cannot commit to their future monetary policy. Chapter 4, entitled "Monetary Delegation and Time-inconsistency in a New Keynesian Two-country Model", studies the effectiveness of delegation in solving the time-inconsistency problem of monetary policy in a micro-founded New Keynesian two-country model. Chapter 5 contains conclusions and policy implications.

Chapter 2

Time Consistency Problem of Monetary Policy under Open Economy: A Survey

2.1 Introduction

Time consistency has been a workhorse in studying the macroeconomic policy. The literature on time consistency is pioneered by Kydland and Prescott (1977) who regard the policymaker as a joint decision maker, maximizing a social utility function. The policy decision is set sequentially instead of once and for all so that they concentrate on the credibility of various policy rules. Kydland and Prescott indicate that policy rules which are optimal at the initial period might not be time consistent because the government can change such policies after private sectors set prices. Hence time inconsistency refers to a lack of credibility. The rational private sector will not believe an optimal but time-inconsistent policy rule unless the government is legally obligated to conduct it. Kydland and Prescott provide an intuitive explanation for policy surprises induced by the government under discretion. The reason is that of the difference in the interest between the government and private sector. Then the government implements its policy to affect the behavior of private sector to achieve its preferred policy plan. Under a dynamic framework, behaviors of the private agents are contingent upon the future policies' expectation. As a result, credibility which features the ability to alter expectation plays an important role in completing a policy successfully. When making policy decision sequentially, the government is subject to an incentive constraint which narrows what the government can obtain and thereby lower the welfare compared to the case in which the commitment technology is available.

2.2 Consequences of time inconsistency

2.2.1 Inflation bias

Kydland and Prescott (1977) demonstrate their argument with a couple of examples in different contexts. Among them the monetary policy model with a trade-off between inflation and unemployment has received much attention due to the popularization by the seminal work of Barro and Gordon (1983a) which analyze an augmented Phillips curve model with nominal wage rigidity. When government introduces an inflation rate which is higher than anticipated, it can cut unemployment below the natural rate. This result arises because the government sets actual policy after nominal wages are contracted. There exists a trade-off between inflation and unemployment. The optimal policy is to balance the marginal cost of a rise in inflation with the marginal benefit of a decrease in unemployment. Because the private sector understands the government's incentive, they adjust their expectation accordingly. As a consequence, equilibrium unemployment still equates to the natural rate whereas there exists an inflationary bias. This inflationary bias is attributed to the lack of credibility of the government.

2.2.2 Expectation trap

The lack of commitment may lead to the expectation trap (Chari et al., 1998; Albanesi et al., 2003). They build a general equilibrium model to reflect the cost-benefit trade-off between unanticipated and realized inflation as in Kydland and Prescott (1977) and Barro and Gordon (1983a)'s setting. This type of trade-off causes multiple equilibria in a general equilibrium model. Private sector can expect high or low inflation, which dictates their defensive behaviors. As a consequence, policymaker is trapped to validate such expectations. Indeed, the monetary authorities might find them optimal to verify a high inflation expectation when a recession arises from not doing that. When the policymaker chooses to validate, this self-fulling expectation is labeled as an expectation trap. Albanesi and Christiano (2001) indicate that expectation trap can generate costly volatility in output and employment allocations which could be used to explain a substantial drop in output of Mexico in 1995 and of several Asian sovereigns in 1998. One of a critical distinction between Chari et al. (1998) and Albanesi et al. (2003) is the source of expectation trap. The former relies on trigger strategies, but this is ruled out in the latter. They both conclude that an institutional setup which provides a partial commitment can help escaping from expectation trap.

2.2.3 Free riding

Chari and Kehoe (2008) show that in a monetary union, a time inconsistency problem causes a kind of fiscal free riding problem in terms of non-monetary policies and this leads to a failure in some monetary unions. The general reason is that free-riding countries set loose non-monetary policies which are prosper-thy-self but beggar-thy-neighbor and thereby dictates the central bank to induce high inflationary policies for the union. When a direct solution to the time inconsistency is not available, solving the free-riding problem also alleviates the time inconsistency problem. The idea to deal with the free-rider problem is to constrain the non-monetary policies unitedly. As long as the limitations on the non-monetary policies exist, the central bank has less incentives to introduce inflation. Note that, such constraints only mitigate the time inconsistency problem and does not remove it. Chari and Kehoe apply their argument to three types of non-monetary policies. In line with this survey, we next summarize how the argument is applied to the fiscal policy. Note that departing from the current literature, the behaviors of forward-looking private sectors trigger the free-riding problem when the central bank cannot commit to their future policy.

A simple model includes many economies belonging to a monetary union. The government in each economy finances its spending by issuing nominal debt to its inhabitants. The union's central bank then sets the uniform inflation rate with consideration on a trade-off between inflation's benefits and costs. An increase in inflation lowers the real debt and hence reduces the tax amounts needed to back up the debt. While this reflects the benefits of inflation, higher inflation also generates cost due to its allocation distortion and output reduction. In this model, the benefits outweigh the costs, and thus the central bank attempts to raise inflation when the debt is higher. When setting its spending, each government only takes into account the costs of higher inflation imposed on its output but not on other countries'. Hence, compared with the cooperation case, each government issues an excessive debt, which causes the central bank to generate excessive inflation. The free-riding problem can be solved by imposing constraints on the level of debt that a government can issue when commitment technology is infeasible.

2.3 Solution to the time inconsistency problem

2.3.1 Rules

Kydland and Prescott (1977) argue that under discretion, although using policy rules cannot achieve the efficient level, it does improve the welfare. This idea is related to earlier contribution by Friedman (1948) who claims that monetary and fiscal policy should comply with simple and stable rules. However, Friedman's contribution is neither analytical nor quantitative. In contrast, Kydland and Prescott's model is built on the concept of credibility, time-inconsistency problem, and a formulation of a game theoretic model. More importantly, the supporting for rules against discretion is based on the understanding of policies' effect in terms of timing and size. Indeed, Kydland and Prescott suggest that when people have the knowledge of the structure of the economy, the government should obey rules rather than discretion. The reason is that under discretion the government chooses a suboptimal policy.

2.3.2 Reputation

Backus and Driffill (1985) extend the Kreps and Wilson (1982)'s model to the credibility problem. In this model, the information is asymmetric in the sense that the private sectors do not know whether the government induces an inflationary policy or not, and in turn, the government does not know how persistent the private agent's wage demand is. The incentive for cheating prevents any players to reveal their actions credibly. They derived a dynamically consistent equilibrium that is the solution to the game between government and private agents. Each player checks the other's behavior until credibility or lack of credibility is formed. The interplay between government and private agent generates the output cost of disinflation which manipulates weak governments as in Barro and Gordon (1983a)'s game to randomize and hence affect the information obtained by the private sector.

2.3.3 Maturity structures of public debt

Recent papers have used the maturity structures of public debt as a commitment device to induce an optimal monetary and fiscal policy. Alvarez et al. (2004) and Persson et al. (2006) put some restrictions on the maturity structure of public debt such that the marginal benefit of unexpected inflation equals to the marginal cost. These restrictions are modeled to balance both the temptation of employing inflationary surprise to support public spending and the temptation of rending a devaluation of the inflation-indexed debt. Liu (2013), however, shows that this method does not work in a stylized small open economy because how leisure is formulated in utility function and what process the productivity follows determine the success of using commitment technology to induce a time consistent solution.

2.3.4 Monetary delegation

Conservative central banker

When a distortion induces a suboptimal market rate of employment, wage setter can increase wage inflation in order to prevent the central bank from raising employment by reducing the real wage below its target level. Rogoff (1985b) points out that this time-consistency problem can be overcome by delegating monetary policy to a central banker which is conservative but not too conservative. The conservative central banker means the central bank will put a higher weight than the society does on the inflation in its welfare function. This appointment lowers the temptation to increase wage inflation whereas distorts the reactions of the central bank to unexpected shocks such as supply shock. The cost arises due to the role of central bank in stabilizing inflation and employment around the mean level determined by the market.

When a central bank imposes an infinite weight on inflation, it can force down the inflation to the optimal level of the society. However, the central bank unsuitably reacts to supply shock and then pass the total shock to employment. Hence, a lower weight helps the society reach a first-order stabilization benefit at the expense of second order inflation. When supply shock matters, instead of inflation rate another monetary target which is the "most highly correlated with the society's ultimate objective function" (Rogoff, 1985b) should be chosen by the central bank.

Targeting rules

Svensson (1997) proposes the targeting rules in which the government imposes some penalty on the central bank for defecting from the targeted variable. Under an inflation-targeting regime, inflation target is explicitly delegated to the central bank whereas employment target and relative weight on employment stabilization are assigned implicitly. In the absence of employment persistence, targeting rules dominate the conservative central banker method in terms of eliminating the inflationary bias. When employment persistence is present, this method cannot completely remove the inflationary bias and stabilization bias by delegating the long-run natural rate.

Contracting method

Walsh (1995) proposes optimal contracts for central bankers based on a principal-agent problem to eliminate the inflation bias under discretion. The central bank is regarded as the government's agent who maximizes an objective function taking as given the government transferring. The existence of a contract removes the inflationary bias of discretionary policy whereas guarantees that inflation reacts optimally to supply disturbances. The interpretation is that since inflationary bias remains constant across the states of the world, a contract only has to induce a constant amount of the marginal cost of inflation while allowing the central bank to freely react to economic shocks under discretion. This contract is a kind of inflation-targeting rule. Equivalently, when the central bank considers both transfer amount and social welfare, the optimal contract mimics the inflation-targeting rule. By contrast, when the central bank considers only its transfer amount, the optimal contract acts like a contingent inflation-targeting rule in which the observed signaling of supply shocks determines the target. This contracting method implies that targeting rules of the type that is frequently observed might help to ease inflation bias whereas restoring the strength of discretionary policy.

However, McCallum (1995) criticizes the monetary delegation method: Delegation cannot resolve the time consistency, but merely postpones it. There are arguments for and against this criticism. Proponents are Jensen (1997) and Bilbiie (2011), while opponents are Driffill and Rotondi (2006) and Basso (2009).

2.4 The role of the open economy in resolving the time inconsistency

There is a vast literature on a two-country game-theoretic approach to address the issue of policy cooperation pioneered by Hamada (1976) who analyzes strategic monetary interdependence under a fixed exchange rate regime. Subsequent studies employed this technique from the static game theory and found that Nash equilibrium can be over-loosening or over-tightening relative to the cooperative equilibrium depending on the policy transmission mechanism, the kind of shocks and the targets of policymakers. The concentration of research shifts to the dynamic game theory with an intertemporal feature (Oudiz and Sachs, 1984; Currie and Levine, 1985; Miller and Salmon, 1985) and also highlights the importance of time-consistency issue. Given the development of literature on the two-country model and the solutions to time consistency, it is of relevance to compare the severity of consequences and the effectiveness of solutions between closed and open economy, but this has received less attention from the existing literature.

Laskar (1989) extends the one-country model of Rogoff (1985b) into a two-country model and documented that conservative central bankers might aggravate the welfare of both countries without monetary cooperation. The explanation is that the fear of the inflationary effects generated by the real depreciation hinders the governments from raising money growth rate high enough. When assigning a higher weight on the inflation objective, conservative central bankers exacerbate this fear and thus increase the inefficiency of equilibrium under non-cooperation. Using a similar model, Alesina and Grilli (1991) also point out a welfare reduction when delegating monetary policy in an international framework.

Moreover, putting together the cooperation problem and time-consistency problem gives rise to counterproductive cooperation as pointed out by Rogoff (1985a). In a closed economy, the forward-looking wage setters foresee the government's incentive to induce inflationary surprise and then demand a higher inflationary wage. When the economy is open, a depreciation potential of exchange rate imposes an additional constraint on the government's decision. However, the cooperation agreement eliminates this exchange rate constraint, and hence both governments end up with high inflation. This causes higher inflation and lower output under cooperation than under non-cooperation.

This literature continues growing by taking into account a microfoundation of economic behaviors as well as modeling of cooperation problem (Arseneau, 2012). Under cooperation, each government maximizes its residents' objective function when seeking for the optimal policy, whereas cooperative governments jointly maximize a weighted average of welfare. Hence, instead of balancing output-inflation trade-off in an ad hoc model, each government now has to balance the trade-off between consumption and leisure which is affected by the movement of the exchange rate. There are two opposing channels in operation when the government sets policy independently. The first is a standard New Keynesian aggregate demand channel due to the interaction between sticky prices and monopolistic distortions which produce an inflationary bias. The second is a strategic terms of trade channel due to a floating nominal exchange rate which generates a deflationary bias. As mentioned above, the cooperation agreement removes the free movement of the exchange rate and switches off the second channel. Thus, cooperative governments end of the global economy with high inflation. As a consequence, welfare under cooperation may be worse than that under non-cooperation. Moreover, the same argument can be used to show that the optimal monetary policy under commitment is less likely to be sustained when governments cooperate than when they do not.

2.5 Conclusion

This chapter aims to survey the role of the open economy in resolving the time inconsistency. In the next two chapters, we will compare the severity of consequences and the effectiveness of solutions between closed and open economy under the microfounded New

Keynesian framework. We will show that the open economy has more ground to mitigate the consequences as well as improve the effectiveness of the solutions to the time inconsistency.

First, in Chapter 3 we compare the inflationary bias and the application of reputation in a model of Ireland (1997) for a closed economy and of Arseneau (2007) for an open economy. In the closed economy, there is a standard New Keynesian aggregate demand channel because of the interaction between sticky prices and monopolistic distortions which produces an inflationary bias. When the economy is open, in addition to the above channel, strategic terms of trade channel arises due to a floating nominal exchange rate which generates a deflationary bias. The existence of two opposite channel reduces each government's incentive to increase inflation unexpectedly. Thus, the inflationary bias under open economy is less severe than that under the closed economy. Moreover, when the second channel dominates the first one, the time consistency problem even vanishes. This result happens when the size of monopolistic distortion is sufficiently small so that the benefit of inflationary surprise is dominated by its cost. With the trigger-type strategies to support the reputation, the optimal policy under commitment, the Friedman rule, is likely to be sustained under cooperation than under non-cooperation. The reason is that the two opposing channels lower the government benefits from deviation thereby dampen government's temptation to defect from optimal equilibrium under commitment when such commitment is not possible.

Second, Arseneau (2012) examines the expectation trap in a two-country model and indicated that under non-cooperation the optimal surprise in one country is independent of other monetary policy. It means that a country cannot borrow credibility from the foreign country to restore the optimal policy under commitment. However, Chapter 3 of this dissertation also shows that when governments work together, unexpected inflation in one country will trigger another country to induce inflationary surprise and the global economy ends of high inflation. In this sense, the expectation trap under open economy is worse than that under closed economy.

Third, Basso (2009) modifies the governmental structure in Ireland (1997)'s model to introduce the costly delegation as a way to escape the time consistency. In Chapter 4 of this dissertation, we will show that the delegation method under open economy is more effective than that under closed economy. In detail, less punishments by the governments under open economy are required to restore central bank from deviating from Friedman rule than under closed economy. We will show that: When governments lack such commitment abilities, (i) international model requires a lower threshold of reappointment cost parameter for central bank appointment being sustained, and (ii) when the reappointment cost is less than the above threshold, the maintenance of central bank is more likely to be sustained in our open-economy model than in the closed economy model of Basso.

Chapter 3

Time Consistency and Counterproductive Monetary Policy Cooperation in a New Keynesian Two-country Model

3.1 Introduction

One of the most well-known results that have appeared from the study of international monetary policy cooperation is the counterproductive cooperation raised by Rogoff (1985a). He extends the Barro and Gordon (1983a)'s model to a two-country framework and showed that cooperation in monetary policy is inflationary while having no effect on employment. Therefore, monetary cooperation only worsens the credibility problem between government and private sectors. This is a situation of counterproductive cooperation. Rogoff's argument is attractive since it seems to fit some stylized facts about policy coordination in three major developed countries during the 1970s and 1980s, say, the United States, Japan, and Germany. The first is the Bonn summit agreement in 1978 in which both Japan and Germany agree with the United States about expanding their economies and soon afterward the global inflation takes place. The second is the Louvre Accord in 1987-1988 in which both Japan and Germany again agree on several loose policies and then the financial bubble happens in Japan as well as the global stock market collapses. Iida (1999) shows that although there is some statistical evidence in support of the Rogoff's result, the evidence of macroeconomic statistics, in general, is not strong. In fact, the money growth rate under the periods of cooperation is higher, but the inflation is not distinctly higher than the average of ordinary years for the period 1973-1996. This evidence can be interpreted that when monetary policy is cooperated, the implementation of the cooperation agreement generates inflation that is not significantly greater than without cooperation. These experiences have been regarded as fundamental mistakes in Japan and Germany refraining them from joining such a cooperation and can be used to explain somewhat the reduction of monetary cooperation among major industrial countries during the 1990s and the first half of the 2000s. However, in recent years, there have been two sources which call for international monetary policy cooperation: First, the 2008-2009 financial crisis gives rise to a need for coordination to recover the world economy; Second, the recent behaviors of major currencies generate growing concern about adverse spillovers from major advance countries into emerging and developing countries. Given the emergence of the demand for international cooperation in monetary policy, we ask whether monetary cooperation may still be counterproductive when inflation under cooperation is indifferent from that under non-cooperation, and if yes, to what extent? Answer to this question seems essential for implementing the international monetary policy cooperation in reality.

We consider a micro-founded two-country New Keynesian model developed by Arseneau (2007, 2012) to revisit the problem of counterproductive cooperation. By cooperation, we mean that the governments of the two countries jointly maximize a weighted average of the welfare functions, whereas under non-cooperation they unilaterally maximize the utilities of their respective households taking the other policy as given. We introduce the new aspect of counterproductive cooperation by using the concept of a sustainable plan proposed by Chari and Kehoe (1990). We find that even when commitment equilibria are indifferent under various regimes and so do the autarky plans after governments' deviation, cooperation may be counterproductive to the extent that the sustainability constraints of optimal policy under commitment, the Friedman rule, is more restrictive under cooperation than under non-cooperation.

This chapter is related to three strands of literature but differs from the existing work in crucial ways. First, this chapter is built on the literature on the counterproductive cooperation which arises when putting the two workhorses in studying the monetary policy-making under the strategic framework where governments behave strategically with each other and with their own private sectors. The first workhorse is the time inconsistency of optimal commitment policy raised by Kydland and Prescott (1977) and contributed by many economists. Among others, Barro and Gordon (1983a) apply this problem in monetary policy with the well-known inflation bias and Woodford (1999) proposes the timeless perspective optimal policy with the stabilization bias. The second workhorse is the degree of international coordination between governments in setting the monetary policy proposed by Hamada (1976) and developed by Benigno (2002) and Benigno and Benigno (2003). Table 3.1 is useful to understand these alternative cases. There are two possibilities for policy regimes. Governments can cooperate to set monetary policy jointly or not. In terms of policy precommitment, there are also two cases in which governments might have the commitment technology or not. Without the time consistency problem, cooperation may be expected to at least benefit all the participants weakly or welfare of case A is at least as good as that of case C. Analogously, the insulated government can never be worse off when making precommitment.

Cooperation regime	Commitment technology	
	Commitment	Discretion
Cooperation	A	B
Non-cooperation	C	D

Table 3.1 Alternative cases of cooperation regime and commitment technology

In his pioneering paper, Rogoff (1985a) compares the equilibrium under discretion and pointed out that inflation under equilibrium coded by B is higher than under equilibrium coded by D so that the former might be dominated by the latter in terms of welfare. Rogoff labels this result counterproductive cooperation. Some following works reinforce Rogoff (1985a)'s result by using firmer theoretical contexts and numerical simulations. For example, Kehoe (1989) studies an optimal tax model with benevolent governments whereas Miller and Salmon (1985) conduct a numerical exercise in an ad hoc open economy framework with over-shooting of a real exchange rate. These studies model policy cooperation in which governments jointly agree to fix the nominal exchange rate so that the counterproductive cooperation was interpreted as follows. The interplay between the workers and the nonbenevolent governments generates an inflationary bias. When the economies are open, and both governments do not cooperate, the effect of a floating exchange rate on inflation constitutes an additional constraint which alters the inflation-output trade-off and thereby alleviates the inflationary bias. When governments establish a cooperative agreement fixing their exchange rate, this agreement eliminates the above constraint faced by each government under noncooperation. The wage setters again demand higher wages to keep their real wages. This demand leads to higher inflation and lower output under cooperation than that under noncooperation. Since the collapse of Bretton Woods system in the late 1960s, there is no reason to be convinced that different governments can be easily involved in such cooperation agreement. We apply a microfounded modeling of cooperation such that both governments jointly maximize the weighted welfare function of the global economy. It, however, turns out that under cooperation the nominal exchange rate is independent of policy variables thereby switching off the strategic terms of trade channel and affecting the trade-off between consumption and leisure. Hence, this mechanism will be used to explain the occurrence of counterproductive cooperation in our model.

Second, the early studies of counterproductive cooperation specify the ad hoc welfare function. Our model is built on the microeconomic foundation along the line of Ireland (1997)'s closed model and Arseneau (2007, 2012)'s open economy model. With such a microfoundation, it is intuitive for assigning values to the main parameters. More importantly, the microfoundation sheds new light on deriving time consistency problem and hence gives more grounds to analyze the new aspect of counterproductive cooperation.

Third, this chapter is related to works that study sustainable equilibrium in which the set of sustainable outcome is characterized as in Chari and Kehoe (1990). In a closed economy, Ireland (1997) and Kurozumi (2008) describe the sustainability condition of Ramsey outcome in a model with inflation bias and the stabilization bias, respectively. In a simple two-country economy without time consistency, Jensen (1994) studies the sustainability of policy cooperation by deriving a set of discount factor that supports the success of cooperation. He finds that the national heterogeneity adversely affects the sustainability of policy cooperation. Both Ireland and Jensen investigate the model parameters to see when the sustainability constraints seem to hold. In the latter, the opposite effects of an increase in national heterogeneity on the terms of trade under various regimes help to explain why sustainable cooperation seems to be successful among comparable countries. In a recent paper, Fujiwara et al. (2016) examine the quantitative properties of sustainable cooperative international monetary policies. They show that a temptation to deviate from cooperation agreement exists when countries are different in a variance of markup shock and countries' size, and the responses of macroeconomic variables under sustainable cooperation regime lie in between those generated by cooperation and non-cooperation regimes.

Because this chapter uses Arseneau's model, it is essential to compare our work with him. In Arseneau (2007), he investigates the role of inflation tax in an open economy with imperfect competition and found that the monopolistic distortion reduces the temptation to introduce inflationary tax strategically and determines whether the welfare gains from cooperation exist or not. Regarding sustainable equilibrium and the time consistency in an open economic environment, Arseneau (2012) is the most related paper to ours. He analyzed the welfare gains from commitment under non-cooperation and the sustainability of the non-cooperative equilibrium under commitment when the commitment technologies are infeasible. We instead focus on the study of counterproductive cooperation, thereby have to examine sustainable equilibrium in cooperative policy regime and utilize those results done by Arseneau (2012). Moreover, we apply the Chari-Kehoe technique to characterize the set of sustainable outcomes in various regimes that is abstracted in Arseneau (2012)'s paper.

The main contribution of this chapter is that the monetary policy cooperation may be counterproductive in terms of sustainable cooperation. This aspect of counterproductive cooperation arises from comparing the restrictiveness of sustainability constraints between cooperation and non-cooperation in which the more restrictive constraint requires a higher discount factor to support the optimal monetary policy under commitment, the Friedman rule, as a solution in the absence of commitment technology. In an open economy, the equilibrium can be sustainable under a lower discount factor because the immediate gain (one period deviation) is lower. The underlying reason is that the coexistence of two opposing channels under non-cooperation dampens the governments' temptation to induce inflationary surprise and thereby lowers the short-run benefits from deviation. Hence, compared with cooperation, a less discount factor is enough to prevent the governments from deviating from the Friedman rule.

The chapter is organized as follows: Section 2 presents the model, and Section 3 and 4 consider the monetary policy with and without commitment, respectively. Sustainable cooperation is analyzed in Section 5. Section 6 offers some concluding remarks. Derivations and proofs are relegated to the appendix.

3.2 Model

The economy environment follows the model of Arseneau (2007, 2012) with two countries, Home and Foreign, and each country consists of (i) a representative household, (ii) a continuum of monopolistic firms indexed by $j \in [0,1]$ for Home and $j^* \in [0,1]$ for Foreign country respectively, and (iii) a government which sets monetary policy. Except for consumption and labor, all lower case variables are defined as the ratio of nominal variables to the domestic money supply. Variables with asterisks denote those of Foreign country. We introduce no shock into the model, so this chapter presents a deterministic analysis.

The representative household maximizes its utility derived from consumption and leisure subject to a cash-in-advance constraint and a standard budget constraint. The Home household has the utility function

$$U_0 = \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\alpha}}{1-\alpha} - l_t \right\}, \tag{3.1}$$

and faces the following constraints:

$$c_{H,t} = \left[\int_0^1 c_{H,t}(j)^{(\theta-1)/\theta} dj \right]^{\theta/(\theta-1)}, \quad c_{F,t} = \left[\int_0^1 c_{F,t}(j^*)^{(\theta-1)/\theta} dj^* \right]^{\theta/(\theta-1)}, \tag{3.2}$$

$$c_t = (c_{H,t})^{\gamma} (c_{F,t})^{1-\gamma}, \tag{3.3}$$

$$l_t = \int_0^1 l_t(j) dj, \tag{3.4}$$

$$p_t c_t \leq m_t + (x_t - 1) + b_t - \frac{b_{t+1} x_t}{R_t}, \tag{3.5}$$

$$p_t c_t + \frac{b_{t+1} x_t}{R_t} + m_{t+1} x_t \leq m_t + (x_t - 1) + b_t + \int_0^1 \zeta_t(j) dj + w_t l_t, \tag{3.6}$$

where c_t is Home's consumption bundle including aggregate consumption of goods produced in Home, $c_{H,t}$ and in Foreign, $c_{F,t}$ (given in (3.2)), l_t is the number of hours supplied, m_{t+1} is the cash holdings to next period and b_{t+1} is the bond holding to next period. Parameter γ is the expenditure share of domestic goods in the Home consumption basket. The utility of Foreign agent j^* is similar to (3.1), but Foreign's consumption bundle is $c_t^* = (c_{H,t}^*)^{\gamma}(c_{F,t}^*)^{1-\gamma}$. Hence, preferences over consumption goods are identical both within and across countries: the elasticity of intertemporal substitution $1/\alpha$. The cash-in-advance constraint (3.5) is used to explain why households hold real balances as well as the welfare cost of expected inflation since agents inefficiently reduce their money holdings.

R_t is the gross interest rate of domestic bond, w_t the wage, $\zeta_t(j)$ the profits of firm j (defined below), and $p_t = P_t/M_t^S$ in which the aggregate price of the good bundle consumed by Home residents is given as follows:

$$P_t = \frac{1}{\gamma^W}(P_{H,t})^{\gamma}(S_t P_{F,t}^*)^{1-\gamma}, \tag{3.7}$$

where

$$P_{H,t} = \left[\int_0^1 P_{H,t}(j)^{1-\theta} dj\right]^{1/(1-\theta)}, \qquad P_{F,t}^* = \left[\int_0^1 P_{F,t}^*(j^*)^{1-\theta} dj^*\right]^{1/(1-\theta)}, \tag{3.8}$$

and $\gamma^W = \gamma^{\gamma}(1-\gamma)^{1-\gamma}$. As in Ireland (1997), the money growth rate is bounded in both sides to guarantee a monetary equilibrium, $x_t \in [\beta, \bar{x}]$ and $x_t^* \in [\beta, \bar{x}^*]$. The lower bound is imposed so the net nominal interest rate $(R_t - 1)$ is non-negative in equilibrium, while the upper bound ensures that private agents never prohibit the use of money simultaneously à la Calvo (1978). Let $\beta \in (0,1)$ denote the subjective discount rate. Let θ denote the elasticity of substitution between goods. We impose $\theta > 1$ to guarantee the existence of equilibria in the presence of imperfect competition. Also, we impose $\alpha \in (0,1)$ to guarantee the concavity of the utility function and a well-defined utility at $c_t = 0$.

Firm j production function is $y(j) = l(j)$. At time $t = 0, 1, 2, ...$ each firm sets $p_{H,t}(j)$ to maximize profits

$$\zeta_t(j) = [p_{H,t}(j) - w_t] y_t^D(j), \tag{3.9}$$

where $y_t^D(j)$ is the global demand for good j

$$y_t^D(j) = c_{H,t}(j) + c_{H,t}^*(j). \tag{3.10}$$

Free trade implies that the law of one price is satisfied for each good, $P_{H,t}(j) = S_t P_{H,t}^*(j)$ and $P_{F,t}(j^*) = S_t P_{F,t}^*(j^*)$. This result together with the same preferences of the individuals across countries indicate that the purchasing power parity (PPP) holds for prices of good bundle in both countries, $P_{H,t} = S_t P_{H,t}^*$ and $P_{F,t} = S_t P_{F,t}^*$, and for the aggregate price

$$P_t = S_t P_t^*. \tag{3.11}$$

We then define the terms of trade, $\tau_t = P_{H,t}/(S_t P_{F,t}^*)$, as the ratio of export price to import price in which prices are measured in Home currency.

3.3 Monetary policy with commitment

When governments can commit to their future policies, the governments make their decisions at the beginning of time before firms choose their prices. Let $x = \{x_t | t = 0, 1, 2, ...\}$ and $x^* = \{x_t^* | t = 0, 1, 2, ...\}$ denote an infinite sequence of the money growth rates in Home and Foreign country respectively, where $x_t \in [\beta, \bar{x}]$ and $x_t^* \in [\beta, \bar{x}^*]$ for all $t = 0, 1, 2, ...$ Combining both of them, let $X(x, x^*) = (x, x^*)$ be world monetary policy. We now define the rules and allocations for Home country; those of Foreign country are analogous. In Home country, firms and households behavior, given a world policy (x, x^*), are characterized by allocation rule $\pi^j(x, x^*)$, $j \in [0, 1]$ and $\omega(x, x^*)$. The Home representative firm's rule $\pi^j(x,x^*)$ dictates how price, $p_{H,t}(j)$, $t = 0,1,2,...$, is chosen for each possible policy pair (x,x^*) . The representative household's rule $\omega(x,x^*)$ dictates choices for $\Omega_t = (c_t, c_{H,t}, c_{F,t}, c_{H,t}(j), c_{F,t}(j^*), l_t, m_{t+1}, b_{t+1})$, $t = 0,1,2,...$ for each possible policy pair (x,x^*). Let π refer to the set of function π^j for all $j \in [0,1]$. Then π and ω map world policy (x,x^*) into allocations (Π,Ω), where $\Pi = \{\Pi^j | j \in [0,1]\}$, $\Pi^j = \{p_{H,t}(j) | t = 0,1,2,...\}$, and $\Omega = \{\Omega_t | t = 0,1,2,...\}$. Hence, together with those from Foreign country the allocation $(\Pi,\Omega,\Pi^*,\Omega^*)$ portrays the sequence of equilibrium prices and quantities that achieves when governments implement the policy pair (x,x^*), private sectors react based on π, ω, π^*, and ω^*.

Given (x,x^*), $\pi^{\hat{j}}(x,x^*)$ for all $\hat{j} \neq j$, $\pi^{*j^*}(x,x^*)$ for all $j^* \in [0,1]$, $\omega(x,x^*)$, and $\omega^*(x,x^*)$, the Home representative firm's choice of $\pi^j(x,x^*)$ solves

(FO) Maximize (3.9) for each $t = 0,1,2,...$, taking w_t, w_t^*, $p_{H,t}$, and $p_{F,t}^*$ as given for all $t = 0,1,2,...$

Given (x,x^*), $\pi^j(x,x^*)$ for all $j \in [0,1]$ and $\pi^{*j^*}(x,x^*)$ for all $j^* \in [0,1]$, the Home representative household's choice of $\omega(x,x^*)$ solves

(HO) Maximize (3.1) subject to (3.2), (3.3), (3.4), (3.5) and (3.6) for each $t = 0,1,2,...$, taking as given $\zeta_t(j)$, $\zeta_t^*(j^*)$, w_t, w_t^*, R_t and R_t^* for all $j, j^* \in [0,1]$ and $t = 0,1,2,...$

In addition, $\omega(x,x^*)$ must correspond with the appropriate market clearing conditions

(MO) $m_{t+1} = 1$, $b_{t+1} = 0$, $y_t = l_t$, $y_t = c_{H,t} + c_{H,t}^*$, and the currency exchange market clearing conditions

$$S_t = \frac{1-\gamma}{\gamma} \frac{x_t}{x_t^*}, \tag{3.12}$$

which comes from the fact that in each country the value of export is equal to that of import denominated in the domestic currency. In the same fashion, for Foreign country we can derive the problem for firm and household, FO^* and HO^*, respectively together with the market clearing condition, MO^*.

An *equilibrium under commitment* consists of a world policy (x, x^*) and an allocation $(\Pi, \Omega, \Pi^*, \Omega^*)$ that satisfy: (i) For every (x, x^*), each Π^j solves (FO) given $\Pi^{\hat{j}}$ for all $\hat{j} \in [0,1]$, $\hat{j} \neq j$, and $(\Omega, \Pi^*, \Omega^*)$. For every (x, x^*), each Π^{*j^*} solves (FO^*) given $\Pi^{*\hat{j}^*}$ for all $\hat{j}^* \in [0,1]$, $\hat{j}^* \neq j^*$, and (Π, Ω, Ω^*); (ii) For every (x, x^*), Ω solves (HO) given (Π, Ω, Π^*). For every (x, x^*), Ω^* solves (HO^*) given (Π, Ω, Π^*); (iii) Ω and Ω^* are consistent with (MO) and (MO^*), respectively. The definition does not require the policy pair to be chosen optimally. Therefore, this definition allows us to derive the optimal monetary policy by first featuring the whole set of equilibria under commitment and then choosing the policy pair that maximizes either separately or jointly the utility functions in both countries.

Arseneau (2007, 2012) derive the necessary and sufficient conditions for a solution to the household problem (HO and HO^*):

$$c_t = \gamma \frac{x_t^{\gamma} x_t^{*(1-\gamma)}}{p_{H,t}^{\gamma} p_{F,t}^{*(1-\gamma)}}, \qquad c_t^* = (1-\gamma) \frac{x_t^{\gamma} x_t^{*(1-\gamma)}}{p_{H,t}^{\gamma} p_{F,t}^{*(1-\gamma)}}, \tag{3.13}$$

$$l_t = \frac{x_t}{p_{H,t}}, \qquad l_t^* = \frac{x_t^*}{p_{F,t}^*}, \tag{3.14}$$

$$w_t = \frac{x_t x_{t+1}}{\beta c_{t+1}^{1-\alpha}}, \qquad w_t^* = \frac{x_t^* x_{t+1}^*}{\beta c_{t+1}^{*(1-\alpha)}}, \tag{3.15}$$

$$\lim_{t\to\infty} \beta^t \frac{c_t^{1-\alpha}}{x_t} = 0, \qquad \lim_{t\to\infty} \beta^t \frac{c_t^{*(1-\alpha)}}{x_t^*} = 0, \tag{3.16}$$

and to the the firm's problem (FO and FO^*):

$$p_{H,t} = \frac{\theta}{\theta - 1} w_t, \qquad p_{F,t}^* = \frac{\theta}{\theta - 1} w_t^*. \tag{3.17}$$

Combining (3.13), (3.14), (3.15), and (3.17) yields a system of difference equations that must be satisfied in any equilibrium under commitment:

$$c_t = \frac{\gamma\beta(\theta-1)}{\theta}\frac{c_{t+1}^{(1-\alpha)\gamma}c_{t+1}^{*(1-\alpha)(1-\gamma)}}{x_{t+1}^{\gamma}x_{t+1}^{*(1-\gamma)}}, \quad c_t^* = \frac{(1-\gamma)\beta(\theta-1)}{\theta}\frac{c_{t+1}^{(1-\alpha)\gamma}c_{t+1}^{*(1-\alpha)(1-\gamma)}}{x_{t+1}^{\gamma}x_{t+1}^{*(1-\gamma)}}. \quad (3.18)$$

Employing the fact that $c_t = \gamma/(1-\gamma)c_t^*$, we can solve these equations forward:

$$\begin{aligned}
\ln(c_t) &= \frac{1}{\alpha}\ln\left(\frac{\rho\beta(\theta-1)}{\theta}\right) - \sum_{i=0}^{\infty}\gamma(1-\alpha)^i\ln(x_{t+1+i}) - \sum_{i=0}^{\infty}(1-\gamma)(1-\alpha)^i\ln(x_{t+1+i}^*),\\
\ln(c_t^*) &= \frac{1}{\alpha}\ln\left(\frac{\rho^*\beta(\theta-1)}{\theta}\right) - \sum_{i=0}^{\infty}\gamma(1-\alpha)^i\ln(x_{t+1+i}) - \sum_{i=0}^{\infty}(1-\gamma)(1-\alpha)^i\ln(x_{t+1+i}^*),
\end{aligned} \quad (3.19)$$

where $\rho = \gamma^{1-(1-\alpha)(1-\gamma)}(1-\gamma)^{(1-\alpha)(1-\gamma)}$ and $\rho^* = \gamma^{(1-\alpha)\gamma}(1-\gamma)^{1-(1-\alpha)\gamma}$. Labors can also be expressed in terms of policy variables

$$l_t = \frac{\beta(\theta-1)}{\theta}\frac{(c_{t+1})^{1-\alpha}}{x_{t+1}}, \qquad l_t^* = \frac{\beta(\theta-1)}{\theta}\frac{(c_{t+1}^*)^{1-\alpha}}{x_{t+1}^*}. \quad (3.20)$$

We next consider how governments set the optimal monetary policy with consideration for the interaction between them.

