

ESSAYS ON WELFARE COSTS OF SHIFTING TREND
INFLATION AND POLICY UNCERTAINTY

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by

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To my beloved ones

Executive Summary

Economists and policy makers alike saw the U.S. economy as being simultaneously buffeted by a sustained rise in inflation and larger-than-usual uncertainty about future policy. To model the sustained increase in inflation, this study uses a highly persistent shock to trend inflation, which can be interpreted as the Federal Reserve's slowly-moving implicit inflation target. Uncertainty is defined as the dispersion of economic shock distributions, thus a joint consideration of structural and volatility shocks. Regarding uncertainty, the focus lies on uncertainty about the monetary policy, the volatility shock arising in a monetary policy shock. We propose three research questions: (i) how large are welfare costs of shifting trend inflation? (ii) what are the roles of time-varying volatility in affecting welfare costs of shifting trend inflation? and (iii) do staggered price or wage contracts more importantly determine welfare costs of shifting trend inflation?

This dissertation is timely and important due to the following reasons. First, the two factors: shifting trend inflation and time-varying volatility, which are salient features of the U.S. economy, have been examined largely in isolation thus far. This dissertation, therefore, is the first to study welfare costs in the economy featuring both shifting trend inflation and policy uncertainty. Second, the literature on shifting trend inflation has so far only considered the sticky price model. We, hence, expand the standard model by discussing transmission mechanisms of shifting trend inflation through two channels: staggered price and wage contracts. They lead to inefficient allocations of both output and labor supply, then create high costs. Without these expansions, one might conclude that the costs of shifting trend inflation are modest and ignore its consequences.

The dissertation analyses the relationship between shifting trend inflation, policy uncertainty and economic welfare. We start the dissertation by describing the motivation, research objectives, methodology, and organization in Chapter 1. In Chapter 2, we illustrate the presence of two main features of the U.S. data and review the literature. In Chapter 3, we develop the DSGE model which incorporates a shock to trend inflation and a time-varying stochastic volatility shock arising in a monetary policy shock. The results show interactions between shifting trend inflation and policy uncertainty. On the one hand, the cost of exogenous variations in trend inflation is larger if there is policy uncertainty. A rise in the variance of shocks to trend inflation decreases welfare not only by increasing volatilities of consumption and leisure, but also by decreasing their average levels. An introduction of uncertainty then signifies these changes to produce greater welfare costs. Further, policy uncertainty itself produces large welfare costs. On the other hand, adverse impacts of policy uncertainty on the economy, documented by impulse response functions of macroeconomic variables to policy uncertainty shocks, become more sizeable when central banks raise their inflation targets.

In Chapter 4, we develop a New Keynesian model featuring staggered price and wage contracts to study welfare costs of exogenous variations in trend inflation. The analyses show that the consequences of constant positive trend inflation and shifting trend inflation are severe, especially when trend inflation is high. Among two channels, staggered wage contracts play a vital role in transmitting adverse impacts of constant and shifting trend inflation into the economy. Without the staggered wage channel, these costs are modest. We also conduct exercises to examine the sensitivity of welfare costs to a wide range of plausible parameters. The results show that if the price and wage friction are sufficiently large, the price and wage indexation level are sufficiently small, or there is upward biased trend inflation process, the welfare costs become larger. Finally, Chapter 5 provides conclusions and policy implications.

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Chapter 1

Introduction

A conventional but empirically unrealistic assumption in the economics is that trend inflation, interpreted as implicit inflation targets of central banks or the long-run inflation expectation of private sectors, is zero. In addition to the fact that the zero trend inflation is exceedingly rare in the real world, this restrictive assumption might lead to misleading conclusions, such as a divine coincidence suggested by Blanchard and Gali (2007)¹, or a highly non-linear and positive slope of the long run New Keynesian Phillips curve (NKPC, henceforth)². Therefore, it is necessary to relax this assumption since its costs outweigh benefits from a simple and tractable resulting model by employing this idea in traditional economics. Further, evidence on changing trend inflation rates for several industrial countries and for the U.S. is provided by Levin and Piger (2003) and Ireland (2007)³, respectively. Therefore, the trend inflation level tends to change over time.

The aforementioned reasons inspire us to study optimizing behavior without making an assumption of zero trend inflation. Although there is increasing work on the model with constant positive trend inflation, recent studies have still not paid

¹Blanchard and Gali (2007) suggest that in the face of preference or technology shocks, any monetary policy rule that stabilizes the inflation rate also stabilizes the output gap. Alves (2014), on the other hand, shows that there is a trade-off between output gap variability and price inflation variability in the standard New Keynesian model and the divine coincidence only holds when the inflation rate is stabilized at zero.

²Ascari and Ropele (2009) indicate that due to a strong price dispersion effect, the slope turns quite rapidly negative for extreme values of trend inflation.

³By using the model that allows inferences concerning the Federal Reserve's inflation targets, Ireland (2007) shows changes from 1959 (1.25 percent) to the late 1970s (8 percent) and in 2004 (2.5 percent).

enough attention to the property of its variation, referred to the concept of shifting trend inflation⁴ in the literature, despite its implications for various aspects of macroeconomic dynamics. Contributions on this issues show necessities of research on shifting trend inflation, such as its implications on the term structure of interest rate by Kozicki and Tinsley (2001), its consequences on output and inflation by Ireland (2007) or changes of estimated parameters by Cogley and Sbordone (2008). However, its welfare analysis has not been fully exploited. The only paper by Nakata (2014) using the model with staggered prices measures welfare costs of shifting trend inflation⁵. The similar spirit of the existing literature is that they model shifting trend inflation by employing a highly persistent trend inflation process, regarded as the central bank’s moving implicit target. Moreover, these papers have so far only discussed one form of rigidities, the sticky-price model with shifting trend inflation.

Besides, economists and policy makers alike realize the other fact that the U.S. economy is buffeted by larger-than-usual uncertainty about future policy⁶ in addition to a sustained rise in inflation. Following to the literature, we define the term “uncertainty”, referred to “objective uncertainty” or “risk” as the dispersion or the spread of economic shock distribution. The fact that the future shocks are widely distributed (time-varying variance) is known by rational consumers and firms and they react to this fact. Accordingly, we clearly classify the structural shocks and the stochastic volatility shocks in the dissertation.

Although previous work has exploited different aspects of policy uncertainty, there still exist gaps. They have so far mainly devoted to aggregate effects of short-run fluctuations of volatility⁷ on different macroeconomic variables. Recently, few

⁴In other words, trend inflation is assumed to be positive and is more likely to shift upward and downward over time.

⁵As in Nakata (2014), these welfare costs can be interpreted as the consequences when central banks lack a commitment to pursue a fixed inflation target or central banks have a tendency to let inflation change.

⁶Born and Pfeifer (2014) argue that the Great Inflation 1970s periods were plagued by high shock volatilities. The evidence for shifts on the variance of innovations is provided by Bernanke and Mihov (1998), Cogley and Sargent (2005) and Primiceri (2005).

⁷Variance measures how far a data set is spread out from its mean, and volatility is the square

authors have paid attention to effects of uncertainty on welfare, such as one-sided movement of volatility (a decrease in the certain level of variance of shocks to zero) as in Lester, Pries, and Sims (2014) and Cho, Cooley, and Kim (2015) or two-sided movement of volatility (a coexistence of structural shocks and volatility shocks) as in Xu (2017) and Bachmann, Bai, Lee, and Zhang (2018). Further, the welfare analysis of policy uncertainty has been examined independently of shifting trend inflation, which is another important feature of the U.S. data, despite the potential impacts of their interaction on the welfare.

To in part fill these gaps, the dissertation primarily concentrates on analysing the relationship between shifting trend inflation, volatility risk and economic welfare. We address the question in empirical macroeconomics: (i) how large are costs of shifting trend inflation? (ii) what are the roles of time-varying volatility in affecting the welfare costs of shifting trend inflation? and (iii) do staggered price or wage contracts more importantly determine the welfare costs of shifting trend inflation?

To seek the answers to these questions, we use the Dynamic Stochastic General Equilibrium (DSGE, henceforth) models with nominal rigidities as in Ireland (2007). The literature is also expanded in the following ways. In Chapter 3, we incorporate both structural shocks and time-varying volatility shocks in the model. Regarding uncertainty, the focus lies on the monetary policy (Policy Rate Risk (PR, henceforth)) that is the volatility shock to the monetary policy shock. The stochastic volatility shock is assumed to follow an AR(1) process as in Shephard (2008) and Fernandez-Villaverde, Guerron-Quintana, and Uribe (2011). We then compare the welfare between the economy with a zero and positive variance of shocks to trend inflation as well as a zero and positive variance of volatility shocks. In Chapter 4, the dissertation develops the model with a Calvo price and wage setting. We investigate how exogenous variations in trend inflation distort the economy by two

root of the variance. The time-varying volatility implies that the volatility is more likely to move upwards or downwards over time. Modelling time-varying volatility can help us to understand the sources of economy fluctuations.

channels: staggered price and staggered wage contracts. We then discuss two special cases: the model with staggered price contracts and completely flexible wage, and the model with completely flexible price and staggered wage contracts. This exercise is helpful to analyse the role of each channel separately in transmitting impacts of exogenous variations in trend inflation into the economy.

In summary, the dissertation makes two main contributions to fill the existing gaps in the literature. First, we investigate interactions between shifting trend inflation and policy uncertainty due to the potential impacts on the welfare of the economy. In addition to an existence of these two features as shown by the U.S data in Chapter 2, Fernandez-Villaverde, Guerron-Quintana, and Rubio-Ramirez (2015) argue that the presence of both stochastic volatility and parameter drifting⁸ successfully models the U.S. economy. Second, the study expands the literature by discussing the mechanism of how shifting trend inflation distorts the economy through two channels: staggered prices and staggered wages. We argue that with staggered prices and staggered wages, shifting trend inflation can induce dispersion in prices across firms and hence inefficient output levels, and induce dispersion in the distribution of employment across households, respectively.

There are some main results. In Chapter 3, we found that there exist the interactions between shifting trend inflation and monetary policy uncertainty. On the one hand, the costs of constant and exogenous variations in trend inflation are larger if there is policy uncertainty. More specifically, welfare reduction caused by shifting trend inflation can be explained by a fall in average levels as well as a growth in volatility of consumption and leisure. These changes are then signified when there exists uncertainty about monetary policies.

On the other hand, effects of monetary policy uncertainty on the economy become more sizeable when the trend inflation level increases. In particular, the

⁸Shifting trend inflation is a type of parameter drifting, in which the steady-state inflation drifts over time.

presence of policy uncertainty generates a small welfare cost by itself. A higher level of trend inflation then drives this cost up. Besides, the impulse response functions documenting the trend and magnitude of effects of policy uncertainty shocks with respect to diverse levels of trend inflation report a decline in output growth as well as a rise in inflation. When trend inflation is higher, these shocks affect the economy more remarkably. To put it differently, the higher inflation targets the central banks propose to raise, the more severe consequences the monetary policy uncertainty generates.

In Chapter 4, we found that shifting trend inflation produces large welfare costs, in which the staggered wage contracts play a vital role in transmitting adverse impacts of this shock into economy. Further, the results illustrating sensitivity of welfare costs to a wide range of plausible parameters show that changes in parameters governing the wage environment cause more substantial movements of welfare costs as opposed to parameters controlling the pricing environment. The result supports the conclusion that the staggered wage contracts more importantly determine welfare costs of shifting trend inflation.

Chapter 5 summarizes the main findings of the dissertation and then provides policy implications based on these findings. We also leave the space for a discussion over costs and benefits of raising inflation targets.

The organisation of the dissertation is as follows. Chapter 2 is an overview of shifting trend inflation and uncertainty. Chapter 3 studies welfare costs in the economy featuring shifting trend inflation and policy uncertainty. Chapter 4 presents our computations for welfare costs of shifting trend inflation in the economy featuring staggered prices and wages. Chapter 5 concludes and provides policy implications.

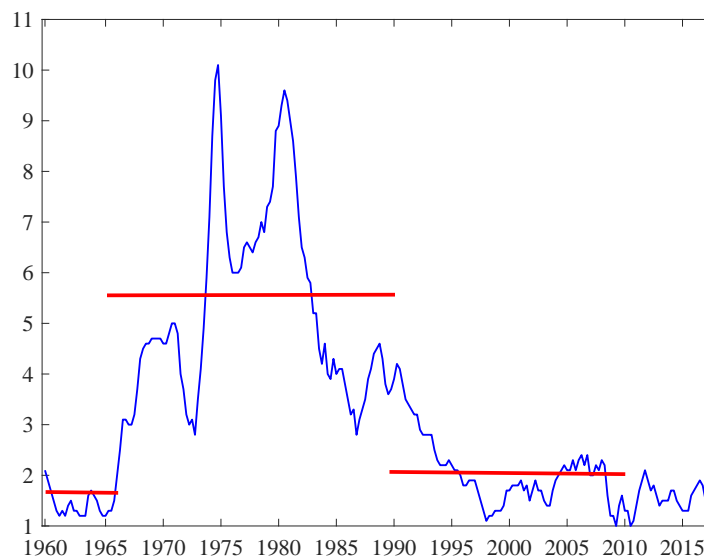
Chapter 2

An Overview of Shifting Trend Inflation and Policy Uncertainty

2.1 The Great Inflation

Figure 2.1 shows the evolution of United States core PCE Inflation from 1960:Q1 to 2015:Q1. The period of Great Inflation is characterized as a sustained rise in inflation starting from the late 1960s to the early 1980s. During the episode of the Great Inflation, the U.S. inflation increased dramatically. Specifically, in contrast to low and stable 2 percent in the first half of 1960s, United States core PCE inflation rose to more than 10 percent by the end of 1970s. Subsequently, inflation fell to low and stable roughly 2 percent over the past two decades. If we compute the average of inflation (it is red lines in Figure 2.1) in different periods, we clearly observe shifts in means of inflation over time. Further, a rise in inflation was a gradual process instead of a sudden increase to a high level. It implies that the rise in inflation was persistent during the Great Inflation. The property of high persistence for inflation is documented by other empirical studies, namely Cogley and Sbordone (2008), Fuhrer (2010), and Nakata (2014).

Figure 2.1: United States Core PCE Inflation: 1960Q1-2015Q1 (annualized quarter-to-quarter percent changes)



Source: FRED. The red lines are the average of inflation in different periods.

In addition, we also show an existence of the time-varying volatility in the U.S. economy. Figure 2.2 and 2.3 plot the absolute deviations of United States real GDP growth and core PCE from their means. There were the big spikes starting from the 1970s to early 1980s. They disappeared since 1984 and did not return even briefly as shown in Figure 2.3. Moreover, the absolute deviations of real GDP growth and inflation rarely crossed 0.2 and 2 point since 1984, while they did it rather often before. The evidence suggests the existence of time-varying volatility in the U.S. data.

The existing literature has also documented a feature of time-varying volatility of the U.S. data, for example, Sims and Zha (2006), Fernandez-Villaverde and Rubio-Ramirez (2007), and Justiniano and Primiceri (2008). Born and Pfeifer (2014) also show that the 1970s were plagued by high shock volatilities in monetary policy shocks. Therefore, the Great Inflation period is also characterized by time-varying volatility.

Figure 2.2: United States Real GDP Growth, Absolute Deviations from Mean: 1960Q1-2015Q1

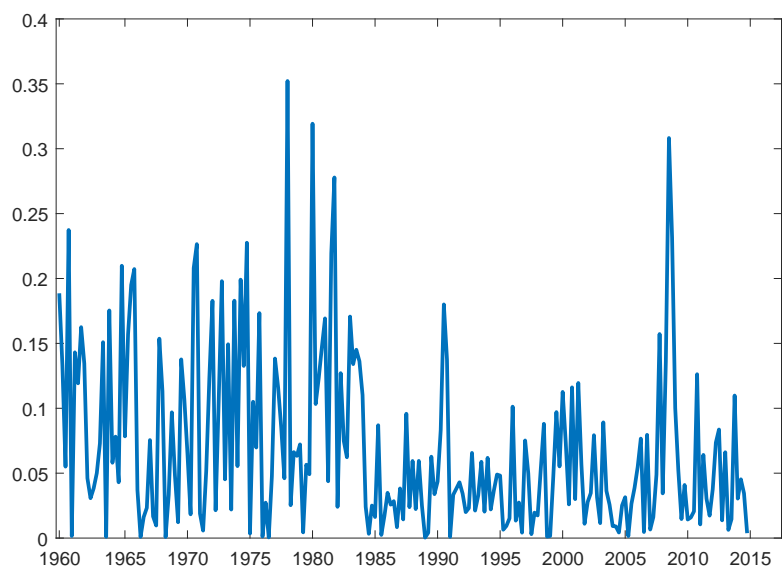
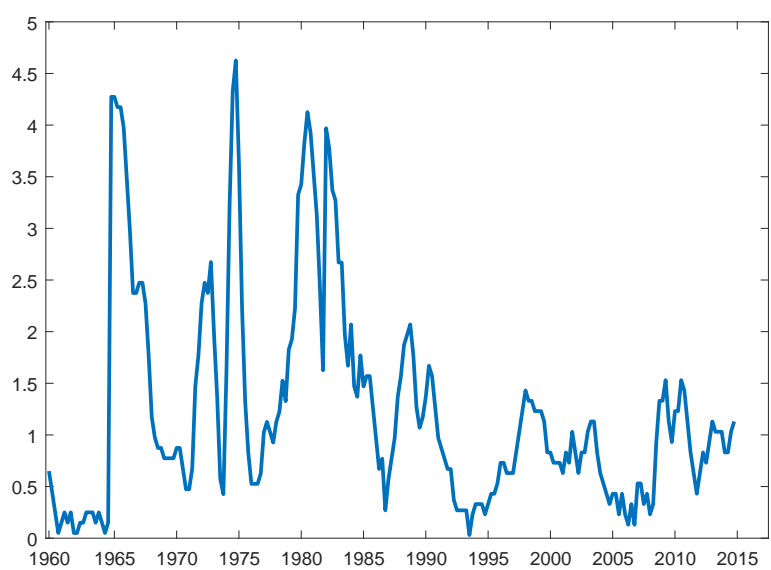


Figure 2.3: United States Core PCE Inflation, Absolute Deviations from Mean: 1960Q1-2015Q1



In short, we document two main features of the U.S. data during recessions, especially the Great Inflation: the sustained rise in inflation and the time-varying volatility.