3.3.1 Optimal non-cooperative monetary policy

We start with the natural benchmark case where governments set monetary growth rate independently. An *optimal non-cooperative equilibrium under commitment* is an *equilibrium under commitment* that satisfies the following: (i) taking as given x^*, the Home government chooses x to maximize the Home welfare function, and (ii) taking as given x, the Foreign government chooses x^* to maximize the Foreign welfare function. We then derive each government's best response to any foreign policies, which corresponds to the characterization of optimal Nash equilibrium policies. Substituting (3.19) and (3.20) into (3.1), we can obtain the indirect utility function for Home country, $U_0(x,x^*)$, as a function of the policy pair that satisfies allocation rules defined by the equilibrium under commitment.

$$
\begin{aligned}
U_0(x,x^*) = \sum_{t=0}^{\infty}\beta^t \Bigg\{ \frac{1}{1-\alpha} \left[\left(\frac{\rho\beta(\theta-1)}{\theta} \right)^{\frac{1}{\alpha}} \prod_{i=0}^{\infty} x_{t+1+i}^{-\gamma(1-\alpha)^i} \prod_{i=0}^{\infty} x_{t+1+i}^{*-(1-\gamma)(1-\alpha)^i} \right] \\
- \frac{\beta(\theta-1)}{\theta x_{t+1}} \left[\left(\frac{\rho\beta(\theta-1)}{\theta} \right)^{\frac{1}{\alpha}} \prod_{i=0}^{\infty} x_{t+2+i}^{-\gamma(1-\alpha)^i} \prod_{i=0}^{\infty} x_{t+2+i}^{*-(1-\gamma)(1-\alpha)^i} \right] \Bigg\},
\end{aligned}
\tag{3.21}
$$

and similarly, the indirect utility function for the Foreign country is that

$$
\begin{aligned}
U_0^*(x,x^*) = \sum_{t=0}^{\infty}\beta^t \Bigg\{ \frac{1}{1-\alpha} \left[\left(\frac{\rho^*\beta(\theta-1)}{\theta} \right)^{\frac{1}{\alpha}} \prod_{i=0}^{\infty} x_{t+1+i}^{-\gamma(1-\alpha)^i} \prod_{i=0}^{\infty} x_{t+1+i}^{*-(1-\gamma)(1-\alpha)^i} \right] \\
- \frac{\beta(\theta-1)}{\theta x_{t+1}^*} \left[\left(\frac{\rho\beta(\theta-1)}{\theta} \right)^{\frac{1}{\alpha}} \prod_{i=0}^{\infty} x_{t+2+i}^{-\gamma(1-\alpha)^i} \prod_{i=0}^{\infty} x_{t+2+i}^{*-(1-\gamma)(1-\alpha)^i} \right] \Bigg\}.
\end{aligned}
\tag{3.22}
$$

In this setting, both countries will engage in a policy competition because of international spillovers (Corsetti and Pesenti, 2001; Tille, 2001). An expansion in Home country imposes a tax on consumption or a subsidy to leisure by raising the nominal interest rate above zero, leading money to be dominated by bond in terms of a rate of return. Home households respond by inefficiently economizing the money balance and substituting out of consumption and into leisure. Under the open economy, the consumption tax burden is shared across countries, and hence consumption in both countries decrease proportionally. Simultaneously, leisure is non-traded good so the Home household will enjoy all the benefits of leisure subsidy. As a consequence, although the ratio of goods consumed in both country is unchanged, Foreign labors have to bear a severer burden compared with Home labors. The Home government is tempted to expand money supply as long as the welfare benefits from leisure subsidy exceed the welfare costs by the consumption tax. This action, however, causes a beggar-thy-neighbor welfare spillover since the Foreign households suffer a proportional consumption tax but benefit nothing from leisure subsidy. As such an expansion reduces Foreign welfare, loose monetary policy in the Foreign country and depreciation of Foreign currency can raise Foreign welfare by lowering the high disutility of work effort. It is worth noting that the assumption $\alpha \in (0,1)$ matters here. Higher inflation in Home country makes the Home-produced goods relatively expensive than Foreign-produced one and therefore affects Foreign imports negatively. Lower imports affect the Foreign consumers' marginal utility of the Foreign good and thereby the choice of optimal policy. In our case where $\alpha < 1$ implies that two goods are complement, lower imports in Foreign country reduce marginal utility of the Foreign good, and Foreign government hence faces a lower marginal cost of anticipated inflation. In effect, under non-cooperation, the Foreign government wants to raise inflation in response to an expansion in Home country.

However, the presence of monopolistic competition lowers the temptation to involve in inflation competition as it pushes the consumption lower than the efficient level, so makes the expected inflation more costly. As we show below, when this dampening impact is strong enough, it will alter the game structure of two governments such that the Friedman rule is chosen irrespective of foreign monetary policy. Appendix A.1 shows that the outcome of optimal non-cooperative equilibrium under commitment can be found by: (i) Home government maximizes (3.21) subject to $x_t \in [\beta, \bar{x}]$ for all $t = 0,1,2,...$; (ii) Foreign government maximizes (3.22) subject to $x_t^* \in [\beta, \bar{x}^*]$ for all $t = 0,1,2,...$

A general characterization of optimal non-cooperative policy under commitment is complicated, so we focus our analysis on a constant money growth rate in each country. Now we assume that the money growth rates are constant. This assumption is theoretically justified as the central banks can follow the Friedman rule. In reality, it seems to fit the stylized facts that some central banks have announced to target a constant money growth rate. Let $x_t = \hat{x}^N$ and $x_t^* = \hat{x}^{*N}$, for all $t = 0,1,...$ be the solution to the Home's optimization problem and the Foreign's, respectively. Let $\hat{X}^N$ be the optimal non-cooperative policy pair and define $\Psi^N = (1 + \frac{1-\gamma}{\gamma}\alpha)$ and $\Psi^{*N} = (1 + \frac{\gamma}{1-\gamma}\alpha)$ as the magnitude of the strategic terms of trade distortion in the Home and Foreign country, respectively, and $\Phi = \frac{\theta}{\theta-1}$ be the monopolistic distortion (see Appendix A.1.2). Superscript N stands for non-cooperation.

The optimal non-cooperative monetary policy in each country includes corner or interior solution contingent on the relative size of monopolistic distortion to that of strategic terms of trade and can be summarized as follows:

(i) If $\Psi^N, \Psi^{*N} \leq \Phi$, $\hat{X}^N = (\beta, \beta)$.

(ii) If $\Psi^N, \Psi^{*N} > \Phi$, $\hat{X}^N = (\beta\Psi^N/\Phi, \beta\Psi^{*N}/\Phi)$.

(iii) If $\Psi^N \leq \Phi$ and $\Psi^{*N} > \Phi$, $\hat{X}^N = (\beta, \beta\Psi^{*N}/\Phi)$.

(iv) If $\Psi^N > \Phi$ and $\Psi^{*N} \leq \Phi$, $\hat{X}^N = (\beta\Psi^N/\Phi, \beta)$.

Arseneau (2007) documents that this optimal policy is a dominant strategy for a given set of parameters in a non-cooperative setting. When the monopolistic distortion is not large so that the dampening effects are small in both countries, the benefits from leisure subsidy exceed the welfare loss from consumption tax. This situation entices governments to enroll in inflation competition and both governments inflate away from the Friedman rule as in case (ii) above. Up to now, our analysis mainly bases on Arseneau (2007). In the next subsection, we extend Arseneau's work to the case of cooperation.

3.3.2 Optimal cooperative monetary policy

It is common that non-cooperative policies may be undesirable in the world economy because each government does not take into account the possibility of negative externality imposed by its policy on other countries. If governments involve in the joint determination of policies, such externalities would be internalized to the benefit of both. So we analyze the situation in which governments cooperate on the choice of their money growth rates. An *optimal cooperative equilibrium under commitment* is an equilibrium under commitment in which governments jointly maximize the equally weighted welfare. The weighted welfare function is defined as follows:

$$U_0(x,x^*) + U_0^*(x,x^*). \tag{3.23}$$

The outcome of optimal cooperative equilibrium under commitment can be found by maximizing (3.23) subject to $x_t \in [\beta, \bar{x}]$ and $x_t^* \in [\beta, \bar{x}^*]$ for all $t = 0, 1, ...$[1]. Let $\hat{X}^C = (\hat{x}^C, \hat{x}^{*C})$ denote the outcome of monetary policy in the optimization problem (3.23) when the money growth rate is constant, $\Psi^C = \gamma^{-\alpha}/(\gamma^{1-\alpha} + (1-\gamma)^{1-\alpha})$ and $\Psi^{*C} = (1-\gamma)^{-\alpha}/(\gamma^{1-\alpha} + (1-\gamma)^{1-\alpha})$[2]. Superscript C stands for cooperation. It is worthwhile to stress that under cooperation the ratio of the optimal money growth rates can be expressed in terms of model parameters only, and so does the nominal exchange rate. The intuition of this result will be discussed in later part of this chapter. We derive the following proposition:

Proposition 3.1 *The optimal cooperative monetary policy is given by:*

(i) *If* Ψ^C, $\Psi^{*C} \leq \Phi$, $\hat{X}^C = (\beta, \beta)$.

(ii) *If* $\Psi^C \leq \Phi < \Psi^{*C}$, $\hat{X}^C = (\beta, \beta\Psi^{*C}/\Phi)$.

(iii) *If* $\Psi^C > \Phi \geq \Psi^{*C}$, $\hat{X}^C = (\beta\Psi^C/\Phi, \beta)$.

It is straightforward to see that when the monopolistic distortion is sufficiently large, such that $\Psi^C, \Psi^N, \Psi^{*C}, \Psi^{*N} \leq \Phi$, the optimal cooperative monetary policy and the optimal non-cooperative monetary policy coincide and both governments set the money growth rate equal to discount rate which is labeled as global Friedman rule. In this case, there is no gain from cooperation, so the cooperative strategy is a self-enforced process. Based on that, we can draw the following proposition which will be used in later part:

[1] Appendix A.1.1 shows how we can obtain the outcome of optimal cooperative equilibrium under commitment

[2] Appendix A.1.2 provides the result of optimal cooperative monetary policy

Proposition 3.2 *When monopolistic distortions are large, $\Phi \geq max[\Psi^C, \Psi^N, \Psi^{*C}, \Psi^{*N}]$, the outcome of optimal cooperative equilibrium under commitment coincides with that of optimal non-cooperative equilibrium under commitment.*

According to Arseneau (2007), under commitment, the monopoly power reduces the government's incentive to increase inflation as its presence causes an inefficiently low level of output together with consumption. The marginal utility of consumption is even lower when the monopolistic distortion increases and thereby makes expected inflation more costly. As a result, when this dampening effect is sufficiently large, it can be used to reinstate the global Friedman rule as an outcome of policy under equilibrium in both policy regimes.

The results in this section bases on the assumption that governments own commitment technology. In reality, governments may lack such an ability and cause a severe consequence. To feature equilibria in this situation, we provide a useful result as follows:

Proposition 3.3 *The worst equilibrium under various regimes is described as follows:*

(i) *When $\Phi \geq max(\Psi^N, \Psi^{*N})$, the worst non-cooperative equilibrium under commitment has $x_t^N = \bar{x}$ and $x_t^{*N} = \bar{x}^*$ for all $t = 0, 1, 2...$*

(ii) *When $\Phi \geq max(\Psi^C, \Psi^{*C})$, the worst cooperative equilibrium under commitment has $x_t^C = \bar{x}$ and $x_t^{*C} = \bar{x}^*$ for all $t = 0, 1, 2...$*

When $\Phi \geq \Psi^N$, the optimal money growth rate without the bounded restriction in Home country, $\hat{x} = \beta\Psi^N/\Phi$, is smaller than β. At $\hat{x} = \beta\Psi^N/\Phi$, the Home welfare achieves the maximum. Taking the Foreign money growth rate as given, for $x_t^N \in [\beta, \bar{x}]$, the Home welfare function is decreasing in the Home money growth rate because of an inverse U-shape of the concave function. Thus, at $x_t^N = \bar{x}$, the Home welfare reaches the worst possible level. When $\bar{x}$ and $\bar{x}^*$ become arbitrarily large, equation (3.19) and its Foreign analog reveal that with $x_t = \bar{x}$ and $x_t^* = \bar{x}^*$ for all $t = 0, 1, ...$, c_t and c_t^* are approaching zero.

3.4 Monetary policy without commitment

In the absence of commitment technologies, the optimal monetary policy under commitment faces a time consistency problem. Since governments can observe the prices set by firms before deciding on policy rates, and since the level of production is insufficiently low due to imperfect competition, the governments have the incentive to induce an unanticipatedly high money growth rate. In equilibrium, firms perceive this incentive and adjust their prices accordingly. The governments' motives to boost production by generating inflationary monetary surprises merely cause higher inflation while producing no effect on the real economy. This is a time consistency problem of monetary policy à la Barro and Gordon (1983a) due to the coexistence of monopolistic competition and sticky price. We will start this section by analyzing the equilibrium under discretion. This analysis has two purposes: first, to show that the counterproductive cooperation as in Rogoff (1985a) still arises under a micro-founded New Keynesian two-country model; second, to derive the one-period deviation money growth rates which are set by governments once they decide to deviate from Friedman rule. These deviation rates will be used to characterize a set of sustainable outcome through the sustainability constraints, and therefore helps to study the new aspect of counterproductive cooperation.

3.4.1 Equilibrium under discretion

The lack of commitment ability leads the governments to set policy sequentially so that under discretion, the governments solve a static problem by taking private expectation as given. Let scripts d denote discretion. Let $X^e = (x^e, x^{*e})$ denote the private expectation of the world policy which is exogenously given to the governments and $X_d = (x_d, x_d^*)$ be the world policy under discretion. An *non-cooperative equilibrium under discretion* is an equilibrium under discretion that satisfies (i) taking as given the private expectation, $X^e = (x^e, x^{*e})$, Home government sets x_d^N such that the private sector in both countries can never be surprised; (ii) taking as given the private expectation, $X^e = (x^e, x^{*e})$, Foreign government sets x_d^{*N} such that the private sector in both countries can never be surprised. On the other hand, an *cooperative equilibrium under discretion* is an equilibrium under discretion in which, taking as given the private expectation, $X^e = (x^e, x^{*e})$, governments jointly choose the money growth rates, $X_d = (x_d^C, x_d^{*C})$, that do not surprise the global private sector. By saying "never be surprised", we mean that the governments validate the private expectation such that $x_d^C = x^e$ and $x_d^{*C} = x^{*e}$. From now on, we assume that the private expectations of inflation are pinned down by the optimal monetary policy under commitment, the global Friedman rule, instead of being exogenously given.

Under non-cooperation, the Home government's problem under discretion is to maximize

$$\tilde{U}(p_{H,t}, p_{F,t}^*) = \max_{\tilde{x}\in[\beta,\bar{x}]} \frac{1}{1-\alpha}\left(\gamma \frac{\tilde{x}^{\gamma}(x^{*N})^{(1-\gamma)}}{p_{H,t}^{\gamma} p_{F,t}^{*(1-\gamma)}}\right)^{1-\alpha} - \frac{\tilde{x}}{p_{H,t}}, \tag{GD}$$

by taking as given the Foreign government policy $\hat{x}^{*N} = \beta$ and the prices $p_{H,t}$ and $p_{F,t}^*$ which are solutions of (FO) and (FO^*), respectively, since the inflation endowments are consistent with the optimal non-cooperative monetary policy under commitment. The solution to the problem (GD) is

$$\tilde{x}^N = \beta(\gamma\Phi)^{\frac{1}{1-\gamma(1-\alpha)}}, \tag{3.24}$$

and to its foreign analog is

$$\tilde{x}^{*N} = \beta((1-\gamma)\Phi)^{\frac{1}{1-(1-\gamma)(1-\alpha)}}. \tag{3.25}$$

One needs to check whether it is worthwhile for the governments to deviate from Friedman rule. As money growth is bounded on by discount factor β, it prevents the government from an deflationary surprise which induces the money growth rate below β but allows for an inflationary surprise. Then we can draw the following proposition:

Proposition 3.4 *When $\Phi \leq min(1/\gamma, 1/(1-\gamma))$, the global Friedman rule is consistent with the non-cooperative monetary policy under discretion.*

This proposition implies that when the monopolistic distortion is large such that $\Phi \leq min(1/\gamma, 1/(1-\gamma))$, neither government has incentive to deviate from the Friedman rule. Hence, the global Friedman rule is time-consistent. In contrast, when this condition is not satisfied, the global Friedman rule is not time consistent. In detail, we can characterize the non-cooperative monetary policy under discretion in other cases.

(i) $1/\gamma > \Phi \geq 1/(1-\gamma)$, $(\hat{x}_d^N, \hat{x}_d^{*N}) = (\bar{x}, \beta)$,

(ii) $1/\gamma \leq \Phi < 1/(1-\gamma)$, $(\hat{x}_d^N, \hat{x}_d^{*N}) = (\beta, \bar{x}^*)$,

(iii) $\Phi > \max(1/\gamma, 1/(1-\gamma))$, $(\hat{x}_d^N, \hat{x}_d^{*N}) = (\bar{x}, \bar{x}^*)$.

The interpretation of these results arises from whether the government has the temptation to induce an inflationary surprise or not. If the government has such a temptation, it will conduct inflationary surprise. Private agents in this country recognize this temptation and adjust expectation to a higher inflation level. This adjustment, in turn, strengthens the incentive for the government to create even bigger monetary surprise, causing inflation expectation even further. The iterative procedure continues until both inflation expectations of the private sector and of the government coincide, and the economy ends up with high inflation. If the government does not have such a temptation, it still chooses the Friedman rule regardless of other government's policy.

Note that our deviation rates are different from those of Arseneau (2012) since he is incorrect to equalize the consumption under commitment and the one achieved after one-shot deviation when substituting into the first-order condition of the problem (GD). Indeed, when governments choose to deviate, the former is less than the latter as the deviation rates in this chapter are higher than Arseneau (2012)'s. Although this difference does not change the main conclusion of his paper, the consequence of expectation trap is severer under this chapter than under Arseneau (2012)'s. The cooperation problem under discretion is given by

$$\tilde{U}(p_{H,t}, p^*_{F,t}) + \tilde{U}^*(p_{H,t}, p^*_{F,t}) = \max_{\tilde{x}\in[\beta,\bar{x}],\tilde{x}^*\in[\beta,\bar{x}^*]} \frac{\gamma^{1-\alpha} + (1-\gamma)^{1-\alpha}}{1-\alpha} \left(\frac{\tilde{x}^{\gamma}\tilde{x}^{*(1-\gamma)}}{p^{\gamma}_{H,t} p^{*(1-\gamma)}_{F,t}} \right)^{1-\alpha} - \left(\frac{\tilde{x}}{p_{H,t}} + \frac{\tilde{x}^*}{p^*_{F,t}} \right), \tag{3.26}$$

where $p_{H,t}$ and $p^*_{F,t}$ are also solutions of (FO) and (FO^*). The solution to the problem (CD) is

$$\tilde{x}^C = \beta \left(\frac{\Phi}{\Psi^C} \left(\frac{1-\gamma}{\gamma} \right)^{(1-\alpha)(1-\gamma)} \right)^{\frac{1}{\alpha}}, \quad \tilde{x}^{*C} = \beta \left(\frac{\Phi}{\Psi^{*C}} \left(\frac{\gamma}{1-\gamma} \right)^{(1-\alpha)\gamma} \right)^{\frac{1}{\alpha}}. \tag{3.27}$$

Contrast to the non-cooperation, the optimal surprise in Home country depends on the monetary policy of Foreign government and vice versa. Hence, the governments can jointly deviate from the global Friedman rule so that the world economy ends up with high inflation. Then, we can derive the following proposition:

Proposition 3.5 *The global Friedman rule is not consistent with cooperative monetary policy under discretion.*

The explanation is that when inflation expectations are pinned down by the optimal monetary policy under commitment, the global Friedman rule, the optimal cooperative monetary policy under discretion is characterized by the high inflation, $(\hat{x}^C_d, \hat{x}^{*C}_d) = (\bar{x}, \bar{x}^*)$.

Propositions 3.4 and 3.5 do not characterize the entire set of equilibrium under discretion with any private expectation; rather it concentrates on the cases that tempt governments to increase money growth unexpectedly. These propositions imply that alternative policy regimes lead to different equilibria under discretion even though optimal monetary policy under commitment and inflation expectation in both policy regime are all represented by the global Friedman rule. We draw the following corollary:

Corollary 3.1 *When $\Phi \in [max(\Psi^N, \Psi^{*N}, \Psi^C, \Psi^{*C}), max(1/\gamma, 1/(1-\gamma))]$, cooperation can be counterproductive.*

If governments lack commitment technology and optimal monetary policy under commitment is the global Friedman rule in both policy regimes, cooperation is counterproductive in terms of welfare when monopolistic distortion lies in the interval, $\Phi \in [\max(\Psi^N, \Psi^{*N}, \Psi^C, \Psi^{*C}), \max(1/\gamma, 1/(1-\gamma))]$.

By focusing on the self-enforced cooperation under commitment, Corollary 3.1 shows that lack of commitment abilities may lead to the counterproductive cooperation in the sense that the welfare under cooperation between governments can be worse than that under non-cooperation (Rogoff, 1985a; Kehoe, 1989). The reason is that the optimal surprise under non-cooperation is a dominant Nash strategy whereas under cooperation, the optimal surprises are jointly set and positively related. Cooperation agreement ensures the welfare gains from deviation in both countries when initially, at least one government has the incentive to introduce inflationary surprise. Moreover, this corollary together with Proposition 3.2 reveal how large the monopolistic distortion is needed for the occurrence of counterproductive cooperation, $\Phi \in [\max(\Psi^N, \Psi^{*N}, \Psi^C, \Psi^{*C}), \max(1/\gamma, 1/(1-\gamma))]$.

Why does the monopolistic distortion play such an important role? As Φ lies inside the interval, this interval is enough to ensure the indifference of equilibrium under commitment. Under discretion, although unexpected monetary expansion is regarded as a subsidy to consumption and therefore boost output toward the efficient level, it is not always worthwhile for the government to do that. Non-cooperative governments face the policy trade-off between a standard New Keynesian aggregate demand channel and a strategic terms of trade channel (Arseneau, 2012). The former channel arises due to the interaction between sticky price and monopolistic distortion and creates an inflationary bias. Meanwhile, a floating nominal exchange in an open economy gives rise to the latter channel which generates a deflationary bias. When Φ is less than the upper bound of the interval, the former channel is dominated by the latter in either or both countries. Thus, at most one government attempts to conduct inflation surprise. In contrast, when Φ is greater than the upper bound of the interval, the former dominates the latter, and therefore both governments boost the economy toward the efficient level by creating surprise inflation simultaneously. When it comes to cooperation regime, such an agreement leads to a fixed nominal exchange rate and hence removes the strategic terms of trade channel. Rogoff (1985a) simply assumes that governments would fix their bilateral exchange rate under cooperation, but it is optimal for them to do that under our framework in which both government jointly maximize a weighted welfare function. As long as the monopolistic distortion exists, $\Phi > 1$, the cooperative governments always introduce high inflation unexpectedly, and then the global economy will end up with high inflation. In brief, the counterproductive cooperative arises when Φ is located in the interval mentioned above.

When governments cannot commit, firms, households, and governments can change their decisions depending on the history of policy variables up to the point at which the decision is made. Only the history of policy variables matters whereas neither firm nor household is aware of the possible effects of its actions on decision making by governments and other private agents. In contrast, the future market expectations are influenced by the current policy decisions, which require governments to take into account their reputation. In the following parts, we apply the Chari and Kehoe (1990)'s approach to feature the set of sustainable equilibrium outcomes by using its worst sustainable equilibrium. We begin by defining sustainable equilibrium in various regimes.

3.4.2 Sustainable equilibrium under non-cooperation

In the absence of commitment, firms, households, and governments revise their plans at each period $t = 0, 1, 2, ...$, depending on the history of the governments' history. For each $t = 0, 1, 2...$, let $\xi_t^N = (\xi_{H,t}, \xi_{F,t})$ denote the history pair of government policy up to time t where $\xi_{H,t} = \{x_s^N | s = 0, 1, ..., t\}$, $\xi_{F,t} = \{x_s^{*N} | s = 0, 1, ..., t\}$, $\xi_{H,-1} = \emptyset$, and $\xi_{F,-1} = \emptyset$. A policy plan for the Home government is a sequence of function $\sigma_H = \{\sigma_{H,t} | t = 0, 1, 2, ...\}$

where $\sigma_{H,t}(\xi_{t-1}^N)$ determines the money growth rate at time t in Home country conditional on the history ξ_{t-1}^N and taking Foreign policy at time t as given; A policy plan for the Foreign government is a sequence of function $\sigma_F = \{\sigma_{F,t}|t = 0,1,2,...\}$ where $\sigma_{F,t}(\xi_{t-1}^N)$ determines the money growth rate at time t in Foreign country conditional on the history ξ_{t-1}^N and taking Home policy at time t as given. Let $(\sigma_{H,t}, \sigma_{F,t})$ define a pair of policy plan. Let σ_H^t and σ_F^t denote the continuation of σ_H and σ_F at t such that $\sigma_H^t = \{\sigma_{H,t+s}|s = 0,1,2...\}$ and $\sigma_F^t = \{\sigma_{F,t+s}|s = 0,1,2...\}$, respectively.

The representative Home firm, given the history ξ_{t-1}^N, set prices for period t. Let the firm's pricing rule be given by $\pi^j = \{\pi_t^j|t = 0,1,2...\}$, where $\pi_t^j(\xi_{t-1}^N)$ determines the choice of $p_{H,t}(j)$ conditional on the history ξ_{t-1}^N. The allocation rule for the representative Home household is a sequence of function $\omega = \{\omega_t|t = 0,1,2...\}$, where $\omega(\xi_t^N)$ determines the choice of Ω_t conditional on ξ_t^N. It is worth emphasizing that Home government decides x_t^N after firms choose their prices for time t and therefore firms' decisions are subject to ξ_{t-1}^N. Households, however, decide their allocations for time t after knowing X_t^N and hence are subject to ξ_t^N. Given the allocation rules π^j and ω, define the continuation rules $\pi^{jt} = \{\pi_{t+s}^j|s = 0,1,2...\}$ and $\omega_t = \{\omega_{t+s}|s = 0,1,2...\}$.

Without commitment, dividends $\zeta_t(j) = \zeta_t(j, \xi_t^N)$, wage rates $w_t = w_t(\xi_t^N)$, interest rates $R_t = R_t(\xi_t^N)$, and the price level $p_{H,t} = p_{H,t}(\xi_{t-1}^N)$ and $p_t = p_t(\xi_{t-1}^N)$ are also functions of the policy history. At any date t and history ξ_{t-1}^N, private agents can use σ_H and σ_F to update the all possible future histories, $\xi_{t+s}^N = (\xi_t^N, \sigma_{H,t+s}(\xi_{t+s-1}^N), \sigma_{F,t+s}(\xi_{t+s-1}^N))$, $s = 0,1,2...$, and their knowledge of the functions $\zeta_{t+s}(j, \xi_{t+s}^N)$, $w_{t+s}(\xi_{t+s}^N)$, $R_t(\xi_{t+s}^N)$, $p_{H,t+s} = p_{H,t+s}(\xi_{t+s-1}^N)$ and $p_{t+s} = p_{t+s}(\xi_{t+s-1}^N)$. We can also define a sequence of functions for firms and household in the Foreign country.

The representative firm enters each period $t = 0,1,2...$ by taking as given ξ_{t-1}^N, σ_H, σ_F, π^j for all $\hat{j} \in [0,1]$, $\hat{j} \neq j$, π^{*j^*} for all $j^* \in [0,1]$, ω, and ω^*, and choose π^{jt} to solve

(FT) Maximize

$$\zeta_{t+s}(j) = [p_{H,t+s}(j) - w_{t+s}]\gamma \left(\frac{p_{H,t+s}(j)}{p_{H,t+s}}\right)^{-\theta} \left(\frac{p_{H,t+s}}{p_{t+s}}\right)^{-1} \left(\frac{x_{t+s}}{p_{t+s}} + \frac{x^*_{t+s}}{p^*_{t+s}}\right), \tag{3.28}$$

for each $s = 0, 1, 2...$ taking $w_{t+s} = w_{t+s}(\xi^N_{t+s})$, $p_{H,t+s} = p_{H,t+s}(\xi^N_{t+s-1})$, $p_{t+s} = p_{t+s}(\xi^N_{t+s-1})$, and $\xi^N_{t+s} = (\xi^N_{t+s-1}, \sigma_{H,t+s}(\xi^N_{t+s-1}), \sigma_{F,t+s}(\xi^N_{t+s-1}))$ as given for all $s = 0, 1, 2...$

In each period $t = 0, 1, 2...$ the representative household takes ξ^N_t, σ_H, σ_F, π^j for all $j \in [0, 1]$, π^{*j^*} for all $j^* \in [0, 1]$ and ω^* as given and make a decision on ω^t to solve

(HT) Maximize

$$\sum_{s=0}^{\infty} \beta^s \left\{ \frac{c^{1-\alpha}_{t+s}}{1-\alpha} - l_{t+s} \right\}, \tag{3.29}$$

subject to

$$c_{H,t+s} = \left[\int_0^1 c_{t+s}(j)^{(\theta-1)/\theta} dj\right]^{\theta/(\theta-1)}, \quad c_{F,t+s} = \left[\int_0^1 c_{t+s}(j^*)^{(\theta-1)/\theta} dj^*\right]^{\theta/(\theta-1)}, \tag{3.30}$$

$$c_{t+s} = (c_{H,t+s})^{\gamma} (c_{F,t+s})^{1-\gamma}, \tag{3.31}$$

$$l_{t+s} = \int_0^1 l_{t+s}(j) dj, \tag{3.32}$$

$$p_{t+s} c_{t+s} \le m_{t+s} + (x_{t+s} - 1) + b_{t+s} - \frac{b_{t+s+1} x_{t+s}}{R_{t+s}}, \tag{3.33}$$

$$p_{t+s} c_{t+s} + \frac{b_{t+s+1} x_{t+s}}{R_{t+s}} + m_{t+s+1} x_{t+s} \le m_{t+s} + (x_{t+s} - 1) + b_{t+s} + \int_0^1 \zeta_{t+s}(j) dj + w_{t+s} l_{t+s}, \tag{3.34}$$

for each $s = 0, 1, 2...$, taking $\zeta_{t+s}(j) = \zeta_{t+s}(j, \xi^N_{t+s})$, $\zeta^*_{t+s}(f) = \zeta^*_{t+s}(j^*, \xi^N_{t+s})$, $w_{t+s} = w_{t+s}(\xi^N_{t+s})$, $w^*_{t+s} = w^*_{t+s}(\xi^N_{t+s})$, $R_{t+s} = R_{t+s}(\xi^N_{t+s})$, $R^*_{t+s} = R^*_{t+s}(\xi^N_{t+s})$, and $\xi^N_{t+s} = (\xi^N_{t+s-1}, \sigma_{H,t+s}(\xi^N_{t+s-1}), \sigma_{F,t+s}(\xi^N_{t+s-1}))$ as given for all $j, j^* \in [0,1]$ and $s = 0, 1, 2...$

Moreover, for each $t = 0, 1, 2, ...$ and ξ^N_t, the continuation policy ω^j must correspond with the market clearing conditions:

(MT) $m_{t+s+1} = 1$, $b_{t+s+1} = 0$, $y_{t+s} = l_{t+s}$, $y_{t+s} = c_{H,t+s} + c^*_{H,t+s}$ and the currency exchange market clearing conditions

$$S_{t+s} = \frac{1-\gamma}{\gamma}\frac{x_{t+s}}{x^*_{t+s}}, \tag{3.35}$$

for all $s = 0, 1, 2...$

In the same manner, for the Foreign country, we can characterize the problem for the firm and household, FT^* and HT^*, respectively together with the market clearing condition, MT^*.

Eventually, at each date $t = 0, 1, ...$, under non-cooperation the Home government takes x^*_t, ξ^N_{t-1}, π^j for all $j \in [0,1]$, π^{*j^*} for all $j^* \in [0,1]$, ω, and ω^* as given and decides a continuation policy σ^t_H to solve

(GT) Maximize

$$\sum_{s=0}^{\infty} \beta^s \left\{ \frac{c^{1-\alpha}_{t+s}}{1-\alpha} - l_{t+s} \right\}, \tag{3.36}$$

where c_{t+s} and l_{t+s} are determined by π^j for all $j \in [0,1]$, π^{*j^*} for all $j^* \in [0,1]$, ω, and ω^* for all $s = 0, 1, ...$

Let π and π^* denote the set of function π^j for all $j \in [0,1]$ and π^{*j^*} for all $j^* \in [0,1]$, respectively. As in Chari and Kehoe (1990), we use the concept of *sustainable equilibrium* to refer outcome that can predominate when governments cannot commit to their future policies.

A *sustainable equilibrium under non-cooperation* is a policy plan $\sigma^N = (\sigma_H, \sigma_F)$ and a set of allocation rules $(\pi, \omega, \pi^*, \omega^*)$ that satisfy: (i) Given σ_H and σ_F, $\pi^{\hat{j}}$ for all $\hat{j} \in [0,1]$, $\hat{j} \neq j$, π^{*j^*} for all $j^* \in [0,1]$, ω, and ω^*, the continuation π^{jt} of each π^j solves (FT) for all $t = 0,1,...$ and ξ^N_{t-1}. Given σ_H and σ_F, π^j for all $j \in [0,1]$, $\pi^{*\hat{j}^*}$ for all $\hat{j}^* \in [0,1]$, $\hat{j}^* \neq j^*$, ω, and ω^*, the continuation π^{*j^*t} of each π^{*j^*} solves (FT^*) for all $t = 0,1,...$ and ξ^N_{t-1}; (ii) Given σ_H and σ_F, π^j for all $j \in [0,1]$, π^{*j^*} for all $j^* \in [0,1]$, and ω^*, the continuation ω^t of ω solves (HT) for all $t = 0,1,...$ and ξ^N_t. Given σ_H and σ_F, π^j for all $j \in [0,1]$, π^{*j^*} for all $j^* \in [0,1]$, and ω, the continuation ω^{*t} of ω^* solves (HT^*) for all $t = 0,1,...$ and ξ^N_t; (iii) the continuation of ω and ω^* are corresponding with (MT) and (MT^*), respectively, for all $t = 0,1,...$ and ξ^N_t; (iv) Given π^j for all $j \in [0,1]$, π^{*j^*} for all $j^* \in [0,1]$, ω, ω^*, and σ_F, the continuation σ^t_H of σ_H solves (GT) for all $t = 0,1,...$ and ξ^N_{t-1}; Given π^j for all $j \in [0,1]$, π^{*j^*} for all $j^* \in [0,1]$, ω, ω^*, and σ_H, the continuation σ^t_F of σ_F solves (GT^*) for all $t = 0,1,...$ and ξ^N_{t-1}.