2.2 Literature Reviews

The present dissertation is closely related to three strands of literature. The first and second strand consists of studies with the idea of constant non-zero and shifting trend inflation, respectively. While the first line has been gained attention by many authors, there are growing focuses on the economy with shifting trend inflation. The third strand is about studies related to policy uncertainty. In this section, we will present a discussion over each strand and an effort of the dissertation to fill the existing gaps in the literature.

2.2.1 Constant Trend Inflation

A log-linearization around the zero inflation steady state is the popular approach in the existing literature⁹. The approach is convenient because it can produce a simple and tractable model but wash out some implications of the microfoundations¹⁰ and can lead to misleading conclusions. Realizing the limitations, Ascari (2004) and Bakhshia, Khanb, Burriel-Llombartc, and Rudolf (2007) make substantial changes in the short-term and long-term properties of model based on the Calvo staggered price model with trend inflation. In particular, Ascari (2004) studies the impulse response of output to the money-growth shock with various levels of trend inflation to analyse the effects of trend inflation on output persistence, while the

⁹See Clarida, Gali, and Gertler (2000), Gali (2002), Woodford (2003) and among others. Furthermore, Woodford (2003) and Goodfriend and King (2009) argue that many specifications of the New Keynesian framework have the 0-percent long-run inflation.

¹⁰Theoretically, some effects produced from the interaction of trend inflation, relative prices and the monopolistic competition framework like price dispersion, marginal mark-up (the ratio of newly adjusted price to marginal cost) and discounting (if firms discount the future more, they less likely worry about an erosion of future mark-up) are eliminated by the zero-steady-state inflation.

effects of non-zero inflation on the slope of the NKPC are investigated by Bakhshia et al. (2007). The early contribution of Ascari (2004) and Bakhshia et al. (2007) are building blocks in the literature for the latter researchers to exploit diverse implications of non-zero inflation. For example, impacts of non-zero inflation on the optimal monetary policy are examined by Ascari and Ropele (2009). The authors show that there is a decreasing effectiveness in controlling inflation as well as a strong influence of trend inflation on the optimal monetary policy if trend inflation is higher. Amano, Ambler, and Rebei (2007) utilize three genres of price setting, namely Calvo pricing, Taylor pricing and truncated-Calvo pricing, to analyse macroeconomic effects of non-zero inflation and then derive the optimal rate of inflation. Their findings emphasize an inverse relationship between trend inflation and welfare as well as the effectiveness of monetary policy.

Furthermore, other authors have examined the relationship between trend inflation and the model's determinacy. For example, Kiley (2007) uses the model with the sticky price to study how trend inflation affects the ability of the monetary authority to guarantee a determinate equilibrium and macroeconomic stability. Kiley (2007) shows that if trend inflation climbs to moderate levels, it leads to the possibility of increased macroeconomic instability, and the equilibrium indeterminacy. An expansion of the indeterminacy region when trend inflation increases is also observed by Ascari and Ropele (2009). Besides, Coibion and Gorodnichenko (2011) provide an alternative interpretation of the Great Inflation, which relies on changes in the determinacy properties of a non-zero trend inflation model. The combined effects of a response to inflation that is close to one, a non-existent response to output growth, a relatively little interest smoothing, and a high trend inflation cause the U.S. economy to be in the determinacy region in the 1970s.

Other researchers have focused exclusively to a relationship between a positive trend inflation and the welfare. In particular, this relationship is investigated by Coibion, Gorodnichenko, and Wieland (2012) in New Keynesian model while em-

bedding the zero lower bound for the nominal interest rates. Following Woodford's approach, they derive the utility-based welfare loss function. Moreover, they also analyse three various channels, namely steady state effects, magnitude of the coefficients in the utility-function approximation, and dynamics of the model, through which the steady state inflation affects welfare. They also show that typical targets adopted by industrialized countries are higher than the optimal level of inflation. However, two conclusions, an independence of variance of log deviation of output around the steady state on the trend inflation and a dependence of the intercept of the approximation and the coefficient of the variance of the log-deviation of inflation from its trend on trend inflation are controversial. It stems from their way of deriving the welfare loss function that does not depend on the trend inflation. Alves (2012) perceive the issues from the approach of Coibion et al. (2012), and then derives the trend inflation welfare-based loss function that merely depends on aggregate variables to avoid the previous approximation pitfall. With this approach, Alves (2012) finds that the trend inflation is inversely related to the relative weight of output gap that contrasts with Coibion et al. (2012). Furthermore, he also derives a way to compute the inefficiency sources that affect the loss function. By using the same approach, Alves (2014) provides an evidence of policy trade-off that happens when trend inflation deviates from zero, thus the divine coincidence no longer holds.

In general, the existing literature indicates that a less-than-4-percent trend inflation rate produces minor impacts on the properties of the standard New-Keynesian model. For example, when central banks raise inflation targets from 2 percent to 4 percent, it creates an additional steady-state output loss of roughly 0.5 percent (Ascari, 2004), or an additional consumption-equivalent welfare loss of less than one percent (Amano, Moran, Murchison, & Rennison, 2009). Ascari, Phaneuf, and Sims (2016) address the welfare of moderate trend inflation in an augmented medium-scale DSGE model. They show that an increase in trend inflation from 2 percent to 4 percent produces a consumption-equivalent welfare loss of nearly 3.7 percent

based on the non-stochastic steady-state and 4.3 percent based on the stochastic mean. They also emphasize that the staggered wage contracts play a vital role determining costs of a constant trend inflation. Our dissertation shares the similar spirit of Ascari, Phaneuf, and Sims (2016) in studying the welfare costs of a non-zero trend inflation in the model with staggered price and wage contracts. Besides, we make an additional contribution by quantifying welfare consequences of shifting trend inflation.

2.2.2 Shifting Trend Inflation

The previous studies have mostly abstracted a time-varying property of inflation target. Just recently, the property has gained attention by authors to exploit its implications for macroeconomic dynamics. One of the pioneers, Kozicki and Tinsley (2001), attempt to analyze impacts of shifting trend inflation on the term structure of interest rates. Subsequently, a new version of NKPC, a log-linear approximation around a shifting trend inflation in Calvo price model, is derived by Cogley and Sbordone (2008) to investigate the implication of shifting trend inflation on estimated parameters. They also seek answers for a question of whether it explains deviations of inflation from that trend. They find that controlling for trend inflation has an important implication in the estimation of the NKPC. Cogley, Primiceri, and Sargent (2009) explore implications of shifting trend inflation for the predictability of inflation by using two models, namely a univariate and multivariate auto-regression with shifting trend inflation.

Recently, Nakata (2014) employs a small-scale New Keynesian DSGE model with nominal rigidities to measure welfare costs of shifting trend inflation. Two models, the one with a staggered Calvo price setting and the other one with the Rotemberg quadratic adjustment cost, are examined to quantify welfare costs for comparison purposes. Alternations in trend inflation cause movements in average

levels and volatilities of consumption and leisure, then have some effects on the welfare of the representative agents. He finds that welfare costs of shifting trend inflation are modest. It comes mostly from changes in the average level of consumption and leisure, whereas its effects on their variance are quantitatively trivial.

It can be seen that the similar spirit of these papers is that trend inflation is assumed to follow a highly persistent AR(1) process to capture its time-varying property. The present dissertation also adapts the same approach to measure welfare costs of shifting trend inflation. The literature, however, is expanded by different ways as in Chapter 3 and Chapter 4 to in part fill the existing gaps.

2.2.3 Policy Uncertainty

Time-varying volatility has become a central topic of discussion for a long time. The work of Haavelmo (1944) perhaps opened the modern understanding of changes in volatility when he taught economists to consider observed time series as a stochastic process. This idea implies that the variance of stochastic process changes over time. However, the procedure to incorporate time-varying volatility lacks a sound theory at that time. Some first work attempts to employ the absolute value of first difference of inflation as in Khan (1977) or by using variance around a moving mean as in Klein (1977) to measure the time component in the variance of inflation. But either has not explicitly connected to theoretical models.

Engle (1982) makes a breakthrough by using the autoregression conditional heteroscedastivity or ARCH to explore the variance evolution of time series, x_t , over time. In his model, the square of innovation on the level of x_t hits an autoregressive process. By estimating an ARCH process for inflation in Britain, he indicates that time-varying components play a crucial role in understanding the dynamics of inflation. Following Engle (1982), others expand the original model, such as General ARCH or GRARCH, Nonlinear GARCH or NGARCH (Engle & Ng, 1993), Expo-

nential GARCH or EGARCH (Nelson, 1991), Quadratic GARCH or WGARCH (Sentana, 1995), Threshold GARCH or TGARCH (Zakoian, 1994).

The most influential papers by C. Kim and Nelson (1999), McConnell and Perez-Quiros (2000), and Blanchard and Simon (2001) have presented the time-varying changes in the volatility of U.S. aggregate fluctuations. The first two studies emphasize a change in volatility around 1984, whereas the great moderation that was characterized as the long-run period of low and stable volatility and only interrupted during the 1970s is indicated by Blanchard and Simon (2001). And this phenomenon is named as the “Great Moderation” by Stock and Watson (2003).

Since then the needs for the development of model to generate time-varying volatility is prerequisite. Sims and Zha (2006) make subsequent improvements by incorporating the Markov-regime switching in autoregressive coefficients as well as in the variance of innovations into a structural vector autoregression (SVAR, henceforth). They show that the model only allowing changes in the variance of innovations fits the data better. They also argue that the evolution of observed volatility cannot be explained by the estimated changes. But the SVAR with time-varying volatility has some limitations as argued by Benati and Surico (2009). With the data created from a simple New-Keynesian DSGE model, Benati and Surico (2009) show that the SVAR approach might misinterpret changes in policies as changes in variances since policy changes have implications for the volatility of endogenous variables.

These papers have provided compelling evidence that a crucial feature of the U.S. data is time-varying volatility but the previous approach on this feature presents its own challenges. Hence, the needs to use different models are required, among which the DSGE model presents its advantages. Fernandez-Villaverde and Rubio-Ramirez (2007) and Justiniano and Primiceri (2008) are the first attempts in this direction, which show that the DSGE model incorporating stochastic volatility on structural shocks fits the data better than the model with homoscedastic

structural shocks. By using a financial accelerator model, Christiano, Motto, and Rostagno (2008) find a significant impact of volatility shocks to productivity of individual firms on the business cycle throughout their changes in the level of leverage. The large impact of uncertainty is also found in Arellano, Bai, and Kehoe (2010). Bloom (2009) and Bloom, Floetotto, Jaimovich, and Terry (2012) show a substantial effect of risk on an investment decision by exploring the real-option effects. The investment in these papers is subjected to frictions such as non-convex adjustment costs or irreversibilities. However, the large effects of uncertainty are not universally accepted. Studies, including Bachmann, Elstner, and Sims (2013), Bachmann and Bayer (2013) and Bekaert, Hoerova, and Duca (2013), report evidence on little impacts.

In general, the existing literature has largely investigated aggregate effects of uncertainty, while little efforts have devoted to other implications. Just recently, few studies have focused on the welfare analysis of uncertainty. For example, Xu (2017) constructs an endogenous growth model with stochastic volatility shocks arising in the capital depreciation rate to investigate the effects of time-varying volatility on welfare. In contrast to the model with one-sided movements in volatility, Xu (2017) studies the welfare effects of two-sided movements in volatility. Accordingly, Xu (2017) finds that the values of model parameters determine the trend of welfare. Further, this paper also shows negligible welfare costs of volatility risk when calibrated to U.S data. The other contribution by Bachmann et al. (2018) also explores the welfare and distributional effects of volatility but they focus on uncertainty arising on productivity and government purchases shocks. By employing a neoclassical stochastic growth model with incomplete market, they also find a fairly small welfare cost.

In short, empirical evidence has indicated that shifting trend inflation and time-varying volatility are important features of the U.S. data. There is an increasing number of work that studies these features but we realize gaps that can be exploited.

First, these two factors have been examined separately so far despite a more possibly severe consequences from their interactions. Second, the exiting literature on policy uncertainty has so far mainly concentrated on aggregate effects of the short-run fluctuations of volatility on macroeconomic variables. Little is investigated on its welfare effects but they mainly analyse one-sided changes in volatility, interpreted as changes of the level shock. Third, previous studies on shifting trend inflation have mainly focused on the model with sticky price.

These gaps inspire us to conduct this dissertation. First, we expand the standard New-Keynesian model by developing the economy featuring shifting trend inflation and monetary policy uncertainty in Chapter 3. We examine roles of staggered price and wage contracts in transmitting effects of shifting trend inflation in Chapter 4. The present dissertation focuses mainly on the welfare analysis.

Chapter 3

Welfare Costs of Shifting Trend Inflation in the Economy with Policy Uncertainty

3.1 Introduction

Economists and policy makers alike saw the U.S. economy as being simultaneously buffeted by a sustained rise in inflation and larger-than-usual uncertainty about future policy¹¹. Although their consequences have subsided, there remains concerns related to the possibility of a persistent increase in inflation as well as the policy uncertainty in the future. This chapter investigates whether either the adverse impacts of increased inflation, which are magnified under a condition of uncertainty, or increased uncertainty about future policy, stuns economic activity. To model a sustained increase of inflation, we use a highly persistent shock to trend inflation, regarded as the central bank's slowly-moving implicit inflation targets as argued by Kozicki and Tinsley (2001), Ireland (2007), Cogley and Sbordone (2008), and Cogley et al. (2009). The term “uncertainty”, referred to as “objective uncertainty” or “risk” in the literature, is defined as the dispersion or spread of economic shocks distribution. Rational consumers and firms then response to the fact that the future shocks are widely distributed (time-varying variance). An essential feature of our specification is a clear classification of structural shocks and stochastic volatility

¹¹Born and Pfeifer (2014) argue that the U.S data were plagued by high shock volatilities since the 1970s. The evidence for shifts on the variance of innovations is also provided by Bernanke and Mihov (1998), Cogley and Sargent (2005), Primiceri (2005) and Sims and Zha (2006).

shocks.

So far, existing literature has primarily concentrated on aggregate effects of the short-run fluctuations in volatility of various macroeconomic variables, while little is investigated on its welfare effect. Few papers exploit the welfare analysis but only consider one-sided movement of volatility (a decrease in the certain level of volatility of level shocks to zero), for example Lester et al. (2014) and Cho et al. (2015). The recent work by Xu (2017), and Bachmann et al. (2018) examines welfare consequences of time-varying volatility. Research by Nakata (2014) also investigates the welfare effects of exogenous alternations in trend inflation. However, these two factors of the U.S data, shifting trend inflation and time-varying volatility, have been examined largely in isolation thus far.

To fill the gap, we analyze the relationship between shifting trend inflation, volatility risk, and economic welfare. This chapter addresses the questions in empirical macroeconomics: what are the roles of time-varying volatility in affecting the welfare costs of shifting trend inflation? We use the Dynamic Stochastic General Equilibrium (DSGE, henceforth) models with nominal rigidities as in Ireland (2007) to seek the answers. We also expand the previous literature as follows. First, trend inflation is not a constant value but a shock that follows a highly persistent AR(1) process. Second, both structural shocks and time-varying volatility shocks jointly participate in the model. Regarding uncertainty, we concentrate on the uncertainty about the monetary policy (Policy Rate Risk (PR, henceforth)), or the volatility shock to the monetary policy. The stochastic volatility shock is assumed to follow an AR(1) process as in Shephard (2008) and Fernandez-Villaverde et al. (2011). The study aims at developing the economy that is characterized by two features: a sustained high inflation (as expressed by shifting trend inflation) and high volatility (by using the time-varying volatility shock). This is the first study that analyses welfare costs of variations in inflation targets under policy uncertainty.

The main findings should be emphasized here. The most striking point are

the interactions between shifting trend inflation and policy uncertainty. On the one hand, a growth in the variance of shocks to trend inflation lowers welfare not only by decreasing an average level of consumption and leisure, but also by increasing their volatility. An incorporation of policy uncertainty then signifies these changes to produce a larger welfare cost. We also conducted exercises to examine the sensitivity of welfare costs to a wide range of plausible parameters. We found that when the price frictions are sufficiently sizeable and there is an upward biased trend inflation process, welfare costs become more significant.

On the other hand, policy uncertainty impacts the economy more significantly when central banks raise their inflation targets. To demonstrate that, we quantified the welfare costs of policy uncertainty considering distinct levels of inflation target. The results indicate that the presence of uncertainty produces negligible welfare costs. Uncertainty also leads to high dynamism of business cycles, which is reflected by decreased mean and increased variance of consumption and leisure. With a higher level of trend inflation, these changes are larger, leading to more significant reduction in welfare. Furthermore, we also documented the impulse response functions illustrating trend and magnitude of policy uncertainty shock with respect to various levels of trend inflation. We found that the monetary policy uncertainty shock adversely impacts the economy by leading to a decrease in output growth as well as an increase in inflation and price dispersion. When the central banks set inflation target to a higher level, these adverse effects of policy uncertainty shock become stronger. In other words, the consequences of policy uncertainty become more severe if there are policy proposals urging central banks to raise inflation targets.

This chapter is related to three strands of the literature. The first strand is about effects of uncertainty on the economy. With various identification schemes, the empirical studies have examined aggregate effects of uncertainty but results are still ambiguous. For example, a block of studies by Alexopoulos and Cohen (2009), Bloom (2009) and Bloom et al. (2012) shows a sizeable effect of uncertainty in

productivity on aggregate variables like GDP and employment, whereas Bachmann et al. (2013), Bachmann and Bayer (2013) and Bekaert et al. (2013) report evidence on little impacts. In addition to inconclusive evidence, previous studies have mainly analysed the adverse impacts of uncertainty on economic activities while abstracting opposite directions. A lack of concentration on other aspect of uncertainty's impacts partly explains these unclear results as argued by Born and Pfeifer (2014).

Secondly, the chapter is mostly related to works examining welfare costs of business cycle. The first one by Lucas (1987) shows that eliminating consumption fluctuations produces a minor gain in welfare. A block of subsequent studies, such as Obstfeld (1994), Campbell and Cochrane (1999), and Krusell, Mukoyama, Sahin, and Smith (1999) relax assumptions in Lucas's computation to present a more considerable welfare cost of business cycle than those in Lucas (1987). Conversely, Lester et al. (2014) and Cho et al. (2015) have recently argued that if agents are able to use uncertainty purposefully in their favor, the welfare can be improved even in a more volatile economy. On the other hand, Xu (2017) discusses that volatility risk can drive welfare up or down depending on the model parameters. It is worth noticing that the previous studies have only considered one-sided movements in volatility, which examine welfare gains from permanently decreasing the certain level of volatility of level shocks to zero. Xu (2017) makes the difference by studying the welfare effects of larger two-sided changes in volatility. The focus of Xu (2017) lies on uncertainty about the capital depreciation rate in an endogenous growth model with recursive preferences. Bachmann et al. (2018) employ a neoclassical stochastic growth model with incomplete markets to study the welfare and distributional effects of fiscal volatility. They allow aggregate uncertainty to arise from both productivity and government purchases shocks. The present chapter also studies welfare effects of uncertainty but differs from Xu (2017) and Bachmann et al. (2018) by focusing on the uncertainty about monetary policy and its interaction with shifting trend inflation. This study's focus on the interactions of uncertainty and

shifting trend inflation can help us to successfully model the U.S economy as argued by Fernandez-Villaverde et al. (2015). In terms of welfare, their interactions potentially generate sizeable consequences. Although there are diverse concerns about uncertainty, this study presents similar findings that the welfare costs of volatility risk are negligible for the U.S economy when central banks set a 2-percent inflation target. These costs, however, become more sizeable when trend inflation is higher.

The third strand of literature is related to research on shifting trend inflation. The work has used a highly persistent shock to trend inflation¹² to capture this trend and to explore distinct sides of macroeconomic dynamics. For example, the implications of shifting trend inflation for the term structure of interest rates are explored by Kozicki and Tinsley (2001) in an early contribution. While Ireland (2007) utilizes an estimated New Keynesian model to analyze the consequences of moving trend inflation, Cogley et al. (2009) attempt to investigate its implications for the predictability of inflation. Most recently, Nakata (2014) exploits welfare consequences of shifting trend inflation.

The literature has so far not concentrated on the economy featuring both shifting trend inflation and policy uncertainty. They examine costs of each channel separately instead of considering their interaction in one model that might cause more severe consequences. Therefore, this chapter attempts to examine the welfare effects of time-alteration in uncertainty in the economy with shifting trend inflation. To conduct this exercise, we develop the model featuring both a sustained increase in inflation (shifting trend inflation) and policy uncertainty (a joint consideration of both structural and volatility shocks) to quantify welfare costs of shifting trend inflation and policy uncertainty. Regarding policy uncertainty, the main focus lies on the monetary policy.

The rest of this chapter is organized as follows. The subsequent section

¹²It can be defined as the central bank's inflation target and private sector's expectation on long-run inflation.

presents the extended model. Section 3.3 explains the method to compute welfare and welfare costs. Structural estimations are presented in Section 3.4 while Section 3.5 shows estimated results. Some conclusions are provided in Section 3.6.

3.2 New Keynesian DSGE with Policy Uncertainty

The New-Keynesian model¹³ is populated by four classes of agents: the household, the final-goods producing firms, a continuum of intermediate-goods producing firms indexed by $i \in [0, 1]$, and the government.

3.2.1 The Household

Households maximize the expected discounted present value of future period utility

$$\sum_{t=0}^{\infty} \beta^t \left(\ln(C_t - \gamma C_{t-1}) - \frac{\omega}{1+v} H_t^{1+v} \right), \quad (3.1)$$

where β and γ denotes the discount factor and the habit formation parameter, which are restricted as $0 < \beta < 1$, $0 \leq \gamma < 1$. v is the inverse Frisch elasticity of labor supply.

The households face the flow budget constraint

$$P_t C_t + \frac{B_t}{R_t} = B_{t-1} - P_t T_t + W_t H_t + D_t. \quad (3.2)$$

At the beginning of each period, the households provide $h_t(i)$ units of labor supplied to each intermediate-goods producing firm $i \in [0, 1]$ to earn $W_t h_t(i)$ where W_t is the nominal wage rate. They also receive a nominal profit, D_t , due to owning the intermediate goods firms. They also save by purchasing the one-period bond, B_t , from the intermediate-goods producers at the price $1/R_t$. A lump-sum tax, T_t , is imposed to finance government spending, G_t . During each period t , households pur-

¹³The detailed information of variables in the model is reported in A2.1.

chase consumption goods, C_t , from the final-goods producing firms at the nominal price, P_t . Therefore, the households choose labor supply, h_t , bond holding, B_t , and consumption, C_t , to maximize the lifetime utility subject to the budget constraint.

The first order conditions for the households utility maximization problem can be represented as

$$\lambda_t P_t = \frac{1}{C_t - \gamma C_{t-1}} - \beta \gamma E_t \left(\frac{1}{C_{t+1} - \gamma C_t} \right), \quad (3.3)$$

$$\lambda_t W_t = H_t^v \omega, \quad (3.4)$$

$$\lambda_t = \beta R_t E_t (\lambda_{t+1}), \quad (3.5)$$

where λ_t is a non-negative Lagrange multiplier on the budget constraint and π_t is the gross inflation rate between t and $t+1$.

3.2.2 The Final-Goods Producing Firm

The final-goods producing firms operate in a competitive environment. In order to manufacture Y_t units of final products, the firms employ $Y_t(i)$ units of intermediate goods $i \in [0, 1]$ according to the constant-returns-to-scale technology as follows

$$\left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} = Y_t, \quad (3.6)$$

where θ is the price elasticity of demand for intermediate goods. The profit maximization problem of the final goods-producing firms is given as

$$P_t \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} - \int_0^1 P_t(i) Y_t(i) di. \quad (3.7)$$

The first order conditions for the final goods-producing firms problem is given as

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\theta} Y_t. \quad (3.8)$$

Since the final goods-producing firms operate in a competitive environment, they have zero profit in the equilibrium. The final good price, therefore, can be rewritten as

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}. \quad (3.9)$$

3.2.3 The Intermediate-Goods Producing Firm

There is a continuum of intermediate-goods producing firms $i \in [0, 1]$ owned by the households. In order to produce $Y_t(i)$ units of intermediate good i , the intermediate-goods producing firms hire $h_t(i)$ units of labor from the household during period t . The property of constant-returns-to-scale technology of the intermediate-goods producing firm can be written as

$$Z_t h_t(i) = Y_t(i). \quad (3.10)$$

The aggregate technology shock follows a stationary stochastic process

$$\ln(Z_t) = \rho_Z \ln(Z_{t-1}) + \epsilon_{Z_t}, \quad (3.11)$$

where ϵ_{Z_t} is the serially uncorrelated innovation, which has a normal distribution with mean zero and standard deviation σ_Z . Intermediate-goods producers are assumed to set nominal prices as in staggered Calvo price fashion. According to Calvo (1983), in each period, there is a fixed fraction $1 - \eta$ of firms re-optimize its nominal price, whereas a fraction η of firms are not able to do so. Firms which cannot optimize their prices can still update their prices. Following Ascari (2004), the way firms update their price is given as

$$P_t(i) = (\pi_{t-1}^\mu \bar{\pi}_t^{1-\mu})^\chi P_{t-1}(i), \quad (3.12)$$

where χ and μ denote a degree of price indexation and the relative weight on lagged inflation, respectively. Intermediate-goods producing firms set the price P_t^* to maximize the expected discounted sum of future profits as

$$E_t \sum_{t=0}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} \eta^j \left[P_t^*(i) (\bar{\pi}_t^{\chi j})^{(1-\mu)} (\pi_{t-1, t+j-1}^\chi)^\mu - \frac{W_{t+j}}{Z_{t+j}} \right] Y_{i, t+s}, \quad (3.13)$$

subject to a sequence of demand functions

$$Y_{t+j}(i) = \left[\frac{P_t^*(i) (\bar{\pi}_t^{\chi j})^{(1-\mu)} (\pi_{t-1, t+j-1}^\chi)^\mu}{P_{t+j}} \right]^{-\theta} Y_{t+j}, \quad (3.14)$$

where $\pi_{t+j-1} = (\frac{P_{t+2}}{P_{t+1}}) \dots (\frac{P_{t+j}}{P_{t+j-1}})$ if $j = 1, 2, 3, \dots$ and λ_t is the same as the Lagrangian multiplier on the household's budget constraints.

The evolution of aggregate price is given as

$$P_t = \left[\int_0^1 P_t^{1-\theta}(i) di \right]^{\frac{1}{1-\theta}} = \left[(1-\eta) P_t^*(i)^{1-\theta} + \eta P_{t-1}^{1-\theta} (\bar{\pi}_t^{\chi(1-\mu)} \pi_{t-1}^{\chi\mu})^{1-\theta} \right]^{\frac{1}{1-\theta}}. \quad (3.15)$$

Following Ascari (2004), this paper introduces the price dispersion that derives from the aggregate output as

$$s_t = \int_0^1 \left[\left(\frac{P_t(i)}{P_t} \right)^{-\theta} \right] di. \quad (3.16)$$

Equation (3.16) dictates the resource costs due to the relative price dispersion under the Calvo price setting. A higher level of s_t means that a firm needs more labors to produce the same level of output. s_t can be rewritten as

$$s_t = (1-\eta) (\phi_t^*(i))^{-\theta} + \eta (\bar{\pi}_t^{-\chi\theta})^{1-\mu} (\pi_{t-1}^{-\chi\theta})^\mu \pi_t^\theta s_{t-1}. \quad (3.17)$$

3.2.4 Authority's policy

Monetary Policy

We modify the standard Taylor rule as

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\bar{\pi}_t}\right)^{\phi_\pi} \left(\frac{y_t}{\bar{y}}\right)^{\phi_y} \right]^{1-\rho_R} \exp(\sigma_t^R \epsilon_{R_t}), \quad (3.18)$$

where $y_t = \frac{Y_t}{Z_t}$, \bar{R}, \bar{y} are the steady state of R_t and Y_t , respectively. The parameter ρ_R illustrates the degree of interest rate smoothing. Finally, ϵ_{R_t} is an i.i.d monetary policy shock and σ_t^R is a volatility shock that allows for the time-varying volatility of a policy shock.

The evolution of trend inflation can be described as a persistent AR(1) process

$$\ln(\bar{\pi}_t) = (1 - \rho_{\bar{\pi}}) \ln(\bar{\pi}^*) + \rho_{\bar{\pi}} \ln(\bar{\pi}_{t-1}) + \epsilon_{\bar{\pi}_t}, \quad (3.19)$$

where $\epsilon_{\bar{\pi}_t}$ is a standard normal and independent of time.

To sum up, changes in the policy rate can be attributed to two elements: shifting trend inflation ($\bar{\pi}_t$), and the time-varying stochastic volatility shock (σ_t^R). While the the shock to trend inflation directly affect the policy rate, there are indirect effects captured by the time-varying stochastic volatility shock through increasing the volatility of corresponding monetary policy shock.

Fiscal Policy

The government expenditure is financed by lump-sum taxes as follows

$$G_t = T_t. \quad (3.20)$$

Government spending is specified as a fraction of output Y_t according to

$$G_t = \left(1 - \frac{1}{g_t}\right) Y_t, \quad (3.21)$$

where g_t , which is greater than unit ¹⁴, is an AR(1) process

$$\ln(g_{t+1}) = (1 - \rho_g) \ln(\bar{g}) + \rho_g \ln(g_t) + \epsilon_{g_t}, \quad (3.22)$$

where $(1 - \frac{1}{\bar{g}})$ is the value of government spending relative to output in the steady state.

3.2.5 Market Clearing Condition

The market clearing condition in the labor market, the final goods market and the government bond can be expressed as

$$H_t = \int H_t(i) di, \quad (3.23)$$

$$Y_t = C_t + G_t, \quad (3.24)$$

$$B_t = 0. \quad (3.25)$$

3.2.6 Policy Uncertainty

As in Fernandez-Villaverde et al. (2011) and Shephard (2008), the standard deviation σ_t^R is assumed to follow an AR(1) process

$$\sigma_t^R = (1 - \rho_{\sigma^R})\sigma^R + \rho_{\sigma^R}\sigma_{t-1}^R + \eta^R \epsilon_t^R, \epsilon_t^R \sim N(0, 1), \quad (3.26)$$

where σ^R is the unconditional mean of σ_t^R , which is the stochastic volatility shock arising in the monetary policy. We call it Policy Rate Risk (PR, henceforth). The shock to the volatility ϵ_t^R is an i.i.d process that is assumed to be independent of the level shock ϵ_t^R . η^R represents one-standard deviation of policy uncertainty shock.

¹⁴To ensure this condition, we set the inverse-gamma distribution for the government spending shocks.

It is worth noting that a one-standard deviation uncertainty shock increases the volatility of the respective shock process by $(\exp(\eta^R) - 1)100$.

3.3 Welfare and Welfare Cost Computation

3.3.1 Welfare Computation

We use the expected utility of the household, $E[\sum_{t=0}^{\infty} \beta^t u(C_t, C_{t-1}, H_t)]$ to measure the welfare. This paper follows Nakata's approach, which use the perturbation method to compute the approximation to the policy functions around the deterministic steady-state, and use these to compute the welfare. With an employment of the second-order Taylor expansion of the household's utility function around the deterministic steady-state, we can decompose the welfare into the different components as given

$$\begin{aligned} E\left[\sum_{t=0}^{\infty} \beta^t u(x_t)\right] &\approx \sum_{t=0}^{\infty} \beta^t u(\bar{x}) + \sum_{t=0}^{\infty} \beta^t Mu(\bar{x})E[x_t - \bar{x}] + \sum_{t=0}^{\infty} \beta^t Nu(\bar{x})E[(x_t - \bar{x}) \otimes (x_t - \bar{x})] \\ &= U_d + U_l + U_v, \end{aligned}$$

where $x_t = [C_t, C_{t-1}, H_t]$; and $Mu(\bar{x})$ and $Nu(\bar{x})$ are vector which contain the first and second derivative of $u(\cdot)$ evaluated at \bar{x} which are the deterministic steady state of x_t . Therefore, the welfare can be decomposed into three components: the deterministic component, $U_d = \sum_{t=0}^{\infty} \beta^t u(\bar{x})$, the level component, $U_l = \sum_{t=0}^{\infty} \beta^t Mu(\bar{x})E[x_t - \bar{x}]$, and the volatility component, $U_v = \sum_{t=0}^{\infty} \beta^t Nu(\bar{x})E[(x_t - \bar{x}) \otimes (x_t - \bar{x})]$. U_d depends on the deterministic steady-state (\bar{x}), U_l depends on the mean of x_t , and U_v depends on the volatility of x_t . It is worth noticing that U_l reflects the nonlinearity of policy function and this term would always be zero in the linear-quadratic form. U_v captures the household's risk aversion. The decomposition is helpful to identify sources of welfare costs of shifting trend inflation and uncer-

tainty¹⁵. Three components are the deterministic steady-state component, the level component and the volatility component. They can, in turn, be interpreted as the deterministic steady-state effect generated by changes in the deterministic steady-state variables, the level effect generated by changes in the mean of variables, and the volatility effect generated by changes in variance of variables. The analyses also show changes in the properties of economy due to constant trend inflation, shifting trend inflation and policy uncertainty shocks. By analysing changes in the welfare components and the business cycle properties, findings provide important inferences to explain the welfare effects as well as the transmission mechanism of shocks.

3.3.2 Welfare Cost Computation

The present study defines the welfare cost as compensating variation in consumption that enhances the welfare of a typical household in one economy to make them as well-off as others in another economy. Mathematically, we can be represented as

$$E \left[\sum_{t=0}^{\infty} \beta^t u \left(\left(1 + \frac{wc}{100}\right) C_{A,t}, \left(1 + \frac{wc}{100}\right) C_{A,t-1}, H_{A,t} \right) \right] = E \left[\sum_{t=0}^{\infty} \beta^t u \left(C_{B,t}, C_{B,t-1}, H_{B,t} \right) \right], \quad (3.27)$$

where $C_{A,t}, H_{A,t}$ are consumption and working hours in the economy with $\sigma_m > 0$ and $C_{B,t}, H_{B,t}$, are in economy with $\sigma_m = 0$. Here, “m” could be the shock to trend inflation or uncertainty shock (the uncertainty arising on the monetary policy shock).

¹⁵Previous studies also decompose the welfare. For example, Ascari et al. (2016) investigate how much welfare loss based on non-stochastic steady-states and stochastic means, while J. Kim and Kim (2018) decompose the welfare gains contingent on tax policies into efficient gains (the mean effects) and stabilization gains (the variance effects). In our study, we follow Nakata’s approach using the pruning algorithm suggested by Kim, Kim, Sims, and Schaumburg (2008) to compute the welfare components. The decomposition identifies how the welfare costs could be signified due to interactions of shifting trend inflation and monetary policy uncertainty.

3.4 Calibration

Table A2.2 reports the parameter values, which we use to quantify the welfare costs of shifting trend inflation and uncertainty in the next step. The set of model parameters is split into two subsets. The first subset includes parameters that we can directly compute them without solving the model or whose values are standard in the literature. As argued by Born and Pfeifer (2014), it is computationally difficult to jointly estimate all parameters in the model featuring time-varying volatility. Therefore, we firstly estimate the time-varying stochastic volatility shocks. For simplicity, we follow Born and Pfeifer (2014) to select parameters for the time-varying stochastic volatility process arising in the monetary policy. The paper shows moderate evidence of time-varying policy uncertainty with a high level of variance (0.363) and persistence (0.921).

Further, the discount factor, β , is equal to 0.9974 and the inverse Frisch elasticity, ν , is set to 1.59 as in Justiniano and Primiceri (2008)¹⁶. The persistence level and standard deviation of structural shocks are also set to values that are common in the literature. For example, we follow Justiniano and Primiceri (2008) to set autoregressive parameters of the productivity shock and the government expenditure shock at 0.32 and 0.98, respectively. Regarding the production sector, we use the estimates from Cogley and Sbordone (2008)¹⁷. The degree of price indexation, χ , and the elasticity of substitution, $\bar{\theta}$, are set to 0 and 10, respectively. The estimations of Cogley and Sbordone (2008) are consistent with those in the literature on constant non-zero trend inflation and imperfect indexation. For the trend inflation process, we set its standard deviation, $\sigma_{\bar{\pi}}$, and its persistence level, $\rho_{\bar{\pi}}$, to 0.0008 and 0.995, respectively as in Cogley et al. (2009).