A sustainable equilibrium under non-cooperation $((\sigma_H, \sigma_F), \pi, \omega, \pi^*, \omega^*)$ induces a *sustainable outcome under non-cooperation*, $(X^N, \Pi, \Omega, \Pi^*, \Omega^*)$ which is defined as follows. From $\xi^N_{-1} = \emptyset$, compute $\xi_H = \{\xi_{H,t} | t = 0,1,...\}$, $\xi_F = \{\xi_{F,t} | t = 0,1,...\}$, and $X^N = \{(x^N_t, x^{*N}_t) | t = 0,1,...\}$ by employing $x^N_t = \sigma_{H,t}(\xi^N_{t-1})$, $x^{*N}_t = \sigma_{F,t}(\xi^N_{t-1})$, $\xi^N_t = (\xi^N_{t-1}, \sigma_H(\xi^N_{t-1}), \sigma_F(\xi^N_{t-1}))$. After that, for all $t = 0,1,...$, compute $\Pi = \{\Pi^j | j \in [0,1]\}$, $\Pi^j = \{p_{H,t}(j) | t = 0,1,...\}$ and $\Omega = \{\Omega_t | t = 0,1,...\}$ using $p_{H,t}(j) = \pi^j_t(\xi^N_{t-1})$ for all $j \in [0,1]$ and $\Omega_t = \omega_t(\xi^N_t)$. Similarly we can get Π^* and Ω^*. Hence, the sustainable outcome $((\sigma_H, \sigma_F), \pi, \omega, \pi^*, \omega^*)$ portrays the sequence of equilibrium prices and quantities that achieves when Home government sets x^N_t and Foreign government sets x^{*N}_t independently and private sectors response optimally.

We next define the set of *autarky plans under non-cooperation* $((\sigma^a_H, \sigma^a_F), \pi^a, \omega^a, \pi^{*a}, \omega^{*a})$. Let $(\sigma^a_H, \sigma^a_F) = \{(\sigma^a_{H,t}, \sigma^a_{F,t}) | t = 0,1,...\}$ have $\sigma^a_{H,t}(\xi^N_{t-1}) = \bar{x}$ and $\sigma^a_{F,t}(\xi^N_{t-1}) = \bar{x}^*$ for all $t = 0,1,...$ and ξ^N_{t-1}. Given (σ^a_H, σ^a_F), let each π^{ja}, $j \in [0,1]$ and π^{*j^*a}, $j^* \in [0,1]$ be the allocation rules such that their continuations π^{jat} and π^{*j^*at} solve (FT) and (FT^*) for all

$t = 0, 1, ...$ and ξ_{t-1}. Let ω^a and ω^{*a} be the allocation rules such that their continuations ω^{at} and ω^{*at} solves (HT) and (HT^*) and are consistent with (MT) and (MT^*), respectively, for all $t = 0, 1, ...$ and ξ_t. It is essential to stress that when $\Phi \geq \max(\Psi^N, \Psi^{*N})$, the worst non-cooperative equilibrium under commitment coincides with the autarky plans under non-cooperation. In addition, when $\Phi \geq \max(1/\gamma, (1/(1-\gamma))$, the autarky plan coincides with non-cooperative equilibrium under discretion. These autarky plans differ from the existing literature in some critical ways. The existing literature (Chari and Kehoe, 1990; Ireland, 1997) sets the autarky plans in a closed economy, so these plans are functions of its own government's history. The autarky plans in our study are functions of world policy history. In addition, one government sets its autarky plan by taking the other government's autarky plan as given. With the definition of sustainable equilibrium, the following proposition indicates that the autarky plans induce a sustainable equilibrium.

Proposition 3.6 $((\sigma_H^a, \sigma_F^a), \pi^a, \omega^a, \pi^{*a}, \omega^{*a})$ *is a sustainable equilibrium under non-cooperation if* $\bar{x} > \beta/(\gamma\Phi)$ *and* $\bar{x}^* > \beta/((1-\gamma)\Phi)$.

By construction, any sustainable outcome must be induced by an equilibrium under commitment. When $\Phi \geq \max(\Psi^N, \Psi^{*N})$, the autarky plan is the worst non-cooperative equilibrium under commitment. Hence, Proposition 3.3 and 3.6 refers that if $\Phi \geq \max[1/\gamma, 1/(1-\gamma)]$, then $((\sigma_H^a, \sigma_F^a), \pi^a, \omega^a, \pi^{*a}, \omega^{*a})$ is a worst sustainable non-cooperative equilibrium. Chari and Kehoe (1990) suggest that policy inducing the reversion to such a worst outcome can be employed to feature the outcome with better levels of welfare.

It is worthwhile to discuss the conditions inside the Proposition 3.6. Given the Foreign government and private sectors following the autarky plan, the Home government may have incentive to deviate by deflating from the autarky plan to obtain one-period gain and revert to the autarky plan after that because the private sectors adjust the inflation expectation accordingly. The conditions, $\bar{x} > \beta/(\gamma\Phi)$ and $\bar{x}^* > \beta/((1-\gamma)\Phi)$, are used to prevent such a deviation.

We now provide the intuition of the revert-to-autarky plans under non-cooperation while the formal setup is put in the appendix. Given an arbitrary world policy $X^N = (x^N, x^{*N})$ and an arbitrary allocation $(\Pi, \Omega, \Pi^*, \Omega^*)$, the revert-to-autarky plans induce the continuation of outcome $((x^N, x^{*N}), \Pi, \Omega, \Pi^*, \Omega^*)$ as long as the world policy (x^N, x^{*N}) has been chosen in the past; if either or both governments choose not to follow the announced policy, this strategy dictates to revert to autarky plans in both countries forever afterward.

Next, let U^{Na} and U^{*Na} denote the constant value of Home and Foreign utility, respectively, generated by the autarky plans $((\sigma_H^a, \sigma_F^a), \pi^a, \omega^a, \pi^{*a}, \omega^{*a})$ with $\hat{x} = \bar{x}$ and $\hat{x}^* = \bar{x}^*$:

$$U^{Na} = \frac{1}{1-\alpha}\left(\frac{\gamma^W \beta}{\Phi \bar{x}}\right)^{\frac{1-\alpha}{\alpha}} - \frac{1}{\gamma^W}\left(\frac{\gamma^W \beta}{\Phi \bar{x}}\right)^{\frac{1}{\alpha}}, \tag{3.37}$$

$$U^{*Na} = \frac{1}{1-\alpha}\left(\frac{\gamma^W \beta}{\Phi \bar{x}^*}\right)^{\frac{1-\alpha}{\alpha}} - \frac{1}{\gamma^W}\left(\frac{\gamma^W \beta}{\Phi \bar{x}^*}\right)^{\frac{1}{\alpha}}. \tag{3.38}$$

Recall that $\tilde{U}(p_{H,t}, p_{F,t}^*)$ is the maximum current-period welfare that Home government achieves by deviating from x^N at time t, given that it has followed x^N in every period before t and Foreign government has chosen x^{*N} until period t when private sector takes the revert-to-autarky plan. Notation $\tilde{U}^*(p_{H,t}, p_{F,t}^*)$ is the maximum current-period welfare that Foreign government achieves by deviating from x^{*N} at time t, given that it has followed x^{*N} in every period before t and Home government has chosen x^N until period t when private sector takes the revert-to-autarky plan.

For an arbitrary policy pair and arbitrary allocation, the next proposition portrays a necessary and sufficient condition for the existence of a sustainable equilibrium whose outcome satisfies:

Proposition 3.7 *Suppose* $\Phi \geq max[1/\gamma, 1/(1-\gamma)]$. *Let* $X^N = (x^N, x^{*N})$ *be an arbitrary policy pair and* $(\Pi, \Omega, \Pi^*, \Omega^*)$ *be an arbitrary allocation. Then* $(X^N, \Pi, \Omega, \Pi^*, \Omega^*)$ *is the outcome of a sustainable equilibrium under non-cooperation if and only if:*

(SN1) $(X^N, \Pi, \Omega, \Pi^*, \Omega^*)$ *is an outcome of equilibrium under commitment;*

(SN2) $(X^N, \Pi, \Omega, \Pi^*, \Omega^*)$ *satisfies*

$$\tilde{U}^N(p_{H,t}, p^*_{F,t}) + \frac{\beta}{1-\beta}U^{Na} \leq \sum_{s=0}^{\infty} \beta^s \left(\frac{c_{t+s}^{1-\alpha}}{1-\alpha} - l_{t+s} \right), \tag{3.39}$$

$$\tilde{U}^{*N}(p_{H,t}, p^*_{F,t}) + \frac{\beta}{1-\beta}U^{*Na} \leq \sum_{s=0}^{\infty} \beta^s \left(\frac{c_{t+s}^{*1-\alpha}}{1-\alpha} - l^*_{t+s} \right), \tag{3.40}$$

for all $t = 0, 1...$, *where* c_t, c^*_t, l_t, *and* l^*_t *are determined by* Ω *and* Ω^*, *and where* $p_{H,t}$ *and* $p^*_{F,t}$ *are found by* Π *and* Π^* *for all* $t = 0, 1...$

Equation (3.39) and (3.40) are labeled as *sustainability constraints* which imply that in each country the welfare generated by any sustainable equilibrium from date t onwards must be at least as large as the one achieved by generating inflationary surprise at date t and reverting to the autarky forever after.

3.4.3 Sustainable equilibrium under cooperation

Although we can sequentially apply the same argument as in the previous subsection to define sustainable equilibrium under cooperation and then to characterize the sustainable outcome under cooperation, these equilibria and outcomes are different in a number of aspects.

For each $t = 0, 1, 2...$, let ξ_t^C denote the history of government policy up to time t where $\xi_t^C = \{X_s^C | s = 0, 1, ..., t\}$ and $\xi_{-1}^C = \emptyset$. Let a world policy plan σ be equal to $(\sigma_1, \sigma_2, ..., \sigma_t)$ where $\sigma_t(\xi_{t-1}^C)$ determines the money growth rates at time t conditional on the history ξ_{t-1}^C and let σ^t denote the continuation of σ at t such that $\sigma^t = \{\sigma_{t+s} | s = 0, 1, 2...\}$. It is clear to see under cooperation a single policy plan determines the money growth rates in both countries whereas under non-cooperation, there are two separate plans each of which dictating the money growth in their own countries taking other country's plan as given. The allocation rule of firm now is conditional on the realization history ξ_{t-1}^C while that of household is conditional on ξ_t^C. The problem of the firm and household, and the market conditions are derived as under non-cooperation. The government problem, however, is that at each date $t = 0, 1, ...$, the governments take ξ^C_{t-1}, π^j for all $j \in [0,1]$, π^{*j^*} for all $j^* \in [0,1]$, ω, and ω^* as given and decides a continuation policy σ^t to solve

(GT^C) Maximize

$$\sum_{s=0}^{\infty} \beta^s \left\{ \frac{c_{t+s}^{1-\alpha}}{1-\alpha} - l_{t+s} \right\} + \sum_{s=0}^{\infty} \beta^s \left\{ \frac{c_{t+s}^{*1-\alpha}}{1-\alpha} - l^*_{t+s} \right\}, \tag{3.41}$$

where c_{t+s}, c^*_{t+s}, l_{t+s}, and l^*_{t+s} are determined by π^j for all $j \in [0,1]$, π^{*j^*} for all $j^* \in [0,1]$, ω, and ω^* for all $s = 0, 1, ...$

Using the same logic, we can define the *sustainable equilibrium under cooperation* including a policy plan σ and a set of allocation rules $(\pi, \omega, \pi^*, \omega^*)$ which, in turn, induces the *sustainable outcome under cooperation*, $(X^C, \Pi, \Omega, \Pi^*, \Omega^*)$.

The set of *autarky plans* $(\sigma^a, \pi^a, \omega^a, \pi^{*a}, \omega^{*a})$ is then defined consistently with the existing literature. Let $\sigma^a = \{\sigma^a_t | t = 0, 1, ...\}$ have $\sigma^a_t(\xi^C_{t-1}) = (\bar{x}, \bar{x}^*)$ for all $t = 0, 1, ...$ and ξ^C_{t-1}. Given σ^a, let each π^{ja}, $j \in [0,1]$ and π^{*j^*a}, $j^* \in [0,1]$ be the allocation rules such that their continuations π^{jat} and π^{*j^*at} solve (FT) and (FT^*) for all $t = 0, 1, ...$ and ξ^C_{t-1}. Let ω^a and ω^{*a} be the allocation rules such that their continuations ω^{at} and ω^{*at} solves (HT) and (HT^*) and are consistent with (MT) and (MT^*), respectively, for all $t = 0, 1, ...$ and ξ^C_t. When $\Phi \geq \max(\Psi^C, \Psi^{*C})$, the autarky plan coincides with the worst cooperative equilibrium under commitment. Moreover, the autarky plan coincides with the cooperative equilibrium under discretion. With the definition of sustainable equilibrium, the following proposition indicates that the autarky plans induce a sustainable equilibrium.

Proposition 3.8 $(\sigma^a, \pi^a, \omega^a, \pi^{*a}, \omega^{*a})$ *is a sustainable equilibrium under cooperation.*

By construction, any sustainable outcome of equilibrium under cooperation must be induced by a cooperative equilibrium under commitment. When $\Phi \geq \max(\Psi^C, \Psi^{*C})$, the autarky plan is the worst cooperative equilibrium under commitment. Thus, both Proposition 3.3 and 3.8 implies that $(\sigma^a, \pi^a, \omega^a, \pi^{*a}, \omega^{*a})$ is a worst sustainable cooperative equilibrium.

Given an arbitrary world policy $X^C = (x^C, x^{*C})$ and an arbitrary allocation $(\Pi, \Omega, \Pi^*, \Omega^*)$, the revert-to-autarky plans under cooperation induce the continuation of outcome $(X^C, \Pi, \Omega, \Pi^*, \Omega^*)$ as long as X^C has been chosen in the past; otherwise, the strategy specifies to adopt the autarky plan forever.

Next, let U^{Ca} and U^{*Ca} denote the constant value of Home and Foreign utility, respectively, generated by the autarky plans $(\sigma^a, \pi^a, \omega^a, \pi^{*a}, \omega^{*a})$, using (3.23) with $\hat{x} = \bar{x}$ and $\hat{x}^* = \bar{x}^*$:

$$U^{Ca} + U^{*Ca} = \frac{1}{1-\alpha}\left(\left(\frac{\gamma^W \beta}{\Phi \bar{x}}\right)^{\frac{1-\alpha}{\alpha}} + \left(\frac{\gamma^W \beta}{\Phi \bar{x}^*}\right)^{\frac{1-\alpha}{\alpha}}\right) - \frac{1}{\gamma^W}\left(\left(\frac{\gamma^W \beta}{\Phi \bar{x}}\right)^{\frac{1}{\alpha}} + \left(\frac{\gamma^W \beta}{\Phi \bar{x}^*}\right)^{\frac{1}{\alpha}}\right). \tag{3.42}$$

Recall that $\tilde{U}^C(p_{H,t}, p^*_{F,t}) + \tilde{U}^{*C}(p_{H,t}, p^*_{F,t})$ is the maximum current-period weighted utilities that the governments jointly achieve by defecting from $\hat{X}^C$ at time t, given that they have followed $\hat{X}^C$ in every period before t, when private sector takes the revert-to-autarky plans accompanied by the outcome $(X^C, \Pi, \Omega, \Pi^*, \Omega^*)$. For an arbitrary policy pair and arbitrary allocation, the next proposition portrays a necessary and sufficient condition for the existence of a sustainable equilibrium whose outcome satisfies:

Proposition 3.9 *Suppose $\Phi \geq max(\Psi^C, \Psi^{*C})$. Let $X^C = (x^C, x^{*C})$ be an arbitrary world policy and $(\Pi, \Omega, \Pi^*, \Omega^*)$ be an arbitrary allocation. Then $(X^C, \Pi, \Omega, \Pi^*, \Omega^*)$ is the outcome of a sustainable equilibrium if and only if:*

(SC1) $(X^C, \Pi, \Omega, \Pi^*, \Omega^*)$ *is an outcome of equilibrium under commitment;*

(SC2) $(X^C, \Pi, \Omega, \Pi^*, \Omega^*)$ *satisfies*

$$\left(\tilde{U}^C(p_{H,t}, p^*_{F,t}) + \tilde{U}^{*C}(p_{H,t}, p^*_{F,t})\right) + \frac{\beta}{1-\beta}\left(U^{Ca} + U^{*Ca}\right) \leq \sum_{s=0}^{\infty} \beta^s \left(\frac{c_{t+s}^{1-\alpha}}{1-\alpha} - l_{t+s} + \frac{c_{t+s}^{*1-\alpha}}{1-\alpha} - l^*_{t+s}\right), \tag{3.43}$$

for all $t = 0, 1...$, *where* c_t, c_t^*, l_t, *and* l_t^* *are determined by* Ω *and* Ω^*, *and where* $p_{H,t}$ *and* $p_{F,t}^*$ *are found by* Π *and* Π^* *for all* $t = 0, 1...$

Equation (3.43) is labeled as a *sustainability constraint* which implies that the weighted welfare generated by any sustainable equilibrium from date t forward must be at least as big as the one obtained by jointly introducing the optimal monetary surprise at date t and reverting to the autarky forever after.

3.5 Sustainability of the global Friedman rule

Proposition 3.7 and 3.9 portray the set of conditions that must be held for the optimal monetary policy under commitment in two types of policy regime to be a sustainable outcome. When the condition in Proposition 3.7, $\Phi \geq \max[1/\gamma, 1/(1-\gamma)]$, is satisfied, the condition in Proposition 3.9, $\Phi \geq \max(\Psi^C, \Psi^{*C})$, is also satisfied as we have $\gamma\Psi^C < 1$ and $(1-\gamma)\Psi^{*C} < 1$. In this part, we consider the new aspect of the counterproductive cooperation by comparing the restrictiveness of different sustainability constraints. We will show that when governments cannot commit to their future policies, the sustainability constraint under cooperation may be more restrictive than the set of two constraints under non-cooperation.

To express our argument in a formal form, we rearrange conditions (3.39) and (3.40) as follows:

$$\mathfrak{B}\left(U^N - U^{Na}\right) \geq \tilde{U}^N - U^N, \tag{3.44}$$

$$\mathfrak{B}\left(U^{*N} - U^{^*Na}\right) \geq \tilde{U}^{*N} - U^{*N}, \tag{3.45}$$

where $\mathfrak{B} = \beta/(1-\beta)$ is an increasing function of the discount factor, β. The left-hand side of (3.44) and (3.45) captures the present value of punishment from deviation whereas the right-hand side of (3.44) and (3.45) captures the current incentive to deviate. These conditions can be reduced further

$$\mathfrak{B} \geq \frac{\tilde{U}^N - U^N}{U^N - U^{Na}} \equiv \underline{\mathfrak{B}}^N(\Phi, \alpha), \tag{3.46}$$

$$\mathfrak{B} \geq \frac{\tilde{U}^{*N} - U^{*N}}{U^{*N} - U^{*Na}} \equiv \underline{\mathfrak{B}}^{*N}(\Phi, \alpha). \tag{3.47}$$

When (3.46) and (3.47) hold, the punishment after a deviation is sufficient to preclude the governments to deviate from the optimal policy under commitment. The higher the incentive to deviate, or the lower the punishment following a deviation, the higher $\underline{\mathfrak{B}}(\Phi, \alpha)$, and hence the higher $\mathfrak{B}$ is required to prevent the governments' deviation. Define $\mathfrak{B}_H$ be the set of the discount factor that satisfies $\mathfrak{B} \geq \underline{\mathfrak{B}}^N(\Phi, \alpha)$ and $\mathfrak{B}_F$ be the set of the discount factor that satisfies $\mathfrak{B} \geq \underline{\mathfrak{B}}^{*N}(\Phi, \alpha)$. Equilibrium under non-cooperation is sustainable if neither government has temptation to deviate:

$$\mathfrak{B}_H \cap \mathfrak{B}_F. \tag{3.48}$$

The condition under cooperation is analogous:

$$\mathfrak{B} \geq \frac{(\tilde{U}^C + \tilde{U}^{*C}) - (U^C + U^{*C})}{(U^C + U^{*C}) - (U^{Ca} + U^{*Ca})} \equiv \underline{\mathfrak{B}}^C(\Phi, \alpha). \tag{3.49}$$

Let $\mathfrak{B}^N$ define a set of discount factor that satisfies (3.48) for an optimal non-cooperative equilibrium under commitment as a solution when government lack of commitment. Also, let $\mathfrak{B}^C$ define a set of discount factor that satisfies (3.49) for an optimal cooperative equilibrium as a solution when government lack of commitment.

We focus on the situation in which the optimal monetary policies under commitment in both policy regimes coincide and are featured by the global Friedman rule. That happens when the monopolistic distortion is large such that $\Phi \geq \Psi^N, \Psi^{*N}, \Psi^C, \Psi^{*C}$. Moreover, when the monopolistic distortion is satisfied, $\Phi > \max(1/\gamma, 1/(1-\gamma))$, high inflation outcomes under discretion are used as punishment after deviating from the global Friedman rule.

If $(\hat{X}^N, \Pi, \Omega, \Pi^*, \Omega^*)$ is a non-cooperative equilibrium under commitment with $\hat{X}^N = (\beta, \beta)$ for all $t = 0, 1, 2, ...$, then (3.13), (3.14), (3.15), and (3.17) imply that this equilibrium has $p_{H,t} = \underline{p}_H$, $p^*_{F,t} = \underline{p}^*_F$, $\hat{c}_t$, and $\hat{c}^*_t$ for all $t = 0, 1, ...$, where

$$\underline{p}_H = \beta(\rho^{\alpha-1}\Phi)^{\frac{1}{\alpha}}, \qquad \underline{p}^*_F = \beta(\rho^{*\alpha-1}\Phi)^{\frac{1}{\alpha}}, \tag{3.50}$$

$$\hat{c} = \left(\frac{\rho}{\Phi}\right)^{\frac{1}{\alpha}}, \qquad \hat{c}^* = \left(\frac{\rho^*}{\Phi}\right)^{\frac{1}{\alpha}}. \tag{3.51}$$

Recall that with $p_{H,t} = \underline{p}_H$ and $p^*_{F,t} = \underline{p}^*_F$, the solution to (GD) and (GD^*) are given by (3.24) and (3.25). For large monopolistic distortion, $\Phi > \max(1/\gamma, 1/(1-\gamma)$, the one-period welfare in Home and Foreign country can be expressed as

$$\tilde{U}^N(\underline{p}_H, \underline{p}^*_F) = \frac{(\tilde{c}^N)^{1-\alpha}}{(1-\alpha)} - \frac{\tilde{c}^N}{\rho}, \tag{3.52}$$

where

$$\tilde{c}^N = \gamma \frac{(\tilde{x}^N)^\gamma (\hat{x}^{*N})^{1-\gamma}}{\underline{p}^\gamma_H \underline{p}^{*(1-\gamma)}_F} \tag{3.53}$$

is the efficient value of consumption in Home country under non-cooperation. Lastly, the welfare under autarky plan is given by

$$U^{Na} = \frac{\underline{c}^{1-\alpha}}{(1-\alpha)} - \frac{\underline{c}}{\rho}, \tag{3.54}$$

where $\underline{c} = \left(\frac{\gamma^W \beta}{\Phi \bar{x}}\right)^{\frac{1}{\alpha}}$. Hence, when X^N is characterized by the global Friedman rule, the (3.39) becomes,

$$(1-\beta)\left(\frac{(\tilde{c}^N)^{1-\alpha}}{(1-\alpha)} - \frac{\tilde{c}^N}{\rho}\right) + \beta\left(\frac{\underline{c}^{1-\alpha}}{(1-\alpha)} - \frac{\underline{c}}{\rho}\right) \leq \frac{\hat{c}^{1-\alpha}}{(1-\alpha)} - \frac{\hat{c}}{\rho}. \tag{3.55}$$

When it comes to the cooperation regime, if $(\hat{X}^C, \Pi, \Omega, \Pi^*, \Omega^*)$ is a cooperative equilibrium under commitment with $X_t^C = (\beta, \beta)$ for all $t = 0, 1, 2, ...$, then the equilibrium outcome of consumption and prices are also given by (3.50) and (3.51). With $p_{H,t} = \underline{p}_H$ and $p^*_{F,t} = \underline{p}^*_F$, the solution to (GD) is given by (3.27). Hence, the weighted welfare in the problem (CD) is

$$\tilde{U}^C(\underline{p}_H, \underline{p}^*_F) + \tilde{U}^{*C}(\underline{p}_H, \underline{p}^*_F) = \frac{(\tilde{c}^C)^{1-\alpha}}{(1-\alpha)\gamma\Psi^C} - \frac{\tilde{c}^C}{\gamma\gamma^W}, \tag{3.56}$$

where $\tilde{c}^C = (\gamma^W/\Psi^C)^{1/\alpha}$ is the efficient value of consumption in Home country under cooperation. The weighted welfare under autarky plan is given by (3.54). Hence, when X^C is characterized by the global Friedman rule, (3.43) becomes,

$$(1-\beta)\left(\frac{(\tilde{c}^C)^{1-\alpha}}{(1-\alpha)} - \frac{\tilde{c}^C}{\rho\kappa}\right) + \beta\left(\frac{\underline{c}^{1-\alpha}}{(1-\alpha)} - \frac{\underline{c}}{\rho\kappa}\right) \leq \frac{\hat{c}^{1-\alpha}}{(1-\alpha)} - \frac{\hat{c}}{\rho\kappa}, \tag{3.57}$$

where $\kappa = \gamma^{\alpha\gamma}(1-\gamma)^{\alpha(1-\gamma)}(\gamma^{1-\alpha} + (1-\gamma)^{1-\alpha}) \geq 1$.

Letting $\bar{x}$ and $\bar{x}^*$ be large, the welfare under autarky plan are all zero. We restrict our attention to the case in which $\gamma = 1/2$, then $\kappa = 1$ and $\underline{\mathfrak{B}}^N \equiv \underline{\mathfrak{B}}^{*N}$ or $\mathfrak{B}^N = \mathfrak{B}_H = \mathfrak{B}_F$. As a result, we only need to show that the one-period welfare that governments obtain by defecting under cooperation is larger than those obtained under non-cooperation. Equivalently, if $\tilde{c}^C > \tilde{c}^N$, then $\underline{\mathfrak{B}}^C > \underline{\mathfrak{B}}^N$ or $\mathfrak{B}^C \subset \mathfrak{B}^N$ which can be interpreted as (3.43) being less likely to hold than (3.55). Based on that, we have the following proposition:

Proposition 3.10 *Suppose $\gamma = 1/2$. If $\Phi > 2$, the cooperation without commitment is counterproductive in the sense that the global Friedman rule is less likely to be sustained under cooperation than under non-cooperation, or $\mathfrak{B}^C \subset \mathfrak{B}^N$.*

The condition $\mathfrak{B}^C \subset \mathfrak{B}^N$ implies that the discount factor under cooperation is required to be higher than that under non-cooperation so as to keep the sustainability constraints hold. An increase in the discount factor makes the revert-to-autarky plans more costly. Since the cooperative governments have higher incentive to deviate, the discount factor under cooperation must be greater in order to rule out deviation. In essence, when governments cooperate, they can jointly set the optimal inflation surprise so as to create short-run production efficiency and then households consume at the maximum level. By contrast, the competition mechanism lowers the short-run benefits from inflationary surprise and hence makes the low-inflation policy under non-cooperation more credible. As a result, governments have less incentives to deviate from the global Friedman rule when they do not cooperate, and this makes the sustainability constraint more likely to hold.

We apply Corsetti and Pesenti (2001)'s logic to explain the mechanism of the above result. An unexpected monetary expansion in Home country causes a depreciation in Home currency. Such a depreciation increases the price of Home imports, thereby raising Home inflation. As prices are sticky, $P_{H,t}$ is fixed within a period, the aggregate price of Home country indexed by P_t goes up by merely a proportion $1-\gamma$ of the increase in the money supply. Hence, the real balance of Home household increases with an unexpected expansion. Meanwhile, since the real interest rate decreases globally in response with a rise in the money supply, consumption increases proportionally in Home and Foreign country. This growth in consumption together with a relatively expensive price of Foreign goods raise the demand for Home goods, improving the Home output. This transmission is labeled as a standard New Keynesian demand channel due to the coexistence of sticky nominal price and imperfect competition in goods market. Briefly, in this channel, an unexpected monetary expansion in Home country causes a consumption subsidy or a leisure tax that improves welfare by boosting output toward the efficient level.

At the same time, an inflationary surprise generates an instantaneous depreciation of the nominal exchange rate due to one-period price rigidity that deteriorates the terms of trade of Home country in the short run. This deterioration leads to an expenditure shifting from Foreign goods to Home goods. Because goods are traded, Home households only enjoy part of consumption subsidy while the remainder is transferred to Foreign households through the dynamics of the short-run terms of trade. However, Home households suffer the whole burden of leisure tax because leisure cannot be traded. Hence, while consumption subsidy benefits households in both countries proportionally, the Home households must work more than the Foreign households. This strategic terms of trade channel creates a deflationary bias. Under cooperation, the New Keynesian demand channel is the only channel in operation because it is optimal for governments to fix the nominal exchange rate thereby turns off the strategic terms of trade channel. As a consequence, the unexpected monetary policy is more expansionary under cooperation where the deflationary bias is absent than under non-cooperation. Thus the short-run benefits from inflationary surprise under cooperation is higher than that under non-cooperation.

3.6 Conclusion

The primary result in this chapter is that cooperation may be counterproductive in terms of credibility. This new aspect of counterproductive cooperation relies on the difference in the restrictiveness of the sustainability constraints. The central message of this result is that when governments lack commitment technology, competition among policymakers is regarded as partial commitment and then might be superior to cooperation.

Chapter 4

Monetary Delegation and Time-inconsistency in a New Keynesian Two-country Model

4.1 Introduction

One of the solutions to the dynamic inconsistency theories of inflation à la Kydland and Prescott (1977) and Barro and Gordon (1983b) is the monetary delegation approach. What delegation implies here is to appoint a conservative central banker whose aversion to inflation outweighs that of society. In line with this research field, Rogoff (1985a) provides a famous prediction that has received great attention by economists: the more open the economy becomes, the less inflation it suffers. The reason is that depreciation in real exchange rate caused by an inflationary surprise will dampen the governments' incentive to conduct such a surprise. Romer (1993) provides strong evidence for a negative relationship between inflation and openness. Based on this observation, it is natural to ask whether or not the openness of economy affects the effectiveness of delegation approach in solving the inflationary bias.

This chapter assesses a micro-founded two-country New Keynesian model. The processes of delegation, reappointment, and monetary policy are formulated explicitly with the primary assumption being that it is costly for the government to reappoint a central bank. The international dimension adds the strategic games across countries in addition to those within a nation. The openness of the economy is captured by the weight put on the foreign goods in the domestic good bundle. A higher weight implies a more open economy. Our findings are two folds: first, a more open economy requires a lower threshold of reappointment cost parameter for the current central bank not to be reappointed. The interpretation is that when the economy becomes more open, this economy enjoys a less consumption subsidy as domestic good accounts for a smaller fraction of domestic expenditure. Therefore, the central bank has less incentive to induce inflationary surprise. As a consequence, a smaller reappointment cost is required to prevent the government from reappointing the current central bank; second, when the reappointment cost is less than such a threshold, the maintenance of central bank is more likely to be sustained in a more open economy. The explanation is that the competition mechanism under a more open economy lowers the short-run benefits from inflationary surprise even further, and hence make the commitment equilibrium more sustainable.

This chapter is related to three strands of literature. First, we build on the monetary delegation approach that is used to reduce or remove the inflation bias caused by time-inconsistency problem. There are three forms of delegation: The first is to appoint a conservative central banker whose inflation aversion is higher than the society's (Rogoff, 1985a); The second is based on targeting rules in which the government imposes a penalty on the central banker for any deviation from the targeted variable (Svensson, 1997); The third is the contracting method (Walsh, 1995; Persson and Tabellini, 1993) in which monetary policy delegation is regarded as a principal-agent problem. While the first two forms can reduce the inflationary bias in general, the bias is not totally eliminated because of a trade-off between inflation and output stabilization. The last form is more appealing as it completely removes the inflation bias as well as the stabilization bias. The underlying reason is that the principal might design a contract such that the agent's income is subject to its actions, thereby altering the agent's incentives. In the context of monetary policy, the institutional structure of the central bank can affect its incentives when conducting policy. Walsh (1995) derives the optimal rewards to the central bank so as to achieve the socially optimal policy. Hence, in this chapter, we study the delegation in the contracting method.

However, this approach is questioned by McCallum (1995). He claims that delegation does not solve the time-consistency problem, but merely postpones it. This verbal argument is then formalized by Jensen (1997) with an introduction of a parameter capturing the cost of reappointing a central bank. Bilbiie (2011) provides a stronger result that optimal delegation is time inconsistent. However, in the same context, Driffill and Rotondi (2006) argue against this result by showing that using a tougher contract with initial delegation parameter being greater than the delegation parameter, delegation helps to ease the time-consistency problem, but it comes at the cost of continually reappointing the central bank. This feature is unrealistic as the central banker should be selected for a couple of periods of time (Waller and Walsh, 1996). Basso (2009) points out that the functional form of reappointment cost plays a important role. Jensen (1997)'s and Driffill and Rotondi (2006)'s model use a quadratic function which has a zero reappointment cost at the margin. The announcement of a central bank contract is regarded as cheap talk and therefore unable to outweigh the government's incentive to reappoint the central bank at each period. In reality, the reappointment can generate non-negligible marginal cost even though the new central bank is almost identical to the previous one. Basso then proposes an absolute value functional form in which the marginal cost of reappointment is not equal to zero. Using this form under a micro-founded model, he proved that there exists a threshold for reappointment cost parameter such that delegation can be used to solve the time-consistency problem without constant reappointments when this parameter is higher than the threshold. Otherwise, a trigger-type strategy is required to sustain the commitment outcome.

Second, we build on the literature on the effect of openness on inflation in the absence of commitment ability. Rogoff (1985a) shows that under non-cooperation an unexpected increase in the money growth rate leads to a depreciation in the real exchange rate discouraging governments from inflating the economies. He claimed further that monetary cooperation might be counterproductive as cooperation puts two countries into a single economy with larger size but less open. This reduction in openness decreases the discouraging effect of the real depreciation on governments' incentives to conduct an inflationary surprise and hence boosts inflation in equilibrium. Laskar (1989) extends the work of Rogoff (1985b) into a two-country model and showed that conservative central bankers might worsen the welfare in both countries in the absence of monetary cooperation. The reason is that the fear of the inflationary effects caused by the real depreciation prevents the governments from expanding monetary policies high enough. When putting a higher weight on the inflation objective, conservative central bankers amplify this fear and hence increase the inefficiency of equilibrium under non-cooperation. Using a similar model, Alesina and Grilli (1991) also find a welfare reduction when delegating monetary policy in an international framework. These papers specify an ad hoc utility function to consider the game between two conservative central bankers. We instead rely on a micro-founded framework to analyze delegation in the form of Walsh's contracting method. We compare the effectiveness of delegation approach under closed economy with that under open economy and provide the interpretation of mechanism behind the results which are different from previous studies in some meaningful ways. In a micro-founded New Keynesian open economy, an unanticipated expansion is regarded as a consumption subsidy or a leisure tax. The costs of leisure tax impose stronger dampening effects on the governments' temptation to run an inflationary surprise as goods are traded across countries so that each state only enjoys a proposition of consumption subsidy whereas the closed economy takes the whole benefits of such a subsidy.

Third, we apply the sustainable plan proposed by Chari and Kehoe (1990) to characterize the set of sustainable outcome. Ireland (1997) and Kurozumi (2008) use this method to feature the optimal policy in a canonical New Keynesian model with inflation bias and stabilization bias, respectively. This chapter investigate the effect of openness on the sustainability constraints and finds a more restrictive constraint under a more open economy

The remaining of this chapter is organized as follows: Section 2 presents the model and Section 3 considers monetary delegation with commitment. Delegation and sustainability of Nash equilibria are analyzed in Section 4. Section 5 offers some concluding remarks. Derivations and proofs are put at the appendix.

4.2 Model

The economic environment is an extension of Basso (2009)'s closed economy to a model with two countries, Home and Foreign, and each country consists of (i) a representative household, (ii) a continuum of monopolistic firms indexed by $j \in [0,1]$ for Home and $j^* \in [0,1]$ for Foreign country respectively, (iii) a governmental structure which sets monetary policy. Except for consumption and labor, all lower case variables are defined as the ratio of nominal variables to the domestic money supply. Variables with asterisks denote for variables of the Foreign country. We introduce no shock into the model, so this chapter presents a deterministic analysis.

The representative household maximizes its utility derived from consumption and leisure subject to a cash-in-advance constraint and a standard budget constraint. The Home household has the utility function

$$U_0 = \sum_{t=0}^{\infty} \beta^t \left\{ \left(\frac{(c_{H,t})^{\alpha} (c_{F,t})^{\delta}}{\alpha^{\alpha} \delta^{\delta}} \right) - l_t \right\}, \tag{4.1}$$

and faces the following constraints:

$$c_{H,t} = \left[\int_0^1 c_t(j)^{(\theta-1)/\theta} dj \right]^{\theta/(\theta-1)}, \qquad c_{F,t} = \left[\int_0^1 c_t(j^*)^{(\theta-1)/\theta} dj^* \right]^{\theta/(\theta-1)}, \tag{4.2}$$

$$l_t = \int_0^1 l_t(j)dj, \tag{4.3}$$

$$\int_0^1 p_{H,t}(j)c_{H,t}(j)dj + \int_0^1 p_{F,t}(j^*)c_{F,t}(j^*)dj^* \leq m_t + (x_t - 1) + b_t - \frac{b_{t+1}x_t}{R_t}, \tag{4.4}$$

$$\int_0^1 p_{H,t}(j)c_{H,t}(j)dj + \int_0^1 p_{F,t}(j^*)c_{F,t}(j^*)dj^* + \frac{b_{t+1}x_t}{R_t} + m_{t+1}x_t \leq m_t + (x_t - 1) + b_t + \int_0^1 \zeta_t(j)dj + w_t l_t, \tag{4.5}$$

where Home's consumption bundle includes goods produced in Home, $c_{H,t}$ and in Foreign, $c_{F,t}$ (given in (4.2)), l_t is the number of hours supplied, m_{t+1} is the cash holdings to next period, and b_{t+1} is the bond holding to next period. Parameters α and δ are weights put on domestic and foreign goods, respectively and they are allowed to be different such that $\alpha \neq \delta$. We assume $\alpha, \delta \in (0,1)$ and $0 < \alpha + \delta < 1$. The Foreign utility function is that

$$U_0^* = \sum_{t=0}^{\infty} \beta^t \left\{ \left(\frac{(c_{F,t}^*)^\alpha (c_{H,t}^*)^\delta}{\alpha^\alpha \delta^\delta} \right) - l_t^* \right\}. \tag{4.6}$$

The cash-in-advance constraint (4.4) is used to explain why households hold real balance, and hence the welfare cost of expected inflation as households inefficiently economize their money holdings in response to an expected increase in the money growth rate which raises net nominal interest rate above zero.

R_t is the gross interest rate of a domestic bond, w_t the wage, $\zeta_t(j)$ the profits of firm j (defined below), and the aggregate price is given as follows:

$$p_{H,t} = \left[\int_0^1 p_{H,t}(j)^{1-\theta} dj \right]^{1/(1-\theta)}, \qquad p_{F,t}^* = \left[\int_0^1 p_{F,t}^*(j^*)^{1-\theta} dj^* \right]^{1/(1-\theta)}. \tag{4.7}$$

As in Ireland (1997), the money growth rate is bounded on both sides to guarantee a monetary equilibrium. The lower bound is imposed, so the net nominal interest rate $(R_t - 1)$ is non-negative in equilibrium, while the upper bound ensures that private agents never prohibit the use of money à la Calvo (1978). Let $\beta \in (0,1)$ be the subjective discount rate and $\theta > 1$ be the elasticity of substitution between goods produced within a country. The elasticity of substitution between domestic and foreign goods is unity.

The utility of Home household can be expressed in a recursive form which will be useful in a later analysis of this chapter:

$$v_H(m_t, x_t) = \left(\frac{(c_{H,t})^{\alpha}(c_{F,t})^{\delta}}{\alpha^{\alpha}\delta^{\delta}} \right) - l_t + \beta v_H(m_{t+1}, x_{t+1}). \tag{4.8}$$

Firm j production function is $y_t(j) = l_t(j)$. At time $t = 0, 1, 2, ...$ each firm sets $p_{H,t}(j)$ to maximize profits

$$\zeta_t(j) = [p_{H,t}(j) - w_t] y_t(j), \tag{4.9}$$

where $y_t(j) = c_H(j) + c_H^*(j)$. Free trade implies that the law of one price is satisfied for each good, $p_{H,t}(j) = S_t p_{H,t}^*(j)$ and $p_{F,t}(j^*) = S_t p_{F,t}^*(j^*)$. We then define the terms of trade, $\tau_t = p_{H,t}/(S_t p_{F,t}^*)$, as the ratio of export price to import price in which prices are measured in Home currency.

Departing from the single unit of governmental structure in Ireland (1997), our structure consists of two authorities in each country: a government and a central bank. The government is responsible for setting an initial delegation contract to the central bank at the beginning of the period. Following Jensen (1997), the government is then allowed to reappoint the central bank altering the initial contract, at a determined cost, after observing the prices set by firms. The initial delegation contracts are summarized by the variable $q_t^a \geq 1$ for the Home country and $q_t^{*a} \geq 1$ for the Foreign country, which determine the costs of the central banks from setting the money growth rate greater than β. The reappointment contract is described by the variables $q_t, q_t^* \geq 1$. If $q_t^a = q_t$ and $q_t^{a*} = q_t^*$, then the central banks are not reappointed while if $q_t^a \neq q_t$ and $q_t^{a*} \neq q_t^*$, the central bank contracts are altered at time t. It is worth to explain that the terminology "reappointment" in this chapter means the current central banker is dismissed and replaced by a new central banker.

Each central bank controls her money supply and makes a lump-sum transfer to the respective representative individual at the beginning of date $t = 0, 1, 2, \ldots$ This transfer is $(x_t - 1)M_t^s$ for the home central bank and $(x^* - 1)M_t^{*s}$ for the foreign central bank, where M_t^s and M_t^{*s} are, respectively, the per capita home money stock and foreign money stock at the beginning of time t and x_t and x_t^* are, respectively, the gross money growth rates in the home and foreign countries. So $M_{t+1}^s = x_t M_t^s$ and $M_{t+1}^{*s} = x_t^* M_t^{*s}$.

In the home country, the reappointment decision is made to maximize the welfare of the representative home household minus a penalty if the central bank is in fact reappointed. Hence, the Home government objective function is

$$V_G = \sum_{t=0}^{\infty} \beta^t \left\{ \left(\frac{(c_{H,t})^{\alpha}(c_{F,t})^{\delta}}{\alpha^{\alpha}\delta^{\delta}} \right) - l_t - \varphi g(q_t, q_t^a) \right\}. \tag{4.10}$$

Function $g(.)$ is a reappointment cost function dependent on the initial delegation contract captured by variable q_t^a, the reappointment contract represented by variable q_t, and the reappointment cost parameter denoted by φ. Parameter φ, which is assumed to be positive, determines the weight of the reappointment cost relative to the economy's welfare in the home government's objective function. A higher φ increases the difficulty in adjusting the monetary conditions after the setting of the nominal prices. This parameter is set to be nonnegative to capture the idea that the government will be punished for any deviation from its announcement.

The central bank sets the actual monetary policy to maximize the household welfare minus a linear adjustment cost, determined by the government, which punishes the central bank if the money supply is higher than the optimal commitment level, $x_t = \beta$. Thus, the central bank's objective function is

$$V_{CB} = \sum_{t=0}^{\infty} \beta^t \left\{ \left(\frac{(c_{H,t})^{\alpha}(c_{F,t})^{\delta}}{\alpha^{\alpha}\delta^{\delta}} \right) - l_t - \frac{q_t - 1}{p_{H,t}}(x_t - \beta) \right\}. \tag{4.11}$$

4.3 Monetary delegation with commitment

Under commitment, the optimization of government and central bank are expressed as follows. The Home government's delegation problem (GRO)) is

$$V_G = \max_{\{q_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left\{ \left(\frac{(c_{H,t})^{\alpha}(c_{F,t})^{\delta}}{\alpha^{\alpha}\delta^{\delta}} \right) - l_t - \varphi g(q_t, q_t^a) \right\}, \tag{4.12}$$

taking k_0^a and p_0 as given. Let $q = \{q_t\}_{t=0}^{\infty}$.

The initial appointment decision is also made to maximize the welfare of the household; however, this decision is made before prices are set. As the government acts strategically, it already predicts that it might reappoint the central bank in the future and takes that into account when setting the initial delegation parameter. Thus, the government's initial delegation problem (GDO) is

$$V_G = \max_{\{q_t^a\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left\{ \left(\frac{(c_{H,t})^{\alpha}(c_{F,t})^{\delta}}{\alpha^{\alpha}\delta^{\delta}} \right) - l_t - \varphi g(q_t, q_t^a) \right\}. \tag{4.13}$$

Let $q^a = \{q_t^a\}_{t=0}^{\infty}$. Define the government action at time t as $Q_t = \{q_t^a, q_t\}$ and $Q = \{Q_t | t = 0, 1, ...\}$.

The central bank's problem in the Home country (CBO) is given by

$$V_{CB} = \max_{\{x_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left\{ \left(\frac{(c_{H,t})^{\alpha}(c_{F,t})^{\delta}}{\alpha^{\alpha}\delta^{\delta}} \right) - l_t - \frac{q_t - 1}{p_{H,t}}(x_t - \beta) \right\}, \tag{4.14}$$

taking the current period delegation, q_0, and prices, $p_{H,0}$, as given. There exist counterpart optimization for the government and central bank in the foreign country.

When governments can commit to their future policies, the governments set policies once and for all and then firms, central banks and households make their decisions sequentially. As a consequence, reappointments do not happen or at time zero the governments can pre-commit to appoint the central banks with delegation parameter $\hat{q} = q_t^a = q_t$ in Home country and $\hat{q}^* = q_t^{*a} = q_t^*$ in Foreign country for all $t = \{0,1,2...\}$. For the central banks, let $x = \{x_t | t = 0,1,2,...\}$ and $x^* = \{x_t^* | t = 0,1,2,...\}$ denote an infinite sequence of the money growth rates in Home and Foreign country respectively, where $x_t \in [\beta, \bar{x}]$ and $x_t^* \in [\beta, \bar{x}]$ for all $t = 0,1,2,...$ Combining all of them, let $G = (x, Q, x^*, Q^*)$ be the world money policy of the entire governmental structure. It is worthwhile to note that when $q_t^a = q_t$ and $q_t^{*a} = q_t^*$, the delegation problem is equivalent to an extension along an international dimension of the Ramsey problem documented by Ireland (1997) in which government sets money growth directly, but in this chapter, it sets delegation parameters to maximize households' objective function.

We now define the rules and allocations for Home country, and those of Foreign country are analogous. In Home country, firms and households behavior, given a world policy $G = (x, Q, x^*, Q^*)$, are characterized by allocation rule $\pi^j(G)$, $j \in [0,1]$ and $\omega(G)$. The Home representative firm's rule $\pi^j(G)$ dictates how price, $p_{H,t}(j)$, $t = 0,1,2,...$, is chosen for each possible set of policy G . The representative household's rule $\omega(G)$ dictates choices for $\Omega_t = (c_{H,t}, c_{F,t}, c_{H,t}(j), c_{F,t}(j^*), l_t, m_{t+1}, b_{t+1})$, $t = 0,1,2,...$ for each possible world policy G. Let π refer to the set of function π^j for all $j \in [0,1]$. Then π and ω map world policy G into allocations (Π, Ω), where $\Pi = \{\Pi^j | j \in [0,1]\}$, $\Pi^j = \{p_{H,t}(j) | t = 0,1,2,...\}$, and $\Omega = \{\Omega_t | t = 0,1,2,...\}$. Hence, together with those from Foreign country the allocation $(\Pi, \Omega, \Pi^*, \Omega^*)$ portrays the sequence of equilibrium prices and quantities that achieve when governments implement the policy set, G, private sectors react based on π, ω, π^*, and ω^*.

Given G, $\pi^{\hat{j}}(G)$ for all $\hat{j} \neq j$, $\pi^{*j^*}(G)$ for all $j^* \in [0,1]$, $\omega(G)$, and $\omega^*(G)$, the Home representative firm's choice of $\pi^j(G)$ solves

(FO) Maximize (4.9) for each $t = 0, 1, 2, ...$, taking w_t, w_t^*, $p_{H,t}$, and $p_{F,t}^*$ as given for all $t = 0, 1, 2, ...$

Given G, $\pi^j(G)$ for all $j \in [0,1]$ and $\pi^{j^*}(G)$ for all $j^* \in [0,1]$, the Home representative household's choice of $\omega(G)$ solves

(HO) Maximize (4.1) subject to (4.2), (4.3), (4.4) and (4.5) for each $t = 0, 1, 2, ...$, taking as given $\zeta_t(j)$, $\zeta_t^*(j^*)$, w_t, w_t^*, R_t and R_t^* for all $j, j^* \in [0,1]$ and $t = 0, 1, 2, ...$

In addition, $\omega(G)$ must correspond with the appropriate market clearing conditions

(MO) $m_{t+1} = 1$, $b_{t+1} = 0$, $y_t = l_t$, $y_t = c_{H,t} + c_{H,t}^*$ and the currency exchange market clearing conditions

$$S_t = \frac{x_t}{x_t^*}, \tag{4.15}$$

which comes from the fact that in each country the value of export is equal to that of import denominated in the domestic currency. In the same fashion, for Foreign country we can derive the problem for firm and household, FO^* and HO^*, respectively together with the market clearing condition, MO^*.

An *equilibrium under commitment* consists for a set of world policy G and an allocation $(\Pi, \Omega, \Pi^*, \Omega^*)$ that satisfy: (i) Given G, $\Pi^{\hat{j}}$ for all $\hat{j} \in [0,1]$, $\hat{j} \neq j$, and $(\Omega, \Pi^*, \Omega^*)$, each Π^j solves (FO). Given G, $\Pi^{*\hat{j}^*}$ for all $\hat{j}^* \in [0,1]$, $\hat{j}^* \neq j^*$, and (Π, Ω, Ω^*), each Π^{*j^*} solves (FO^*); (ii) Given G and (Π, Π^*, Ω^*), Ω solves (HO). Given G and (Π, Ω, Π^*), Ω^* solves (HO^*); (iii) Ω and Ω^* correspond with (MO) and (MO^*), respectively. The definition does not require the policy set to be chosen optimally. Therefore, this definition allows us to derive the optimal monetary policy by first featuring the whole set of equilibria under commitment and then choosing the policy set that independently maximizes the utility functions in each country.

The necessary and sufficient conditions for a solution to the Household problem (HO and HO^*) collapse to:[1]

$$c_{H,t} = \frac{\alpha}{\alpha+\delta}\frac{x_t}{p_{H,t}}, \qquad c_{F,t} = \frac{\delta}{\alpha+\delta}\frac{x_t}{p_{F,t}}, \tag{4.16}$$

$$c^*_{F,t} = \frac{\alpha}{\alpha+\delta}\frac{x^*_t}{p^*_{F,t}}, \qquad c^*_{H,t} = \frac{\delta}{\alpha+\delta}\frac{x^*_t}{p^*_{H,t}}, \tag{4.17}$$

$$l_t = \frac{x_t}{p_{H,t}}, \qquad l^*_t = \frac{x^*_t}{p^*_{F,t}}, \tag{4.18}$$

$$w_t = \frac{x_t x_{t+1}}{\beta(\alpha+\delta)/(\alpha^\alpha\delta^\delta)c^\alpha_{H,t+1}c^\delta_{F,t+1}}, \quad w^*_t = \frac{x^*_t x^*_{t+1}}{\beta(\alpha+\delta)/(\alpha^\alpha\delta^\delta)c^{*\alpha}_{F,t+1}c^{*\delta}_{H,t+1}}, \tag{4.19}$$

$$\lim_{t\to\infty}\beta^t\frac{(\alpha+\delta)/(\alpha^\alpha\delta^\delta)(c_{H,t})^\alpha(c_{F,t})^\delta}{x_t} = 0, \quad \lim_{t\to\infty}\beta^t\frac{(\alpha+\delta)/(\alpha^\alpha\delta^\delta)(c^*_{F,t})^\alpha(c^*_{H,t})^\delta}{x^*_t} = 0. \tag{4.20}$$

For the firm's problem (FO and FO^*), the unique solution means that all firms charge the same of a fixed markup of price over marginal cost so that

$$p_{H,t} = \frac{\theta}{\theta-1}w_t, \qquad p^*_{F,t} = \frac{\theta}{\theta-1}w^*_t. \tag{4.21}$$

We now study the natural benchmark where governments set monetary policy independently. An *optimal non-cooperative equilibrium under commitment* is an *equilibrium under commitment* that satisfies the following: (i) taking as given q^{*a}, the Home government chooses q^a to maximize the Home welfare function minus initial delegation cost; taking as given q^*, the Home government chooses q to optimize the Home welfare function minus reappointment cost; taking as given x^*, the Home central bank chooses x to optimize the Home welfare function minus linear adjustment cost, and (ii) taking as given q^a, the Foreign

[1] see Appendix B.1 for derivation

government chooses q^{*a} to maximize the Foreign welfare function minus initial delegation cost; taking as given q, the Foreign government chooses q^* to maximize the Foreign welfare function minus reappointment cost; taking as given x, the Foreign central bank chooses x^* to maximize the Foreign welfare function minus linear adjustment cost.

Plugging the household's first-order conditions (4.16) and (4.18) into the Home central bank's objective function yields

$$V_{CB} = \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{(\alpha+\delta)^{\alpha+\delta}} \left(\frac{x_t}{p_{H,t}} \right)^{\alpha} \left(\frac{x_t^*}{p_{F,t}^*} \right)^{\delta} - \frac{x_t}{p_{H,t}} - \frac{q_t - 1}{p_{H,t}} (x_t - \beta) \right\}. \qquad (4.22)$$

The money growth rate in Home country at time $t = 0, 1, 2, ...$ is set such that

$$x_t = \left((\alpha+\delta)^{\alpha+\delta} / \alpha q_t \right)^{\frac{1}{\alpha-1}} \left(\frac{p_{F,t}^*}{x_t^*} \right)^{\frac{\delta}{\alpha-1}} p_{H,t}, \text{ for } \beta \leq x_t \leq \bar{x}. \qquad (4.23)$$

Similarly, the money growth in the Foreign country is as follows:

$$x_t^* = \left((\alpha+\delta)^{\alpha+\delta} / \alpha q_t^* \right)^{\frac{1}{\alpha-1}} \left(\frac{p_{H,t}}{x_t} \right)^{\frac{\delta}{\alpha-1}} p_{F,t}^*, \text{ for } \beta \leq x_t^* \leq \bar{x}. \qquad (4.24)$$

We assume that under commitment central banks set a constant money growth rate, $x_t = \hat{x}$ and $x_t^* = \hat{x}^*$ for all $t = 0, 1, 2, \ldots$. This assumption is theoretically justified as the central banks can follow the Friedman rule. In reality, it seems to fit the stylized facts that some central banks have announced to target a constant money growth rate. This assumption with (4.19) and (4.21) give the following difference equations:

$$p_{H,t} = \frac{\theta(\alpha+\delta)^{\alpha+\delta-1}}{(\theta-1)\beta} \hat{x}^{2-\alpha} \hat{x}^{*-\delta} p_{H,t+1}^{\alpha} p_{F,t+1}^{*\delta}, \qquad (4.25)$$

$$p_{F,t}^* = \frac{\theta(\alpha+\delta)^{\alpha+\delta-1}}{(\theta-1)\beta} \hat{x}^{*2-\alpha} \hat{x}^{-\delta} p_{F,t+1}^{*\alpha} p_{H,t+1}^{\delta}. \qquad (4.26)$$

Solving these first-order difference equations yields

$$p_{H,t} = \left(\frac{\theta(\alpha+\delta)^{\alpha+\delta-1}}{(\theta-1)\beta}\right)^{\frac{1}{1-\alpha-\delta}} \hat{x}^{1+\frac{1-\alpha}{(1-\alpha-\delta)(1-\alpha+\delta)}} \hat{x}^{*\frac{\delta}{(1-\alpha-\delta)(1-\alpha+\delta)}}, \tag{4.27}$$

$$p^*_{F,t} = \left(\frac{\theta(\alpha+\delta)^{\alpha+\delta-1}}{(\theta-1)\beta}\right)^{\frac{1}{1-\alpha-\delta}} \hat{x}^{*1+\frac{1-\alpha}{(1-\alpha-\delta)(1-\alpha+\delta)}} \hat{x}^{\frac{\delta}{(1-\alpha-\delta)(1-\alpha+\delta)}}. \tag{4.28}$$

Substituting (4.27) and (4.28) into (4.23) and (4.24), we obtain the optimal money growth rates

$$\hat{x} = \hat{q}\frac{(\theta-1)(1+\delta/\alpha)\beta}{\theta}, \qquad \hat{x}^* = \hat{q}^*\frac{(\theta-1)(1+\delta/\alpha)\beta}{\theta}, \tag{4.29}$$

in which the optimal money growth rate in Home country is independent of the Foreign country's money growth rate and vice versa. It is essential to stress that although the policy actions implemented by Home government affect the welfare of the Foreign agents, they do not affect the optimal money growth rate in the Foreign country.

A combination of the optimal money growth rate given by (4.29) and the central banks' utility functions with adjustment cost reveal the trade-off facing the governments when setting the delegation parameters. A higher delegation parameter leads to lower incentive of the central bank to deviate since it is more costly for them to do so in terms of welfare. Meanwhile, according to (4.29), the central banks set the money growth rates subject to the current level of delegation parameter. Hence, the governments have to balance the trade-off when designing the contract with the central banks.

Under commitment, an increase in Home openness, δ, causes Home central bank to inflate more because it can export the inflation cost abroad through the expenditure switching channel in which households in both countries will substitute away from the Home goods into the Foreign goods as the former becomes relatively expensive than the latter. When $\delta = 0$, optimal monetary growth rate coincides with Basso (2009)'s result. Basso argues that the optimal delegation parameter should be set equal to the monopolistic distortion which is the markup price to dampen the central bank's incentive to raise output and thereby restore the Friedman rule. He explains further that a larger monopolistic distortion in the goods market causes a severer inefficiency and tempts central bank to increase money supply. However, we have different explanation such that the monopolistic distortion reduces the central bank's incentive to run inflation. Arseneau (2007) claims that because the output and consumption are already far below the efficient level, the marginal utility of consumption is relatively high compared with the case of perfect competition, causing expected inflation more costly.

As the money growth rates are bounded on both sides, (4.29) implies that delegation parameters are too. Moreover, the money growth rate has a linear and positive relationship with the level of delegation parameter so that government cannot set an arbitrarily large value of the delegation since it dictates central bank to set a high money growth rate or even hit the upper bound with significantly high inflation. To guarantee the central banks set $\hat{x} = \beta$ and $\hat{x}^* = \beta$, the optimal choice to maximize consumption, the optimal delegation parameter need not be uniquely determined as in Basso (2009). Rather, it can be any number whose value lies between the lower bound and $\theta/[(\theta-1)(1+\delta/\alpha)]$. When δ approaches zero, the upper bound approaches the markup so that to achieve the Friedman rule, the government cannot set the delegation parameter exceeding the markup level which is the maximum of dampening effect caused by monopolistic competition. A higher openness, $\delta > 0$, narrows the range value of delegation parameter thereby lowers the government's power to design a contract with a central bank since a part of the monopolistic distortion is used to dampen the central bank's incentive to inflate economy strategically. In what follows, we simply set the optimal delegation parameter equal to the highest possible value restoring the Friedman rule such that $\hat{q} = \theta/[(\theta-1)(1+\delta/\alpha)]$ in Home country and $\hat{q}^* = \theta/[(\theta-1)(1+\delta/\alpha)]$ in Foreign country. The following proposition is summarized the above discussion:

Proposition 4.1 *The maximum of the optimal delegation parameter under commitment decreases in the openness of economy such that*

$$\frac{\partial \hat{q}}{\delta}, \frac{\partial \hat{q}^*}{\delta} < 0.$$

Because we substitute the first order conditions of households and firms into the central banks' and governments' problems, the allocation set $(\Pi, \Omega, \Pi^*, \Omega^*)$ that refers $\hat{q}$, $\hat{q}^*$, and $\hat{x} = \hat{x}^* = \beta$ is an equilibrium under commitment. As long as $\hat{q} = \theta/[(\theta-1)(1+\delta/\alpha)]$ and $\hat{q}^* = \theta/[(\theta-1)(1+\delta/\alpha)]$, central banks sequentially choose not to deviate from $\hat{x} = \hat{x}^* = \beta$. Based on that, we achieve a two-country economy version of the Ramsey outcome under commitment along the line of Ireland (1997) which includes a Friedman rule such that $x_t = x_t^* = \beta$ and zero net nominal interest rates $R_t = R_t^* = 1$ in both countries. In what follows, whenever we mention the Ramsey outcome, we refer to the outcome generated by the optimal commitment policy with no reappointment.

When governments cannot commit to their appointments, firms, households, central banks, and governments reoptimize their plans at each date $t = 0, 1, 2, ...$, based on the history of policy variables. While neither firms nor households aware the possible effects of their actions on decision making by governmental institutes and other private agents, the private sectors' expectations are influenced by the current policy decisions, which require the governments to take into account their reputation. In our framework, the source of time inconsistency problem shifts from the monetary policy decision to the delegation and therefore the idea of using delegation to eliminate the time-inconsistency problem was criticized by McCallum (1995) that this method only relocates the problem to the delegation decision making. Jensen (1997) is the first to formally model and verify the McCallum (1995)'s argument, but in the same context, Driffill and Rotondi (2006) prove that a tougher appointment strategy could remove the time-consistency problem at the expense of a constant reappointment. Basso (2009) develops a micro-founded model and confirms the Driffill and Rotondi (2006)'s unrealistic result in which he claims that a quadratic reappointment cost function with zero marginal cost of reappointing causes a constant reappointment when dealing with the time-consistency problem. To overcome this, Basso introduced an absolute functional form which creates a small jump in reappointing. The next section focuses on Basso's absolute value reappointment cost function in analyzing the effectiveness of delegation under the open economy environment.

4.4 Monetary delegation without commitment

In this section, we first define the concept of sustainable equilibrium. We then study the equilibrium under discretion by considering whether the monetary delegation approach can solve the time inconsistency problem, what problems are remaining, and what additional conditions are needed.

4.4.1 Sustainable equilibrium

We model the timing schedule with five stages as follows. First, the initial delegation parameters are chosen by the governments. Second, firms set prices. Third, the governments decide to whether reappoint the central bank or not. Fourth, central banks decide the actual money growth rate and fifth the households choose their allocations. In the absence of commitment technology, the optimal delegation policy under commitment suffers a time consistency problem. Since governments can observe the prices set by firms before deciding the reappointment and since the economy productions are sufficiently low due to the imperfect competition, the governments have incentives to set a higher delegation parameter than the optimal commitment one. After firms make price decisions, the government has an incentive to reappoint a central bank who will set the money growth rate higher than the commitment level. In equilibrium, firms understand these incentives and adjust their price accordingly. Hence, such a government's action only generates an inflationary bias. This mechanism is a modified version of time consistency problem pioneered by Barro and Gordon (1983a) due to the presence of monopolistic competition and sticky price.

For each $t = 0, 1, 2...$, denote the history pair of government policy up to time t by $\xi_t = (\xi_{H,t}, \xi_{F,t})$ where $\xi_{H,t} = \{(Q_s, x_s)|s = 0, 1, ..., t\}$, $\xi_{F,t} = \{(Q^*_s, x^*_s)|s = 0, 1, ..., t\}$, $\xi_{H,-1} = \emptyset$, and $\xi_{F,-1} = \emptyset$. Let a pair of of government delegation rule $(\varsigma_{H,t}, \varsigma_{F,t})$ be equal to $(\varsigma_{H,1}, \varsigma_{F,1})$, $(\varsigma_{H,2}, \varsigma_{F,2})$, ..., $(\varsigma_{H,t}, \varsigma_{F,t})$ where $\varsigma_{H,t}(\xi_{t-1})$ determines Home government delegation q^a_t at time t in Home country conditional on the history ξ_{t-1} and taking Foreign monetary policy at time t as given, and $\varsigma_{F,t}(\xi_{t-1})$ determines Foreign government delegation q^{*a}_t at time t in Foreign country conditional on the history ξ_{t-1} and taking Home monetary policy at time t as given.

In Home country, the representative Home firm, given the history ξ_{t-1}, q^a_t, and q^{*a}_t set prices for period t in the second stage. Let the firm's pricing rule be given by $\pi^j = \{\pi^j_t | t = 0, 1, 2...\}$, where $\pi^j_t(\xi_{t-1}, q^a_t, q^{*a}_t)$ determines the choice of $p_{H,t}(j)$ conditional on the history ξ_{t-1}, q^a_t, and q^{*a}_t. Given ξ_{t-1}, q^a_t, q^{*a}_t, and $p_{H,t}$, the Home government decides the new contract parameter for time t, q_t by using the reappointment rule $\rho_{H,t}(\xi_{t-1}, q^a_t, q^{*a}_t) = q_t$. After that, the Home central bank decides the Home monetary policy conditional on ξ_{t-1}, q^a_t, q^{*a}_t, q_t, and q^*_t, and taking the Foreign monetary policy as given. Denote the Home monetary policy rule be $\sigma_{H,t}(\xi_{t-1}, Q_t, Q^*_t) = x_t$. The future policy histories from $\xi_{t-1} = (\xi_{H,t-1}, \xi_{F,t-1})$ are achieved by using the Home policy plan ς^t_H, ρ^t_H, and σ^t_H to update $\xi_{H,t-1}$, and its Foreign counterparts to update $\xi_{F,t-1}$. Given the new policy history ξ_t, the allocation rule for the representative Home household is a sequence of function $\omega = \{\omega_t | t = 0, 1, 2...\}$, where $\omega(\xi_t)$ determines the choice of Ω_t conditional on ξ_t. Let a policy plan $(\varsigma_H, \rho_H, \sigma_H)$ be equal to $(\varsigma_{H,0}, \rho_{H,0}, \sigma_{H,0}, \varsigma_{H,1}, \rho_{H,1}, \sigma_{H,1}, ...)$, and denote $(\varsigma^t_H, \rho^t_H, \sigma^t_H)$ as the continuation of each policy at time t. Similar definitions apply for allocation rules π and ω.

Without commitment, dividends $\zeta_t(j) = \zeta_t(j, \xi_t)$, wage rates $w_t = w_t(\xi_t)$, interest rates $R_t = R_t(\xi_t)$, and the price level $p_{H,t} = p_{H,t}(\xi_{t-1}, q^a_t, q^{*a}_t)$ is also function of the policy history. At any date t and history ξ_{t-1}, private agents can use $(\varsigma_H, \rho_H, \sigma_H)$ and $(\varsigma_F, \rho_F, \sigma_F)$

to update the all possible future histories, $\xi_{H,t+s}$, $\xi_{F,t+s}$, $s = 0,1,2...$, and their knowledge of the functions $\zeta_{t+s}(j,\xi_{t+s})$, $w_{t+s}(\xi_{t+s})$, $R_t(\xi_{t+s})$, and $p_{H,t+s} = p_{H,t+s}(\xi_{t+s-1}, q^a_{t+s}, q^{*a}_{t+s})$. Analogously, we can obtain policy, price, and allocation rules in Foreign country.

The representative firm enters each period $t = 0,1,2...$ by taking as given ξ_{t-1}, q^a_t, q^{*a}_t, $(\varsigma_H, \rho_H, \sigma_H)$, $(\varsigma_F, \rho_F, \sigma_F)$, $\pi^{\hat{j}}$ for all $\hat{j} \in [0,1]$, $\hat{j} \neq j$, $\pi^{*j^*}(x,x^*)$ for all $j^* \in [0,1]$, ω, and ω^*, and makes a choice of π^{jt} to solve

(FT) Maximize

$$\zeta_{t+s}(j) = [p_{H,t+s}(j) - w_{t+s}]y_{t+s}(j), \tag{4.30}$$

for each $s = 0,1,2...$ taking $w_{t+s} = w_{t+s}(\xi_{t+s})$, $p_{H,t+s} = p_{H,t+s}(\xi_{t+s-1})$, $p_{t+s} = p_{t+s}(\xi_{t+s-1})$, and $\xi_{t+s} = (\xi_{t+s-1}, Q_{t+s}, Q^*_{t+s}, x_{t+s}, x^*_{t+s})$ where $q^a_{t+s} = \varsigma_{H,t+s}(\xi_{t+s-1})$, $q_{t+s} = \rho_{H,t+s}(\xi_{t+s-1}, q^a_{t+s}, q^{*a}_{t+s})$, $x_{t+s} = \sigma_{H,t+s}(\xi_{t+s-1}, Q_{t+s}, Q^*_{t+s})$, $q^{*a}_{t+s} = \varsigma_{F,t+s}(\xi_{t+s-1})$, $q^*_{t+s} = \rho_{F,t+s}(\xi_{t+s-1}, q^{*a}_{t+s}, q^a_{t+s})$, and $x^*_{t+s} = \sigma_{F,t+s}(\xi_{t+s-1}, Q_{t+s}, Q^*_{t+s})$ as given for all $s = 0,1,2...$

In each period $t = 0,1,2...$ the representative household takes ξ_t, $(\varsigma_H, \rho_H, \sigma_H)$, $(\varsigma_F, \rho_F, \sigma_F)$, π^j for all $j \in [0,1]$, π^{*j^*} for all $j^* \in [0,1]$ and ω^* as given and make a decision on ω^t to solve

(HT) Maximize

$$\sum_{s=0}^{\infty} \beta^s \left\{ \left(\frac{(c_{H,t+s})^{\alpha}(c_{F,t+s})^{\delta}}{\alpha^{\alpha}\delta^{\delta}} \right) - l_{t+s} \right\}, \tag{4.31}$$

subject to

$$c_{H,t+s} = \left[\int_0^1 c_{t+s}(j)^{(\theta-1)/\theta} dj \right]^{\theta/(\theta-1)}, \quad c_{F,t+s} = \left[\int_0^1 c_{t+s}(j^*)^{(\theta-1)/\theta} dj^* \right]^{\theta/(\theta-1)}, \tag{4.32}$$

$$l_{t+s} = \int_0^1 l_{t+s}(j) dj, \tag{4.33}$$

$$\int_0^1 p_{H,t+s}(j)c_{H,t+s}(j)dj + \int_0^1 p_{F,t+s}(j^*)c_{F,t+s}(j^*)dj^* \leq m_{t+s} + (x_{t+s}-1) + b_{t+s} - \frac{b_{t+s+1}x_{t+s}}{R_{t+s}}, \tag{4.34}$$

$$\int_0^1 p_{H,t+s}(j)c_{H,t+s}(j)dj + \int_0^1 p_{F,t+s}(j^*)c_{F,t+s}(j^*)dj^* \leq m_{t+s} + (x_{t+s}-1) + b_{t+s} + \int_0^1 \zeta_{t+s}(j)dj + w_{t+s}l_{t+s} - \frac{b_{t+s+1}x_{t+s}}{R_{t+s}} - m_{t+s+1}x_{t+s}, \tag{4.35}$$

for each $s = 0,1,2...$, taking $\zeta_{t+s}(j) = \zeta_{t+s}(j,\xi_{t+s})$, $\zeta^*_{t+s}(j^*) = \zeta^*_{t+s}(j^*,\xi_{t+s})$, $w_{t+s} = w_{t+s}(\xi_{t+s})$, $w^*_{t+s} = w^*_{t+s}(\xi_{t+s})$, $R_{t+s} = R_{t+s}(\xi_{t+s})$, $R^*_{t+s} = R^*_{t+s}(\xi_{t+s})$, and $\xi_{t+s} = (\xi_{t+s-1}, Q_{t+s}, Q^*_{t+s}, x_{t+s}, x^*_{t+s})$ where $q^a_{t+s} = \varsigma_{H,t+s}(\xi_{t+s-1})$, $q_{t+s} = \rho_{H,t+s}(\xi_{H,t+s-1}, q^a_{t+s}, q^{*a}_{t+s})$, $x_{t+s} = \sigma_{H,t+s}(\xi_{t+s-1}, Q_{t+s}, Q^*_{t+s})$, $q^{*a}_{t+s} = \varsigma_{F,t+s}(\xi_{t+s-1})$, $q^*_{t+s} = \rho_{F,t+s}(\xi_{t+s-1}, q^a_{t+s}, q^{*a}_{t+s})$, and $x^*_{t+s} = \sigma_{F,t+s}(\xi_{t+s-1}, Q_{t+s}, Q^*_{t+s})$ as given for all $j \in [0,1]$ and $s = 0,1,2...$

Moreover, for each $t = 0,1,2,...$ and ξ_t, the continuation policy ω^j must correspond with the market clearing conditions:

(MT) $m_{t+s+1} = 1$, $b_{t+s+1} = 0$, $y_{t+s} = l_{t+s}$, $y_{t+s} = c_{H,t+s} + c^*_{H,t+s}$ and the currency exchange market clearing conditions

$$S_{t+s} = \frac{x_{t+s}}{x^*_{t+s}}, \tag{4.36}$$

for all $s = 0,1,2,...$

In the same manner, for Foreign country we can characterize the problem for the firm and household, FT^* and HT^*, respectively together with the market clearing condition, MT^*.