¹⁶Justiniano and Primiceri (2008) is the first attempt that uses the DSGE model incorporating the stochastic volatility shocks.

¹⁷Since that is the only study to estimate the parameters of a Calvo model relaxing the assumption of perfect indexation.

The second subset consists of remaining parameters for the habit formation, γ , the probability of non-optimization, η , and parameters related to Taylor rule, including the smoothing parameter, ρ_R , the coefficient on inflation, ϕ_π , and the coefficient on output growth, ϕ_y , which are calibrated jointly to match selected moments in the 1954Q3-2015Q1 U.S. data. Details about the data source can be found in the Appendix 1. We follow Ruge-Murcia (2012) to employ Simulated Method of Moments (SMM)¹⁸ approach to fit the model to the data. With this approach, we attempt to minimize the weighted distance between the simulated moments computed from artificial data and the empirical moments.

Note that we need to solve the model non-linearly before simulating the data. Perturbation method is used to approximate the policy function around the deterministic steady-state. To capture effects of volatility shocks, we need to go beyond the first order approximation. In this study, we stop at the second order approximation¹⁹ due to the following reasons. First, we include two main mechanisms: volatility shocks and parameter drifting (shifting trend inflation) in the present study. This differs from Fernandez-Villaverde et al. (2011) and Born and Pfeifer (2014) who only consider the model with stochastic volatility shocks. With these two mechanisms, the higher-order approximation would make the evaluations of model challenging for models with a number of state variables because of dimensionality issues as argued by Fernandez-Villaverde et al. (2015). Second, the second-order approximation is preferred because it still captures the implications of stochastic volatility and is convenient because it is accurate and computationally cheap. Further, Ruge-Murcia (2012) shows two caveats when using the third-order approximation. First, the third-order approximation is computationally expensive. Second, the stability of the second order system is insured by using the pruning algorithm suggested by

¹⁸We use Matlab for this exercise. We strictly follow the procedure proposed by Ruge-Murcia (2012) for this exercise. Details about SMM method are presented in Appendix 6.

¹⁹To analyse impacts of higher order approximation on parameter values, we employ the third order approximation as well. The results of these exercises are reported in Appendix 5.2.

Kim, Kim, Sims, and Schaumburg (2008), whereas this is no longer the case when the algorithm is applied to a third-order solution.

The four selected moments include output volatility (σ_Y), volatility of consumption relative to output (σ_C/σ_Y), a correlation between output and consumption ($\rho(Y, C)$), and a correlation between output and labor ($\rho(Y, N)$). These moments are vital for the subsequent welfare and welfare cost computations since they reflect the dynamic behavior of consumption and working hours. Accordingly, the habit formation, γ , and the probability of non-optimization, η , are calibrated at 0.81 and 0.70, respectively. Three parameters in Taylor rule, including ρ_R , ϕ_π and ϕ_y are calibrated at 0.81, 1.68 and 0.08, which are consistent to Justiniano and Primiceri (2008).

Table 3.1: Moments

| | σ_Y | σ_C/σ_Y | $\rho(Y, C)$ | $\rho(Y, N)$ |
|-------------|------------|---------------------|--------------|--------------|
| Data | 0.002 | 0.56 | 0.79 | 0.87 |
| Calibration | 0.003 | 0.78 | 0.79 | 0.34 |

Note: moments in the second row are obtained from HP-filtered U.S data (1954Q3-2015Q1). The last row is the moments from simulations for the calibrated model.

Table 3.1 compares the moments created by the calibrated model with moments computed by the data. The reported volatility and correlation statistics represent the HP-filtered U.S data during 1954Q3-2015Q1 period. Table 3.1 shows that the model does a good job for matching the selected moment, especially the volatility of output and the correlation between output and consumption. In short, the key features of the data are captured reasonably well by the calibrated model. Therefore, the calibrated model can provide an appropriate laboratory for the subsequent welfare analysis.

3.5 Welfare and Welfare Cost Results

This study firstly conducts exercises to observe an interaction between shifting trend inflation and uncertainty by using the calibrated parameters. In the first step, we quantify welfare costs of constant and shifting trend inflation and observe how they change when incorporating the volatility shock. Welfare costs of uncertainty with different trend inflation levels are presented in the next part²⁰.

We then perform some sensitivity analysis that shows how welfare costs respond to changes in relevant parameters. For example, Fernandez-Villaverde et al. (2010) and Guerron-Quintana (1999) show a larger value for η and μ . In particular, the range $\eta = [0.6, 0.65, 0.7, 0.75]$ and $\mu = [0.00, 0.33, 0.67, 1.00]$ are respectively taken into account to compare with the baseline model. To explore the role of trend inflation process, we consider different values for the steady state level of trend inflation, the various levels of persistence and standard deviation of trend inflation process as follows: $[1.02^{0.25} \dots 1.06^{0.25}]$; $[0.9900 \dots 0.9999]$, $[0.100; 0.075; 0.05; 0.025; 0]$ ²¹, respectively.

3.5.1 Welfare Costs of Constant Positive Trend Inflation

Table 3.2 compares the welfare of two economies featuring by distinct positive levels of trend inflation, 0 and 6 annualized percent. First, we look at the baseline model (an economy without policy uncertainty) to observe the welfare differences due to constant trend inflation. Clearly, a large welfare cost (5.53%) in the baseline model stems from a higher level of trend inflation. Three components, especially

²⁰For welfare and welfare costs computation, we use Matlab program with the method suggested by Kim et al. (2008). Appendix 7 is the detail about method. We greatly appreciate Prof. Taisuke Nakata for a huge support as well as advice to help me to write the codes for these exercises. We use Schmitt-Grohe and Uribe's toolbox for a second-order approximation to policy rules. The codes are provided by the author upon request.

²¹These parameters are taken from the literature. The first one comes from Ireland (2007), while the others are taken from Cogley et al (2009) for 1960-1979 subsample, 1982-2007 subsample, respectively.

the level and volatility components mainly explain the difference of welfare when central banks raise inflation targets. A higher level of trend inflation causes price dispersion to increase as non-optimizing firms cannot index their prices, and leads to less efficient aggregate production. As a consequence, the households need to supply more labors to maintain the same level of consumption, and output. Less consumption but more working hours lead to substantial welfare reduction.

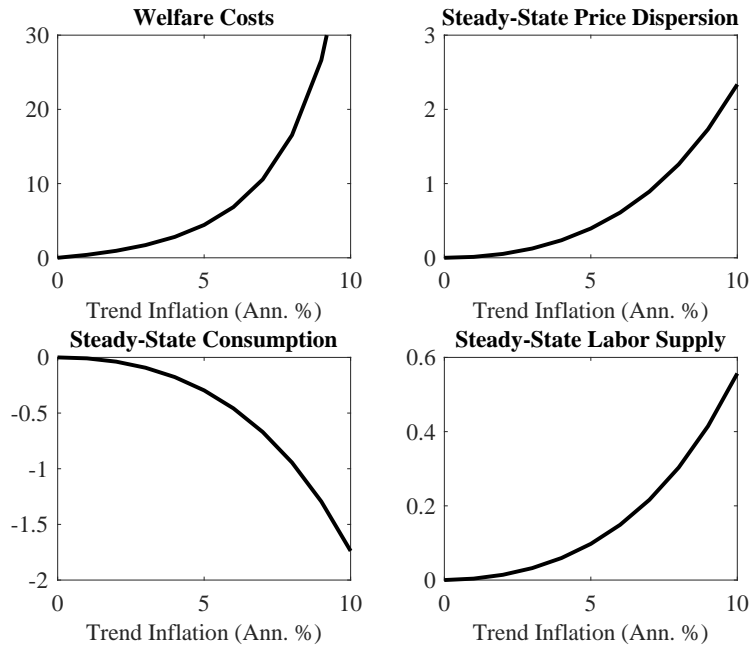
Table 3.2: Welfare Costs of Constant Trend Inflation

| | W/O Uncertainty ($\eta^R = 0$) | | Policy Risks (PR) ($\eta^R > 0$) | |
|----------------|-------------------------------------|---------------------|---------------------------------------|---------------------|
| | $\bar{\pi}^* = 0\%$ | $\bar{\pi}^* = 6\%$ | $\bar{\pi}^* = 0\%$ | $\bar{\pi}^* = 6\%$ |
| Welfare Cost | | 5.53% | | 7.85% |
| Welfare | -895.98 | -939.85 | -897.21 | -959.97 |
| U_d | -889.96 | -896.30 | -889.96 | -896.30 |
| U_l | -2.06 | -28.20 | -3.17 | -46.21 |
| U_v | -3.96 | -15.35 | -4.08 | -17.46 |
| Steady-state C | 0.821 | 0.812 | 0.821 | 0.812 |
| Steady-state H | 1.066 | 1.071 | 1.066 | 1.071 |
| E(C)(*) | -0.004 | -0.058 | -0.007 | -0.095 |
| E(H)(*) | 0.0001 | 0.0013 | 0.0001 | 0.0020 |
| $100\sigma_C$ | 1.32 | 2.56 | 1.32 | 2.73 |
| $100\sigma_H$ | 1.74 | 2.56 | 1.49 | 3.16 |

Note: (*) expressed as percentage deviation from the deterministic steady-state. U_d , U_l and U_v are the deterministic steady-state, level and volatility component, respectively.

In the following exercise, we incorporate the time-varying stochastic volatility shock into the model. In general, the volatility shocks lead to larger reduction of welfare. The welfare differences due to constant trend inflation in the model with policy uncertainty mainly come from reduction in the level and volatility components as in the baseline model but the role of these two components become increasingly important. In particular, once central banks raise inflation targets from 0% to 6%, a large amount of welfare cost is generated (7.85%). This cost is mainly accounted by the changes in the level component (-3.17 to -46.21) and the volatility component

Figure 3.1: Welfare Costs, Steady State versus Policy Risks

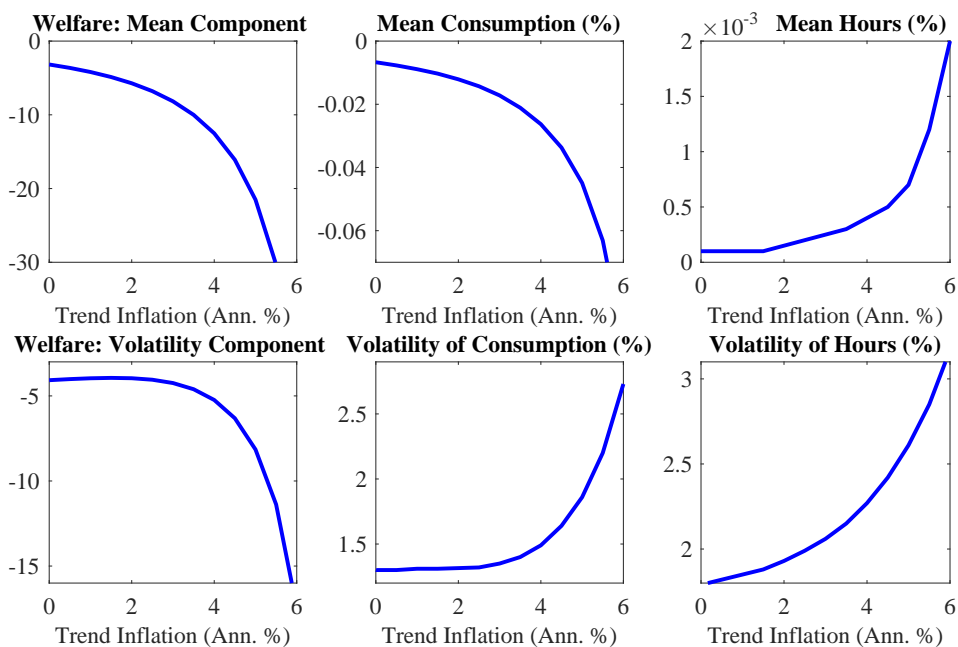


(-4.08 to -17.46). Changes in the deterministic component remains unchanged as in the model without uncertainty. The results imply that the policy uncertainty signifies welfare costs of constant trend inflation by causing larger changes in the level and volatility components. These changes are reflected by more substantial movements of mean and standard deviation of consumption and hours worked. The higher inflation targets lead to a decrease in mean consumption and a rise in mean working hours, while their variance increases.

Figure 3.1 and 3.2 respectively depict movements of deterministic steady-state component, and the level and volatility components corresponding to the different levels of trend inflation. Regarding the deterministic component, the top left panel of Figure 3.1 shows how welfare costs of the economy with monetary policy risk vary with the diverse levels of trend inflation. The most striking feature is the non-linear relationship between trend inflation and welfare cost. The top right panel in part explains this relationship. A given percentage point increase in trend inflation produces a greater rise in the steady-state price dispersion when trend inflation is

higher. Hence, the consumption increases while the working hours decreases as reflected by the bottom panel of Figure 3.1. Both figures for steady-state consumption and working hours exhibit the non-linear relationship with the trend inflation. In the model without policy uncertainty, the deterministic component is the main factor explaining changes in welfare, thus such nonlinearity in the steady-state quantities causes the relationship between welfare costs and trend inflation levels.

Figure 3.2: Mean and Volatility Welfare



Note that in the model with time-varying stochastic shocks, the level and volatility components play a vital role in explaining any welfare difference. Therefore, it is important to observe movements of these components with respect to different levels of trend inflation. Figure 3.2 provides more intuitions to explain significant reductions of welfare. Both mean and volatility components of welfare fall dramatically, especially at a high level of trend inflation. The figures for consumption and working hours explain this relationship. In particular, the mean consumption and leisure decline while their variance increase with respect to a higher level of trend inflation. All figures exhibit the non-linear relationship with trend inflation, which a

given percentage point rise in trend inflation leads to greater changes in consumption and leisure when trend inflation is high. Such nonlinearity in the mean and variance quantities are the key factor in understanding the aforementioned relationship between trend inflation and welfare if there exists policy uncertainty.

3.5.2 Welfare Costs of Shifting Trend Inflation

The section compares welfare costs of exogenous alternation in trend inflation in three economies, including the economy without shifting trend inflation and policy uncertainty ($\sigma_{\bar{\pi}} = 0, \eta^R = 0$), the economy without policy uncertainty but with shifting trend inflation ($\sigma_{\bar{\pi}} > 0, \eta^R = 0$), and the economy with shifting trend inflation and monetary policy uncertainty ($\sigma_{\bar{\pi}} > 0, \eta^R > 0$). We set the steady-state inflation to 2 annualized percent. Table 3.3 dictates changes in the welfare cost of shifting trend inflation and properties of an economy due to monetary policy uncertainty. Several points should be emphasized. First, the model without monetary policy uncertainty shows how shifting trend inflation affects welfare. Welfare costs of shifting trend inflation are 0.24 percent if there does not exist the monetary policy uncertainty, in which the welfare difference comes from reduction of level component. The changes in level component are reflects by changes in properties of economy. The shifting trend inflation leads to a decline in mean consumption and an increase in the mean hours worked.

Since the policy functions for consumption and labor supply are respectively concave and convex functions of trend inflation, a mean-preserving spread in the shock distribution lowers mean consumption and increases mean labor supply. As argued by Nakata (2014), a shock to trend inflation can be interpreted as taking the economy to a new steady-state with a various level of trend inflation if trend inflation is highly persistent. The concave policy function for consumption implies that a positive shock to trend inflation leads to reduction in consumption by an

amount greater than its increase caused by a negative shock. The convex policy function for labor supply implies that a positive shock to trend inflation leads to an increase in labor supply by an amount greater than its reduction caused by a negative shock. Hence, consumption declines and labor supply rises due to a higher variance of shock to trend inflation.

Table 3.3: Welfare Costs of Shifting Trend Inflation

| | W/O Uncertainty | | With Uncertainty |
|----------------|--|--|--|
| | $(\sigma_{\bar{\pi}} = 0, \eta^R = 0)$ | $(\sigma_{\bar{\pi}} > 0, \eta^R = 0)$ | $(\sigma_{\bar{\pi}} > 0, \eta^R > 0)$ |
| Welfare Cost | | 0.24% | 0.51% |
| Welfare | -896.09 | -898.04 | -900.11 |
| U_d | -890.42 | -890.42 | -890.42 |
| U_l | -1.98 | -3.71 | -5.72 |
| U_v | -3.68 | -3.91 | -3.96 |
| Steady-state C | 0.821 | 0.821 | 0.821 |
| Steady-state H | 1.067 | 1.067 | 1.067 |
| E(C)(*) | -0.0042 | -0.0078 | -0.0121 |
| E(H)(*) | 0.0001 | 0.0001 | 0.0001 |
| $100\sigma_C$ | 1.26 | 1.30 | 1.30 |
| $100\sigma_H$ | 1.85 | 1.85 | 1.93 |

Note:(*) expressed as percentage deviation from the deterministic steady-state. U_d , U_l and U_v are the deterministic steady-state, level and volatility component, respectively.