In the first stage, at each date $t = 0,1,...$, under non-cooperation the Home government takes q^{*a}_t, ξ_{t-1}, π^j for all $j \in [0,1]$, π^{*j^*} for all $j^* \in [0,1]$, ω, and ω^* as given and decides a continuation policy ς^t_H to solve

(GDT) Maximize

$$\sum_{s=0}^{\infty}\beta^{s}\left\{\left(\frac{(c_{H,t+s})^{\alpha}(c_{F,t+s})^{\delta}}{\alpha^{\alpha}\delta^{\delta}}\right)-l_{t+s}-\varphi g(q_{t+s},q_{t+s}^{a})\right\}, \tag{4.37}$$

where c_{t+s} and l_{t+s} are determined by π^{j} for all $j\in[0,1]$, $\pi^{*j^{*}}$ for all $j^{*}\in[0,1]$, ω, and ω^{*} for all $s=0,1,2,...$

In the third stage, at each date $t=0,1,2,...$, under non-cooperation the Home government takes q_{t}^{*}, q_{t}^{a}, q_{t}^{*a}, ξ_{t-1}, π^{j} for all $j\in[0,1]$, $\pi^{*j^{*}}$ for all $j^{*}\in[0,1]$, ω, and ω^{*} as given and decides a continuation policy ρ_{H}^{t} to solve

(GRT) Maximize

$$\sum_{s=0}^{\infty}\beta^{s}\left\{\left(\frac{(c_{H,t+s})^{\alpha}(c_{F,t+s})^{\delta}}{\alpha^{\alpha}\delta^{\delta}}\right)-l_{t+s}-\varphi g(q_{t+s},q_{t+s}^{a})\right\}, \tag{4.38}$$

where c_{t+s} and l_{t+s} are determined by π^{j} for all $j\in[0,1]$, $\pi^{*j^{*}}$ for all $j^{*}\in[0,1]$, ω, and ω^{*} for all $s=0,1,2,...$

In the fourth stage, at each date $t=0,1,2,...$, under non-cooperation the Home central bank takes x_{t}^{*}, q_{t}^{a}, q_{t}^{*a}, q_{t}, q_{t}^{*}, ξ_{t-1}, π^{j} for all $j\in[0,1]$, $\pi^{*j^{*}}$ for all $j^{*}\in[0,1]$, ω, and ω^{*} as given and decides a continuation policy σ_{H}^{t} to solve

(CBT) Maximize

$$\sum_{s=0}^{\infty}\beta^{s}\left\{\left(\frac{(c_{H,t+s})^{\alpha}(c_{F,t+s})^{\delta}}{\alpha^{\alpha}\delta^{\delta}}\right)-l_{t+s}-\frac{q_{t+s}-1}{p_{H,t+s}}(x_{t+s}-\beta)\right\}, \tag{4.39}$$

where c_{t+s} and l_{t+s} are determined by π^{j} for all $j\in[0,1]$, $\pi^{*j^{*}}$ for all $j^{*}\in[0,1]$, ω, and ω^{*} for all $s=0,1,2,...$

Let π and π^{*} denote the set of function π^{j} for all $j\in[0,1]$ and $\pi^{*j^{*}}$ for all $j^{*}\in[0,1]$, respectively. As in Chari and Kehoe (1990), we use the concept of *sustainable equilibrium* to refer outcome that can predominate when governments cannot commit to their future policies. A *sustainable equilibrium* includes a set of policy plan $\sigma = (\varsigma_H, \varsigma_F, \rho_H, \rho_F, \sigma_H, \sigma_F)$ and a set of allocation rules $(\pi, \omega, \pi^*, \omega^*)$ that satisfy: (i) Given σ, $\pi^{\hat{j}}$ for all $\hat{j} \in [0,1]$, $\hat{j} \neq j$, π^{*j^*} for all $j^* \in [0,1]$, ω, and ω^*, the continuation π^{jt} of each π^j solves (FT) for all $t = 0,1,...$ and ξ_{t-1}. Given σ, π^j for all $j \in [0,1]$, $\pi^{*\hat{j}^*}$ for all $\hat{j}^* \in [0,1]$, $\hat{j}^* \neq j^*$, ω, and ω^*, the continuation π^{*j^*t} of each π^{*j^*} solves (FT^*) for all $t = 0,1,...$ and ξ_{t-1}; (ii) Given σ, π^j for all $j \in [0,1]$, π^{*j^*} for all $j^* \in [0,1]$, and ω^*, the continuation ω^t of ω solves (HT) for all $t = 0,1,...$ and ξ_t. Given σ, π^j for all $j \in [0,1]$, π^{*j^*} for all $j^* \in [0,1]$, and ω, the continuation ω^{*t} of ω^* solves (HT^*) for all $t = 0,1,...$ and ξ_t; (iii) the continuation of ω and ω^* are corresponding with (MT) and (MT^*), respectively, for all $t = 0,1,...$ and ξ_t; (iv) Given $(\varsigma_F, \rho_H, \rho_F, \sigma_H, \sigma_F)$, π^j for all $j \in [0,1]$, π^{*j^*} for all $j^* \in [0,1]$, ω, and ω^*, the continuation ς_H^t solves (GDT) for all $t = 0,1,...$ and ξ_{t-1}; Given $(\varsigma_H, \rho_H, \rho_F, \sigma_H, \sigma_F)$, π^j for all $j \in [0,1]$, π^{*j^*} for all $j^* \in [0,1]$, ω, and ω^*, the continuation ς_F^t solves (GDT^*) for all $t = 0,1,...$ and ξ_{t-1}; (v) Given $(\varsigma_H, \varsigma_F, \rho_F, \sigma_H, \sigma_F)$, π^j for all $j \in [0,1]$, π^{*j^*} for all $j^* \in [0,1]$, ω, and ω^*, the continuation ρ_H^t solves (GRT) for all $t = 0,1,...$ and ξ_{t-1}; Given $(\varsigma_H, \varsigma_F, \rho_F, \sigma_H, \sigma_F)$, π^j for all $j \in [0,1]$, π^{*j^*} for all $j^* \in [0,1]$, ω, and ω^*, the continuation ρ_F^t solves (GRT^*) for all $t = 0,1,...$ and ξ_{t-1}; (vi) Given $(\varsigma_H, \varsigma_F, \rho_H, \rho_F, \sigma_F)$, π^j for all $j \in [0,1]$, π^{*j^*} for all $j^* \in [0,1]$, ω, and ω^*, the continuation σ_H^t solves (CBT) for all $t = 0,1,...$ and ξ_{t-1}; Given $(\varsigma_H, \varsigma_F, \rho_H, \rho_F, \sigma_H)$, π^j for all $j \in [0,1]$, π^{*j^*} for all $j^* \in [0,1]$, ω, and ω^*, the continuation σ_F^t solves (CBT^*) for all $t = 0,1,...$ and ξ_{t-1};

A sustainable equilibrium under non-cooperation $(\sigma, \pi, \omega, \pi^*, \omega^*)$ induces a *sustainable outcome*, $(G, \Pi, \Omega, \Pi^*, \Omega^*)$ which is defined as follows. From $\xi_{-1} = \emptyset$, compute $\xi_t = (\xi_{H,t}, \xi_{F,t})$ where $\xi_H = \{\xi_{H,t} | t = 0,1,...\}$ and $\xi_F = \{\xi_{F,t} | t = 0,1,...\}$, and compute $G = \{G_t | t = 0,1,...\}$ where $G_t = (x_t, Q_t, x_t^*, Q_t^*)$ by employing $q_t^a = \varsigma_{H,t}(\xi_{t-1})$, $q_t^{*a} = \varsigma_{F,t}(\xi_{t-1})$, $q_t = \rho_{H,t}(\xi_{t-1}, q_t^a, q_t^{*a})$, $q_t^* = \rho_{F,t}(\xi_{t-1}, q_t^a, q_t^{*a})$, $x_t = \sigma_{H,t}(\xi_{t-1}, Q_t, Q_t^*)$, $x_t^* = \sigma_{F,t}(\xi_{t-1}, Q_t, Q_t^*)$, $\xi_{H,t} = (\xi_{H,t-1}, q_t^a, q_t, x_t)$, and $\xi_{F,t} = (\xi_{F,t-1}, q_t^{*a}, q_t^*, x_t^*)$. After that, for all $t = 0,1,...$, compute $\Pi = \{\Pi^j | j \in [0,1]\}$, $\Pi^j = \{p_{H,t}(j) | t = 0,1,...\}$ and $\Omega = \{\Omega_t | t =$

$0, 1, \ldots\}$ using $p_{H,t}(j) = \pi_t^j(\xi_{t-1}, q_t^a, q_t^{*a})$ for all $j \in [0,1]$ and $\Omega_t = \omega_t(\xi_t)$. Similarly we can get Π^* and Ω^*. Hence, the sustainable outcome $(\sigma, \pi, \omega, \pi^*, \omega^*)$ portrays the sequence of equilibrium prices and quantities that achieves when governments sets policy independently and private sectors response optimally.

4.4.2 Equilibrium under discretion

We now examine whether the monetary delegation approach can solve the time inconsistency problem under discretion. In addition to the sustainable equilibrium, we employ the concept of Markov equilibrium contributed by the work of Albanesi et al. (2003) to analyze the monetary policy under discretion because in the absence of state variables as in our model, the Markov equilibrium coincides with an equilibrium under discretion. The two elegant characteristics of Markov equilibrium simplify the analysis. The link between the two equilibrium concepts are given by the following proposition:

Proposition 4.2 *The Markov equilibrium is also a sustainable equilibrium.*

This proposition is proved by Basso (2009) in a closed economy. The same logic is applied to prove for the two-country model as we focus on Nash games among policymakers.

The optimization problem of the governmental institute is static because we concentrate on Markov equilibria and state variables are absent in the economy of our model. In the absence of the state variables, the policymakers simply choose the current policy variable to maximize the current period welfare. To derive the Markov equilibrium, we adjust the Home government's and the Home central bank's problem in which they maximize the value function of their own households, $v_H(m_t, x_t)$, minus the reappointment cost for the Home government and punishment by a deviation for the Home central bank such that:

$$\max_{q_t^a} \left(v_H(1, x_t) - \varphi g(q_t^a, q_t) \right), \tag{4.40}$$

$$\max_{q_t} \left(v_H(1, x_t) - \varphi g(q_t^a, q_t) \right), \tag{4.41}$$

$$\max_{x_t} \left(v_H(1, x_t) - \frac{q_t - 1}{p_{H,t}} (x_t - \beta) \right). \tag{4.42}$$

A *Markov outcome* is a set of $\{x, Q, \Pi, \Omega, x^*, Q^*, \Pi^*, \Omega^*\}$ such that $\{\Pi, \Omega, \Pi^*, \Omega^*\}$ satisfies conditions (4.16) - (4.21), and the monetary policy decisions of the whole governmental structure, $\{x, Q, x^*, Q^*\}$, solves (4.40), (4.41), (4.42), and their Foreign analog. A *Markov equilibrium* includes a set of strategies for the governments at stages 1 and 3, for the central banks, for the households, and for the firms, which generates a Markov outcome.

It is worthwhile to stress the two characteristics of a Markov equilibrium without state variables. First, households take as given the set of future policy choices $\{x, Q, x^*, Q^*\}_{t+1}^{\infty}$ and so do the governmental structure when they maximize $v_{H,t}(1, x_t)$ and $v_{F,t}(1, x_t)$. Second, the current policy $\{q^a, q_t, x_t, q^{*a}, q_t^*, x_t^*\}$ has no effect on $\{x_{t+1}, x_{t+1}^*\}$ and thereby has no effect on the lifetime utility of the households from period $t+1$ onwards. As a result, in a Markov equilibrium, each institute within the governmental structure maximizes the current one-period utility of its own household minus its corresponding adjustment cost.

We follow Basso (2009) to investigate the monetary delegation problem with two different functional forms, quadratic and absolute value, under the two-country model.

Quadratic reappointment cost function

We define the reappointment cost function for the home country (the foreign country is same) as $g(q_t, q_t^a) = (z_t - z_t^a)^2/2$, where $z_t^a = (\alpha + \delta)(q_t^a(1 + \delta/\alpha))^{1/(\alpha+\delta-1)}$ and $z_t = (\alpha + \delta)(q_t(1 + \delta/\alpha))^{1/(\alpha+\delta-1)}$. Variable z_t is the real money growth in terms of domestic goods $x_t/p_{H,t}$. In equilibrium, z_t can also be interpreted as the level of labor supply and the outcome of domestic economy. This mapping shows a negative relationship between the delegation parameter and the real money growth. The function $g(q_t, q_t^a)$ may seem to imply that only the distance between q_t^a and q_t matters so that the government can set an arbitrarily high value of q_t^a to prevent the central bank from deviation and then slightly reduces the delegation parameter to q_t. However, the significantly high level of initial delegation parameter makes firms to expect a more contractionary policy than the Friedman rule, demand less labor and then produce even further below the efficient level. The central bank has no incentive to boost the economy because any deviation from the Friedman rule is very costly. Hence, although the government successfully prevents the central bank's deviation, the economy becomes worse. For some arbitrary delegation parameter, we have the following proposition:

Proposition 4.3 *The outcome under discretionary equilibrium obtained by an implementation of the Friedman rule, $x_t = x_t^* = \beta$, is a Markov outcome and therefore a sustainable outcome.*

The sustainable Markov outcome described in Proposition 4.3 is identical to the Ramsey outcome except for the initial delegation parameters of the governments in stage 1. In particular, while the reappointment delegation parameters are identical, $q_t = q_t^* = \theta/[(\theta - 1)(1+\delta/\alpha)]$, the initial delegation parameters are higher than those under commitment

$$q_t^a = q_t^{*a} = \left(\left(\frac{\theta\alpha}{(\theta-1)(\alpha+\delta)} \right)^{1/(\alpha+\delta-1)} + \frac{1}{\varphi(\alpha+\delta)} \left(\frac{\alpha}{\alpha+\delta} \right)^{1/(\alpha+\delta-1)} \left(1 - \frac{\theta\alpha}{(\theta-1)(\alpha+\delta)} \right) \right)^{\alpha+\delta-1} . \tag{4.43}$$

As $q_t^a > q_t$ and $q_t^{*a} > q_t^*$, Proposition 4.3 indicates that with the quadratic reappointment cost function, the governments decide a greater initial delegation than the optimal one under commitment, and hence they will reappoint the central banks by resetting the reappointment parameter equal to the optimal level of commitment after firms fix their prices. A decrease in q^a and q^{*a} will lower the reappointment cost, but it comes at the cost of higher aggregate prices and therefore reduces the governments' welfare. If the initial delegation is set equal to the optimal level under commitment, this is lower than needed to obtain the commitment equilibrium. In essence, in our type of models, the equilibrium money growth rate equals either lower or upper bound due to the existence of expectation trap (Albanesi et al., 2003; Basso, 2009; Arseneau, 2012). Smaller initial delegations tempt governments to reappoint central banks who will set the money growth rates beyond the commitment level. As a consequence, the global economy ends up with high inflation. When governments move first by setting q^a and q^{*a} as in Proposition 4.3, this is enough to dictate their behaviors in the reappointment stage such that they have no incentive to reappoint central banks who tend to boost output up to the efficient level after prices are set. We do not need the additional constraint imposed on the reappointment cost parameter, φ.

Even though the Ramsey outcome can be obtained with the absence of a commitment technology, it is unrealistic that the central banks are reappointed in each period. This unrealistic feature happens because the quadratic reappointment cost function is a continuous function, so the marginal cost of reappointing is approximately zero as q_t^a and q_t are close together. However, in reality, even the new central bank seems to similar to its predecessor, there is still a negligible cost to the first-order approximation which can be treated as a fixed cost of reappointment.

To capture such a cost, we consider the reappointment cost that has a non-zero marginal cost. The absolute value reappointment cost function owns this property, and therefore using this form will deal with the constant reappointment problem.

Absolute value reappointment cost function

Let $g(q_t, q_t^a) = z_t^a |z_t - z_t^a|$, where $z_t^a = (\alpha + \delta)(q_t^a(1 + \delta/\alpha))^{1/(\alpha+\delta-1)}$ and $z_t = (\alpha + \delta)(q_t(1 + \delta/\alpha))^{1/(\alpha+\delta-1)}$. Substituting (4.18) and (4.23) into the the problem of Home government at stage 3, then using the definition of z_t and z_t^* gives us

$$\max_{z_t} \frac{1}{(\alpha+\delta)^{\alpha+\delta}} z_t^{\alpha} z_t^{*\delta} - z_t - \varphi z_t^a |z_t - z_t^a|, \tag{4.44}$$

which yields the following first-order condition due to the discontinuity of the objective function at $z_t = z_t^a$

$$z_t = \begin{cases} \left(\dfrac{(1+\varphi z_t^a)(\alpha+\delta)^{\alpha+\delta}}{\alpha z_t^{*\delta}} \right)^{1/(\alpha-1)}, \text{ if } z_t > z_t^a \\ z_t^a \; otherwise. \end{cases} \tag{4.45}$$

For a given value of φ and initial delegation parameter q_t^a, the reappointment cost may outweigh the welfare gain from inflation surprise. Hence, it is optimal for Home government to maintain the contract with the Home central bank, $q_t = q_t^a$. This case happens when φ is bounded below by $\underline{\varphi} = (\theta/(\theta-1))^{1/(1-\alpha-\delta)}(\theta/[(\theta-1)(1+\delta/\alpha)]-1)/(\alpha+\delta)$:

Proposition 4.4 *If $\varphi > \underline{\varphi}$ and delegation contract such that $q_t^a = q_t = \theta/[(\theta-1)(1+\delta/\alpha)]$, and $q_t^{*a} = q_t^* = \theta/[(\theta-1)(1+\delta/\alpha)]$, the outcome under discretionary equilibrium obtained by an implementation of the Friedman rule, $x_t = x_t^* = \beta$, is a Markov outcome and hence a sustainable outcome.*

Compared with Proposition 4.3, the sustainable Markov outcome described in Proposition 4.4 is precisely identical to the Ramsey outcome so that the welfare of governments, central banks, firms, and households are equal to the respective levels under commitment. Proposition 4.4 provides a formal setup to Persson and Tabellini (1999)'s argument that a sufficient cost caused by a change in institutional setup might help to overcome the McCallum (1995)'s criticism. If $\varphi > \underline{\varphi}$, the commitment outcome is obtained even under discretion. Moreover, this equilibrium is a self-sustained process without the support of any trigger-type strategy. Similar to the case of quadratic reappointment cost, delegation here keeps monetary institutions away from inducing optimal surprise since the governments' welfare function is discontinuous, which implies that the reappointment may bear a fixed cost. Hence, there is a threshold of reappointment parameter such that reappointment costs are high enough to prevent any reappointment and so we obtain the commitment outcome. This result is also pointed out in Basso (2009)'s closed economy with the lower bound of reappointment cost parameter, $\underline{\varphi}^{closed} = (\theta/(\theta-1))^{1/(1-\alpha)}(\theta/(\theta-1)-1)/\alpha$. We can show that $\underline{\varphi} < \underline{\varphi}^{closed}$. Furthermore, we can express the relationship between openness and such a threshold in the following proposition:

Proposition 4.5 *Holding $\alpha+\delta$ fixed, the more open the economy becomes, the less the minimum threshold of reappointment cost parameter is required for Ramsey outcome being a sustainable outcome.*

The fixed $\alpha+\delta$ condition is reasonable since for a given income, an increase in expenditure for the foreign goods implies a decrease in the money spent for the domestic goods.

Because the monopolistic distortion causes the economy to produce at a suboptimally low level, the central bank has a temptation to run inflationary surprise to boost the economy. Unexpected inflation acts as a consumption subsidy that improves welfare by raising output closer to the efficient level. Alternatively, unexpected inflation is regarded as a tax on domestic leisure, causing households to work more. This mechanism is labeled as a standard New Keynesian aggregate demand channel (Arseneau, 2012) due to the coexistence of sticky price and monopolistic competition. In essence, an unexpected expansion in Home country induces a depreciation in Home currency which raises the price of imports in Home country and contribute to inflation. Because of nominal price rigidities, the aggregate prices of Home goods are fixed within a period, the aggregate prices of Home consumption goods bundle increase only a fraction of share of expenditure on Foreign goods. The Home household's real money balance, thus, grows with the unexpected expansion. Simultaneously, as an increase in money supply dampens the real interest rate, consumption grows proportionally in both countries. This rise in consumption plus a relatively expensive price of Foreign goods boost the demand for Home goods, raising the Home output.

As consumption goods are traded among countries, the benefit of consumption subsidy is shared across borders proportionally. Meanwhile, when a central bank induces unanticipated inflation, its residents have to bear the entire burden of leisure tax as leisure is a non-traded good and hence work harder than the foreign counterpart to consume an additional good. This transmission can be labeled as strategic terms of trade channel which dampens the central banks' incentive to inflate the economy unexpectedly. The more open the economy is, the less consumption subsidy this economy can enjoy since domestic good accounts for a smaller part of domestic expenditure, and hence the less the incentive the central bank induces inflationary surprise. As a result, a smaller reappointment cost is needed to prevent the government from reappointing the central bank.

When $\varphi < \underline{\varphi}$, governments find them optimal to set $q_t < q_t^a$ and $q_t^* < q_t^{*a}$, and reappoint the central banks. With such a low φ, there exists no optimal value of the initial delegation parameter, q_t^a and q_t^{*a}, that would dictate prices being set subject to an expectation of monetary policy which is truly verified by the central banks except for $x_t = x_t^* = \bar{x}$. The driving force of this result is the existence of the expectation trap in our model that induces a Markov equilibrium to happen at the bounds of the money supply. In particular, the central bank always has incentive to raise the money growth rate and firms form the expectations monetary policy $x_t = x_t^* = \bar{x}$ for all periods. The unique Markov equilibrium occurs at the high money growth rate regardless of whether the model uses the monetary delegation or not. As a consequence, the delegation approach itself cannot help to remove the time inconsistency. In this case, we employ the method of Chari and Kehoe (1990) to characterize the set of sustainable outcome. They showed that any sustainable outcome could be obtained by the support of reputational equilibrium in which the expectations of private sectors obey the trigger-like behavior: when government unilaterally deviates from its promised plan, economy reverts to the worst possible outcome forever afterward. Therefore, finding the worst possible outcome is a critical step in characterizing the set of sustainable outcome. This step requires a specific form of governments' utility function which explains why we put this analysis here within the analysis of the absolute value reappointment cost rather than right after the definition of sustainable outcome in the previous subsection.

We start by defining the set of *autarky plans under non-cooperation* $(\sigma^A, \pi^A, \omega^A, \pi^{*A}, \omega^{*A})$ where $\sigma^A = (\varsigma_H^A, \varsigma_F^A, \rho_H^A, \rho_F^A, \sigma_H^A, \sigma_F^A)$. Let $\sigma^A = \{\sigma_t^A | t = 0, 1, ...\}$ have $\varsigma_{H,t}^A(\xi_{t-1}) = 1$, $\varsigma_{F,t}^A(\xi_{t-1}) = 1$, $\rho_{H,t}^A(\xi_{t-1}) = 1$, $\rho_{F,t}^A(\xi_{t-1}) = 1$, $\sigma_{H,t}^A(\xi_{t-1}) = \bar{x}$ and $\sigma_{F,t}^A(\xi_{t-1}) = \bar{x}$ for all $t = 0, 1, ...$ and $\xi_{t-1} = (\xi_{H,t-1}, \xi_{F,t-1})$. Given σ^a, let each π^{jA}, $j \in [0,1]$ and π^{*j^*A}, $j^* \in [0,1]$ be the allocation rules such that their continuations π^{jAt} and π^{*j^*At} solve (FT) and (FT^*) for all $t = 0, 1, ...$ and $\xi_{t-1} = (\xi_{H,t-1}, \xi_{F,t-1})$. Let ω^A and ω^{*A} be the allocation rules such that their continuations ω^{At} and ω^{*At} solves (HT) and (HT^*) and are consistent with (MT) and (MT^*), respectively, for all $t = 0, 1, ...$ and $(\xi_{H,t}, \xi_{F,t})$. With the definition of sustainable equilibrium, the following proposition indicates that the autarky plans induce a sustainable equilibrium.

Proposition 4.6 *The autarky equilibrium under non-cooperation* $(\sigma^A, \pi^A, \omega^A, \pi^{*A}, \omega^{*A})$ *is a Markov equilibrium, and therefore, a sustainable equilibrium if* $\bar{x} > \beta(\theta - 1)(\alpha + \delta)/(\theta\alpha)$.

As $\varphi < \underline{\varphi}$, it is optimal for governments to reappoint the current central banks to less conservative ones. Hence, central banks always have the incentive to introduce inflationary surprise. Private sectors react by adjusting their expectation accordingly, and then central banks tempt to raise the money growth rates even higher. The resulting monetary policy under discretion is $(\bar{x}, \bar{x})$. When $\bar{x}$ becomes arbitrarily large, both consumption and labor are approaching zero. Basso (2009) proves the following result: The autarky equilibrium is the worst sustainable equilibrium for all players. We extend this result to our two-country framework. Chari and Kehoe (1990) suggest that policy inducing the reversion to such a worst outcome can be employed to feature the outcome with better levels of welfare.

We now provide the intuition of the revert-to-autarky plans under non-cooperation while the formal setup is put in the appendix. Given an arbitrary world policy $G = (x, Q, x^*, Q^*)$ and an arbitrary allocation $(\Pi, \Omega, \Pi^*, \Omega^*)$, the revert-to-autarky plans induces the continuation of outcome $(G, \Pi, \Omega, \Pi^*, \Omega^*)$ as long as the world policy (x, Q, x^*, Q^*) has been chosen in the past. If governments choose not to follow the announced policy, the strategy dictates to revert to autarky plan in both countries forever.

Proposition 4.7 *Let $G = (x, Q, x^*, Q^*)$ be an arbitrary world policy and $(\Pi, \Omega, \Pi^*, \Omega^*)$ be an arbitrary allocation. Then $(G, \Pi, \Omega, \Pi^*, \Omega^*)$ is the outcome of a sustainable equilibrium if and only if*

(i) $(G, \Pi, \Omega, \Pi^*, \Omega^*)$ *is an outcome of equilibrium under commitment.*

(ii) $(G, \Pi, \Omega, \Pi^*, \Omega^*)$ *satisfies the following four inequalities for every t*

$$V_G^S \geq U_{G,t}^d + \beta V_G^A, \tag{4.46}$$

$$V_G^{*S} \geq U_{G,t}^{*d} + \beta V_G^{*A}, \tag{4.47}$$

where V_G^S and V_G^{*S} equal to the total utility under the sustainable equilibrium from time t onward, $U_{G,t}^d$ and $U_{G,t}^{*d}$ equal to the gain from deviating from the sustainable equilibrium in the current period t and V_G^A and V_G^{*A} equal to the total utility under the autarky equilibrium from time $t+1$ onward for the governments.

It is worth discussing some main points. Firstly, since Proposition 4.7 characterizes the sustainability of non-cooperative outcome under commitment, we do not need the incentive compatibility constraint for central banks. If inequalities (4.46) and (4.47) hold, it is optimal for the central banks to confirm the sustainable outcome. In detail, given $q_t = \theta/[(\theta-1)(1+\delta/\alpha)]$, the best response of the Home central bank at time t is to set $x_t = \beta$, validating the sustainable outcome. Put it differently, deviation is not a welfare improvement for the central bank at period t. Secondly, Proposition 4.7 does not only characterize the set of sustainable outcomes induced by a trigger-like behavior but also captures the outcome given by Proposition 4.3 and 4.4. Lastly, the main feature of the revert-to-autarky plan is that government defection triggers the autarky plan. With this plan, after the governments' defection at stage 3, the central banks will confirm the defection by setting $x_t = x_t^* = \bar{x}$.

Given this general characterization of sustainable outcomes one can now verify under what conditions the commitment equilibrium is sustainable. In the home country the commitment outcome, as shown in Section 4.3, is $c_{H,t} = \bar{c}_H$, $c_{F,t} = \bar{c}_F$, $x_t = x_t^* = \beta$, $p_{H,t} = p_{F,t}^* = \underline{p}_H = \beta/(\alpha+\delta)(\theta/(\theta-1))^{1/(1-\alpha-\delta)}$, and $q_t^a = q_t = \theta/[(\theta-1)(1+\delta/\alpha)]$. That way, $V_G^S = (1/(1-\beta))(\bar{c}_H^{\alpha}\bar{c}_F^{\delta}/(\alpha^{\alpha}\delta^{\delta}) - (\alpha+\delta)/\alpha\bar{c}_H$. Under autarky the utility is given by $V_G^A = (1/(1-\beta))(\underline{c}_H^{\alpha}\underline{c}_F^{\delta}/(\alpha^{\alpha}\delta^{\delta}) - (\alpha+\delta)/\alpha\underline{c}_H$. Note that the reappointment costs are zero in both V_G^S and V_G^A since $q_t^a = q_t$. Finally, the gain from deviation, $U_{G,t}^d$, is equal to $\left[\tilde{z}_t^{\alpha}z_t^{*\delta}/(\alpha+\delta)^{\alpha+\delta} - \tilde{z}_t - \varphi z_t^a|(\tilde{z}_t - z_t^a)|\right]$, where $\tilde{z}_t$ is set according to (4.45) taking $z_t^a = z_t^* = (\alpha+\delta)(\theta/(\theta-1))^{1/(\alpha+\delta-1)}$ as given. The necessary and sufficient condition for not reappointing is, letting $\bar{x}$ be arbitrarily large,

$$\begin{aligned}\frac{1}{1-\beta} \geq \left(\frac{\theta}{(\theta-1)(\alpha+\delta)} - 1\right)^{-1} \frac{\theta\alpha}{(\theta-1)(\alpha+\delta)} \left(\frac{\theta\alpha}{(\theta-1)(\alpha+\delta)\left(1+\varphi(\alpha+\delta)\left(\frac{\theta}{\theta-1}\right)^{\frac{1}{\alpha+\delta-1}}\right)}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{1}{\alpha} - 1\right) \\ + \left(\frac{\theta}{(\theta-1)(\alpha+\delta)} - 1\right)^{-1} \varphi(\alpha+\delta)\left(\frac{\theta}{\theta-1}\right)^{\frac{1}{\alpha+\delta-1}},\end{aligned} \tag{4.48}$$

which gives rise to the following proposition.

Proposition 4.8 *Holding $\alpha+\delta$ fixed, the more open the economy, the less likely the central bank to be reappointed.*

When the economy becomes more open, δ is increasing, the right-hand side of the constraint (4.48) decreases, making this constraint more likely to hold. Alternatively, when δ decreases, a smaller discount factor, β, is required to keep the sustainability constraint (4.48) to be satisfied. A fall in β reduces the cost of reverting to the autarky plan forever.

Proposition 4.8 implies that the maintenance of the contract between the current central bank and government is more credible or more sustainable when the economy becomes more open. The reason is that the competition mechanism under open economy lowers the short-run benefits from inflationary surprise, and thus make the equilibrium under commitment more sustainable when the commitment technologies are absent. In essence, there are two opposite mechanism channels under open economy as described in the previous section: standard New Keynesian demand channel causes an inflationary bias whereas strategic terms of trade channel creates a deflationary bias. When the economy is more open, the effects of the former channel are relatively weak compared with those of the latter. As δ is bigger, the increase in prices of imports accounts for a larger fraction of the aggregate prices of domestic consumption bundle, lowering the increase in the real money balance and lessening the decrease in the real interest rate. Consumption, therefore, grows at a lower level. In brief, an increase in openness reduces the effect of unexpected expansion transmitted through the standard New Keynesian demand channel.

Simultaneously, a higher openness exacerbates the deflationary bias under the strategic terms of trade channel as the Home households enjoy a less consumption subsidy but must work more than before. Putting those channels together, when the economy becomes more open, the central bank has a less incentive to induce inflationary surprise, and in terms of welfare, the short-run benefits from deviation become smaller. As a consequence, the maintenance of central bank becomes more likely to be sustainable in a more open economy.

4.5 Conclusion

This chapter extends Basso (2009)'s one-country model to two-country model to study the effects of openness on the effectiveness of monetary delegation as a solution to time consistency problem in a micro-founded New Keynesian. As the economy becomes more open, the delegation approach is easier to achieve the first best solution, the Ramsey outcome. First, to ensure the Ramsey outcome to be a sustainable outcome, it requires a lower value of minimum threshold of reappointment . Second, when the reappointment cost parameter is less than the threshold, the central bank is less likely to be reappointed.

The fundamental message of this chapter is that international linkage can help improve the effectiveness of delegation in solving the commitment problem. Indeed, the coexistence of two opposing policy transmission channels under open economy dampens the central banks' temptation to raise inflation unexpectedly, and therefore lowers the benefits of any defection. Taking this argument into account, any other adjustments in an economic framework that generates a trade-off of an inflationary surprise would also help to enhance the effectiveness of delegation.

Chapter 5

Conclusion and Policy Implications

This dissertation analyzes the time-consistency problem in the context of open economy framework with strategic behavior among policymakers. We aim to contribute to the study of counterproductive cooperation and of the effectiveness of delegation in resolving the time inconsistency of international monetary policy. In particular, we attempt to answer two research questions:

1) Is the global Friedman rule less likely to be sustained under cooperation than under non-cooperation when the governments lack commitment technology?

2) Is delegation more effective to solve the time consistency when the economy becomes more open?

Our main findings suggest that:

1) Cooperation is counterproductive in the sense that the global Friedman rule is less credible under cooperation than under non-cooperation.

2) We show that openness enhances the effectiveness of delegation approach in solving the time consistency problem.

The general conclusion of Chapter 2 is that the consequences of time consistency is less severer and the solution to this problem is more effective under the open economy than under the closed economy. The first finding is in favor of non-cooperative setting between policymakers in different sovereigns due to the coexistence of two opposing channels. Competition among policymakers is regarded as partial commitment and then might be superior to cooperation. This finding can be applied for the developing countries in which the competitiveness is still low. Such a low competitiveness induces a large market distortion which makes the counterproductive cooperation be likely to appear. The second finding implies the more open the economy, the more effective the delegation approach. The transmission of monetary policy via the terms of trade channel dampen the monetary authority's incentive to induce inflationary surprise and therefore enhance the effectiveness of the monetary delegation approach.