Subsequently, we observe how welfare costs of shifting trend inflation change when incorporating the stochastic volatility shock. The fourth column of Table 3.3 illustrates costs in the model with the monetary policy uncertainty ($\sigma_{\bar{\pi}} > 0, \eta^R > 0$). It is worth noting that welfare declines more considerably due to shifting trend inflation and monetary policy uncertainty. Welfare costs of exogenous variance in trend inflation in the economy with monetary policy risk is about 0.51%. Larger changes in the mean and volatility of consumption and working hours²² are mainly attributed to higher welfare costs. The results suggest that monetary policy uncer-

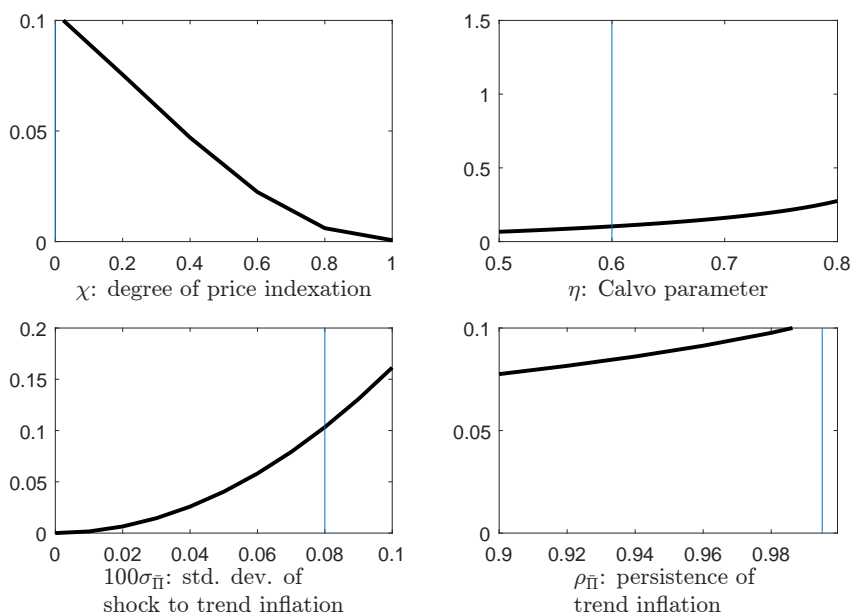
²²Welfare differences come from significant reduction in the level and volatility component, in which the level component makes up the greatest proportion.

tainty amplifies costs of shifting trend inflation by causing greater changes in mean and volatility of consumption and working hours.

The time-varying volatility tells a story around changing size of the variance of structural shocks that hit the economy. In the baseline model, the impacts of exogenous variations in trend inflation are trivial. It might not precisely explain sources of movements. While we struggle with times of high volatility, we might dwell in joy with a low volatility. The study emphasizes that the changing size of the variance of structural shocks might bring about the severe welfare consequences. The higher volatilities directly reduce welfare through the volatility component, but also indirectly lower welfare by effects on the average levels of consumption and hours worked as indicated in Table 3.3.

Sensitivity Analysis

Figure 3.3: Welfare Costs of Shifting Trend Inflation under Uncertainty



The study also conducts some exercises to examine how welfare costs of shifting trend inflation in the economy with the presence of uncertainty vary with respect to changes in parameter values, for instance, the steady state level of trend inflation ($\bar{\pi}^*$), Calvo parameters (a combination of χ and η), a persistence level ($\rho_{\bar{\pi}}$) and

a standard deviation of shock to trend inflation (σ_{π}). Figure 3.3 expresses movements of welfare costs corresponding to parameter changes. Two messages can be identified. First, the parameters controlling the pricing environment significantly determine these welfare costs. The top panel of Figure 3.3 shows changes in welfare costs with degree of price indexation and the frequency of price adjustment. Notice that a greater level of price indexation implies that prices are less dispersed at any trend inflation rate because this index allows non-optimizing firms to catch up with prices set by optimizing firms. Therefore, both consumption and labor supply are less impacted by the different levels of trend inflation, thus a higher degree of price indexation produces lower welfare costs of shifting trend inflation. On the other hand, the lower price adjustment frequency causes welfare costs to increase. As can be seen from Figure 3.3, welfare costs are high corresponding to a high price indexation level and a low price adjustment frequency. Table A4.3 in Appendix 4 shows the detailed computation for welfare costs with alternative values of price indexation and Calvo parameters.

Second, the bottom panel of Figure 3.3 also illustrates how welfare costs vary with changes in shifting trend inflation shock properties. In particular, either the higher variance levels or higher persistence levels of shock generate larger welfare costs. It is worth noting that the nonlinear relationship between welfare costs and the standard deviation levels of shock to trend inflation implies that the higher the variance levels of shock to trend inflation are, the larger welfare costs are produced. Table A4.1 and A4.2 in Appendix 4 report details about computation for the sensitivity analysis. It also indicates how welfare costs of shifting trend inflation change if there is a presence of uncertainty. A common point in the models with and without uncertainty is that a given amount of increase in these parameters leads to a larger change in welfare when these parameters are high. For example, when the standard deviation level rises from 0.075 to 0.1, welfare costs increase from 0.22 to 0.39% and from 0.49 to 0.66%, while there is a little change when the standard deviation level

moves from 0.025 to 0.05 in the model without and with policy uncertainty, respectively. Therefore, if the economy is featured by the high standard deviation level of shock, especially if policy uncertainties are presented, changes in inflation-targeting policy can create a severe issue.

3.5.3 Welfare Costs of Policy Uncertainty

This section documents the welfare costs of policy uncertainty in terms of welfare and welfare costs corresponding to a 2% and 4% trend inflation. Subsequently, impacts of uncertainty on the economy reported by the impulse response function of monetary policy risk shocks are presented. Some main facts should be emphasized in Table 3.4. The focus firstly lies on the 2-percent-trend-inflation economy. Adding the stochastic volatility shocks dampens the welfare negligibly. The welfare diminishes from -898.04 to -900.11 (or by 0.25%) due to the monetary policy uncertainty. The result indicates a small welfare consequence caused by the uncertainty about monetary policy. Moreover, the welfare differences are derived mostly from the changes in the level and volatility component, in which the level component accounts for the largest proportion. Changes in properties of these two economies reflect these facts by a fall in mean consumption (from -0.0078 and -0.0121) and an increase in mean labor supply from (0.0001 to 0.0002). Their standard deviation, on the other hand, grows up by a large amount. By analysing changes in the welfare components and the business cycle properties, the findings provide important intuitions to explain the welfare effects as well as the transmission mechanism of the stochastic volatility shocks.

It is worth noting that the total costs in the individual exercises, which we quantify welfare costs of shifting trend inflation as in the third column of Table 3.3 and the welfare costs of monetary policy uncertainty as in the third column of Table 3.4, is smaller than welfare costs in the economy with both shifting trend inflation

and policy uncertainty as in the fourth column of Table 3.3. The results of these exercises imply that the simultaneous existence of shifting trend inflation and the monetary policy uncertainty can lead to more severe consequences.

Table 3.4: Welfare Costs of Policy Uncertainty

| | $\bar{\pi}^* = 2\%$ | | $\bar{\pi}^* = 4\%$ | |
|---------------|------------------------------------|------------------------|------------------------------------|------------------------|
| | No Uncertainty ($\eta^R = 0$) | PR ($\eta^R > 0$) | No Uncertainty ($\eta^R = 0$) | PR ($\eta^R > 0$) |
| Welfare Cost | | 0.25% | | 0.59% |
| Welfare | -898.04 | -900.11 | -905.34 | -909.96 |
| U_d | -890.42 | -890.42 | -892.21 | -892.21 |
| U_l | -3.71 | -5.72 | -7.98 | -12.51 |
| U_v | -3.91 | -3.96 | -5.15 | -5.24 |
| $E(C)(*)$ | -0.0078 | -0.0121 | -0.0167 | -0.0263 |
| $E(H)(*)$ | 0.0001 | 0.0001 | 0.0003 | 0.0004 |
| $100\sigma_C$ | 1.30 | 1.30 | 1.49 | 1.49 |
| $100\sigma_H$ | 1.85 | 1.92 | 2.04 | 2.27 |

Note: PR is the economy with the policy rate risk. (*) expressed as percentage deviation from the deterministic steady-state. U_d , U_l and U_v are the deterministic steady-state, level and volatility component, respectively.

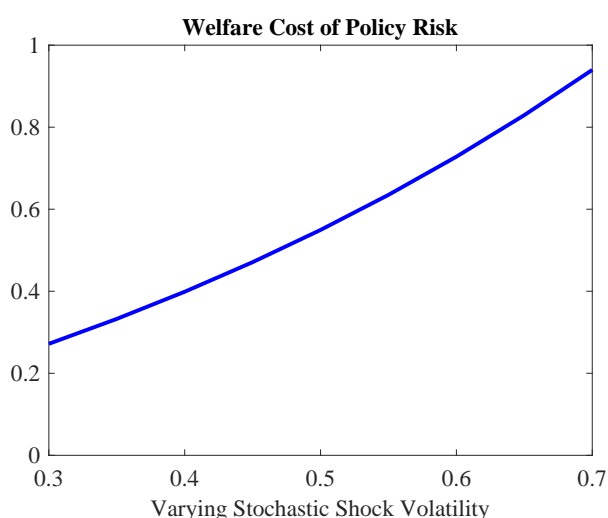
Further, Table 3.4 also illustrates that the consequences of uncertainty become more severe if the monetary authorities set a high level of trend inflation. Take 4% trend inflation as a given. From the fourth column of Table 3.4, the model with policy risk produces greater welfare costs (0.59%) when trend inflation is high. It suggests that the costs of policy uncertainty have a tendency to rise when the central banks raise their inflation targets. The welfare distinction between the model with and without policy uncertainty stems from changes in the level and volatility components. It is reflected by corresponding changes in properties of this economy. Significant changes in both mean and standard deviation of consumption and labor supply are presented in Table 3.4. The welfare difference due to policy uncertainty can be explained not only by reduction of consumption and leisure, but also by a growth in their volatility. In short, the results suggest that welfare costs of monetary policy uncertainty are higher if central banks raise their inflation targets, which

induces more significant reduction of the level and volatility component of welfare.

Sensitivity Analysis

Figure 3.4 shows an exercise to see how welfare costs of uncertainty depend on the uncertainty shock volatility (η_R). The figure suggests that if the time-varying stochastic volatility shock gets more volatile, it creates more significant reduction in welfare. As a result, costs of uncertainty are higher.

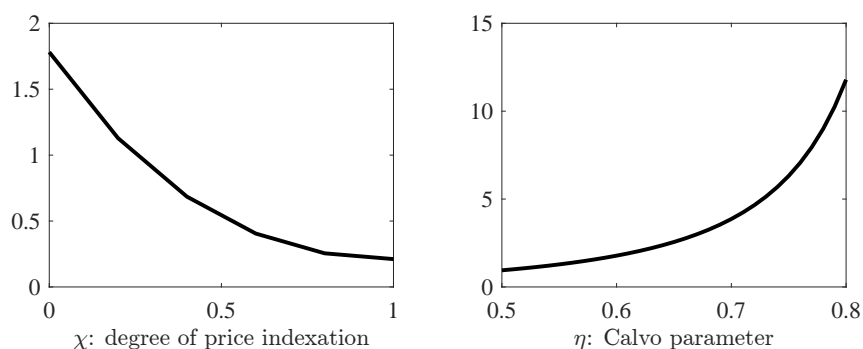
Figure 3.4: Welfare Costs of Policy Risk



Further, the study also conducts exercises to report how welfare costs of uncertainty change with a variety of parameters governing the pricing environment. Figure 3.5 shows changes in welfare costs with respect to changes in these relevant parameters. First, both price indexation level and the price adjustment frequency lead to moves of costs as expected. A higher chance that non-optimizing firms are able to catch up with price changes by optimizing firms makes costs of uncertainty less severe. A larger η implying less frequent price adjustment generates large welfare costs.

Detailed computations for sensitivity analysis are reported in Table A4.4. The table is useful to help us understand the nonlinear curve of welfare costs of policy risks corresponding to changes in these relevant parameters. Table A4.4 shows

Figure 3.5: Welfare Costs of Uncertainty: Monetary Policy Risk



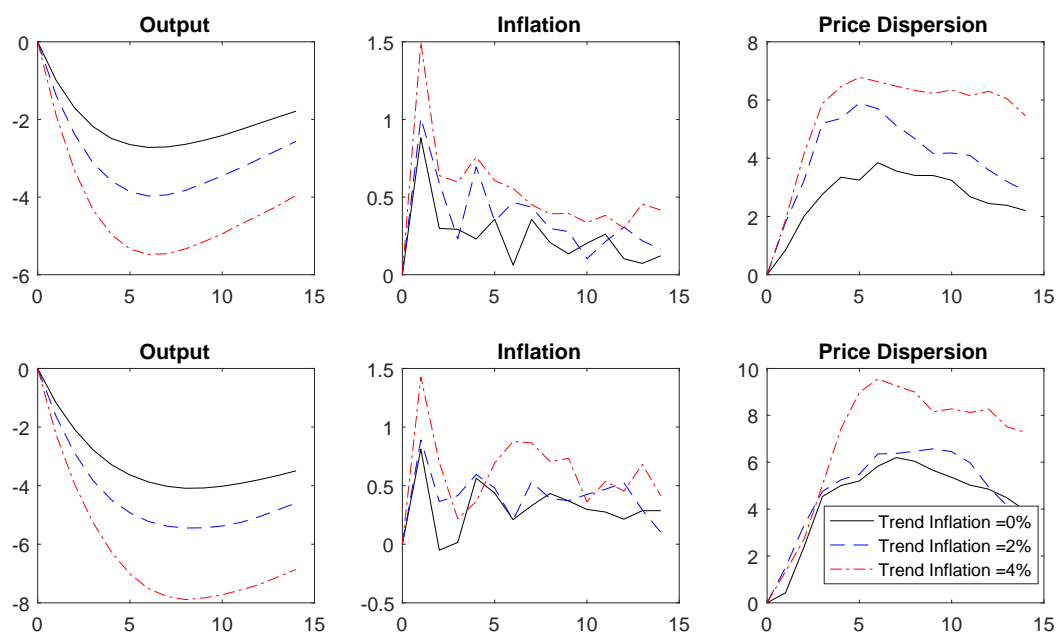
welfare costs of stochastic volatility shocks caused by moves in their own standard deviations. In general, a given amount of increase in the standard deviation leads to a greater fall in welfare when the standard deviation is high.

The Cyclical Effects of Trend Inflation

In the next exercise, this study discusses the trend and magnitude of effects of policy uncertainty shocks on the economy. Figure 3.6 is the simulation documenting impulse responses of key macroeconomic variables to policy uncertainty shocks with a consideration of various levels of trend inflation: 0%, 2%, and 4%. The upper panel is a simulation when the shock persistence is equal to 0.9, and the bottom panel corresponds to the case when we increase the shock persistence to 0.95. Some messages could be mentioned here. First, it can be seen that these shocks adversely impact the economy. In particular, a reduction of output as well as a rise in inflation, and price dispersion are the direct effects of these shocks. What is more, when central banks set a higher level of trend inflation, the magnitude of effect has become stronger. In other words, the consequences of policy uncertainty become more severe when trend inflation is higher.

Moreover, when the policy risk shocks become more persistent, they distort the economy more significantly. In particular, output decrease as well as inflation increases more significantly when policy uncertainty shocks become more persistent. A rise in price dispersion also plays a crucial role in explaining the welfare reductions.

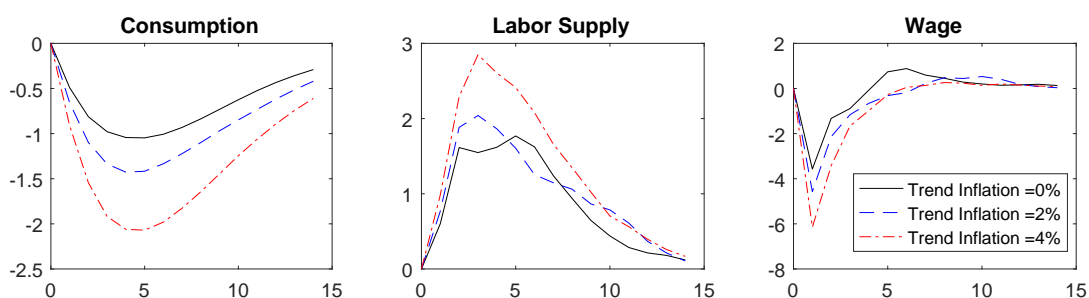
Figure 3.6: Impulse Response Functions to Policy Uncertainty Shocks



Note: The upper figure: the shock persistence level is 0.9 while the bottom figure: when the shock persistence level is 0.95.

Figure 3.7 depicts that the policy risk shocks induce households to consume less and work more. An increase in labor supply then creates a pressure in the labor market, thus lowers wages²³. When trend inflation is higher, the impacts of policy risk shocks become more considerable, thus greater welfare costs.

Figure 3.7: Impulse Response Functions to Policy Uncertainty Shocks



In short, the study indicates the important role of trend inflation mechanism²⁴

²³Born and Pfeifer (2014) and Basu and Bundick (2017) have documented the same effects.

²⁴In the Appendix 5.1, we conduct additional analyses that compare the welfare between economy with and without trend inflation mechanism to advocate this argument.

as well as the interaction between shifting trend inflation and monetary policy uncertainty. In the one hand, the existence of policy uncertainty signifies the welfare costs of constant and shifting trend inflation. On the other hand, the adverse impacts of policy uncertainty on the economy is more sizeable when central banks raises their inflation targets.

3.6 Conclusions

The study emphasized the interactions between shifting trend inflation and policy uncertainty. To illustrate that, a number of exercises were conducted in this chapter. In particular, we quantified the welfare costs of alterations in trend inflation in the presence of uncertainty before showing how the policy uncertainty distorts the welfare by itself. The focus of uncertainty lay on the monetary policy. We found that the welfare costs of raising trend inflation are larger if there is monetary policy uncertainty. The costs of shifting trend inflation also show a significant difference when we did and did not incorporate monetary policy uncertainty shocks. The results imply that the welfare costs of shifting trend inflation can be magnified if there exists the monetary policy uncertainty. These welfare differences due to shifting trend inflation mainly come from reductions of the level and volatility component. They are reflected by a decreasing trend in mean and an increasing trend in volatility of consumption and leisure due to shifting trend inflation. The monetary policy uncertainty then signifies these changes to produce the higher welfare costs.

Further, the presence of uncertainty itself produces a negligible welfare cost, whose welfare differences mainly come from the fall in its mean and volatility component. The costs of policy uncertainty can be accounted not only by reduction of mean consumption and leisure, but also by a rise in their volatility. It is worth noting that these costs are greater in the high-trend-inflation economy. More importantly, we found that a simultaneous existence of shifting trend inflation and monetary

policy uncertainty in the economy brings about more serious consequences. We also documented impulse responses of key macroeconomic variables to policy risk shocks on the economy with various levels of trend inflation. The results show that the adverse impacts of policy uncertainty shock can be signified when trend inflation is higher.

APPENDIX

Appendix 1 Data for Simulated Method of Moment

All data are from the Bureau of Economic Analysis (BEA)'s NIPA Tables and available in quarterly frequency from 1954Q3 until 2015Q1. The detail of data is given as

Output. It is the nominal GDP (Table 1.1.5 line 1, Series GDPC96) divided by the GDP deflator (Table 1.1.4 line 1, Series GDPDEF).