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Appendix A

Appendix of Chapter 3

A.1 Derivation of equilibrium

A.1.1 Results of equilibrium under commitment

Assuming that both CIA constraint and budget constraint hold with equality, the Lagrangian of Home household's problem (HO) is

$$
\begin{aligned}
L = & \sum_{t=0}^{\infty} \beta^t \left\{ \left(\frac{(c_{H,t})^{\gamma(1-\alpha)} (c_{F,t})^{(1-\gamma)(1-\alpha)}}{1-\alpha} \right) - l_t \right\} \\
& + \sum_{t=0}^{\infty} \beta^t \mu_t \left\{ m_t + (x_t - 1) + b_t - \frac{x_t b_{t+1}}{R_t} - \left(\int_0^1 p_{H,t}(j) c_{H,t}(j) dj + \int_0^1 p_{F,t}(j^*) c_{F,t}(j^*) dj^* \right) \right\} \\
& + \sum_{t=0}^{\infty} \beta^t \lambda_t \Bigg\{ m_t + (x_t - 1) + b_t + w_t l_t + \int_0^1 \zeta_t(j) dj - \frac{x_t b_{t+1}}{R_t} - x_t m_{t+1} \\
& - \left(\int_0^1 p_{H,t}(j) c_{H,t}(j) dj + \int_0^1 p_{F,t}(j^*) c_{F,t}(j^*) df \right) \Bigg\}.
\end{aligned}
\tag{A.1}
$$

The first-order conditions follow from differentiating with respect to $c_{H,t}(h)$, $c_{F,t}(h)$, l_t, m_{t+1}, and b_{t+1}:

$$(\mu_t + \lambda_t) p_{H,t}(j) = \gamma (c_{H,t})^{\gamma(1-\alpha)-1} (c_{F,t})^{(1-\gamma)(1-\alpha)} c_{H,t}^{1/\theta} (c_{H,t}(j))^{-1/\theta}, \tag{A.2}$$

$$(\mu_t + \lambda_t) p_{F,t}(j^*) = (1-\gamma)(c_{H,t})^{\gamma(1-\alpha)} (c_{F,t})^{(1-\gamma)(1-\alpha)-1} c_{F,t}^{1/\theta} (c_{F,t}(j))^{-1/\theta}, \tag{A.3}$$

$$1 = \lambda_t w_t, \tag{A.4}$$

$$\beta(\mu_{t+1} + \lambda_{t+1}) = \lambda_t x_t, \tag{A.5}$$

$$\beta(\mu_{t+1} + \lambda_{t+1}) = (\mu_t + \lambda_t) \frac{x_t}{R_t}, \tag{A.6}$$

$$\lim_{t \to \infty} \beta^t (\mu_t + \lambda_t) m_t = 0, \tag{A.7}$$

where the market clearing conditions of money market is $m_t = 1$, of asset market is $b_t = b_{t+1} = 0$, and of the currency exchange market is

$$S_t = [(1-\gamma)/\gamma] x_t / x_t^*. \tag{A.8}$$

Taking the integral to the power of $(1-\theta)$ on both sides of equation (A.2) and (A.3) with respect to h and f, respectively:

$$\gamma (c_{H,t})^{\gamma(1-\alpha)-1} (c_{F,t})^{(1-\gamma)(1-\alpha)} = (\mu_t + \lambda_t) p_{H,t}, \tag{A.9}$$

$$(1-\gamma)(c_{H,t})^{\gamma(1-\alpha)} (c_{F,t})^{(1-\gamma)(1-\alpha)-1} = (\mu_t + \lambda_t) p_{F,t}. \tag{A.10}$$

Multiplying both sides of (A.2) and (A.3) by $c_{H,t}(j)$ and $c_{F,t}(j^*)$, respectively, taking integral, and then adding them together:

$$(c_{H,t})^{\gamma(1-\alpha)} (c_{F,t})^{(1-\gamma)(1-\alpha)} = (\mu_t + \lambda_t) x_t, \tag{A.11}$$

where the binding CIA in Home country implies

$$x_t = \left(\int_0^1 p_{H,t}(h) c_{H,t}(j) dj + \int_0^1 p_{F,t}(j^*) c_{F,t}(j^*) dj^* \right) = p_t c_t, \tag{A.12}$$

where the second equality comes from the definition of aggregate price and consumption given by (3.2) and (3.8). Substituting (A.11) back into (A.9) and (A.10) yields

$$c_{H,t} = \gamma \frac{x_t}{p_{H,t}}, \qquad c_{F,t} = (1-\gamma) \frac{x_t}{p_{F,t}}, \tag{A.13}$$

$$c_{H,t}(j) = \gamma \frac{x_t}{p_{H,t}} \left(\frac{p_{H,t}(j)}{p_{H,t}} \right)^{-\theta}, \qquad c_{F,t}(j^*) = (1-\gamma) \frac{x_t}{p_{F,t}} \left(\frac{p_{F,t}(j^*)}{p_{F,t}} \right)^{-\theta}. \tag{A.14}$$

Using these results together with the exchange rate condition, we obtain

$$c_t = \gamma \frac{x_t^{\gamma} x_t^{*(1-\gamma)}}{p_{H,t}^{\gamma} p_{F,t}^{*(1-\gamma)}}. \tag{A.15}$$

Combining (A.4), (A.5), (A.11), and (A.12) yields

$$w_t = \frac{x_t x_{t+1}}{\beta (c_{t+1})^{1-\alpha}}. \tag{A.16}$$

Finally, (A.6), (A.11), and (A.12) imply

$$R_t = \frac{x_{t+1} (x_t/p_t)^{1-\alpha}}{\beta (x_{t+1}/p_{t+1})^{1-\alpha}}. \tag{A.17}$$

Similarly, for the representative agent in Foreign country, the counterpart optimization problem (HO^*) can be solved such that:

$$c^*_{H,t} = \gamma \frac{x^*_t}{p^*_{H,t}}, \qquad c^*_{F,t} = (1-\gamma) \frac{x^*_t}{p^*_{F,t}}, \tag{A.18}$$

$$c^*_{H,t}(j) = \gamma \frac{x^*_t}{p^*_{H,t}} \left(\frac{p^*_{H,t}(j)}{p^*_{H,t}} \right)^{-\theta}, \qquad c^*_{F,t}(j^*) = (1-\gamma) \frac{x^*_t}{p^*_{F,t}} \left(\frac{p^*_{F,t}(j^*)}{p^*_{F,t}} \right)^{-\theta}, \tag{A.19}$$

$$c_t^* = (1-\gamma)\frac{x_t^{\gamma} x_t^{*(1-\gamma)}}{p_{H,t}^{\gamma} p_{F,t}^{*(1-\gamma)}}, \tag{A.20}$$

$$w_t^* = \frac{x_t^* x_{t+1}^*}{\beta (c_{t+1}^*)^{1-\alpha}}, \tag{A.21}$$

$$R_t^* = \frac{x_{t+1}^* (x_t^*/p_t^*)^{1-\alpha}}{\beta (x_{t+1}^*/p_{t+1}^*)^{1-\alpha}}. \tag{A.22}$$

We next solve the Home firms' problem (FO): maximize (3.9), taking w_t, w_t^*, p_t, p_t^*, x_t, and x_t^* as given to obtain the unique solution:

$$p_{H,t}(j) = \theta/(\theta-1) w_t. \tag{A.23}$$

In equilibrium, all firms have the same prices, so that

$$p_{H,t} = p_{H,t}(j) = \theta/(\theta-1) w_t, \tag{A.24}$$

for all $j \in [0,1]$. Similarly, solving (FO^*) yields:

$$p_{F,t}^* = p_{F,t}^*(j^*) = \theta/(\theta-1) w_t^*, \tag{A.25}$$

for all $j^* \in [0,1]$. Combining (A.24), (A.25), the PPP condition, and (A.8), p_t can be computed as:

$$p_t = \frac{1}{\gamma^W} (p_{H,t})^{\gamma} (p_{F,t})^{1-\gamma}. \tag{A.26}$$

The labor market clearing condition in Home country is simply

$$l_t = c_{H,t} + c_{H,t}^*. \tag{A.27}$$

Using (A.8), (A.13), and (A.18) gives

$$l_t = \frac{x_t}{p_{H,t}}. \tag{A.28}$$

Analogously, the labor used by Foreign firm is

$$l_t^* = \frac{x_t^*}{p_{F,t}^*}. \tag{A.29}$$

Combining (A.7), (A.11), and $m_t = 1$ yields the transversality condition:

$$\lim_{t\to\infty} \beta^t \frac{c_t^{1-\alpha}}{x_t}, \tag{A.30}$$

and similarly in the Foreign country we have

$$\lim_{t\to\infty} \beta^t \frac{c_t^{*1-\alpha}}{x_t^*}. \tag{A.31}$$

The necessary and sufficient conditions for equilibrium under commitment can be summarized by (A.13)–(A.22) and (A.28)–(A.31) which correspond to (3.13)–(3.17) in the main text.

Combing (A.15), (A.16), (A.20), and (A.21) yield the difference equations:

$$\begin{aligned}\ln(c_t) = \ln\left(\frac{\gamma\beta(\theta-1)}{\theta}\right) + (1-\alpha)\gamma\ln(c_{t+1}) + (1-\alpha)(1-\gamma)\ln(c_{t+1}^*) \\ - \gamma\ln(x_{t+1}) - (1-\gamma)\ln(x_{t+1}),\end{aligned} \tag{A.32}$$

$$\begin{aligned}\ln(c_t^*) = \ln\left(\frac{(1-\gamma)\beta(\theta-1)}{\theta}\right) + (1-\alpha)\gamma\ln(c_{t+1}) + (1-\alpha)(1-\gamma)\ln(c_{t+1}^*) \\ - \gamma\ln(x_{t+1}) - (1-\gamma)\ln(x_{t+1}).\end{aligned} \tag{A.33}$$

Employing the fact that $c_t = \gamma/(1-\gamma)c_t^*$, we can solve the above difference equation forward to obtain

$$
\begin{aligned}
c_t &= \left(\frac{\rho\beta(\theta-1)}{\theta}\right)^{\frac{1}{\alpha}} \prod_{i=0}^{\infty} x_{t+1+i}^{-\gamma(1-\alpha)^i} \prod_{i=0}^{\infty} x_{t+1+i}^{*-(1-\gamma)(1-\alpha)^i}, \\
c_t^* &= \left(\frac{\rho^*\beta(\theta-1)}{\theta}\right)^{\frac{1}{\alpha}} \prod_{i=0}^{\infty} x_{t+1+i}^{-\gamma(1-\alpha)^i} \prod_{i=0}^{\infty} x_{t+1+i}^{*-(1-\gamma)(1-\alpha)^i},
\end{aligned} \tag{A.34}
$$

where c_t and c_t^* have to satisfy (A.30) and (A.31). Plugging (A.16), (A.21), (A.24), and (A.25) into (A.28) and (A.29) yields

$$
l_t = \frac{\beta(\theta-1)}{\theta}\frac{(c_{t+1})^{1-\alpha}}{x_{t+1}}, \qquad\qquad l_t^* = \frac{\beta(\theta-1)}{\theta}\frac{(c_{t+1}^*)^{1-\alpha}}{x_{t+1}^*}. \tag{A.35}
$$

Therefore, given an optimal monetary policy $X_t = (x_t, x_t^*)$ that satisfy (A.34), we can construct the values for c_t and c_t^*. Based on those optimal values, we can further compute the sequences for $c_{H,t}$, $c_{F,t}$, $c_{H,t}(j)$, $c_{F,t}(j^*)$, l_t, m_{t+1}, b_{t+1}, $p_{H,t}(j)$, $p_{H,t}$, p_t, w_t, $\zeta_t(j)$, R_t, $c_{H,t}^*$, $c_{F,t}^*$, $c_{H,t}^*(j)$, $c_{F,t}^*(j^*)$, l_t^*, m_{t+1}^*, b_{t+1}^*, $p_{F,t}^*(j^*)$, $p_{F,t}^*$, p_t^*, w_t^*, $\zeta_t^*(j^*)$, and R_t^*.

Equations (3.2)–(3.6) and (A.2)–(A.7) imply that these sequences for $c_{H,t}$, $c_{F,t}$, $c_{H,t}(j)$, $c_{F,t}(j^*)$, l_t, m_{t+1}, and b_{t+1} solve (HO) and $c_{H,t}^*$, $c_{F,t}^*$, $c_{H,t}^*(j)$, $c_{F,t}^*(j^*)$, l_t^*, m_{t+1}^*, and b_{t+1}^* solve (HO^*). Equations (A.24) and (A.25) indicate that this sequence for $p_{H,t}(j)$ solves (FO) and $p_{F,t}^*(j^*)$ solves (FO^*). Based on the construction, these sequences also lead to (MO) and (MO^*), respectively.

Now consider how to choose the optimal monetary policy. There are two cases: under non-cooperation, each government sets the money growth rate to maximize their own welfare function, taking other monetary policy as given; under cooperation, governments jointly choose the money growth rate to maximize the weighted welfare function. Substituting consumption, (A.34), and labor, (A.35), into the welfare function, we can express the welfare in terms of policy variables, $U(x,x^*)$ and $U^*(x,x^*)$ for Home and Foreign country, respectively.

For non-cooperation, given Foreign monetary policy, x^*, the Home government sets the optimal policy by solving

$$\max_{x} U(x, x^*) \text{ s.t. } \beta \leq x \leq \bar{x}, \tag{A.36}$$

and given x the Foreign government solves

$$\max_{x^*} U^*(x, x^*) \text{ s.t. } \beta \leq x^* \leq \bar{x}^*. \tag{A.37}$$

Let $X_t^N = (x_t^N, x_t^{*N})$ denote the optimal monetary policy under non-cooperation. Given X_t^N, we can construct allocations for households and firms in both countries. It is straightforward that X_t^N satisfies (A.34), hence sequences for $c_{H,t}$, $c_{F,t}$, $c_{H,t}(j)$, $c_{F,t}(j^*)$, l_t, m_{t+1}, and b_{t+1} solve (HO) and $c_{H,t}^*$, $c_{F,t}^*$, $c_{H,t}^*(j)$, $c_{F,t}^*(j^*)$, l_t^*, m_{t+1}^*, and b_{t+1}^* solve (HO^*). Also, sequence for $p_{H,t}(j)$ solves (FO) and $p_{F,t}^*(j^*)$ solves (FO^*). Based on the construction, these sequences are also corresponding with (MO) and (MO^*), respectively.

Hence, maximizing (A.36) subject to $x_t \in [\beta, \bar{x}]$ and maximizing (A.37) subject to $x_t^* \in [\beta, \bar{x}^*]$ for all $t = 0, 1, ...$, we can obtain the outcome of optimal non-cooperative equilibrium under commitment. This completes the proof of non-cooperative outcome.

For cooperation, the optimization problem reduces to finding the optimal monetary policy as follows

$$\max_{x_t, x_t^*} U(x_t, x_t^*) + U^*(x_t, x_t^*) \text{ s.t } \beta \leq x_t \leq \bar{x}, \beta \leq x_t^* \leq \bar{x}^*. \tag{A.38}$$

Let $X_t^C = (x_t^C, x_t^{*C})$ denote the optimal monetary policy under cooperation. Given X_t^C, we can construct the values for c_t and c_t^*. Based on those optimal values, we can further compute the sequences for $c_{H,t}$, $c_{F,t}$, $c_{H,t}(j)$, $c_{F,t}(j^*)$, l_t, m_{t+1}, b_{t+1}, $p_{H,t}(j)$, $p_{H,t}$, p_t, w_t, $\zeta_t(j)$, R_t, $c_{H,t}^*$, $c_{F,t}^*$, $c_{H,t}^*(j)$, $c_{F,t}^*(j^*)$, l_t^*, m_{t+1}^*, b_{t+1}^*, $p_{F,t}^*(j^*)$, $p_{F,t}^*$, p_t^*, w_t^*, $\zeta_t^*(j^*)$, and R_t^* when governments cooperate. It is clear that X_t^C satisfies (A.34), so that household allocations solve (HO)

and HO^*), firm allocations solve (FO) and (FO^*), and these allocations satisfy (MO) and (MO^*), respectively.

Thus, maximizing (A.38) subject to $x_t \in [\beta, \bar{x}]$ and $x_t^* \in [\beta, \bar{x}^*]$ for all $t = 0, 1, ...$, we can obtain the outcome of optimal cooperative equilibrium under commitment.

A.1.2 Outcome of non-cooperative equilibrium under commitment

A general characterization of optimal monetary policy equilibria under commitment is complicated whether governments cooperate or not, so we concentrate on the study of the equilibria that have constant the money growth rates. By this assumption, consumption and labor reduce to

$$\hat{c} = \left(\frac{\rho\beta(\theta-1)}{\theta\hat{x}^{\gamma}\hat{x}^{*(1-\gamma)}}\right)^{\frac{1}{\alpha}}, \qquad \hat{c}^* = \left(\frac{\rho^*\beta(\theta-1)}{\theta\hat{x}^{\gamma}\hat{x}^{*(1-\gamma)}}\right)^{\frac{1}{\alpha}}, \tag{A.39}$$

$$\hat{l} = \frac{\beta(\theta-1)\hat{c}^{1-\alpha}}{\theta\hat{x}}, \qquad \hat{l}^* = \frac{\beta(\theta-1)\hat{c}^{*1-\alpha}}{\theta\hat{x}^*}. \tag{A.40}$$

Under non-cooperation, the Home government's problem reduces to:

$$\max_{\hat{x}\in[\beta,\bar{x}]} \hat{U} = \left(\frac{\rho\beta(\theta-1)}{\theta\hat{x}^{\gamma}\hat{x}^{*(1-\gamma)}}\right)^{\frac{1-\alpha}{\alpha}} \left(\frac{1}{1-\alpha} - \frac{\beta(\theta-1)}{\theta\hat{x}}\right). \tag{A.41}$$

The relative size of monopolistic distortions to strategic terms of trade distortion dictates the prevalence of optimal monetary policy under noncooperation given a parameter space. When $\Phi \geq \Psi^N$ and $\Phi \geq \Psi^{*N}$, $\hat{x}^N = \beta$ and $\hat{x}^{*N} = \beta$, so that $\hat{c} = (\rho/\Phi)^{1/\alpha}$ and $\hat{c} = (\rho^*/\Phi)^{1/\alpha}$. When $\Psi^{*N} > \Phi \geq \Psi^N$, $\hat{x}^N = \beta$ and $\hat{x}^{*N} = \beta\Psi^{*N}/\Phi$, then $\hat{c} = \left(\rho/(\Phi^{\gamma}\Psi^{*N(1-\gamma)})\right)^{1/\alpha}$ and $\hat{c}^* = \left(\rho^*/(\Phi^{\gamma}\Psi^{*N(1-\gamma)})\right)^{1/\alpha}$. When $\Psi^{*N} \leq \Phi < \Psi^N$, $\hat{x} = \beta\Psi^N/\Phi$ and $\hat{x}^{*N} = \beta$, then $\hat{c} = \left(\rho/(\Phi^{1-\gamma}\Psi^{N\gamma})\right)^{1/\alpha}$ and $\hat{c}^* = \left(\rho^*/(\Phi^{1-\gamma}\Psi^{N\gamma})\right)^{1/\alpha}$. In each case, we can find p_H and p_F^*, and then substitute them plus the policy variables into (A.28) and (A.29) to find labors used by firms in Home and Foreign country, $\hat{l}$ and $\hat{l}^*$.

A.1.3 Revert-to-autarky plans under non-cooperation

We define *revert-to-autarky* plans $((\sigma_H^r, \sigma_F^r), \pi^r, \omega^r, \pi^{*r}, \omega^{*r})$ as follows. For all $t = 0, 1, ...$, let $\sigma_H^r(\xi_{t-1}^N) = x_t^N$ for history $\xi_{t-1}^N = \{(x_s^N, x_s^{*N}) | s = 0, 1, ..., t-1\}$; let $\sigma_H^r(\xi_{t-1}^N) = \bar{x}$ otherwise. For all $t = 0, 1, ...$, let $\sigma_F^r(\xi_{t-1}^N) = x_t^{*N}$ for history $\xi_{t-1}^N = \{(x_s^N, x_s^{*N}) | s = 0, 1, ..., t-1\}$; let $\sigma_F^r(\xi_{t-1}^N) = \bar{x}^*$ otherwise. For all $j \in [0,1]$, let $\pi_t^{jr}(\xi_{t-1}^N) = p_{H,t}(j)$ for $\xi_{t-1}^N = \{(x_s^N, x_s^{*N}) | s = 0, 1, ..., t-1\}$; let $\pi_t^{jr}(\xi_{t-1}^N) = \pi^{ja}(\xi_{t-1}^N)$ otherwise. For all $j^* \in [0,1]$, let $\pi_t^{*j^*r}(\xi_{t-1}^N) = p_{F,t}^*(j^*)$ for $\xi_{t-1}^N = \{(x_s^N, x_s^{*N}) | s = 0, 1, ..., t-1\}$; let $\pi_t^{*j^*r}(\xi_{t-1}^N) = \pi^{*j^*a}(\xi_{t-1}^N)$ otherwise. Let $\omega_t^r(\xi_t^N) = \Omega_t$ and $\omega_t^{*r}(\xi_t^N) = \Omega_t^*$ for $\xi_t^N = \{(x_s^N, x_s^{*N}) | s = 0, 1, ..., t\}$. If $\xi_t^N = (\xi_{t-1}^N, \tilde{x}^N, \tilde{x}^{*N})$ is such that $\xi_{t-1}^N = \{(x_s^N, x_s^{*N}) | s = 0, 1, ..., t-1\}$ but $\tilde{x}^N \neq x_t^N$ and/or $\tilde{x}^{*N} \neq x_t^{*N}$, let $\omega_t^r(\xi_t^N)$ be given by the continuation rule ω^t that solve (HT), given (σ_H^r, σ_F^r), π^{jr} for all $j \in [0,1]$ and π^{*j^*r} for all $j^* \in [0,1]$; let $\omega_t^{*r}(\xi_t^N)$ be given by the continuation rule ω^{*t} that solve (HT^*), given (σ_H^r, σ_F^r), π^{jr} for all $j \in [0,1]$ and π^{*j^*r} for all $j^* \in [0,1]$. If $\xi_t^N = (\xi_{t-1}^N, \tilde{x}^N, \tilde{x}^{*N})$ is such that $\xi_{t-1}^N \neq \{(x_s^N, x_s^{*N}) | s = 0, 1, ..., t-1\}$, let $\omega_t^r(\xi_t^N) = \omega^a(\xi_t^N)$ and $\omega_t^{*r}(\xi_t^N) = \omega^{*a}(\xi_t^N)$.

A.1.4 Revert-to-autarky plans under cooperation

Given an arbitrary world policy $X^C = (x^C, x^{*C})$ and an arbitrary allocation $(\Pi, \Omega, \Pi^*, \Omega^*)$, construct the definition of *revert-to-autarky* plans $(\sigma^r, \pi^r, \omega^r, \pi^{*r}, \omega^{*r})$ as follows. For all $t = 0, 1, ...$, let $\sigma^r(\xi_{t-1}^C) = X_t^C$ for history $\xi_{t-1}^C = \{X_s^C | s = 0, 1, ..., t-1\}$; let $\sigma^r(\xi_{t-1}^C) = \bar{X}$ otherwise. For all $j \in [0,1]$, let $\pi_t^{jr}(\xi_{t-1}^C) = p_{H,t}(j)$ for $\xi_{t-1}^C = \{X_s^C | s = 0, 1, ..., t-1\}$; let $\pi_t^{jr}(\xi_{t-1}^C) = \pi^{ja}(\xi_{t-1}^C)$ otherwise. For all $j^* \in [0,1]$, let $\pi_t^{*j^*r}(\xi_{t-1}^C) = p_{F,t}^*(j^*)$ for $\xi_{t-1}^C = \{X_s^C | s = 0, 1, ..., t-1\}$; let $\pi_t^{*j^*r}(\xi_{t-1}^C) = \pi^{*j^*a}(\xi_{t-1}^C)$ otherwise. Let $\omega_t^r(\xi_t^C) = \Omega_t$ and $\omega_t^{*r}(\xi_t^C) = \Omega_t^*$ for $\xi_t^C = \{X_s^C | s = 0, 1, ..., t\}$. If $\xi_t^C = (\xi_{t-1}^C, \tilde{X}^C)$ is such that $\xi_{t-1}^C = \{X_s^C | s = 0, 1, ..., t-1\}$ but $\tilde{X}^C \neq X_t^C$, let $\omega_t^r(\xi_t^C)$ and $\omega_t^{*r}(\xi_t^C)$ be given by the continuation rule ω^t and ω^{*t} that solve (HT) and (HT^*), respectively, given σ^r, π^{jr} for all $j \in [0,1]$ and π^{*j^*r} for all $j^* \in [0,1]$. If $\xi_t^C = (\xi_{t-1}^C, \tilde{X}^C)$ is such that $\xi_{t-1}^C \neq \{X_s^C | s = 0, 1, ..., t-1\}$, let $\omega_t^r(\xi_t^C) = \omega^a(\xi_t^C)$ and $\omega_t^{*r}(\xi_t^C) = \omega^{*a}(\xi_t^C)$.

A.2 Proof of Proposition and Corollary

A.2.1 Proposition 3.1

For cooperation, governments jointly maximize the weighted welfare function:

$$\max_{\hat{x},\hat{x}^*} \hat{U}(\hat{x},\hat{x}^*) + \hat{U}^*(\hat{x},\hat{x}^*) \text{ s.t } \beta \le \hat{x} < \bar{x}, \beta \le \hat{x}^* < \bar{x}^*. \tag{A.42}$$

The first-order conditions are given by:

$$\frac{\gamma(1-\alpha)}{\alpha}\left(\frac{1}{1-\alpha} - \frac{\beta}{\Phi\hat{x}} + \left(\frac{1-\gamma}{\gamma}\right)^{1-\alpha}\left(\frac{1}{1-\alpha} - \frac{\beta}{\Phi\hat{x}^*}\right)\right) = \frac{\beta}{\Phi\hat{x}}, \tag{A.43}$$

$$\frac{(1-\gamma)(1-\alpha)}{\alpha}\left(\frac{1}{1-\alpha} - \frac{\beta}{\Phi\hat{x}} + \left(\frac{1-\gamma}{\gamma}\right)^{1-\alpha}\left(\frac{1}{1-\alpha} - \frac{\beta}{\Phi\hat{x}^*}\right)\right) = \left(\frac{1-\gamma}{\gamma}\right)^{1-\alpha}\frac{\beta}{\Phi\hat{x}^*}, \tag{A.44}$$

which yields the relationship $\frac{\hat{x}}{\hat{x}^*} = \left(\frac{1-\gamma}{\gamma}\right)^{\alpha}$. Plugging this result back into the first-order condition yields

$$\hat{x} = \beta\frac{\Psi^C}{\Phi}, \qquad \hat{x}^* = \beta\frac{\Psi^{*C}}{\Phi}. \tag{A.45}$$

When $\Phi \ge \Psi^C$ and $\Phi \ge \Psi^{*C}$, $\hat{x}^C = \beta$ and $\hat{x}^{*C} = \beta$, so that $\hat{c} = (\rho/\Phi)^{1/\alpha}$ and $\hat{c} = (\rho^*/\Phi)^{1/\alpha}$. When $\Psi^{*C} > \Phi \ge \Psi^C$, $\hat{x}^C = \beta$ and $\hat{x}^{*C} = \beta\Psi^{*C}/\Phi$, then $\hat{c} = \left(\rho/(\Phi^{\gamma}\Psi^{*C(1-\gamma)})\right)^{1/\alpha}$ and $\hat{c}^* = \left(\rho^*/(\Phi^{\gamma}\Psi^{*C(1-\gamma)})\right)^{1/\alpha}$. When $\Psi^{*C} \le \Phi < \Psi^C$, $\hat{x}^C = \beta\Psi^C/\Phi$ and $\hat{x}^{*C} = \beta$, then $\hat{c} = \left(\rho/(\Phi^{1-\gamma}\Psi^{C\gamma})\right)^{1/\alpha}$ and $\hat{c}^* = \left(\rho^*/(\Phi^{1-\gamma}\Psi^{C\gamma})\right)^{1/\alpha}$. In each case, we can find p_H and p_F^*, then substitute them plus the policy variables into (A.28) and (A.29), we can find labors used by firms in Home and Foreign country, $\hat{l}$ and $\hat{l}^*$. $\square$

A.2.2 Proposition 3.2

The proof of Proposition 3.2 comes directly from the previous subsection of the appendix and subsection A.1.2. When monopolistic distortions are sufficiently large such that $\Phi \geq \Psi^C, \Psi^N, \Psi^{*C}, \Psi^{*N}$, the optimal policy pair is characterized by the global Friedman rule that is independent of policy regime. The consumption in both cases are identical and given by $\hat{c} = (\rho/\Phi)^{1/\alpha}$ and $\hat{c} = (\rho^*/\Phi)^{1/\alpha}$. This implies that the households' and firms' allocation computed by using these consumption levels in both policy regimes are also identical. In conclusion, the outcome of equilibrium under commitment when governments cooperate is identical to that when governments do not cooperate. □

A.2.3 Proposition 3.3

First, we prove the worst non-cooperative equilibrium under commitment for Home country at $\hat{x}^N = \bar{x}$ and $\hat{x}^{*N} = \bar{x}^*$. The Home utility in terms of policy variables is given by

$$\hat{U}(\hat{x}, \hat{x}^*) = \left(\frac{\rho\beta}{\Phi \hat{x}^{\gamma} \hat{x}^{*1-\gamma}} \right)^{\frac{1-\alpha}{\alpha}} \left(\frac{1}{1-\alpha} - \frac{\beta}{\Phi \hat{x}} \right). \tag{A.46}$$

As shown in subsection A.1.2, $\partial^2 \hat{U}/\partial \hat{x}^2 < 0$ for all $\hat{x} \in [\beta, \bar{x})$ and $\partial \hat{U}/\partial \hat{x} = 0$ at $\hat{x}^N = \beta \Psi^N/\Phi$. As $\Psi^N < \Phi$ by assumption, $\partial \hat{U}/\partial \hat{x} < 0$ for all $\hat{x} \in [\beta, \bar{x}]$. Hence, given $\hat{x}^*$, $\hat{U}(\hat{x}, \hat{x}^*)$ reaches the minimum at $\hat{x} = \bar{x}$. The proof for Foreign country is analogous. Combining them yields the first case of proposition 3.3.

Next, we prove the worst cooperative equilibrium under commitment. Substituting the first-order conditions into (A.34) yields

$$\hat{c} = \left(\frac{\gamma^W \beta}{\Phi \hat{x}} \right)^{\frac{1}{\alpha}}, \qquad \hat{c}^* = \left(\frac{\gamma^W \beta}{\Phi \hat{x}^*} \right)^{\frac{1}{\alpha}}. \tag{A.47}$$

From this expression, it is straightforward that the constraints $x_t \in [\beta, \bar{x}]$, and $x_t^* \in [\beta, \bar{x}^*]$ can be transformed into the consumption's constraints $c_t \in [\underline{c}, \bar{c}]$ and $c_t^* \in [\underline{c}^*, \bar{c}^*]$ where

$$\underline{c} = \left(\frac{\gamma^W \beta}{\Phi \bar{x}}\right)^{\frac{1}{\alpha}}, \qquad \underline{c}^* = \left(\frac{\gamma^W \beta}{\Phi \bar{x}^*}\right)^{\frac{1}{\alpha}}, \tag{A.48}$$

$$\bar{c} = \left(\frac{\gamma^W}{\Phi}\right)^{\frac{1}{\alpha}}, \qquad \bar{c}^* = \left(\frac{\gamma^W}{\Phi}\right)^{\frac{1}{\alpha}}. \tag{A.49}$$

Hence, the problem of finding the outcome of optimal cooperation equilibrium under commitment collapses to one of finding a sequence for $\hat{c}_t$ and $\hat{c}_t^*$ to maximize

$$\sum_{t=0}^{\infty} \beta^t \left(U(\hat{c}_t) + U^*(\hat{c}_t^*)\right), \tag{A.50}$$

subject to $\hat{c}_t \in [\underline{c}, \bar{c}]$ and $\hat{c}_t^* \in [\underline{c}^*, \bar{c}^*]$ for all $t = 0, 1, ...$, where

$$U(\hat{c}) + U^*(\hat{c}^*) = \frac{\hat{c}^{1-\alpha}}{(1-\alpha)\gamma \Psi^C} - \frac{\hat{c}}{\gamma \gamma^W} \equiv U^C(\hat{c}), \tag{A.51}$$

or

$$U(\hat{c}) + U^*(\hat{c}^*) = \frac{\hat{c}^{*(1-\alpha)}}{(1-\alpha)(1-\gamma)\Psi^{*C}} - \frac{\hat{c}^*}{(1-\gamma)\gamma^W} \equiv U^C(\hat{c}^*). \tag{A.52}$$

For further analysis, we derive the following lemma:

Lemma 1 *The reduced forms of the weighted welfare function, $U^C(\hat{c})$ and $U^C(\hat{c}^*)$, have the properties:*

(i) If $\Psi^C \leq \Phi$, $U^C(\hat{c})$ is strictly increasing on $[\underline{c}, \bar{c}]$.

(ii) If $\Psi^{*C} \leq \Phi$, $U^C(\hat{c}^*)$ is strictly increasing on $[\underline{c}^*, \bar{c}^*]$.

Proof. Note that $\partial^2 U^C(\hat{c})/\partial \hat{c}^2 < 0$ for all $\hat{c} > 0$ and $\partial^2 U^C(\hat{c}^*)/\partial \hat{c}^{*2} < 0$ for all $\hat{c}^* > 0$. Moreover, $\partial U^C(\hat{c})/\partial \hat{c} = 0$ where $\hat{c}_{max} = \left(\gamma^W/\Psi^C\right)^{1/\alpha}$ and $\partial U^C(\hat{c}^*)/\partial \hat{c}^* = 0$ where $\hat{c}^*_{max} = \left(\gamma^W/\Psi^{*C}\right)^{1/\alpha}$.

If $\Phi \geq \Psi^C$, $\hat{c}_{max} > \bar{c}$, then $\partial U^C(\hat{c})/\partial \hat{c} > 0$ for all $\hat{c} \in [\underline{c}, \bar{c}]$. Similarly, If $\Phi \geq \Psi^{*C}$, $\hat{c}^*_{max} > \bar{c}^*$, then $\partial U^C(\hat{c}^*)/\partial \hat{c}^* > 0$ for all $\hat{c}^* \in [\underline{c}^*, \bar{c}^*]$. $\square$

The Lemma 1 indicates that weighted utility is minimized with $\hat{c}_t = \underline{c}$ and $\hat{c}^*_t = \underline{c}^*$ for all $t = 0, 1, 2, ...$ if $\Phi \geq \max(\Psi^C, \Psi^{*C})$. Hence, the world policy, $\hat{x}_t = \bar{x}$ and $\hat{x}^*_t = \bar{x}^*$ for all $t = 0, 1, 2, ...$ yield the worst cooperative equilibrium under commitment when $\Phi \geq \max(\Psi^C, \Psi^{*C})$. $\square$

A.2.4 Proposition 3.4

Since the optimal surprise in Home country is independent of Foreign monetary policy, we only need to prove for the case of Home country. When $\Phi = 1/\gamma$, Home government has no incentive to conduct a monetary surprise because there is no gains from doing so, whereas when $\Phi < 1/\gamma$, Home government has incentive to conduct a deflationary surprise, but it cannot cut the inflation further because of the lower bound of the money growth rate. In both cases, regardless of Foreign monetary policy, $x^N_d = \beta$ is a Nash equilibrium monetary policy for the Home country.

When $\Phi > 1/\gamma$, the Home government has temptation to generate an inflationary surprise. The Home private sector perceives this temptation and adopts its inflation expectation accordingly. This leads to a temptation to create a higher inflationary surprise. The iteration process continues until both private sector's expectation, and government's actual policy coincide at the upper bound of the policy variable. The Nash policy of Home government under discretion is $x^N_d = \bar{x}$ no matter what Foreign government responds. Similarly, when $\Phi \leq 1/(1-\gamma)$, $x^{*N}_d = \beta$, otherwise $x^{*N}_d = \bar{x}^*$. Combining Home and Foreign Nash discretionary policy yields four cases described in Proposition 3.4. $\square$

A.2.5 Proposition 3.5

Given that private expectations of money growth at the beginning of the current period are $X^e = (\hat{x}^C, \hat{x}^{*C}) = (\beta, \beta)$, substituting X^e into the optimal surprises yields

$$\Delta^C = \beta\left\{\left(\frac{\Phi}{\Psi^C}\left(\frac{1-\gamma}{\gamma}\right)^{(1-\alpha)(1-\gamma)}\right)^{\frac{1}{\alpha}} - 1\right\}, \quad \Delta^{*C} = \beta\left\{\left(\frac{\Phi}{\Psi^{*C}}\left(\frac{\gamma}{1-\gamma}\right)^{(1-\alpha)\gamma}\right)^{\frac{1}{\alpha}} - 1\right\}. \tag{A.53}$$

It is easy to see that at least one of the optimal surprise is positive. If both Δ^C and Δ^{*C} are greater than zero, then the governments have temptation to set the actual money growth at $x_t = \hat{x}^C + \varepsilon_0$ and $x_t^* = \hat{x}^{*C} + \varepsilon_0^*$ where $0 < \varepsilon_0, \varepsilon_0^* < \infty$. And if private agents react by adapting their expectations to $\hat{x}^C + \varepsilon_0$ in Home and $\hat{x}^{*C} + \varepsilon_0^*$ in Foreign country, then governments have temptations to set the money growth rate even higher, at $x_t = \hat{x}^C + \varepsilon_0 + \varepsilon_1$ and $x_t^* = \hat{x}^{*C} + \varepsilon_0^* + \varepsilon_1^*$ where $0 < \varepsilon_1, \varepsilon_1^* < \infty$. This iteration continues until x_t and x_t^* hit the upper bound of policy space, $\bar{x}$ and $\bar{x}^*$.