Consumption. Is a sum of personal consumption expenditures for non-durable goods (Table 1.1.5 line 5, Series PCND) and services (Table 1.1.5 line 6) divided by the GDP deflator (Table 1.1.4 line 1, Series PCESV).

Inflation. It is computed as the log-difference of the GDP deflator (Table 1.1.4 line 1, Series GDPDEF).

Hours. It is hours of all persons (index 2009=100) (St.Louis FED - FRED Database, Series HOABS).

Interest Rate. It is an effective Federal Funds rate (St.Louis FED - FRED Database, Series FEDFUNDS).

Appendix 2 Variables and Parameters

Table A2.1: Description of Variables

| Variables | | |
|------------------------------------|---|--------------------------------|
| $h_t(i)$ | Units of labor provided to an intermediate-goods firm i | |
| H_t | Labor supply | |
| W_t | Nominal wage rate | |
| D_t | Nominal dividend | |
| B_t | Units of bonds | |
| T_t | Lump-sum tax | |
| G_t | Government spending | $G_t = (1 - \frac{1}{g_t})Y_t$ |
| $Y_t(i)$ | Units of the intermediate goods i | |
| Y_t | Units of the final goods | |
| C_t | Units of the consumption goods | |
| λ_t | Non-negative Lagrange multiplier on the budget constrain | |
| π_t | Gross inflation rate | $\pi_t = P_t/P_{t-1}$ |
| $\bar{\pi}_t$ | Trend inflation | |
| $P_t(i)$ | Nominal price of intermediate goods i | |
| P_t | Nominal price of final goods | |
| Z_t | Aggregate technology | |
| R_t | Gross interest rate | |
| Calvo Price | | |
| P_t^* | An optimizing price of intermediate-goods producing firm | |
| ζ_t | A numerator of equation of the optimizing price | |
| ψ_t | A denominator of equation of the optimizing price | |
| \wp_t^* | A relative optimal price | |
| s_t | A price dispersion | |
| Exogenous Shocks | | |
| ϵ_{R_t} | Monetary shocks | |
| ϵ_{g_t} | Government spending shocks | |
| ϵ_{Z_t} | Technology shocks | |
| $\epsilon_{\bar{\pi}_t}$ | Trend inflation shocks | |
| Stochastic Volatility Shock | | |
| σ_t^i | Time-varying volatility of shocks i | $i = R$ |
| ϵ_t^i | Shocks to volatility | $i = R$ |

Table A2.2: Calibration

| Parameter | Description | Calibrated Value |
|-------------------------------------|---|--|
| β | Discount factor | 0.9974 |
| γ | Consumption habit | 0.81 |
| ω | Labor supply disutility | 1.00 |
| ν | Inverse Frisch elasticity of labor supply | 1.59 |
| θ | Elasticity of substitution | 10.0 |
| $1 - \bar{g}^{-1}$ | Steady state share of government expenditure | 0.26 |
| ρ_Z | AR(1) coefficient for technology shock | 0.32 |
| ρ_g | AR(1) coefficient for government spending shock | 0.98 |
| $100\sigma_Z$ | Standard deviation of technology shock | 1.00 |
| $100\sigma_g$ | Standard deviation of government spending shock | 0.55 |
| Monetary Policy | | |
| ϕ_π | Taylor coefficient on the inflation gap | 1.68 |
| ϕ_y | Taylor coefficient on the output gap | 0.08 |
| ρ_R | AR(1) coefficient for monetary shock | 0.81 |
| $100\sigma_R$ | Standard deviation of monetary shock | 0.25 |
| Calvo Price Setting | | |
| η | Probability of not being able to optimize | [0.6 ,0.65,0.7,0.75] |
| χ | Degree of indexation | [0.0 ,0.33,0.67,1.0] |
| μ | Weight on lagged inflation | 1.00 |
| Shifting Trend Inflation | | |
| $\bar{\pi}^*$ | Steady-state level of trend inflation | [1.00 ^{0.25} ... 1.06 ^{0.25}] |
| ρ_π | Persistence level of shocks to trend inflation | [0.99 ... 0.995 ... 0.9999] |
| $100\sigma_\pi$ | Standard deviation level of shocks to trend inflation | [0.1, 0.075 ,0.05,0.025,0] |
| Stochastic volatility shocks | | |
| ρ_{σ^R} | Persistence coefficient of volatility shocks | 0.921 |
| σ^R | Conditional mean of volatility shocks | -6.551 |
| η^R | One-standard deviation volatility shocks | 0.363 |

Appendix 3 The Model

Appendix 3.1 A List of Nonlinear Equilibrium Conditions in Stationary Variables

$$\tilde{\lambda}_t = \frac{1}{C_t - \gamma C_{t-1}} - \beta \gamma E_t \left(\frac{1}{C_{t+1} - \gamma C_t} \right) \quad (\text{NL1})$$

$$\tilde{\lambda}_t w_t = H_t^v \omega \quad (\text{NL2})$$

$$\tilde{\lambda}_t = \beta R_t E_t(\tilde{\lambda}_{t+1}) \frac{1}{\pi_{t+1}} \quad (\text{NL3})$$

$$Z_t H_t = Y_t s_t \quad (\text{NL4})$$

$$\wp_t^* = \frac{\theta}{\theta - 1} \frac{\zeta_t}{\psi_t} \quad (\text{NL5})$$

$$\zeta_t = w_t + \beta \eta (\bar{\pi}_t^{-\chi \theta})^{1-\mu} (\pi_t^{-\chi \theta})^\mu E_t[\pi_{t+1}^\theta \zeta_{t+1}] \quad (\text{NL6})$$

$$\psi_t = 1 + \beta \eta (\bar{\pi}_t^{-\chi(1-\theta)})^{1-\mu} (\pi_t^{\chi(1-\theta)})^\mu E_t[\pi_{t+1}^{\theta-1} \psi_{t+1}] \quad (\text{NL7})$$

$$1 = \left[(1 - \eta) \wp_{i,t}^*{}^{1-\theta} + \eta \pi_t^{\theta-1} \left(\bar{\pi}_{t-1}^{\chi(1-\mu)} \pi_{t-1}^{\chi \mu} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad (\text{NL8})$$

$$s_t = (1 - \eta) (\wp_{i,t}^*)^{-\theta} + \eta (\bar{\pi}_t^{-\chi \theta})^{1-\mu} (\pi_{t-1}^{-\chi \theta})^\mu \pi_t^\theta s_{t-1} \quad (\text{NL9})$$

$$C_t = \frac{1}{g_t} Y_t \quad (\text{NL10})$$

$$\ln(Z_t) = \rho_Z \ln(Z_{t-1}) + \epsilon_{Z_t} \quad (\text{NL11})$$

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}} \right)^{\rho_R} \left[\left(\frac{\pi_t}{\bar{\pi}_t} \right)^{\phi_\pi} \left(\frac{y_t}{\bar{y}} \right)^{\phi_y} \right]^{1-\rho_R} \exp(\sigma_t^R \epsilon_{R,t}) \quad (\text{NL12})$$

$$\ln \bar{\pi}_t = (1 - \rho_{\bar{\pi}}) \ln \bar{\pi}^* + \rho_{\bar{\pi}} \ln \bar{\pi}_{t-1} + \epsilon_{\bar{\pi}_t} \quad (\text{NL13})$$

$$\ln(g_{t+1}) = (1 - \rho_g) \ln(\bar{g}) + \rho_g \ln(g_t) + \epsilon_{g_t} \quad (\text{NL14})$$

The stochastic volatility shock

$$\sigma_t^R = (1 - \rho_{\sigma^R}) \sigma^R + \rho_{\sigma^R} \sigma_{t-1}^R + \eta^R \epsilon_t^R, \epsilon_t^R \sim N(0, 1) \quad (\text{NL15})$$

Appendix 3.2 A List of Steady-State Variables

$$\wp_t^*(\bar{\pi}) = \left[\frac{1 - \eta \bar{\pi}^{(1-\chi)(\theta-1)}}{1 - \eta} \right]^{\frac{1}{1-\theta}} \quad (\text{ss1})$$

$$s(\bar{\pi}) = \frac{1 - \eta}{1 - \eta \bar{\pi}^{(1-\chi)\theta}} (\wp_t^*(\bar{\pi}))^{-\theta} \quad (\text{ss2})$$

$$w(\bar{\pi}) = \frac{\theta - 1}{\theta} \frac{1 - \eta \beta \bar{\pi}^{(1-\chi)(\theta)}}{1 - \eta \beta (\bar{\pi})^{(1-\chi)(\theta-1)}} \wp_t^*(\bar{\pi}) \quad (\text{ss3})$$

$$\zeta(\bar{\pi}) = \frac{w(\bar{\pi})}{1 - \eta \beta (\bar{\pi})^{(1-\chi)(\theta)}} \quad (\text{ss4})$$

$$\psi(\bar{\pi}) = \frac{1}{1 - \eta \beta (\bar{\pi})^{(1-\chi)(\theta-1)}} \quad (\text{ss5})$$

$$y(\bar{\pi}) = \left\{ \frac{w(\bar{\pi})}{\omega s(\bar{\pi})} \left(\frac{z - \beta \gamma}{z - \gamma} \right) \bar{g} \right\}^{\frac{1}{1+\nu}} \quad (\text{ss6})$$

$$c(\bar{\pi}) = \frac{y(\bar{\pi})}{\bar{g}} \quad (\text{ss7})$$

$$\lambda(\bar{\pi}) = \frac{z - \beta \gamma}{c(\bar{\pi})(1 - \gamma)} \quad (\text{ss8})$$

Appendix 4 Sensitivity Analysis

Appendix 4.1 Welfare Costs of Shifting Trend Inflation

Table A4.1: Welfare Costs of Shifting Trend Inflation: Shock Volatility Level

| $100 * \sigma_{\bar{\pi}}$ | Without Uncertainty | | | | | With Uncertainty | | | | |
|----------------------------|---------------------|--------|--------|-------|-------|------------------|--------|--------|-------|-------|
| | wc | U | U_d | U_l | U_v | wc | U | U_d | U_l | U_v |
| 0.000 | | -896.1 | -890.4 | -1.9 | -3.6 | | -896.1 | -890.4 | -1.9 | -3.6 |
| 0.025 | 0.02 | -896.3 | -890.4 | -2.1 | -3.7 | 0.28 | -898.3 | -890.4 | -4.2 | -3.7 |
| 0.050 | 0.10 | -896.8 | -890.4 | -2.6 | -3.8 | 0.36 | -898.9 | -890.4 | -4.6 | -3.8 |
| 0.075 | 0.22 | -897.8 | -890.4 | -3.5 | -3.9 | 0.49 | -899.9 | -890.4 | -5.5 | -3.9 |
| 0.100 | 0.39 | -899.1 | -890.4 | -4.6 | -4.0 | 0.66 | -901.2 | -890.4 | -6.7 | -4.1 |

Note: The welfare costs (wc) are measured in percent. U, U_d, U_l , and U_v denote the welfare, the deterministic steady-state component, the level component, and the volatility component of welfare, respectively.

Table A4.2: Welfare Costs of Shifting Trend Inflation: Shock Persistence Level

| $\rho_{\bar{\pi}}$ | Without Uncertainty | | | | | With Uncertainty | | | | |
|--------------------|---------------------|--------|--------|-------|-------|------------------|--------|--------|-------|-------|
| | wc | U | U_d | U_l | U_v | wc | U | U_d | U_l | U_v |
| 0.900 | 0.01 | -896.2 | -890.4 | -2.0 | -3.6 | 0.27 | -898.2 | -890.4 | -4.0 | -3.7 |
| 0.950 | 0.02 | -896.2 | -890.4 | -2.1 | -3.7 | 0.28 | -898.3 | -890.4 | -4.1 | -3.7 |
| 0.990 | 0.12 | -897.0 | -890.4 | -2.8 | -3.8 | 0.39 | -899.1 | -890.4 | -4.8 | -3.8 |
| 0.995 | 0.24 | -898.0 | -890.4 | -3.7 | -3.9 | 0.51 | -900.1 | -890.4 | -5.7 | -3.9 |
| 0.999 | 1.28 | -906.1 | -890.4 | -10.8 | -4.9 | 1.54 | -908.2 | -890.4 | -12.8 | -5.0 |

Note: The welfare costs (wc) are measured in percent. U, U_d, U_l , and U_v denote the welfare, the deterministic steady-state component, the level component, and the volatility component of welfare, respectively.

Table A4.3: Welfare Costs of Shifting Trend Inflation:
Calvo Parameters

| | Without Uncertainty | | | | With Uncertainty | | | |
|---------------|---------------------|------|------|------|------------------|------|------|------|
| χ η | 0.6 | 0.65 | 0.7 | 0.75 | 0.6 | 0.65 | 0.7 | 0.75 |
| 0.00 | 0.09 | 0.15 | 0.25 | 0.49 | 0.23 | 0.33 | 0.51 | 0.88 |
| 0.33 | 0.04 | 0.05 | 0.08 | 0.15 | 0.09 | 0.14 | 0.19 | 0.30 |
| 0.67 | 0.01 | 0.01 | 0.02 | 0.03 | 0.03 | 0.05 | 0.06 | 0.08 |
| 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

Note: The welfare costs are measured in percent. χ and η represent the price indexation degree and the probability of not being able to optimize the price, respectively.

Appendix 4.2 Welfare Costs of Uncertainty

Table A4.4: Welfare Costs of Uncertainty:
Stochastic Shock Volatility

| Monetary Policy Risk | | | | | |
|----------------------|-------------|--------|--------|-------|-------|
| η^R | wc (unit:%) | U | U_d | U_l | U_v |
| 0.00 | | -898.0 | -890.4 | -3.7 | -3.9 |
| 0.25 | 0.15 | -899.2 | -890.4 | -4.9 | -3.9 |
| 0.5 | 0.42 | -901.3 | -890.4 | -6.9 | -4.0 |
| 0.75 | 0.86 | -904.7 | -890.4 | -10.2 | -4.1 |
| 1.00 | 1.57 | -910.4 | -890.4 | -15.7 | -4.2 |

Note: The welfare costs (wc) are measured in percent. U, U_d, U_l , and U_v denote the welfare, the deterministic steady-state component, the level component, and the volatility component of welfare, respectively. η^R is the volatility level of monetary policy uncertainty shocks.

Appendix 5 The Additional Analyses

Appendix 5.1 The Economy with and without Trend Inflation Mechanism

To emphasize the role of trend inflation mechanism, we conduct additional exercises that estimate parameters and quantify welfare costs of the economy with and without trend inflation. In particular, we first re-estimate the model with $\bar{\pi}^* = 1.00^{0.25}$ by using the SMM method. Table A4.5 reports changes in parameters when we set $\bar{\pi}^* = 1.00^{0.25}$ and $\bar{\pi}^* = 1.02^{0.25}$. The results show that there are changes in $\phi_{\bar{\pi}}, \phi_y, \eta$, while those in ρ_R, γ are modest.

Table A4.5: Calibration: Model with $\bar{\pi}^* = 1.00^{0.25}$ and $\bar{\pi}^* = 1.02^{0.25}$

| Parameter | ρ_R | $\phi_{\bar{\pi}}$ | ϕ_y | η | γ |
|-----------------------------|----------|--------------------|----------|--------|----------|
| $\bar{\pi}^* = 1.00^{0.25}$ | 0.81 | 1.69 | 0.19 | 0.17 | 0.81 |
| $\bar{\pi}^* = 1.02^{0.25}$ | 0.81 | 1.68 | 0.08 | 0.7 | 0.81 |

We then perform two additional exercises. First, we conduct the same exercise as in the main part of dissertation when the case is $\bar{\pi}^* = 1.00^{0.25}$ (using the recalibrated parameters). In particular, we quantify the welfare costs of constant trend inflation (when central banks raise inflation targets from 0 to 6 percent), the welfare costs of shifting trend inflation (we compare the welfare of two hypothetical economies: one with zero variance of shock to trend inflation and one with positive variance of shock to trend inflation) and the welfare costs of monetary policy uncertainty by itself. Table A4.6 and A4.7 compare the results when $\bar{\pi}^* = 1.00^{0.25}$ (using recalibrated parameters) versus when $\bar{\pi}^* = 1.02^{0.25}$ (using the calibrated parameters in the main part of dissertation). In general, the welfare costs when $\bar{\pi}^* = 1.00^{0.25}$ in these exercises are highly trivial as compared with those when $\bar{\pi}^* = 1.02^{0.25}$. However, the main conclusions of interactions between shifting trend inflation and

monetary policy uncertainty in Chapter 3 still hold when we consider that central banks set a 0-percent inflation target.

Table A4.6: Welfare Costs of Constant and Shifting Trend Inflation: $\bar{\pi}^* = 1.00^{0.25}$ versus $\bar{\pi}^* = 1.02^{0.25}$

| | WC of Constant Trend Inflation | | WC of Shifting Trend Inflation | |
|-----------------------------|---|---|--|--|
| | W/O Uncer | W Uncer | W/O Uncer | W Uncer |
| | $\bar{\pi}^* = 1.00^{0.25} \rightarrow 1.06^{0.25}$ | $\bar{\pi}^* = 1.00^{0.25} \rightarrow 1.06^{0.25}$ | $(\sigma_{\bar{\pi}} > 0, \eta^R = 0)$ | $(\sigma_{\bar{\pi}} > 0, \eta^R > 0)$ |
| $\bar{\pi}^* = 1.00^{0.25}$ | 0.020 | 0.023 | 0.004 | 0.015 |
| $\bar{\pi}^* = 1.02^{0.25}$ | 5.53 | 7.85 | 0.24 | 0.51 |

Note: WC is the welfare costs that are measured in percent.

Table A4.7: Welfare Costs of Policy Uncertainty: $\bar{\pi}^* = 1.00^{0.25}$ versus $\bar{\pi}^* = 1.02^{0.25}$

| Welfare Costs of Policy Uncertainty (%) | |
|---|------|
| $\eta_R = 0 \rightarrow \eta_R > 0$ | |
| $\bar{\pi}^* = 1.00^{0.25}$ | 0.01 |
| $\bar{\pi}^* = 1.02^{0.25}$ | 0.25 |

In the second exercise, we compare the welfare to two hypothetical economies: the one with $\bar{\pi}^* = 0$ and the one with $\bar{\pi}^* = 2$. Table A4.8 reports the welfare of two economies with different sets of parameters. In general, the welfare is smaller when trend inflation is 2 percent.

Table A4.8: A Comparison of Welfare: $\bar{\pi}^* = 1.00^{0.25}$ versus $\bar{\pi}^* = 1.02^{0.25}$

| | Welfare | |
|-----------------|-----------------------------|-----------------------------|
| | $\bar{\pi}^* = 1.00^{0.25}$ | $\bar{\pi}^* = 1.02^{0.25}$ |
| W/O Uncertainty | -893.9 | -898.0 |
| W Uncertainty | -894.0 | -900.1 |

Appendix 5.2 The Second Order versus The Third Order Approximation

In order to analyse impacts of a higher order approximation on parameter values as well as the welfare cost computation, we employ the third order approximation to the policy function. The relevant parameters are adjusted to minimize the moments computed from the model and the empirical moments. Table A4.9 shows changes in their values when using the second order and the third order approximation. It can be seen that the parameter values change significantly when we employ the third order approximation. Subsequently, we quantify the welfare costs of shifting trend inflation and monetary policy uncertainty by itself. Table A4.10 reports results in each case. In general, the welfare costs when we use the parameters obtained from the third order approximation are smaller than those obtained from the second order approximation.

Table A4.9: Calibration: Second Order versus Third Order Approximation

| Parameter | ρ_R | $\phi_{\bar{\pi}}$ | ϕ_y | η | γ | v |
|--------------|----------|--------------------|----------|--------|----------|------|
| Second Order | 0.81 | 1.68 | 0.08 | 0.70 | 0.81 | 1.59 |
| Third Order | 0.81 | 1.51 | 0.19 | 0.60 | 0.95 | 1.00 |

Table A4.10: Welfare Costs: Second Order versus Third Order Approximation

| | WC of Shifting Trend Inflation | | WC of Policy Uncertainty | |
|--------------|---|---|--------------------------|--|
| | W/O Uncer ($\sigma_{\bar{\pi}} > 0, \eta^R = 0$) | W Uncer ($\sigma_{\bar{\pi}} > 0, \eta^R > 0$) | W/O Uncer | W Uncer $\eta_R = 0 \rightarrow \eta_R > 0$ |
| Second Order | 0.24 | 0.51 | | 0.25 |
| Third Order | 0.16 | 0.24 | | 0.08 |

Note: WC is the welfare costs that are measured in percent.

Appendix 6 Simulated Method of Moment

The idea of Simulated Method of Moments (SMM) is presented in the following. Let denote κ be a (qx1) vector of structural parameters, $h(x_i)$, be a (px1) vector of empirical observations on whose moments are of our concerns, and $h(x_j(\kappa))$ is the (px1) vector of variables analogous to $h(x_i)$ but based on the data simulated from the model using parameter values. The SMM estimator, $\hat{\kappa}_s$, is obtained by solving

$$\hat{\kappa}_s = \min_{\kappa \in K} \mathbf{H}(\kappa)' \mathbf{W} \mathbf{H}(\kappa), \quad (\text{A6.1})$$

where

$$H(\kappa) = \left\{ (1/T) \sum_{i=1}^T h(x_i) - (1/\tau T) \sum_{j=1}^{\tau T} h(x_j(\kappa)) \right\}, \quad (\text{A6.2})$$

τT is the length of simulated sample, \mathbf{W} is a (qxq) weighting matrix. In words, the SMM estimator is the value of κ that make the distance between the moments implied by the model and those obtained from the observed data as small as possible. The regularity condition argued by Duffie and Singleton (1993) for SMM estimators to be consistent and asymptotically normal states that

$$\sqrt{T}(\hat{\kappa} - \kappa_0) \rightarrow N(0, (1 + 1/\tau)(S'WS)^{-1}S'W\mathbf{R}WS(S'WS)^{-1}), \quad (\text{A6.3})$$

where

$$R = \lim_{T \rightarrow \infty} \text{Var} \left(\frac{1}{\sqrt{T}} \sum_{i=1}^T h(x_i) \right), \quad (\text{A6.4})$$

$S = \partial E h(\kappa) / \partial \kappa$ is a (pxq) matrix of full column rank.

Estimation Procedures

We perform a two-step estimation procedure. As argued by Born and Pfeifer (2014), it is computationally difficult to jointly estimate all parameters because of the non-linear solution of the model required to capture uncertainty effects and the high-dimensional state space. Therefore, we firstly estimate the time-varying

stochastic volatility shocks. For simplicity, we follow Born and Pfeifer (2014) to select parameter values for the time-varying stochastic volatility process arising in the monetary policy. In the second step, this process is fed into the model and remaining parameters are estimated by using the SMM.

Regarding SMM, the model is first solved and estimated using a second-order approximation. The data series are output, consumption, inflation, hours worked and interest rate for the period 1954:Q3 to 2015:Q1. There are parameter values fixed prior to the estimation, including β , v , χ , $\bar{\theta}$, \bar{g} , ρ_g , σ_g , ρ_Z , σ_Z , σ_R , $\bar{\pi}^*$, $\sigma_{\bar{\pi}}$, and $\rho_{\bar{\pi}}$. In this application, κ consists of γ , η , ρ_R , ρ_{π} , and ρ_y . The four selected moments include the output volatility, the volatility of consumption relative to output, a correlation between output and consumption, and a correlation between output and labor. Following Ruge-Murcia (2012), we use the identity matrix as the weighting matrix in the statistical objective function. The number of simulated observations is five times larger than the sample size, that is $\tau = 5$.

Appendix 7 Second-order Approximation of Welfare Computation Method

The method suggested by J. Kim, Kim, Sims, and Schaumburg (2008) to measure welfare is described here. The coefficients matrix is assumed to follow the given form

$$\begin{aligned} y_t &= F_c + F_f x_t + \frac{1}{2} F_s [x_t \otimes x_t], \\ x_{t+1} &= P_c + P_f x_t + \frac{1}{2} P_s [x_t \otimes x_t] + Q e_{t+1}, \end{aligned}$$

while y_t comprises of forward looking variables, x_t and e_t include exogenous and predetermined variables; and all exogenous shock in the economy, respectively. The standard deviation of e_t is expressed by Q .

Kim et al. (2008) argue that the first-order accurate is necessarily computed to obtain the second-order accurate $[x_t \otimes x_t]$. It can be written as given

$$\begin{aligned} \begin{pmatrix} y_t^{(2)} \\ y_t^{(1)} \otimes y_t^{(1)} \end{pmatrix} &= \begin{pmatrix} F_f & \frac{1}{2} F_s \\ 0 & F_f \otimes F_f \end{pmatrix} \begin{pmatrix} x_t^{(2)} \\ x_t^{(1)} \otimes x_t^{(1)} \end{pmatrix} + \eta_{s,t} \Leftrightarrow Y_{s,t} = A_{y,s} X_{s,t} + \eta_{s,t+1}, \\ \begin{pmatrix} x_t^{(2)} \\ x_t^{(1)} \otimes x_t^{(1)} \end{pmatrix} &= \begin{pmatrix} P_f & \frac{1}{2} P_s \\ 0 & P_f \otimes P_f \end{pmatrix} \begin{pmatrix} x_t^{(2)} \\ x_t^{(1)} \otimes x_t^{(1)} \end{pmatrix} + \epsilon_{s,t} \Leftrightarrow X_{s,t+1} = A_{x,s} X_{s,t} + \epsilon_{s,t+1}, \end{aligned}$$

where $x_t^{(i)}$ and $y_t^{(i)}$ express i -th order accurate solution to x_t and y_t . Let denote $T_{y,s} = E_0[\eta_{s,t+1}]$, whose elements are given

$$\begin{aligned} T_{y,s,1} &= F_c, \\ T_{y,s,2} &= 0, \end{aligned}$$

and elements of $T_{x,s} = E_0[\epsilon_{s,t+1}]$ are given

$$T_{x,s,1} = P_c,$$

$$T_{x,s,2} = (Q \otimes Q)vec(I_{n_\epsilon}).$$

And this study uses VAR representation to achieve a second order approximation as follows

$$\begin{aligned}
& E\left[\sum_{t=0}^{\infty} \beta^t u(y_t x_t)\right] \approx \\
& = \frac{u(\bar{y}, \bar{x})}{1 - \beta} + E \sum_{t=0}^{\infty} \beta^t [Mu_y(\bar{y}, \bar{x})y_t^{(2)} + \frac{1}{2}Nu_y(\bar{y}, \bar{x})(y_t^{(1)} \otimes y_t^{(1)})] \\
& \quad + E \sum_{t=0}^{\infty} \beta^t [Mu_x(\bar{y}, \bar{x})x_t^{(2)} + \frac{1}{2}Nu_x(\bar{y}, \bar{x})(x_t^{(1)} \otimes x_t^{(1)})] \\
& = \frac{u(\bar{y}, \bar{x})}{1 - \beta} + E_0\left[\begin{pmatrix} Mu_y(\bar{y}, \bar{x}) \\ \frac{1}{2}Nu_y(\bar{y}, \bar{x}) \end{pmatrix}' Y_{s,t}\right] + E_0\left[\begin{pmatrix} Mu_x(\bar{y}, \bar{x}) \\ \frac{1}{2}Nu_x(\bar{y}, \bar{x}) \end{pmatrix}' X_{s,t}\right] \\
& = \frac{u(\bar{y}, \bar{x})}{1 - \beta} + \begin{pmatrix} Mu_y(\bar{y}, \bar{x}) \\ \frac{1}{2}Nu_y(\bar{y}, \bar{x}) \end{pmatrix}' \sum_{t=0}^{\infty} E_0 Y_{s,t} + \begin{pmatrix} Mu_x(\bar{y}, \bar{x}) \\ \frac{1}{2}Nu_x(\bar{y}, \bar{x}) \end{pmatrix}' \sum_{t=0}^{\infty} E_0 X_{s,t} \\
& = \frac{u(\bar{y}, \bar{x})}{1 - \beta} + \begin{pmatrix} Mu_y(\bar{y}, \bar{x}) \\ \frac{1}{2}Nu_y(\bar{y}, \bar{x}) \end{pmatrix}' \sum_{t=0}^{\infty} E_0 [A_{y,s} X_{s,t-1} + \eta_t] \\
& \quad + \begin{pmatrix} Mu_x(\bar{y}, \bar{x}) \\ \frac{1}{2}Nu_x(\bar{y}, \bar{x}) \end{pmatrix}' \sum_{t=0}^{\infty} E_0 (I - \beta A_{x,s})^{-1} \frac{\beta}{1 - \beta} T_{x,s} \\
& = \frac{u(\bar{y}, \bar{x})}{1 - \beta} + \begin{pmatrix} Mu_y(\bar{y}, \bar{x}) \\ \frac{1}{2}Nu_y(\bar{y}, \bar{x}) \end{pmatrix}' [A_y (I - \beta A_{x,s})^{-1} \frac{\beta}{1 - \beta} T_{x,s} + \frac{\beta}{1 - \beta} T_{y,s}] \\
& \quad + \begin{pmatrix} Mu_x(\bar{y}, \bar{x}) \\ \frac{1}{2}Nu_x(\bar{y}, \bar{x}) \end{pmatrix}' (I - \beta A_{x,s})^{-1} \frac{\beta}{1 - \beta} T_{x,s}.
\end{aligned}$$

Chapter 4

Welfare Costs of Shifting Trend Inflation: Staggered Wage and Price Contracts

4.1 Introduction

The New Keynesian Phillips curve (NKPC, henceforth), a key element of Dynamic Stochastic General Equilibrium (DSGE, henceforth) models, has been used widely for theoretical, empirical and monetary policy analysis. However, the assumptions of NKPC are sometimes restrictive that either trend inflation²⁵ must be zero or firms must index their prices to past inflation or target inflation²⁶. First, central banks in the real world have invariably selected positive inflation targets, thus the assumption of zero inflation target is exceedingly rare. Second, misleading conclusions could be drawn when we assume the zero steady-state inflation. For instance, a divine coincidence suggested by Blanchard and Gali (2007)²⁷ or a highly non-linear and positive slope of the long run NKPC²⁸ might not be true in the case

²⁵It can be interpreted as central bank's implicit inflation target and private sector's long-run inflation expectation.

²⁶Yun (1996) derives the indexation rule. When firms cannot optimize their prices, they are still able to update their price according to this indexation rule.

²⁷Blanchard and Gali (2007) indicate that under a price stickiness, a monetary policy rule might be able to simultaneously stabilize the inflation rate and the output gap in the face of preference or technology shocks. Alves (2014), however, shows that this divine coincidence only holds when the inflation rate is stabilized at zero. Otherwise, there is a trade-off between stabilizing inflation and stabilizing output gap.

²⁸Ascari and Ropele (2009) indicate that due to a strong price dispersion effect, the slope turns quite rapidly negative for extreme values of trend inflation.

of positive trend inflation. Moreover, Levin and Piger (2003) and Ireland (2007)²⁹ indicate that trend inflation has the tendency to vary over time.

The aforementioned reasons motivate the case for studying optimizing behavior without making an assumption of zero trend inflation. However, most existing versions of NKPC have focused on constant positive trend inflation³⁰, while they have not paid enough attention to the property of its time-varying variation (shifting trend inflation). Few papers have recently investigated it to show necessities of the research on this field. For example, Kozicki and Tinsley (2001) explore implications of shifting trend inflation for the term structure of interest rates, while its effects on output and overall inflation are investigated by Ireland (2007). Furthermore, Cogley and Sbordone (2008) and Cogley et al. (2009) conduct research to examine implications of shifting trend inflation for estimated parameters and a predictability of inflation, respectively. Most recently, welfare consequences of shifting trend inflation are measured by Nakata (2014). The similar spirit of the existing literature is that the previous studies employ a highly persistent shock to trend inflation, regarded as the central bank's moving implicit targets to model shifting trend inflation and they adapt only one form of rigidities. So far, these papers only discuss the sticky-price model with shifting trend inflation.

To fill the gap in the literature, this chapter also investigates welfare consequences of shifting trend inflation but we argue that the trend inflation process can adversely impact the economy by distorting output and labor allocations through staggered price and wage contracts. In particular, the Calvo mechanism generates the price and wage dispersion. They in turn induce a gap between the average price level and the price set by resetting firms, and a gap between the average wage level and the wage set by resetting households. The following reasons inspire an inclu-

²⁹By using the model that allows inferences concerning the Federal Reserve's inflation target, Ireland (2007) shows changes from 1959 (1.25 percent) to the late 1970s (8 percent) and in 2004 (2.5 percent).

³⁰For example, we can see in Ascari (2004), Amano et al. (2007), Ascari and Ropele (2007), and Coibion and Gorodnichenko (2011).

sion of both price and wage rigidities. First, a tendency to discuss price and wage rigidities as independent phenomena seems to be inappropriate at the macro-level as argued by Basu and House (2015). The price rigidities depend significantly on the wage rigidities since most models assume that target prices are set as a constant mark-up on nominal marginal cost. The inertia of the price levels then depends on the sluggish adjustment of marginal costs, in which wage accounts for the greatest proportion. As a result, wage stickiness reinforces price stickiness. Second, the empirical New-Keynesian model incorporating the wage rigidities is more successful in explaining cyclical fluctuations as argued by Christiano, Eichenbaum, and Evans (2005).

Furthermore, the staggered wages are an important assumption for welfare analysis. With models that staggered prices are the sole form of nominal rigidity, economists show that monetary authorities only face tradeoffs in stabilizing price inflation and output gap to achieve the Pareto-optimal welfare. However, recent analysis of dynamic general equilibrium models indicates that volatility of aggregate price inflation, and aggregate wage inflation induce dispersion in prices across firms, thus inefficient dispersion in output levels, and inefficient dispersion in the distribution of employment across households, respectively. Therefore, a zero output gap and a complete stabilization of price inflation and wage inflation are necessary to achieve the Pareto-optimal equilibrium. As argued by Erceg, Henderson, and Levin (2000), the Pareto-optimal equilibrium is only achieved where either prices or wages are completely flexible. It is infeasible since the constant nominal price inflation and wage inflation require that the actual real wage is continuously at its Pareto-optimal level but it could never change without the price and wage adjustment. Therefore, the monetary authorities tradeoff between price inflation variability, wage inflation variability and output gap variability. In the scope of this study, we examine whether wage rigidities are quantitatively important in explaining welfare costs of shifting trend inflation. Erceg et al. (2000) employ the model with both price and wage stick-

iness to show that variability of growth rates of nominal wages implies misalignment of wages, thus an inefficient utilization of labors. The inefficient utilization of labor then is magnified by constant positive trend inflation (Ascari et al., 2016).

So far, researchers have not investigated impacts of shifting trend inflation in the model with staggered price and wage contracts. This chapter, thus, expands the existing literature by incorporating both price and wage rigidities to quantify costs of shifting trend inflation. We address the following questions: (i) how large welfare costs of shifting trend inflation are? and (ii) do staggered price or wage contracts more importantly determine the welfare costs of shifting trend inflation?

To solve these questions, we study the same model as Ireland (2007) but considerably expand by developing the model with a Calvo staggered price setting and a Calvo staggered wage setting. While the Calvo price setting is popular in literature, we follow Erceg, Henderson, and Levin (2000) for the Calvo wage setting. Trend inflation is supposed to follow an AR(1) process to model a sustained rise in inflation. We argue that exogenous variations in trend inflation can distort the economy by two channels: staggered prices and staggered wages. Moreover, we discuss two special cases: the model with staggered price contract and completely flexible wage, and the model with completely flexible price and staggered wage contract. This discussion is useful to analyse the role of each mechanism individually in transmitting impacts of exogenous alternations in trend inflation into the economy.

With these extensions, we showed that the consequences of constant positive trend inflation and shifting trend inflation are severe, especially when trend inflation is high. Among two channels, staggered wage contracts play a vital role in transmitting adverse impacts of constant and shifting trend inflation into economy. This conclusion is aligned with findings in Ascari et al. (2016) who also measure welfare costs of trend inflation in the model with both staggered price and wage contracts. Our study provides other evidence that the costs of shifting trend inflation are signified when considering the staggered wage contracts. Without the

staggered wage channel, welfare costs of constant and shifting trend inflation are modest as in Nakata (2014).