When $\Delta^C > 0$ and $\Delta^{*C} < 0$, the Home government has incentive to set $x_t = \beta + \varepsilon_0$ while $x_t^* = \beta + \varepsilon_0^*$ where $\varepsilon_0 > 0$ and $\varepsilon_0^* = 0$. And if private agents react by adapting their expectations to $\beta + \varepsilon_0$ in Home and $\hat{x}^{*C} + \varepsilon_0^*$ in Foreign country, then Home government has temptations to set the money growth rate even higher, at $x_t = \hat{x}^C + \varepsilon_0 + \varepsilon_1$ where $0 < \varepsilon_1 < \infty$, so on; simultaneously, because deviation rate in Foreign country is increasing in Home monetary policy, after some iterations, the Foreign government starts to introduce the inflationary surprise $x_t^* = \hat{x}^{*C} + \varepsilon_0^* + \varepsilon_1^* + ... + \varepsilon_s^*$ where $\varepsilon_1^* = ... = \varepsilon_{s-1}^* = 0$ and $0 < \varepsilon_s^* < \infty$. The resulting cooperative monetary policy under discretion is $(\bar{x}, \bar{x}^*)$. Using the same argument we can prove for the remaining cases. Note that for simplicity, in the main text we assume that ε_0 is large enough to force Foreign government set $\varepsilon_1^* > 0$. □

A.2.6 Proposition 3.6

When governments do not cooperate, to prove that (σ_H^a, σ_F^a) is a sustainable equilibrium, we have to show that it is optimal for each government to follow this plan if other government and private sector all follow the plan.

Taking $x_t = \bar{x}$ and $x_t^* = \bar{x}^*$ as given, plug (3.13) and (3.17) into (3.15) to achieve a system of difference equations

$$\ln(P_{H,t}) = -\ln\left(\frac{\beta(\theta-1)\gamma^{1-\alpha}}{\theta}\right) + (2-\gamma(1-\alpha))\ln(\bar{x}) - (1-\gamma)(1-\alpha)\ln(\bar{x}^*) + \gamma(1-\alpha)\ln(P_{H,t+1}) + (1-\gamma)(1-\alpha)\ln(P^*_{F,t+1}), \tag{A.54}$$

$$\ln(P^*_{F,t}) = -\ln\left(\frac{\beta(\theta-1)(1-\gamma)^{1-\alpha}}{\theta}\right) + (2-\gamma(1-\alpha))\ln(\bar{x}^*) - (1-\gamma)(1-\alpha)\ln(\bar{x}) + \gamma(1-\alpha)\ln(P_{H,t+1}) + (1-\gamma)(1-\alpha)\ln(P^*_{F,t+1}), \tag{A.55}$$

with solution $p_{H,t} = \bar{p}_H$ and $p^*_{F,t} = \bar{p}^*_F$ for all $t = 0, 1, ...$, where

$$\bar{p}_H = \left(\frac{\theta}{\beta(\theta-1)\gamma^{1-\alpha}}\left(\frac{\gamma}{1-\gamma}\right)^{(1-\alpha)^2(1-\gamma)} \bar{x}^{2\alpha+\gamma(1-\alpha)}\bar{x}^{*(1-\gamma)(1-\alpha)}\right)^{\frac{1}{\alpha}}, \tag{A.56}$$

$$\bar{p}^*_F = \left(\frac{\theta}{\beta(\theta-1)(1-\gamma)^{1-\alpha}}\left(\frac{1-\gamma}{\gamma}\right)^{(1-\alpha)^2\gamma} \bar{x}^{\gamma(1-\alpha)}\bar{x}^{*2\alpha+(1-\gamma)(1-\alpha)}\right)^{\frac{1}{\alpha}}. \tag{A.57}$$

Given $\bar{p}_H$ and $\bar{p}^*_F$ above, and $x_t^* = \bar{x}^*$ for all t, we aim to show that $x_t = \bar{x}$ for all t maximizes the one-period welfare of the Home household:

$$U_t = \frac{1}{1-\alpha}\left(\gamma\frac{x_t^\gamma \bar{x}^{*(1-\gamma)}}{\bar{p}_H^\gamma \bar{p}_F^{*(1-\gamma)}}\right)^{1-\alpha} - \frac{x_t}{\bar{p}_H}. \tag{A.58}$$

It is clear to see that $\partial^2 U_t / \partial x_t^2 < 0$ and

$$\frac{\partial U_t}{\partial x_t} = \frac{1}{\bar{p}_H}\left(\gamma^{2-\alpha}\left(\frac{1-\gamma}{\gamma}\right)^{(1-\alpha)^2(1-\gamma)} x_t^{\gamma(1-\alpha)-1}\bar{x}^{2(1-\gamma)(1-\alpha)}\bar{x}^{*-(1-\gamma)(1-\alpha)}\bar{p}_H^{\alpha} - 1\right), \tag{A.59}$$

whose value at $x_t = \bar{x}$ is given by

$$\frac{1}{\bar{p}_H}\left(\frac{\gamma\Phi\bar{x}}{\beta} - 1\right). \tag{A.60}$$

As long as $\frac{\gamma\Phi\bar{x}}{\beta} > 1$, the optimal policy of Home government is $x_t = \bar{x}$ for all t if the Foreign government and private sectors follow the autarky plan under non-cooperation. Similarly, when $\frac{(1-\gamma)\Phi\bar{x}^*}{\beta} > 1$, the optimal policy of Foreign government is $x_t^* = \bar{x}^*$ for all t if the Home government and private sectors follow the autarky plan under non-cooperation. $\square$

A.2.7 Proposition 3.7

In the following proof, it is enough to consider the Home government's problem. That of Foreign government is analogous. The proof of proposition 3.7 includes two parts: First, assume that $(X^N, \Pi, \Omega, \Pi^*, \Omega^*)$ is a sustainable outcome, accompanied with some sustainable equilibrium $((\sigma_H, \sigma_F), \pi, \omega, \pi^*, \omega^*)$. As (FT), (FT^*), (HT), (HT^*), (MT), and (MT^*), at $t = 0$ coincide with (FO), (FO^*), (HO), (HO^*), (MO), and (MO^*), conditions (i)-(iii) in the definition of a sustainable equilibrium mean that $(X^N, \Pi, \Omega, \Pi^*, \Omega^*)$ is an outcome of equilibrium under commitment.

Since ω is part of a sustainable equilibrium, its continuation ω^t must address (HT) and satisfy (MT) for $t = 0, 1, 2, \ldots$ For each $t = 0, 1\ldots$, the necessary and sufficient conditions for (HT) are identical with those for (HO) and the market clearing (MT) are identical with those for (MO). It is analogous to ω^*. Thus, $\omega_t(\xi_t^N)$ and $\omega_t^*(\xi_t^N)$ must have

$$c_t = \gamma \frac{x_t^\gamma x_t^{*(1-\gamma)}}{p_{H,t}^\gamma p_{F,t}^{*(1-\gamma)}}, \qquad c_t^* = (1-\gamma)\frac{x_t^\gamma x_t^{*(1-\gamma)}}{p_{H,t}^\gamma p_{F,t}^{*(1-\gamma)}},$$

$$l_t = \frac{x_t}{p_{H,t}}, \qquad l_t^* = \frac{x_t^*}{p_{F,t}^*},$$

for all $t = 0,1,2,...$ and ξ_t^N.

Then, given t and ξ_{t-1}^N, the Home government could defect from x^N at date t, achieve the one-period welfare $\tilde{U}^N(p_{H,t}, p_{F,t}^*)$ defined by (GD), and revert to the autarky plan σ_H^a after that. This defection generates the welfare given by the left-hand side of Equation (3.39). Since condition (iv) in the definition of a sustainable equilibrium under non-cooperation prevents such a temptation to defect, (3.39) need to be satisfied.

Second, let $(X^N, \Pi, \Omega, \Pi^*, \Omega^*)$ be an outcome of equilibrium under commitment that satisfies the constraint (3.39), and let $((\sigma_H^r, \sigma_F^r), \pi^r, \omega^r, \pi^{*r}, \omega^{*r})$ be the corresponding revert-to-autarky plans. We need to show that $((\sigma_H^r, \sigma_F^r), \pi^r, \omega^r, \pi^{*r}, \omega^{*r})$ is a sustainable equilibrium.

Given the history ξ_{t-1}^N in which governments have never defected from X^N, allocation $(X^N, \Pi, \Omega, \Pi^*, \Omega^*)$ is an outcome of equilibrium under commitment. Choice Ω solves (HO) and satisfies (MO) given X^N, Π^j for all $j \in [0,1]$, and Π^{*j^*} for all $j^* \in [0,1]$, while choice Ω^* solves (HO^*) and satisfies (MO^*) given X^N, Π^h for all $j \in [0,1]$, and Π^{*j^*} for all $j^* \in [0,1]$. Each $\Pi^{\hat{j}}$ solves (FO), given X^N, $\Pi^{\hat{j}}$ for all $\hat{j} \in [0,1]$, $\hat{j} \neq j$, and $(\Omega, \Pi^*, \Omega^*)$, whereas each $\Pi^{*\hat{j}^*}$ solves (FO^*), given X^N, $\Pi^{*\hat{j}^*}$ for all $\hat{j}^* \in [0,1]$, $\hat{j}^* \neq j^*$, and (Π, Ω, Ω^*). Therefore, the continuation of Ω solves (HT) and satisfies (MT) at date t given X^N, Π^j for all $j \in [0,1]$, and Π^{*j^*} for all $j^* \in [0,1]$, the continuation of Ω^* solves (HT^*) and satisfies (MT^*) at date t given X^N, Π^j for all $j \in [0,1]$, and Π^{*j^*} for all $j^* \in [0,1]$, the continuation of Π^j solves (FT) at date $t-1$ given X^N, $\Pi^{\hat{j}}$ for all $\hat{j} \in [0,1]$, $\hat{j} \neq j$, and $(\Omega, \Pi^*, \Omega^*)$, and the continuation of Π^{*j^*} solves (FT) at date $t-1$ given X^N, $\Pi^{*\hat{j}^*}$ for all $\hat{j}^* \in [0,1]$, $\hat{j}^* \neq j$, and (Π, Ω, Ω^*). Both household and firm have no temptation to defect after ξ_{t-1}^N.

Now consider the Home governments' temptation to defect from x^N from date t following ξ_{t-1}^N. Since Ω solves (HO) and satisfies (MO), and Ω^* solves (HO^*) and satisfies (MO^*) for all ξ_t^N, $\omega^r(\xi_t^N)$ and $\omega^{*r}(\xi_t^N)$ have

$$c_t = \gamma \frac{\tilde{x}^\gamma \hat{x}_t^{*(1-\gamma)}}{p_{H,t}^\gamma p_{F,t}^{*(1-\gamma)}}, \qquad\qquad l_t = \frac{\tilde{x}_t}{p_{H,t}}.$$

Utilizing the proof of Proposition 3.6, the best governments can achieve by defecting is to set $\tilde{x}^N$ to solve (GD) at date t and, follow the autarky plans $((\sigma_H^a, \sigma_F^a), \pi^a, \omega^a, \pi^{*a}, \omega^{*a})$ forever after. This best defection produces welfare given by the left-hand side of Equation (3.39). Because Equation (3.39) holds, the governments have no temptation to defect from x^N after ξ_{t-1}^N.

For histories ξ_{t-1}^N in which there are defections from x^N by Home government, the revert-to-autarky plans dictate that in Home country government, firms, and households all choose the autarky plans in every subsequent period. As proved by Proposition 3.6, no one has a temptation to defect from these plans.

Finally, $((\sigma_H^r, \sigma_F^r), \pi^r, \omega^r, \pi^{*r}, \omega^{*r})$ is a sustainable equilibrium, and $(X^N, \Pi, \Omega, \Pi^*, \Omega^*)$ is the accompanied sustainable outcome. $\square$

A.2.8 Proposition 3.8

When governments cooperate, under σ^a, the representative firm takes ξ_{t-1} as given at each date $t = 0, 1, ...$ and solve (FT) in Home and (FT^*) in Foreign country, assuming that $x_{t+s} = \bar{x}$ and $x_{t+s}^* = \bar{x}^*$ for all $s = 0, 1, ...$ Also, the representative household takes ξ_t as given at each date $t = 0, 1, ...$ and solve (HT) in Home and (HT^*) in Foreign country, assuming that $x_{t+s+1} = \bar{x}$ and $x_{t+s+1}^* = \bar{x}^*$ for all $s = 0, 1, ...$

For $t = 0, 1, ...$, the necessary and sufficient conditions for solutions to (FT), (HT), (FT^*), and (HT^*) are identical to those for (FO), (HO), (FO^*), and (HO^*). Hence, plug (3.13) and (3.17) into (3.15) with $x_t = x_{t+1} = \bar{x}$ and $x_t^* = x_{t+1}^* = \bar{x}^*$ to achieve a system of difference equations

$$\ln(P_{H,t}) = -\ln\left(\frac{\beta(\theta-1)\gamma^{1-\alpha}}{\theta}\right) + (2-\gamma(1-\alpha))\ln(\bar{x}) - (1-\gamma)(1-\alpha)\ln(\bar{x}^*)$$
$$+\gamma(1-\alpha)\ln(P_{H,t+1}) + (1-\gamma)(1-\alpha)\ln(P^*_{F,t+1}), \tag{A.61}$$
$$\ln(P^*_{F,t}) = -\ln\left(\frac{\beta(\theta-1)(1-\gamma)^{1-\alpha}}{\theta}\right) + (2-\gamma(1-\alpha))\ln(\bar{x}^*) - (1-\gamma)(1-\alpha)\ln(\bar{x})$$
$$+\gamma(1-\alpha)\ln(P_{H,t+1}) + (1-\gamma)(1-\alpha)\ln(P^*_{F,t+1}), \tag{A.62}$$

with solution $p_{H,t} = \bar{p}_H$ and $p^*_{F,t} = \bar{p}^*_F$ for all $t = 0, 1, ...$, where

$$\bar{p}_H = \left(\frac{\theta}{\beta(\theta-1)\gamma^{1-\alpha}}\left(\frac{\gamma}{1-\gamma}\right)^{(1-\alpha)^2(1-\gamma)} \bar{x}^{2\alpha+\gamma(1-\alpha)}\bar{x}^{*(1-\gamma)(1-\alpha)}\right)^{\frac{1}{\alpha}}, \tag{A.63}$$
$$\bar{p}^*_F = \left(\frac{\theta}{\beta(\theta-1)(1-\gamma)^{1-\alpha}}\left(\frac{1-\gamma}{\gamma}\right)^{(1-\alpha)^2\gamma} \bar{x}^{\gamma(1-\alpha)}\bar{x}^{*2\alpha+(1-\gamma)(1-\alpha)}\right)^{\frac{1}{\alpha}}. \tag{A.64}$$

It follows that each of π^{ja}, $j \in [0,1]$ and π^{*j^*a}, $j^* \in [0,1]$ have $\pi^{ja}_t(\xi^C_{t-1}) = \bar{p}_H$ and $\pi^{*j^*a}_t(\xi^C_{t-1}) = \bar{p}^*_F$ for all $t = 0, 1, ...$ and ξ^C_{t-1}. Plugging these solutions into (3.13) and (3.14) leads to $\omega^a(\xi^C_t)$ and $\omega^{*a}(\xi^C_t)$ that must satisfy

$$c_t = \gamma\frac{x_t^{\gamma}x_t^{*(1-\gamma)}}{\bar{p}_H^{\gamma}\bar{p}_F^{*(1-\gamma)}}, \qquad c^*_t = (1-\gamma)\frac{x_t^{\gamma}x_t^{*(1-\gamma)}}{\bar{p}_H^{\gamma}\bar{p}_F^{*(1-\gamma)}}, \tag{A.65}$$
$$l_t = \frac{x_t}{\bar{p}_H}, \qquad l^*_t = \frac{x^*_t}{\bar{p}^*_F}. \tag{A.66}$$

for all $t = 0, 1, ...$ and ξ^C_t.

By construction, π^{ja} for all $j \in [0,1]$, π^{*j^*a} for all $j^* \in [0,1]$, ω^a, and ω^{*a} satisfy conditions (i)–(iii) in the definition of a sustainable equilibrium. We only need to verify condition (iv). Given π^{ja} for all $j \in [0,1]$, π^{*j^*a} for all $j^* \in [0,1]$, ω^a, and ω^{*a}, the continuation σ^{at} of σ^a must deal with (GT) for all $t = 0, 1, ...$ and ξ^C_{t-1}. In view of (A.65) and (A.66), this condition is satisfied as long as

$$(\bar{x}, \bar{x}^*) = \arg \max_{\tilde{x} \in [\beta, \bar{x}], \tilde{x}^* \in [\beta, \bar{x}^*]} \frac{\gamma^{1-\alpha} + (1-\gamma)^{1-\alpha}}{1-\alpha} \left(\frac{\tilde{x}^{\gamma} \tilde{x}^{*(1-\gamma)}}{\bar{p}_H^{\gamma} \bar{p}_F^{*(1-\gamma)}} \right)^{1-\alpha} - \left(\frac{\tilde{x}}{\bar{p}_H} + \frac{\tilde{x}^*}{\bar{p}_F^*} \right). \quad \text{(A.67)}$$

With $\bar{p}_H$ and $\bar{p}_F^*$ given by (A.63) and (A.64), we have that (A.67) holds, and so does condition (iv). Hence, $(\sigma^a, \pi^a, \omega^a, \pi^{*a}, \omega^{*a})$ is a sustainable equilibrium. □

A.2.9 Proposition 3.9

The proof of proposition 3.9 includes two parts. First, assume that $(X^C, \Pi, \Omega, \Pi^*, \Omega^*)$ is a sustainable outcome, accompanied with some sustainable equilibrium $(\sigma, \pi, \omega, \pi^*, \omega^*)$. As (FT), (FT^*), (HT), (HT^*), (MT), and (MT^*), at $t = 0$ coincide with (FO), (FO^*), (HO), (HO^*), (MO), and (MO^*), conditions (i)–(iii) in the definition of a sustainable equilibrium imply that $(X, \Pi, \Omega, \Pi^*, \Omega^*)$ is a outcome of equilibrium under commitment.

Since ω is part of a sustainable equilibrium, its continuation ω^t must address (HT) and satisfy (MT) for $t = 0, 1, 2, ...$ Since each $t = 0, 1...$, the necessary and sufficient conditions for (HT) are identical with those for (HO) and the market clearing (MT) are identical with those for (MO). It is analogous to ω^*. Thus, $\omega_t(\xi_t^C)$ and $\omega_t^*(\xi_t^C)$ must have

$$c_t = \gamma \frac{x_t^{\gamma} x_t^{*(1-\gamma)}}{p_{H,t}^{\gamma} p_{F,t}^{*(1-\gamma)}}, \qquad c_t^* = (1-\gamma) \frac{x_t^{\gamma} x_t^{*(1-\gamma)}}{p_{H,t}^{\gamma} p_{F,t}^{*(1-\gamma)}},$$

$$l_t = \frac{x_t}{p_{H,t}}, \qquad l_t^* = \frac{x_t^*}{p_{F,t}^*},$$

for all $t = 0, 1, 2, ...$ and ξ_t^C.

Then, given t and ξ_{t-1}^C, the governments could defect from X^C at date t, achieve the one-period weighted welfare $\left(\tilde{U}^C(p_{H,t}, p_{F,t}^*) + \tilde{U}^{*C}(p_{H,t}, p_{F,t}^*)\right)$ defined by (CD), and revert to the autarky plan σ^a after that. This defection generates the total weighted welfare given by the left-hand side of Equation (3.43). Since condition (iv) in the definition of a sustainable equilibrium prevents such a temptation to defect, (3.43) needs to be satisfied.

Second, let $(X^C, \Pi, \Omega, \Pi^*, \Omega^*)$ be an outcome of equilibrium under commitment that satisfied the constraint (3.43), and let $(\sigma^r, \pi^r, \omega^r, \pi^{*r}, \omega^{*r})$ be the corresponding revert-to-autarky plans. We need to show that $(\sigma^r, \pi^r, \omega^r, \pi^{*r}, \omega^{*r})$ is a sustainable equilibrium.

Given the history ξ_{t-1}^C in which governments have never defected from X^C, allocation $(X^C, \Pi, \Omega, \Pi^*, \Omega^*)$ is an outcome of equilibrium under commitment. Choice Ω solves (HO) and satisfies (MO) given X^C, Π^j for all $j \in [0,1]$, and Π^{*j^*} for all $j^* \in [0,1]$, while choice Ω^* solves (HO^*) and satisfies (MO^*) given X^C, Π^j for all $j^* \in [0,1]$, and Π^{*j^*} for all $j^* \in [0,1]$. Each Π^j solves (FO), given X^C, $\Pi^{\hat{j}}$ for all $\hat{j} \in [0,1]$, $\hat{j} \neq j$, and $(\Omega, \Pi^*, \Omega^*)$, whereas each Π^{*j^*} solves (FO^*), given X^C, $\Pi^{*\hat{j}^*}$ for all $\hat{j}^* \in [0,1]$, $\hat{j}^* \neq j^*$, and (Π, Ω, Ω^*). Therefore, the continuation of Ω solves (HT) and satisfies (MT) at date t given X^C, Π^j for all $j \in [0,1]$, and Π^{*j^*} for all $j^* \in [0,1]$, the continuation of Ω^* solves (HT^*) and satisfies (MT^*) at date t given X^C, Π^j for all $j \in [0,1]$, and Π^{*j^*} for all $j^* \in [0,1]$, the continuation of Π^h solves (FT) at date $t-1$ given X^C, $\Pi^{\hat{j}}$ for all $\hat{j} \in [0,1]$, $\hat{j} \neq j$, and $(\Omega, \Pi^*, \Omega^*)$, and the continuation of Π^{*j^*} solves (FT) at date $t-1$ given X^C, $\Pi^{*\hat{j}^*}$ for all $\hat{j}^* \in [0,1]$, $\hat{j}^* \neq j^*$, and (Π, Ω, Ω^*). Both household and firm have no temptation to defect after ξ_{t-1}^C.

Now consider the governments' temptation to defect from X^C from date t following ξ_{t-1}^C. Since Ω solves (HO) and satisfies (MO), and Ω^* solves (HO^*) and satisfies (MO^*) for all ξ_t^C, $\omega^r(\xi_t^C)$ and $\omega^{*r}(\xi_t^C)$ have

$$c_t = \gamma \frac{\tilde{x}^\gamma \tilde{x}_t^{*(1-\gamma)}}{p_{H,t}^\gamma p_{F,t}^{*(1-\gamma)}}, \qquad c_t^* = (1-\gamma) \frac{\tilde{x}_t^\gamma \tilde{x}_t^{*(1-\gamma)}}{p_{H,t}^\gamma p_{F,t}^{*(1-\gamma)}}, \tag{A.68}$$

$$l_t = \frac{\tilde{x}_t}{p_{H,t}}, \qquad l_t^* = \frac{\tilde{x}_t^*}{p_{F,t}^*}. \tag{A.69}$$

Utilizing the proof of Proposition 3.8, the best the government can achieve by defecting is to set $\tilde{X}^C$ to solve (CD) at date t and, follow to the autarky plans $(\sigma^a, \pi^a, \omega^a, \pi^{*a}, \omega^{*a})$ forever after. This best defection produces total weighted welfare given by the left-hand side of Equation (3.43). Because Equation (3.43) holds, the governments have no temptation to defect from X^C after ξ_{t-1}^C.

For histories ξ_{t-1}^{C} in which there are defections from X^{C} by governments, the revert-to-autarky plans dictate that the governments, firms, and households all choose the autarky plans in every subsequent period. As proved by Proposition 3.8, no one has a temptation to defect from these plans.

Finally, $(\sigma^{r}, \pi^{r}, \omega^{r}, \pi^{*r}, \omega^{*r})$ is a sustainable equilibrium, and $(X^{C}, \Pi, \Omega, \Pi^{*}, \Omega^{*})$ is the accompanied sustainable outcome. $\square$

A.2.10 Proposition 3.10

We need to prove that the one-period welfare from deviation under cooperation is larger than that under non-cooperation. As function $f(\tilde{c})$ (defined below) is concave and reach the maximum at $\tilde{c}^{C}$, we only have to find the condition so that $\tilde{c}^{C} > \tilde{c}^{N}$.

Let $f(\tilde{c}) = \frac{(\tilde{c})^{1-\alpha}}{(1-\alpha)} - 2\tilde{c}$. We have $f''(\tilde{c}) < 0$ so $f(\tilde{c})$ achieves the maximum when $f'(\tilde{c}) = 0$. Solving this first-order condition yields $\tilde{c} = (1/2)^{1/\alpha}$ which is consistent with $f(\tilde{c}^{C})$.

We next compute $\tilde{c}^{N} = (1/2)^{1/\alpha}(1/(2^{\alpha}\Phi))^{1/(\alpha(1+\alpha))} < \tilde{c}^{C}$. $\square$

A.2.11 Corollary 3.1

The optimal monetary policy under commitment is the global Friedman rule when $\Phi \geq \max(\Psi^{N}, \Psi^{*N}, \Psi^{C}, \Psi^{*C})$.

The cooperative monetary policy under discretion is $(\hat{x}_{d}^{C}, \hat{x}_{d}^{*C}) = (\bar{x}, \bar{x}^{*})$. When condition $\Phi \geq \max(1/\gamma, 1/(1-\gamma))$ does not hold, the non-cooperative monetary policy under discretion is given by:

(i) $\Phi \leq \min(1/\gamma, 1/(1-\gamma))$, $(x_{d}^{N}, x_{d}^{*N}) = (\beta, \beta)$,

(ii) $1/\gamma > \Phi \geq 1/(1-\gamma)$, $(x_{d}^{N}, x_{d}^{*N}) = (\bar{x}, \beta)$,

(iii) $1/\gamma \leq \Phi < 1/(1-\gamma)$, $(x_{d}^{N}, x_{d}^{*N}) = (\beta, \bar{x}^{*})$.

Because $1/\gamma > \Psi^{C}, \Psi^{N}$ and $1/(1-\gamma) > \Psi^{*C}, \Psi^{*N}$, we can form an interval $[\max(\Psi^{N}, \Psi^{*N}, \Psi^{C}, \Psi^{*C}), \max(1/\gamma, 1/(1-\gamma))]$.

When $\Phi > \max(\Psi^N, \Psi^{*N}, \Psi^C, \Psi^{*C})$, $\hat{U}(\hat{x}, \hat{x}^*)$ and $\hat{U}(\hat{x}, \hat{x}^*)$ are strictly decreasing in both $\hat{x}$ and $\hat{x}^*$ for $\hat{x} \in [\beta, \bar{x}]$ and $\hat{x}^* \in [\beta, \bar{x}^*]$. It is straightforward to see that in all cases, $\hat{U}_d^N > \hat{U}_d^C$ and $\hat{U}_d^{*N} > \hat{U}_d^{*C}$. $\square$

Appendix B

Appendix of Chapter 4

B.1 Derivation of equilibrium

B.1.1 Equilibrium under commitment

Assuming that both CIA constraint and budget constraint hold with equality, the Lagrangian of Home household's problem (HO) is

$$\begin{aligned}
L = & \sum_{t=0}^{\infty} \beta^t \left\{ \left(\frac{(c_{H,t})^{\alpha} (c_{F,t})^{\delta}}{\alpha^{\alpha} \delta^{\delta}} \right) - l_t \right\} \\
& + \sum_{t=0}^{\infty} \beta^t \mu_t \left\{ m_t + (x_t - 1) + b_t - \frac{x_t b_{t+1}}{R_t} - \left(\int_0^1 p_{H,t}(j) c_{H,t}(j) dj + \int_0^1 p_{F,t}(j^*) c_{F,t}(j^*) dj^* \right) \right\} \\
& + \sum_{t=0}^{\infty} \beta^t \lambda_t \left\{ m_t + (x_t - 1) + b_t + w_t l_t + \int_0^1 \zeta_t(j) dj - \frac{x_t b_{t+1}}{R_t} - x_t m_{t+1} \right. \\
& \left. - \left(\int_0^1 p_{H,t}(j) c_{H,t}(j) dj + \int_0^1 p_{F,t}(j^*) c_{F,t}(j^*) dj^* \right) \right\}.
\end{aligned} \tag{B.1}$$

The first-order conditions follow from differentiating with respect to $c_{H,t}(j)$, $c_{F,t}(j)$, l_t, m_{t+1}, and b_{t+1}:

$$(\mu_t+\lambda_t)p_{H,t}(j) = (\alpha^{\alpha-1}\delta^{\delta})^{-1}(c_{H,t})^{\alpha-1}(c_{F,t})^{\delta}c_{H,t}^{1/\theta}(c_{H,t}(j))^{-1/\theta}, \tag{B.2}$$

$$(\mu_t+\lambda_t)p_{F,t}(j^*) = (\alpha^{\alpha}\delta^{\delta-1})^{-1}(c_{H,t})^{\alpha}(c_{F,t})^{\delta-1}c_{F,t}^{1/\theta}(c_{F,t}(j^*))^{-1/\theta}, \tag{B.3}$$

$$1 = \lambda_t w_t, \tag{B.4}$$

$$\beta(\mu_{t+1}+\lambda_{t+1}) = \lambda_t x_t, \tag{B.5}$$

$$\beta(\mu_{t+1}+\lambda_{t+1}) = (\mu_t+\lambda_t)\frac{x_t}{R_t}, \tag{B.6}$$

$$\lim_{t\to\infty}\beta^t(\mu_t+\lambda_t)m_t = 0, \tag{B.7}$$

where the market clearing conditions of money market is $m_t = 1$, of asset market is $b_t = b_{t+1} = 0$, and of the currency exchange market is

$$S_t = \frac{x_t}{x_t^*}. \tag{B.8}$$

Taking the integral to the power of $(1-\theta)$ on both sides of (B.2) and (B.3) with respect to j and j^*, respectively:

$$(\alpha^{\alpha-1}\delta^{\delta})^{-1}(c_{H,t})^{\alpha-1}(c_{F,t})^{\delta} = (\mu_t+\lambda_t)p_{H,t}, \tag{B.9}$$

$$(\alpha^{\alpha}\delta^{\delta-1})^{-1}(c_{H,t})^{\alpha}(c_{F,t})^{\delta-1} = (\mu_t+\lambda_t)p_{F,t}. \tag{B.10}$$

Multiplying both sides of (B.2) and (B.3) by $c_{H,t}(j)$ and $c_{F,t}(j^*)$, respectively, taking integral, and then adding them together, we have

$$(\alpha+\delta)/(\alpha^{\alpha}\delta^{\delta})(c_{H,t})^{\alpha}(c_{F,t})^{\delta} = (\mu_t+\lambda_t)x_t, \tag{B.11}$$

where the binding CIA in Home country implies

$$x_t = \left(\int_0^1 p_{H,t}(j) c_{H,t}(j) dj + \int_0^1 p_{F,t}(j^*) c_{F,t}(j^*) dj^* \right). \tag{B.12}$$

Substituting (B.11) back into (B.9) and (B.10) yields

$$c_{H,t} = \frac{\alpha}{\alpha+\delta} \frac{x_t}{p_{H,t}}, \qquad c_{F,t} = \frac{\delta}{\alpha+\delta} \frac{x_t}{p_{F,t}}, \tag{B.13}$$

$$c_{H,t}(j) = \frac{\alpha}{\alpha+\delta} \frac{x_t}{p_{H,t}} \left(\frac{p_{H,t}(j)}{p_{H,t}} \right)^{-\theta}, \quad c_{F,t}(j^*) = \frac{\delta}{\alpha+\delta} \frac{x_t}{p_{F,t}} \left(\frac{p_{F,t}(j^*)}{p_{F,t}} \right)^{-\theta}. \tag{B.14}$$

Combining (B.4), (B.5), (B.11), and (B.12) yields

$$w_t = \frac{x_t x_{t+1}}{\beta(\alpha+\delta)/(\alpha^\alpha \delta^\delta) c_{H,t+1}^\alpha c_{F,t+1}^\delta}. \tag{B.15}$$

Finally, (B.6), (B.11), and (B.12) imply

$$R_t = \frac{x_{t+1} c_{H,t}^\alpha c_{F,t}^\delta}{\beta c_{H,t+1}^\alpha c_{F,t+1}^\delta}. \tag{B.16}$$

Similarly, for the representative agent in Foreign country, the counterpart optimization problem (HO^*) can be solved such that:

$$c_{F,t}^* = \frac{\alpha}{\alpha+\delta} \frac{x_t^*}{p_{F,t}^*}, \qquad c_{H,t}^* = \frac{\delta}{\alpha+\delta} \frac{x_t^*}{p_{H,t}^*}, \tag{B.17}$$

$$c_{F,t}^*(j^*) = \frac{\alpha}{\alpha+\delta} \frac{x_t^*}{p_{F,t}^*} \left(\frac{p_{F,t}^*(j^*)}{p_{F,t}^*} \right)^{-\theta}, \quad c_{H,t}^*(j) = \frac{\delta}{\alpha+\delta} \frac{x_t^*}{p_{H,t}^*} \left(\frac{p_{H,t}^*(j)}{p_{H,t}^*} \right)^{-\theta}, \tag{B.18}$$

$$w_t^* = \frac{x_t^* x_{t+1}^*}{\beta(\alpha+\delta)/(\alpha^\alpha \delta^\delta) c_{F,t+1}^{*\alpha} c_{H,t+1}^{*\delta}}, \tag{B.19}$$

$$R_t^* = \frac{x_{t+1}^* c_{F,t}^{*\alpha} c_{H,t}^{*\delta}}{\beta c_{F,t+1}^{*\alpha} c_{H,t+1}^{*\delta}}. \tag{B.20}$$

We next solve the Home firms' problem (FO) by maximizing (4.9), taking w_t, w_t^*, p_t, p_t^*, and G_t as given to obtain the unique solution

$$p_{H,t}(j) = \theta/(\theta-1)w_t. \tag{B.21}$$

In equilibrium, all firms have the same prices, so that

$$p_{H,t} = p_{H,t}(j) = \theta/(\theta-1)w_t, \tag{B.22}$$

for all $h \in [0,1]$. Similarly, solving (FO^*) yields

$$p_{F,t}^* = p_{F,t}^*(j^*) = \theta/(\theta-1)w_t^*, \tag{B.23}$$

for all $f \in [0,1]$.

The labor market clearing condition in Home country is simply

$$l_t = c_{H,t} + c_{H,t}^*. \tag{B.24}$$

Using (B.8), (B.13), and (B.17) gives

$$l_t = \frac{x_t}{p_{H,t}}. \tag{B.25}$$

Analogously, the labor used by Foreign firm is

$$l_t^* = \frac{x_t^*}{p_{F,t}^*}. \tag{B.26}$$

Combining (B.7), (B.11), and $m_t = 1$ yields the transversality condition

$$\lim_{t\to\infty} \beta^t \frac{(\alpha+\delta)/(\alpha^\alpha \delta^\delta)(c_{H,t})^\alpha (c_{F,t})^\delta}{x_t} = 0, \tag{B.27}$$

and similarly in the Foreign country we have

$$\lim_{t\to\infty} \beta^t \frac{(\alpha+\delta)/(\alpha^\alpha \delta^\delta)(c^*_{F,t})^\alpha (c^*_{H,t})^\delta}{x^*_t} = 0. \tag{B.28}$$

B.1.2 Proof of worst sustainable equilibrium

This part extends the work of Basso (2009) to prove the worst sustainable equilibrium in open economy model. Given that under autarky consumption and labor approach zero, the welfare of household in both countries approach zero. As $c_{H,t}$, $c_{F,t}$, $c^*_{H,t}$ and $c^*_{F,t}$ approach zero, so do y_t, y^*_t and firms' profits. Given that the utility functions of the households are always greater or equal to zero, and the utilities are non-negative under a competitive equilibrium, any other sustainable equilibrium will deliver greater or equal levels of payoff for household and firms. However, the governments and central banks objective functions consist of the economies' welfare plus a non-positive adjustment. Therefore, we must ensure that there is no sustainable equilibrium that delivers negative utility for the governments and the central banks.

Assuming that there is a sustainable equilibrium with strategy $\hat{\sigma}_{2H}$ that delivers a negative discounted sum of payoffs to the Home government from time t onwards, this implies $q_\tau < q^a_\tau$ for some $\tau > t$. However, at time t, the Home government would do better to set a strategy $\hat{\hat{\sigma}}_{2H}$ with $\hat{\hat{q}}_\tau = q^a_\tau$ for all $\tau > t$ and achieve a discounted sum of payoffs greater than or equal to zero, hence, $\hat{\sigma}_{2H}$ does not solve the reappointment's problem at time t, invaliding condition (v) of a sustainable equilibrium.

Finally, assuming that $\beta \le x_t \le \bar{x}$, if the Home central bank sets monetary policy optimally at period t, we have $x_t = p_{H,t}(\alpha+\delta)(q_t(1+\delta/\alpha))^{-1/(1-\delta-\alpha)}$. Plugging that into the home central bank objective function, we find that the maximized value of the current period utility is given by

$$\left(\frac{\alpha+\delta}{\alpha}\right)^{1/(\alpha+\delta-1)} \frac{\alpha+\delta}{\alpha}\left((q^a_t)^{\frac{\alpha}{\alpha+\delta-1}}(q^{*a}_t)^{\frac{\delta}{\alpha+\delta-1}} - (q_t)^{\frac{\alpha+\delta}{\alpha+\delta-1}}\right) + \frac{(q_t-1)\beta}{p_{H,t}},$$

which is positive as $0 < \alpha, \alpha < 1$ and $q_t = q_t^* \geq 1$. Hence, if the home central bank maximizes the period by period utility its total payoff would be positive. In a sustainable equilibrium, the central bank might not set x_t optimally at each period t, playing strategically, only to ensure a greater payoff. Therefore, in all other sustainable equilibria but the autarky equilibria, the central bank utility must be greater than zero. If the policy constraint, $\beta \leq x_t \leq \bar{x}$, binds above, then $x_t = \bar{x}$. In that case, we are back at the autarky equilibrium. □

B.1.3 Revert-to-autarky plan

Given an arbitrary world policy $G = (x, Q, x^*, Q^*)$ and an arbitrary allocation $(\Pi, \Omega, \Pi^*, \Omega^*)$, we define *revert-to-autarky* plans $(\sigma^r, \pi^r, \omega^r, \pi^{*r}, \omega^{*r})$ as follows. For all $t = 0, 1, ...,$ let $\varsigma_H^r(\xi_{t-1}) = q_t^a$, $\rho_H^r(\xi_{t-1}) = q_t$, and $\sigma_H^r(\xi_{t-1}) = x_t$ for history $\xi_{t-1} = \{q_s^a, q_s, x_s, q_s^{*a}, q_s^*, x_s^* | s = 0, 1, ..., t-1\}$; let $\varsigma_H^r(\xi_{t-1}) = 1$, $\rho_H^r(\xi_{t-1}) = 1$, $\sigma_H^r(\xi_{t-1}) = \bar{x}$ otherwise. For all $t = 0, 1, ...,$ let $\varsigma_F^r(\xi_{t-1}) = q_t^{*a}$, $\rho_F^r(\xi_{t-1}) = q_t^*$, and $\sigma_F^r(\xi_{t-1}) = x_t^*$ for history $\xi_{t-1} = \{q_s^a, q_s, x_s, q_s^{*a}, q_s^*, x_s^* | s = 0, 1, ..., t-1\}$; let $\varsigma_F^r(\xi_{t-1}) = 1$, $\rho_F^r(\xi_{t-1}) = 1$, $\sigma_F^r(\xi_{t-1}) = \bar{x}$ otherwise. For all $j \in [0, 1]$, let $\pi_t^{jr}(\xi_{t-1}) = p_{H,t}(j)$ for $\xi_{t-1} = \{q_s^a, q_s, x_s, q_s^{*a}, q_s^*, x_s^* | s = 0, 1, ..., t-1\}$; let $\pi_t^{jr}(\xi_{t-1}) = \pi^{jA}(\xi_{t-1})$ otherwise. For all $j^* \in [0, 1]$, let $\pi_t^{*j^*r}(\xi_{t-1}) = p_{F,t}^*(j^*)$ for $\xi_{t-1} = \{q_s^a, q_s, x_s, q_s^{*a}, q_s^*, x_s^* | s = 0, 1, ..., t-1\}$; let $\pi_t^{*j^*r}(\xi_{t-1}) = \pi^{*j^*A}(\xi_{t-1})$ otherwise. Let $\omega_t^r(\xi_t) = \Omega_t$ and $\omega_t^{*r}(\xi_t) = \Omega_t^*$ for $\xi_t = \{q_s^a, q_s, x_s, q_s^{*a}, q_s^*, x_s^* | s = 0, 1, ..., t\}$. If $\xi_t = (\xi_{H,t-1}, \tilde{q}^a, \tilde{q}, \tilde{x}, \tilde{q}^{*a}, \tilde{q}^*, \tilde{x}^*)$ is such that $\xi_{t-1} = \{q_s^a, q_s, x_s, q_s^{*a}, q_s^*, x_s^* | s = 0, 1, ..., t-1\}$ but $\tilde{q}^a \neq q_t^a$, $\tilde{q} \neq q_t$, $\tilde{x} \neq x_t$, $\tilde{q}^{*a} \neq q_t^{*a}$, $\tilde{q}^* \neq q_t^*$, and $\tilde{x}^* \neq x_t^*$, let $\omega_t^r(\xi_t)$ be given by the continuation rule ω^t that solve (HT), given σ^r, π^{jr} for all $j \in [0, 1]$ and π^{*j^*r} for all $j^* \in [0, 1]$; let $\omega_t^{*r}(\xi_t)$ be given by the continuation rule ω^{*t} that solve (HT^*), given σ^r, π^{jr} for all $j \in [0, 1]$ and π^{*j^*r} for all $j \in [0, 1]$. If $\xi_t = (\xi_{t-1}, \tilde{q}^a, \tilde{q}, \tilde{x}, q_s^{*a}, q_s^*, x_s^*)$ is such that $\xi_{t-1} \neq \{q_s^a, q^s, x_s, q_s^{*a}, q_s^*, x_s^* | s = 0, 1, ..., t-1\}$, let $\omega_t^r(\xi_t) = \omega^A(\xi_t)$ and $\omega_t^{*r}(\xi_t) = \omega^{*A}(\xi_t)$. Therefore, the revert-to-autarky plans induce the continuation of outcome $(G, \Pi, \Omega, \Pi^*, \Omega^*)$ as long as the policy (x, Q, x^*, Q^*) has been chosen in the past; if governments choose not to follow the announced policy, the strategy dictates to revert to autarky plan in both countries forever. □

B.2 Proof of Proposition

B.2.1 Proposition 4.1

We have $\hat{q} = \hat{q}^* = \theta/[(\theta-1)(1+\delta/\alpha)]$. Taking the first-order condition of $\hat{q}$ and $\hat{q}^*$ with respect to δ yields

$$\frac{\partial \hat{q}}{\delta} = \frac{\partial \hat{q}^*}{\delta} = -\frac{1}{\alpha}\frac{\theta}{(\theta-1)(1+\delta/\alpha)^2}. \tag{B.29}$$

□

B.2.2 Proposition 4.2

We extend the proof of Basso (2009) to the two-country model by directly applying the one-period deviation rule for two reasons: (i) observed behavior of the strategic players and (ii) the continuous game because of discounted and bounded payoffs. Given $\{\Pi, \Omega, \Pi, \Omega^*\}$, the governments and the central banks would not defect from the policy plan that induces a Markov equilibrium, for history ξ_{t-1} that validates such an equilibrium, since any defection causes a reduction in their one-period payoffs. Eventually, given $\{x, Q, x^*, Q^*\}$, allocation $\{\Pi, \Omega, \Pi^*, \Omega^*\}$ is a competitive equilibrium, hence market's conditions are clearing and rule $(\pi, \omega, \pi^*, \omega^*)$ solves the problems of household and firm. □

B.2.3 Proposition 4.3

We first show that given the decisions of the governments and central banks, allocation $(\Pi, \Omega, \Pi^*, \Omega^*)$ is part of a non-cooperative equilibrium under commitment. Using (4.19), (4.21), and $x_t = x_t^* = \beta$, for all t, we achieve $p_{H,t} = p^*_{F,t} = \underline{p}_H$, and using these results together with (4.16) and (4.17), we achieve $c_{H,t} = c^*_{F,t} = \bar{c}_H$ and $c_{H,t} = c^*_{F,t} = \bar{c}_F$.

We consider histories $\xi_{t-1} = (\xi_{H,t-1}, \xi_{F,t-1})$ along which the central banks have not defected from $(x, x^*) = (\beta, \beta)$. Since allocation $(x, \Pi, \Omega, x^*, \Pi^*, \Omega^*)$ is part of a non-cooperative equilibrium under commitment, choice Ω solves (HO) given $(x, x^*\Pi, \Pi^*, \Omega^*)$ and satisfies (MO) whereas choice Ω^* solves (HO^*) given $(x^*, \Pi, \Omega, \Pi^*)$ and satisfies (MO^*). Given (x, x^*), $\Pi^{\hat{j}}$ for all $\hat{j} \in [0,1]$, $\hat{j} \neq j$, and $(\Omega, \Pi^*, \Omega^*)$, each Π^j solves (FO), while given (x, x^*), $\Pi^{*\hat{j}^*}$ for all $\hat{j}^* \in [0,1]$, $\hat{j}^* \neq æ^*$, and (Π, Ω, Ω^*), each Π^{*j^*} solves (FO^*). Therefore, the continuation of Ω solves (HT) and satisfies (MT) at time t given $(x, x^*\Pi, \Pi^*, \Omega^*)$, the continuation of Ω^* solves (HT^*) and satisfies (MT^*) at time t given $(x, x^*, \Pi, \Omega, \Pi^*)$; the continuation of Π^j, $j \in [0,1]$ solves (FT) at time $t-1$ given (x, x^*), $\Pi^{\hat{j}}$ for all $\hat{j} \in [0,1]$, $\hat{j} \neq j$, and $(\Omega, \Pi^*, \Omega^*)$, and the continuation of $\Pi^{*æ^*}$, $j^* \in [0,1]$ solves (FT^*) at time $t-1$ given (x, x^*), $\Pi^{*\hat{j}^*}$ for all $\hat{j}^* \in [0,1]$, $\hat{j}^* \neq j^*$, and (Π, Ω, Ω^*). Neither the households nor the firms have temptations to defect after $(\xi_{H,t-1}, \xi_{F,t-1})$.

Now consider the Home central bank's incentive to defect from $x = \beta$ at time t following the history ξ_{t-1}. Since Ω solves (HO) and satisfies (MO) while Ω^* solves (HO^*) and satisfies (MO^*), the Home central bank solves the following problem

$$V_{CB} = \frac{1}{(\alpha+\delta)^{\alpha+\delta}} \left(\frac{x_t}{p_{H,t}}\right)^{\alpha} \left(\frac{x_t^*}{p_{F,t}^*}\right)^{\delta} - \frac{x_t}{p_{H,t}} - \frac{q_t - 1}{p_{H,t}}(x_t - \beta), \tag{B.30}$$

taking as given $x_t^* = \beta$, $q_t = q_t^* = \theta/[(\theta-1)(1+\delta/\alpha)]$, and $(\Pi, \Omega, \Pi^*, \Omega^*)$. It is easy to see that $\partial^2 V_{CB}/\partial x_t^2 < 0$ and $\frac{\partial V_{CB}}{\partial x_t} = 0$ at $x_t = \beta$. Hence, the Home central bank has no incentive to deviate from $x = \beta$ after ξ_{t-1}.

The problem of Home government at stage 3 using (4.18) and (4.23) becomes

$$\max_{z_t} \frac{1}{(\alpha+\delta)^{\alpha+\delta}} z_t^{\alpha} z_t^{*\delta} - z_t - \frac{\varphi}{2}(z_t - z_t^a)^2. \tag{B.31}$$

The first-order condition is

$$\frac{\alpha}{(\alpha+\delta)^{\alpha+\delta}} z_t^{\alpha-1} z_t^{*\delta} = 1 + \varphi(z_t - z_t^a). \tag{B.32}$$

Using (B.32), $z^* = (\alpha\delta\theta/(\theta-1))^{-1/\alpha}$, and initial delegation,

$$q_t^a = \left(\left(\frac{\theta\alpha}{(\theta-1)(\alpha+\delta)}\right)^{1/(\alpha+\delta-1)} + \frac{1}{\varphi(\alpha+\delta)}\left(\frac{\alpha}{\alpha+\delta}\right)^{1/(\alpha+\delta-1)}\left(1-\frac{\theta\alpha}{(\theta-1)(\alpha+\delta)}\right)\right)^{\alpha+\delta-1},$$

it is optimal for the Home government to set $q_t = \theta/[(\theta-1)(1+\delta/\alpha)]$.

Lastly, at stage 1 the Home government has to set the delegation parameter q_t^a. If firms equilibrium prices are given by (4.19) and (4.21), the central banks decide monetary policy according to (4.23) and (4.24), and the reappointment decision is made according to (B.32), then

$$p_{H,t} = \left(\frac{\theta(\alpha+\delta)^{\alpha+\delta-1}}{(\theta-1)\beta}\right)^{\frac{1}{1-\alpha}} (z_t p_{H,t})^{\frac{2-\alpha}{1-\alpha}} z_t^{*-\frac{\delta}{1-\alpha}} = \frac{(\theta-1)\beta}{\theta(\alpha+\delta)^{\alpha+\delta-1}} z_t^{\alpha-2} z_t^{*\delta}. \quad \text{(B.33)}$$

We have $x_t = z_t p_{H,t}$, so

$$x_t = \frac{(\theta-1)\beta}{\theta(\alpha+\delta)^{\alpha+\delta-1}} z_t^{\alpha-1} z_t^{*\delta}. \quad \text{(B.34)}$$

As the money growth rate is bounded, $\beta \le x_t \le \bar{x}$, and $z^* = (\alpha+\delta)(\theta/(\theta-1))^{1/(\alpha+\delta-1)}$, it follows that

$$\left(\frac{\beta}{\bar{x}}\right)^{\frac{1}{(1-\alpha)}} (\alpha+\delta)\left(\frac{\theta-1}{\theta}\right)^{\frac{1}{1-\alpha-\delta}} \le z_t \le (\alpha+\delta)\left(\frac{\theta-1}{\theta}\right)^{\frac{1}{1-\alpha-\delta}}. \quad \text{(B.35)}$$

When taking as given the firms and households allocation rules, the central banks' policy, governments' reappointment rule, and Foreign government' initial delegation, the Home government's delegation problem becomes

$$\max_{z_t} \frac{1}{(\alpha+\delta)^{\alpha+\delta}} z_t^{\alpha} z_t^{*\delta} - z_t - \frac{1}{2\varphi}\left(\frac{\alpha}{(\alpha+\delta)^{\alpha+\delta}} z_t^{\alpha-1} z_t^{*\delta} - 1\right)^2, \quad \text{(B.36)}$$

subject to (B.35). Given $0 < \alpha, \delta < 1$ and $\theta/[(\theta-1)(1+\delta/\alpha)] > 1$, the Home government's welfare function is increasing in z_t (because using (B.34) and $x_t \ge \beta$ we can show that

$\alpha/(\alpha+\delta)^{\alpha+\delta} z_t^{\alpha-1} z_t^{*\delta} - 1 > 0$. Since the upper bound of z_t is $(\alpha+\delta)((\theta-1)/\theta)^{1/(1-\alpha-\delta)}$, it is optimal to set z_t^a such that $z_t = (\alpha+\delta)((\theta-1)/\theta)^{1/(1-\alpha-\delta)}$ or $q_t = \theta/[(\theta-1)(1+\delta/\alpha)]$. Substituting this result into (B.32) yields the optimal delegation parameter for Home government at stage 1

$$q_t^a = \left(\left(\frac{\theta\alpha}{(\theta-1)(\alpha+\delta)}\right)^{1/(\alpha+\delta-1)} + \frac{1}{\varphi(\alpha+\delta)}\left(\frac{\alpha}{\alpha+\delta}\right)^{1/(\alpha+\delta-1)}\left(1-\frac{\theta\alpha}{(\theta-1)(\alpha+\delta)}\right)\right)^{\alpha+\delta-1}.$$

Similarly, we can show that the Foreign government at stage 1 and 3 and Foreign central bank at stage 4 have no incentives to deviate from q^{*a}, q^* and $x^* = \beta$, respectively, at time t following history ξ_t. In summary, the outcome we have found is a Markov outcome, and hence, a sustainable outcome under non-cooperation. □

B.2.4 Proposition 4.4

The proof mainly follows Proposition 4.3. Considering histories $\xi_{t-1} = (\xi_{H,t-1}, \xi_{F,t-1})$ along which the central banks have not defected from $(x, x^*) = (\beta, \beta)$, neither households nor firms have incentives to deviate after $(\xi_{H,t-1}, \xi_{F,t-1})$. If the Home central bank is given a contract $q_t = \theta\alpha/(\theta-1)$, then it is optimal to set $x_t = \beta$. Hence, the Home central bank does not deviate from $x_t = \beta$ after $\xi_{H,t-1}$ and so does the Foreign central bank.

At stage 3 in the Home country, if the Home central bank is not reappointed, the Home government's welfare function must be decreasing in z_t for $z_t > z_t^a$. Therefore,

$$\begin{aligned} &\frac{\alpha}{(\alpha+\delta)^{\alpha+\delta}} z_t^{\alpha-1} z_t^{*\delta} - 1 - \varphi z_t^a < 0, \ \textit{or} \\ &\varphi > (z_t^a)^{-1}\left(\frac{\alpha}{(\alpha+\delta)^{\alpha+\delta}} z_t^{\alpha-1} z_t^{*\delta} - 1\right). \end{aligned} \tag{B.37}$$

Given the definition of (z_t^a, z_t, z_t^*) and the delegation contract, reappointment does not occur if

$$\varphi > (\theta/(\theta-1))^{1/(1-\alpha-\delta)}(\theta/[(\theta-1)(1+\delta/\alpha)]-1)/(\alpha+\delta) \equiv \underline{\varphi}. \tag{B.38}$$

Alternatively, as long as (B.38) holds and other agents follow the Ramsey outcome, the Home government at stage 3 has no incentive to deviate from a contract with Home central bank.

Finally, at stage 1 the Home government must set the delegation parameter q_t^a to maximize $\left((c_{H,t})^{\alpha}(c_{F,t})^{\delta}\right)/\left(\alpha^{\alpha}\delta^{\delta}\right) - l_t - \varphi z_t^a |z_t - z_t^a|$ given q_t^{*a}. If firms equilibrium prices are given by (4.19) and (4.21), the central banks decide monetary policy according to (4.23) and (4.24), the reappointment decision is made according to (4.45), and $q_t^{*a} = \theta/[(\theta-1)(1+\delta/\alpha)]$, we have

$$x_t = \beta\left(\frac{q_t^a(\theta-1)(\alpha+\delta)}{\theta\alpha}\right)^{\frac{1-\alpha}{1-\alpha-\delta}}. \tag{B.39}$$

As $\beta \leq x_t \leq \bar{x}$, so (B.39) becomes

$$\frac{\theta\alpha}{(\theta-1)(\alpha+\delta)} \leq q_t^a \leq \left(\frac{\bar{x}}{\beta}\right)^{\frac{1-\alpha-\delta}{1-\alpha}}\frac{\theta\alpha}{(\theta-1)(\alpha+\delta)}. \tag{B.40}$$

When taking as given the firms and households allocation rules, the central banks' policy, governments' reappointment rule, and Foreign government' initial delegation, the Home government's delegation problem becomes

$$\max_{q_t^a}\left(\frac{\alpha+\delta}{\alpha}\right)^{1/(\alpha+\delta-1)}\frac{\alpha+\delta}{\alpha}(q_t^a)^{\frac{\alpha}{\alpha+\delta-1}}(q_t^{*a})^{\frac{\delta}{\alpha+\delta-1}} - \left(\frac{\alpha+\delta}{\alpha}\right)^{1/(\alpha+\delta-1)}(q_t^a)^{\frac{1}{\alpha+\delta-1}}, \tag{B.41}$$

subject to (B.40). Given that the objective function (B.41) is decreasing in q_t^a as $q_t^a > 1$ and $q_t^{*a} > 1$, it is optimal for the Home government to set the q_t^a equal to the lower bound such that $q_t^a = \theta/[(\theta-1)(1+\delta/\alpha)]$.

Analogously, we can show that the Foreign government at stage 1 and 3 and Foreign central bank at stage 4 have no incentives to deviate from $q^{*a} = q^* = \theta/[(\theta-1)(1+\delta/\alpha)]$ and $x^* = \beta$, respectively, at time t following history $\xi_{F,t}$. In brief, the Ramsey outcome is a Markov outcome, and hence, a sustainable outcome under non-cooperation when $\varphi > \underline{\varphi}$. $\square$

B.2.5 Proposition 4.5

We have

$$\underline{\varphi} = \left(\frac{\theta}{\theta-1}\right)^{\frac{1}{1-\alpha-\delta}} \left(\frac{\theta\alpha}{(\theta-1)(\alpha+\delta)} - 1\right) \frac{1}{\alpha+\delta}. \tag{B.42}$$

Since $\alpha+\delta$ is fixed, $\partial\underline{\varphi}/\partial\alpha > 0$. An increase in δ means a decrease in α which reduces the value of $\underline{\varphi}$. $\square$

B.2.6 Proposition 4.6

Under σ^A, the Home representative firm takes ξ_{t-1} as given at each date $t = 0, 1, 2, ...$ and solves (FT) assuming $x_{t+s} = \bar{x}$ and $x_{t+s}^* = \bar{x}$ for all $s = 0, 1, 2, ...$ Similarly, the Home representative household takes ξ_t as given at each date $t = 0, 1, 2, ...$ and solves (HT) assuming $x_{t+s+1} = \bar{x}$ and $x_{t+s+1}^* = \bar{x}$ for all $s = 0, 1, 2, ...$ The Foreign firm and household are analogous.

For $t = 0, 1, 2, ...$, the necessary and sufficient conditions for solutions to (FT), (HT), (FT^*), and (HT^*) coincide with those for (FO), (HO), (FO^*), and (HO^*), respectively. Taking $x_t = \bar{x}$ and $x_t^* = \bar{x}$ as given, we plug (4.16), (4.17), and (4.21) into (4.19) to achieve a system of difference equations with solution $p_{H,t} = \bar{p}_H$ and $p_{F,t}^* = \bar{p}_F^*$ for all $t = 0, 1, ...$, where

$$\bar{p}_H = \left(\frac{\theta(\alpha+\delta)^{\alpha+\delta-1}}{(\theta-1)\beta} \right)^{\frac{1}{1-\alpha-\delta}} \bar{x}^{1+1/(1-\alpha-\delta)}, \tag{B.43}$$

$$\bar{p}_F^* = \left(\frac{\theta(\alpha+\delta)^{\alpha+\delta-1}}{(\theta-1)\beta} \right)^{\frac{1}{1-\alpha-\delta}} \bar{x}^{1+1/(1-\alpha-\delta)}. \tag{B.44}$$

It follows that each π^{jA}, $j \in [0,1]$ has $\pi_t^{jA}(\xi_{t-1}) = \bar{p}_H$ and π^{*j^*A}, $j^* \in [0,1]$ has $\pi_t^{*j^*A}(\xi_{t-1}) = \bar{p}_F^*$ for all $t = 0,1,2,...$ and ξ_{t-1}. Then we can drive the solution to $\omega_t^A(\xi_t)$

$$c_{H,t} = \frac{\alpha}{\alpha+\delta}\frac{x_t}{\bar{p}_H}, \qquad c_{F,t} = \frac{\delta}{\alpha+\delta}\frac{x_t}{\bar{p}_F}, \tag{B.45}$$

$$c_{F,t}^* = \frac{\alpha}{\alpha+\delta}\frac{x_t^*}{\bar{p}_F^*}, \qquad c_{H,t}^* = \frac{\delta}{\alpha+\delta}\frac{x_t^*}{\bar{p}_H^*}, \tag{B.46}$$

$$l_t = \frac{x_t}{\bar{p}_H}, \qquad l_t^* = \frac{x_t^*}{\bar{p}_F^*}, \tag{B.47}$$

for all $t = 0,1,2,...$ and ξ_t.

By construction, π^{jA} for all $j \in [0,1]$, π^{*j^*A} for all $j^* \in [0,1]$, ω^A, and ω^{*A} satisfy conditions (i)–(iii) in the definition of a sustainable equilibrium under non-cooperation. We have to verify the rest of conditions (iv)–(vi). First, for condition (vi): Given σ_F, σ_{iH} and σ_{iF} for $i = \{1,2\}$, π^j for all $j \in [0,1]$, π^{*j^*} for all $j^* \in [0,1]$, ω, and ω^*, the continuation σ_H^t solves (CBT) for all $t = 0,1,...$ and ξ_{t-1}; Given σ_H, σ_{iH} and σ_{iF} for $i = \{1,2\}$, π^j for all $j \in [0,1]$, π^{*j^*} for all $j^* \in [0,1]$, ω, and ω^*, the continuation σ_F^t solves (CBT^*) for all $t = 0,1,...$ and ξ_{t-1}. In light of $\bar{p}_H$ and $\bar{p}_F^*$ as above, $q_t = 1$, and $x_t^* = \bar{x}$ for all t, we aim to show that $x_t = \bar{x}$ for all t maximizes the one-period welfare of the Home household

$$V_{CB,t} = \frac{1}{(\alpha+\delta)^{\alpha+\delta}}\frac{x_t^\alpha \bar{x}^\delta}{\bar{p}_H^\alpha \bar{p}_F^{*\delta}} - \frac{x_t}{\bar{p}_H}. \tag{B.48}$$

It is clear to see that $\partial^2 V_{CB,t}/\partial x_t^2 < 0$ and

$$\frac{\partial V_{CB,t}}{\partial x_t} = \frac{1}{\bar{p}_H}\left(\frac{\alpha}{(\alpha+\delta)^{\alpha+\delta}}\frac{x_t^{\alpha-1}\bar{x}^{\delta}}{(\bar{p}_H)^{\alpha+\delta-1}} - 1\right), \tag{B.49}$$

whose value at $x_t = \bar{x}$ is given by

$$\frac{1}{\bar{p}_H}\left(\frac{\alpha\theta\bar{x}}{\beta(\alpha+\delta)(\theta-1)} - 1\right). \tag{B.50}$$

As long as $\frac{\alpha\theta\bar{x}}{\beta(\alpha+\delta)(\theta-1)} > 1$, the optimal policy of Home central bank is $x_t = \bar{x}$ for all t if the Foreign central bank and other agents follow the autarky plan under non-cooperation. Similarly, when $\frac{\alpha\theta\bar{x}}{\beta(\alpha+\delta)(\theta-1)} > 1$, the optimal policy of Foreign central bank is $x_t^* = \bar{x}$ for all t if the Home central bank and other agents follow the autarky plan under non-cooperation.

Next, we verify condition (v) by using the contraction. We check the first-order condition (4.45) for the Home government's problem at stage 3. Given $z_t^a = z_t^* = (\alpha+\delta)((\alpha+\delta)/\alpha)^{1/(\alpha+\delta-1)}$, suppose that $z_t > z_t^a$ holds, we can compute z_t as follows

$$z_t = \left(\frac{1}{1+\varphi z_t^a}\right)^{\frac{1}{1-\alpha}} z_t^a < z_t^a. \tag{B.51}$$

Therefore, it is optimal for Home government to set $z_t = z_t^a$ or $q^t = 1$ given that other agents follow the autarky plan.

Lastly, we verify condition (iv) for governments at stage 1. When taking as given the firms and households allocation rules, the central banks' policy, governments' reappointment rule, and Foreign government' initial delegation, $q_t^{*a} = 1$, the Home government's delegation problem becomes

$$\max_{q_t^a} \frac{\alpha+\delta}{\alpha}(q_t^a)^{\frac{\alpha}{\alpha+\delta-1}} - (q_t^a)^{\frac{1}{\alpha+\delta-1}}, \tag{B.52}$$

subject to $q_t^a \geq 1$. Given that the objective function (B.52) is decreasing in q_t^a as $q_t^a > 1$, it is optimal for the Home government to set the q_t^a equal to the lower bound such that $q_t^a = 1$. $\square$

B.2.7 Proposition 4.7

In the following proof, it is enough to consider the Home government's problem. That of Foreign government is analogous. The proof of proposition 4.7 includes two parts: First, assume that $(G, \Pi, \Omega, \Pi^*, \Omega^*)$ is a sustainable outcome, accompanied with some sustainable equilibrium $(\sigma, \pi, \omega, \pi^*, \omega^*)$. As (FT), (FT^*), (HT), (HT^*), (MT), and (MT^*), at $t = 0$ coincide with (FO), (FO^*), (HO), (HO^*), (MO), and (MO^*), conditions (i)–(iii) in the definition of a sustainable equilibrium mean that $(G, \Pi, \Omega, \Pi^*, \Omega^*)$ is an outcome of equilibrium under commitment. It is enough to consider the Home government's incentive.

Since ω is part of a sustainable equilibrium, its continuation ω^t must address (HT) and satisfy (MT) for $t = 0, 1, 2, \ldots$ Since each $t = 0, 1\ldots$, the necessary and sufficient conditions for (HT) are identical with those for (HO) and the market clearing (MT) are identical with those for (MO). It is analogous to ω^*. Thus, $\omega_t(\xi_t)$ and $\omega_t^*(\xi_t)$ must have

$$c_{H,t} = \frac{\alpha}{\alpha+\delta}\frac{x_t}{p_{H,t}}, \qquad c_{F,t} = \frac{\delta}{\alpha+\delta}\frac{x_t}{p_{F,t}}, \tag{B.53}$$

$$c_{F,t}^* = \frac{\alpha}{\alpha+\delta}\frac{x_t^*}{p_{F,t}^*}, \qquad c_{H,t}^* = \frac{\delta}{\alpha+\delta}\frac{x_t^*}{p_{H,t}^*}, \tag{B.54}$$

$$l_t = \frac{x_t}{p_{H,t}}, \qquad l_t^* = \frac{x_t^*}{p_{F,t}^*}, \tag{B.55}$$

$$w_t = \frac{x_t x_{t+1}}{\beta(\alpha+\delta)/(\alpha^\alpha \delta^\delta) c_{H,t+1}^\alpha c_{F,t+1}^\delta}, \qquad w_t^* = \frac{x_t^* x_{t+1}^*}{\beta(\alpha+\delta)/(\alpha^\alpha \delta^\delta) c_{F,t+1}^{*\alpha} c_{H,t+1}^{*\delta}}, \tag{B.56}$$

for all $t = 0, 1, 2, \ldots$ and ξ_t.

Then, given t and ξ_{t-1}, the Home governments at stage 3 could defect from q at date t, so that achieve the one-period weighted welfare $U_G^d(p_{H,t}, p_{F,t}^*)$ and revert to the autarky plan ρ_H^A after that. This defection generates the total weighted welfare given by the left-hand side of (4.46). Since the condition (v) in the definition of a sustainable equilibrium prevents such a temptation to defect, (4.46) need to be satisfied.

Second, let $(G, \Pi, \Omega, \Pi^*, \Omega^*)$ be an outcome of equilibrium under commitment that satisfied the constraint (4.46), and let $(\sigma^r, \pi^r, \omega^r, \pi^{*r}, \omega^{*r})$ be the corresponding revert-to-autarky plans. We need to show that $(\sigma^r, \pi^r, \omega^r, \pi^{*r}, \omega^{*r})$ is a sustainable equilibrium.

Given the history ξ_{t-1} in which governments have never defected from G, allocation $(G, \Pi, \Omega, \Pi^*, \Omega^*)$ is an outcome of equilibrium under commitment. Choice Ω solves (HO) and satisfies (MO) given G, Π^j for all $j \in [0,1]$, and Π^{*j^*} for all $j^* \in [0,1]$, while choice Ω^* solves (HO^*) and satisfies (MO^*) given G, Π^j for all $j \in [0,1]$, and Π^{*j^*} for all $j^* \in [0,1]$. Each Π^j solves (FO), given G, $\Pi^{\hat{j}}$ for all $\hat{j} \in [0,1]$, $\hat{j} \neq j$, and $(\Omega, \Pi^*, \Omega^*)$, whereas each Π^{*j^*} solves (FO^*), given G, $\Pi^{*\hat{j}^*}$ for all $\hat{j}^* \in [0,1]$, $\hat{j}^* \neq j^*$, and (Π, Ω, Ω^*). Therefore, the continuation of Ω solves (HT) and satisfies (MT) at date t given G, Π^j for all $j \in [0,1]$, and Π^{*j^*} for all $j^* \in [0,1]$, the continuation of Ω^* solves (HT^*) and satisfies (MT^*) at date t given X, Π^j for all $j \in [0,1]$, and Π^{*j^*} for all $j^* \in [0,1]$, the continuation of Π^j solves (FT) at date $t-1$ given G, $\Pi^{\hat{j}}$ for all $\hat{j} \in [0,1]$, $\hat{j} \neq j$, and $(\Omega, \Pi^*, \Omega^*)$, and the continuation of Π^{*j^*} solves (FT) at date $t-1$ given G, $\Pi^{*\hat{j}^*}$ for all $\hat{j}^* \in [0,1]$, $\hat{j}^* \neq j^*$, and (Π, Ω, Ω^*). Both household and firm have no temptation to defect after ξ_{t-1}.

Now we consider the Home governments' temptation to defect from q from date t following ξ_{t-1}. Since Ω solves (HO) and satisfies (MO), and Ω^* solves (HO^*) and satisfies (MO^*) for all ξ_t, $\omega^r(\xi_t)$ and $\omega^{*r}(\xi_t)$ have

$$c_{H,t} = \frac{\alpha}{\alpha+\delta}\frac{\tilde{x}_t}{p_{H,t}}, \qquad c_{F,t} = \frac{\delta}{\alpha+\delta}\frac{\tilde{x}_t}{p_{F,t}}, \tag{B.57}$$

$$l_t = \frac{\tilde{x}_t}{p_{H,t}}. \tag{B.58}$$

Utilizing Appendix B.1.2, the best the Home government at stage 3 can achieve by defecting is to set $\tilde{q}$ at date t and, follow the autarky plans $(\sigma^a, \pi^a, \omega^a, \pi^{*a}, \omega^{*a})$ forever after. This best defection produces welfare given by the left-hand side of (4.46). Because (4.46) holds, the governments have no temptation to defect from q after ξ_{t-1}.

For histories ξ_{t-1} in which there are defections from q by Home government, the revert-to-autarky plans dictate that in Home country the government, firms, and households all choose the autarky plans in every subsequent period. As proved in Appendix B.1.2, no one has a temptation to defect from these plans.

Finally, $(\sigma^r, \pi^r, \omega^r, \pi^{*r}, \omega^{*r})$ is a sustainable equilibrium, and $(G, \Pi, \Omega, \Pi^*, \Omega^*)$ is the accompanied sustainable outcome. □

B.2.8 Proposition 4.8

Holding $\alpha + \delta$ fixed, we need to prove the right-hand side of the constraint (4.48) is decreasing in δ or increasing in α. As $\alpha + \delta$ is constant, so do some parts of the right-hand side of the constraint (4.48). Hence, the problem reduces to show a function $f(\alpha) = \alpha^{\alpha/(1-\alpha)}(1-\alpha)$ is increasing in α.

Taking log of $f(\alpha)$ and then taking derivative yields $\partial \ln(f)/\partial \alpha > 0$. We can derive the first-order of $f(\alpha)$ with respect to α as follows:

$$\frac{\partial f(\alpha)}{\alpha} = \frac{\partial e^{\ln(f)}}{\ln(f)} \frac{\partial \ln(f)}{\partial \alpha} \tag{B.59}$$

$$= e^{\ln(f)} \frac{\partial \ln(f)}{\partial \alpha} > 0. \tag{B.60}$$

□