We also conduct exercises to examine the sensitivity of welfare costs to a range of plausible parameters. The sensitivity analysis results show that if either the price and wage friction are sufficiently large or the price and wage indexation level are sufficiently negligible, welfare costs become more sizeable. The welfare also declines dramatically if there is an upward biased trend inflation process. Moreover, changes in parameters governing the wage environment cause substantial movements of welfare costs as opposed to parameters controlling the pricing environment. The results of sensitivity analysis provide more compelling evidence that the staggered wage contracts are important factor determining welfare costs of shifting trend inflation.

Regarding welfare analysis, this chapter is related to two strands of literature. First, it is related to a literature on non-zero trend inflation. The plurality of models has so far adapted the sticky-price model with positive trend inflation, such as Ascari(2004), Amano et al. (2007), Ascari and Ropele (2007), and Coibion and Gorodnichenko (2011). These papers show that a less-than-4-percent trend inflation rate has a minor impact on the economy. In particular, Ascari (2004) finds that an additional steady-state output loss of 0.5 percent is created when increasing trend inflation from 2 to 4 percent, while a compensation-equivalent welfare loss is less than one percent as found in Amano et al. (2009). Alves (2014) develops an approach to derive the welfare loss function that is only contingent upon aggregate variables to avoid the previous approximation pitfall. Ascari et al. (2016) study a medium-scale model with staggered prices and staggered wages to show significant welfare costs of constant trend inflation. They also emphasize the role of staggered wage contracts on these costs.

The other strand is related to the model with shifting trend inflation. By employing the second perturbation approximation method suggested by Kim et al.

(2008), Nakata (2014) attempts to quantify welfare consequences of shifting trend inflation. In his model, he argues that the negative impacts of exogenous variations in trend inflation are transmitted into the economy solely by the staggered price contracts. With this consideration, he shows trivial welfare costs of shifting trend inflation. In this chapter, we follow the similar approach as in Nakata (2014) to measure welfare consequences of shifting trend inflation. However, we add an additional channel that trend inflation distorts the relative allocation of labors across households throughout a staggered wage contract as discussed by Ascari et al. (2016), thus changes in welfare. Without considering two channels jointly, different conclusions could be drawn.

The rest of this chapter is organized as follows. The extended model will be discussed in section 4.2. Section 4.3 explains how to compute welfare and welfare costs. Calibrated parameters are presented in section 4.4 while section 4.5 shows main results. Some conclusion is provided in section 4.6.

4.2 The Model

The model³¹ is populated by five classes of agents: the final-goods producing firms, a continuum of intermediate-goods producing firms indexed by $i \in [0, 1]$, employment agencies, a continuum of household indexed by $j \in [0, 1]$, and the government. The final-goods producing firms employ a continuum of intermediate goods from monopolistic intermediate goods producers. Households consume the final goods while supplying differentiated labor services to competitive “employment agencies”. Finally, the authority conducts the monetary and fiscal policies.

³¹The detailed description of variables in the model is reported in B1.1.

4.2.1 The Final-Goods Producing Firm

In each period t , perfectly competitive firms manufacture Y_t units of final consumption product by using $Y_t(i)$ units of intermediate goods at the nominal price $P_t(i)$ according to the constant-return-to-scale technology as follows

$$\left[\int_0^1 Y_t(i)^{\frac{\theta_p-1}{\theta_p}} di \right]^{\frac{\theta_p}{\theta_p-1}} = Y_t, \quad (4.1)$$

where θ_p denotes price elasticity of demand for intermediate goods. Profit maximization and the zero profit condition imply the demand function of intermediate good i is given as

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\theta_p} Y_t, \quad (4.2)$$

and that the price of the final good, P_t is a CES aggregate of the prices of the intermediate goods, $P_t(i)$

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta_p} di \right]^{\frac{1}{1-\theta_p}}. \quad (4.3)$$

4.2.2 The Intermediate-Goods Producing Firm

Monopolistic firms produce the intermediate good i using the following production function

$$Y_t(i) = Z_t h_t(i), \quad (4.4)$$

where $h_t(i)$ denotes the labor input for the production good i and Z_t is an exogenous stochastic process capturing the productivity effects. In particular, $\ln(Z_t)$ follows stationary AR(1) process

$$\ln(Z_t) = \rho_Z \ln(Z_{t-1}) + \epsilon_{Z_t}, \quad (4.5)$$

where ϵ_{Z_t} is the serially uncorrelated innovation, which has a normal distribution with mean zero and standard deviation σ_Z . The intermediate-goods producers are assumed to set nominal prices as in staggered Calvo price fashion. According to Calvo (1983), a fixed fraction η_p of firms, which cannot re-optimize its nominal prices, still set their prices according to the indexation rule. Following Ascari (2004), the way that firms reset the price is given as

$$P_t(i) = (\pi_{t-1}^{\mu_p} \bar{\pi}_t^{1-\mu_p})^{\chi_p} P_{t-1}(i), \quad (4.6)$$

where χ_p and μ_p denote a degree of price indexation and the relative weight on lagged inflation, respectively. The inflation, π_t , is defined as $\frac{P_t}{P_{t-1}}$ and $\bar{\pi}_t$ is interpreted as the central bank's inflation target. Subject to the usual cost minimization condition, re-optimizing firms maximize the present value of future profits by choosing their price P_t^*

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \eta_p^s \left[P_t^*(i) (\bar{\pi}_t^{\chi_p s})^{(1-\mu_p)} (\pi_{t-1, t+s-1}^{\chi_p})^{\mu_p} - \frac{W_{t+s}}{Z_{t+s}} \right] Y_{t+s}(i), \quad (4.7)$$

such that

$$Y_{t+s}(i) = \left[\frac{P_t^*(i) (\bar{\pi}_t^{\chi_p s})^{(1-\mu_p)} (\pi_{t-1, t+s-1}^{\chi_p})^{\mu_p}}{P_{t+s}} \right]^{-\theta_p} Y_{t+s}, \quad (4.8)$$

where $\pi_{t+s-1} = \left(\frac{P_{t+2}}{P_{t+1}}\right) \dots \left(\frac{P_{t+s}}{P_{t+s-1}}\right)$ if $s = 1, 2, 3, \dots$, λ_t is the same as the Lagrangian multiplier on the household's budget constraints, and W_t denotes the nominal wage.

4.2.3 The Employment Agency

Firms are owned by a continuum of households indexed by $j \in [0, 1]$. As assumed by Erceg et al. (2000), each household is a monopolistic supplier of specialized labor, $H_t(j)$. A large number of competitive "employment agencies" combines this specialized labor into a homogeneous labor input as given

$$H_t = \left[\int_0^1 H_t(j)^{\frac{\theta_w-1}{\theta_w}} dj \right]^{\frac{\theta_w}{\theta_w-1}}, \quad (4.9)$$

where θ_w denotes the desired mark-up of the wage over the household's marginal rate of substitution. The labor demand function is obtained by solving a profit maximization for the perfectly competitive employment agencies as given

$$H_t(j) = \left[\frac{W_t(j)}{W_t} \right]^{-\theta_w} H_t, \quad (4.10)$$

where $W_t(j)$ is the wage received from employment agencies by the supplier of labor of type j while the wage paid by the intermediate firms for their homogeneous labor input is expressed as

$$W_t = \left[\int_0^1 W_t(j)^{1-\theta_w} \right]^{\frac{1}{1-\theta_w}}. \quad (4.11)$$

4.2.4 The Household

Households maximize the expected discounted utility sum of future period utility

$$\sum_{t=0}^{\infty} \beta^t \left(\ln(C_t - \gamma C_{t-1}) - \frac{\omega}{1+v} H_t^{1+v} \right), \quad (4.12)$$

where β and γ denotes the discount factor and the habit formation parameter, which are restricted as $0 < \beta < 1$, $0 \leq \gamma < 1$ and v is the inverse Frisch elasticity of labor supply.

The households budget constraint is given by

$$P_t C_t + \frac{B_t}{R_t} = B_{t-1} - P_t T_t + W_t H_t + D_t. \quad (4.13)$$

At the beginning of each period, the households provide $h_t(i)$ units of labor to each intermediate-goods producing firm $i \in [0, 1]$ to earn $W_t h_t(i)$. They also receive a nominal profit (D_t) due to owning the intermediate goods firms. They also save by purchasing the one-period bond, B_t , from the intermediate goods producers at the price $1/R_t$. A lump-sum tax (T_t) is imposed to finance government spending

(G_t). During each period t , households purchase consumption goods, (C_t), from the final-goods producing firms at the nominal price, P_t . Therefore, the households choose labor supply, h_t , bond holding, B_t , and consumption, C_t to maximize the lifetime utility subject to the budget constraint.

The first order conditions for the households utility maximization problem can be represented

$$\lambda_t P_t = \frac{1}{C_t - \gamma C_{t-1}} - \beta \gamma E_t \left(\frac{1}{C_{t+1} - \gamma C_t} \right), \quad (4.14)$$

$$\lambda_t = \beta R_t E_t(\lambda_{t+1}), \quad (4.15)$$

where λ_t is a non-negative Lagrange multiplier on the budget constrain.

In term of wage setting, we follow Erceg et al. (2000) to assume that there is a fraction, η_w , of firms who cannot freely set their wage but still can update their wage as follows

$$W_t(j) = (\pi_{t-1}^{\mu_w} \bar{\pi}_t^{1-\mu_w})^{\chi_w} W_{t-1}(j). \quad (4.16)$$

The remaining fraction of firms can choose an optimal wage by maximizing

$$E_t \sum_{s=0}^{\infty} \eta_w^s \beta^s \left\{ -\omega \frac{H_{t+s}(j)^{1+v}}{1+v} \right\}, \quad (4.17)$$

subject to the labor demand function

$$H_t(j) = \left[\frac{W_t(j)}{W_t} \right]^{-\theta_w} H_t. \quad (4.18)$$

4.2.5 Authority's policy

Monetary Policy

The authority sets the short-term nominal interest rates following a Taylor rule. In particular, the rule allows for interest rate smoothing and interest rate

responses to deviations of inflation from the central bank's inflation target and deviations of output from the steady state

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\bar{\pi}_t}\right)^{\phi_\pi} \left(\frac{y_t}{\bar{y}}\right)^{\phi_y} \right]^{1-\rho_R} \exp(\epsilon_{R_t}), \quad (4.19)$$

where \bar{R}, \bar{y} are the steady state of R_t and Y_t , respectively. The parameter ρ_R illustrates the degree of interest rate smoothing. ϵ_{R_t} is an i.i.d monetary policy shock.

The evolution of trend inflation is described as a persistent AR(1) process as

$$\ln(\bar{\pi}_t) = (1 - \rho_\pi) \ln(\bar{\pi}^*) + \rho_\pi \ln(\bar{\pi}_{t-1}) + \epsilon_{\bar{\pi}_t}, \quad (4.20)$$

where ρ_π denotes the degree of shock persistence and $\epsilon_{\bar{\pi}_t}$ is a standard normally distributed shock which is independent of time.

Fiscal Policy

The public spending is given by

$$G_t = \left(1 - \frac{1}{g_t}\right) Y_t, \quad (4.21)$$

where g_t is an exogenous disturbance following the stochastic process

$$\ln(g_{t+1}) = (1 - \rho_g) \ln(\bar{g}) + \rho_g \ln(g_t) + \epsilon_{g_t}, \quad (4.22)$$

where $(1 - \frac{1}{g})$ represents the steady-state value of government spending relative to output.

4.2.6 Market Clearing Condition

The market clearing condition in the labor market, the goods market and the bond can be expressed in turn as

$$H_t = \int H_t(i) di, \quad (4.23)$$

$$Y_t = C_t + G_t, \quad (4.24)$$

$$B_t = 0. \quad (4.25)$$

4.3 Welfare and Welfare Cost Computation

Following Chapter 3, we also use the perturbation method to compute the approximation to the policy functions around the deterministic steady-state, and use these to compute the welfare. We decompose the welfare into the three different components as given

$$\begin{aligned} E\left[\sum_{t=0}^{\infty} \beta^t u(x_t)\right] &\approx \sum_{t=0}^{\infty} \beta^t u(\bar{x}) + \sum_{t=0}^{\infty} \beta^t Mu(\bar{x})E[x_t - \bar{x}] + \sum_{t=0}^{\infty} \beta^t Nu(\bar{x})E[(x_t - \bar{x}) \otimes (x_t - \bar{x})] \\ &= U_d + U_l + U_v, \end{aligned}$$

where $x_t = [C_t, C_{t-1}, H_t]$; and $Mu(\bar{x})$ and $Nu(\bar{x})$ are vector which contain the first and second derivative of $u(\cdot)$ evaluated at \bar{x} which are the deterministic steady state of x_t . Three components consist of: the deterministic component, $U_d = \sum_{t=0}^{\infty} \beta^t u(\bar{x})$, the level component, $U_l = \sum_{t=0}^{\infty} \beta^t Mu(\bar{x})E[x_t - \bar{x}]$, and the volatility component, $U_v = \sum_{t=0}^{\infty} \beta^t Nu(\bar{x})E[(x_t - \bar{x}) \otimes (x_t - \bar{x})]$.

Then the welfare cost can be computed as

$$E\left[\sum_{t=0}^{\infty} \beta^t u\left(\left(1 + \frac{wc}{100}\right)C_{A,t}, \left(1 + \frac{wc}{100}\right)C_{A,t-1}, H_{A,t}\right)\right] = E\left[\sum_{t=0}^{\infty} \beta^t u\left(C_{B,t}, C_{B,t-1}, H_{B,t}\right)\right], \quad (4.26)$$

where $C_{A,t}, H_{A,t}$ are consumption and labor supply in the economy with $\sigma_{\bar{\pi}} > 0$ and $C_{B,t}, H_{B,t}$, are in economy with $\sigma_{\bar{\pi}} = 0$.

4.4 Calibration

Table B1.2 lists the baseline parameters, which are calibrated for the U.S. data during the 1954Q3-2015Q1 period. There are two subsets of parameters. The first subset consists of parameter values that we can compute by using the data or parameter values are standard in the literature. In particular, the standard calibration bases on the steady-state target, such as the steady-state inflation ($\bar{\pi}^*$), the steady-state share of government expenditure ($1 - \frac{1}{g}$). Some parameters are taken from the literature. For example, the discount factor, β , and the inverse Frisch elasticity, ν , are set to 0.9974 and 1.00, respectively. The parameters θ_p and θ_w are the elasticities for goods and labors which are both set at 10, common values in these models.

Moreover, we follow Justiniano and Primiceri (2008) to set the values for parameters related to persistence level and standard deviation of structural shocks. Accordingly, the autoregressive parameters of productivity shock and the government expenditure shock are set to 0.4 and 0.98, respectively. We also base on Justiniano and Primiceri (2008) to select parameter values for Taylor rule, including the smoothing parameter, ρ_R , the coefficient on inflation, ϕ_π , and the coefficient on output growth, ϕ_y . They are in turn 0.81, 1.91 and 0.08, which are also common in the literature. Regarding the shock to trend inflation process, we set its persistence level, $\rho_{\bar{\pi}}$, and standard deviation, $\sigma_{\bar{\pi}}$, to 0.995 and 0.0008 as in Cogley et al. (2009).

The second subset includes parameters that are calibrated jointly to match selected moments in the 1954Q3-2015Q1 U.S. data. These parameters are the habit formation, γ , the probability of non-optimization for prices, η_p , the degree of price indexation, χ_p , the probability of non-optimization for wages, η_w , and the degree of wage indexation, χ_w . The five selected moments include the consumption volatility (σ_C), the volatility of consumption relative to output (σ_C/σ_Y), the volatility of

labor relative to output (σ_N/σ_Y), a correlation between output and consumption ($\rho(Y, C)$), and a correlation between output and labor ($\rho(Y, N)$). These moments are important for the subsequent welfare analysis because they closely reflect the dynamic behavior of consumption and labor supply. Based on the moment matching approach, the habit formation, γ , is set to 0.91. Both the probability of non-optimization for prices and wages, η_p and η_w , are calibrated at 0.69. That is also similar to Bils and Klenow (2004) who find that the median duration of U.S. prices ranges between 4.3 and 5.5 months with a dataset covering the frequency of price changes for 350 categories of consumer goods and services. It is worth noting that the literature has offered different values of these parameters. In particular, these values are broadly consistent with the macro estimate of Christiano, Eichenbaum, and Evans (2005), while Justiniano, Primiceri, and Tambalotti (2010, 2011) and Barattieri, Basu, and Gottschalk (2014) show higher micro estimated values. In the exercise of quantifying welfare costs of trend inflation, Ascari et al. (2016) also set the similar values to these parameters. There is also a fair degree of price and wage indexation (0.5), which is the same but smaller than Fernandez-Villaverde, Guerron-Quintana, and Rubio-Ramirez (2010) (between 0.62-0.63).

Table 4.1: Moments

| | σ_C | σ_C/σ_Y | σ_N/σ_Y | $\rho(Y, C)$ | $\rho(Y, N)$ |
|-------------|------------|---------------------|---------------------|--------------|--------------|
| Data | 0.008 | 0.56 | 1.42 | 0.79 | 0.87 |
| Calibration | 0.007 | 0.62 | 1.42 | 0.77 | 0.75 |

Note: Moments in the second row are obtained from HP-filtered U.S. data (1954Q3-2015Q1). The last row is the moments from simulations for the calibrated model.

Table 4.1 compares the moments generated by the parameterized model with moments computed by the data. The reported volatility and correlation statistics are for the HP-filtered U.S. data during 1954Q3-2015Q1 period. Table 4.1 shows that the model does a good job for matching the volatility of consumption, the variance

of consumption, labor to output, and the correlation between consumption, labor and output. In sum, the key features of the data are captured reasonably well by the calibrated model. Therefore, the calibrated model can provide an appropriate laboratory for the subsequent welfare analysis.

Measuring welfare costs of constant and shifting trend inflation as well as sensitivity analysis exercises are conducted in the following step. Regarding the sensitivity analysis, we base on the discussion of Fernandez-Villaverde et al. (2010) and Guerron-Quintana (2011) to consider a range of various values for η_p , χ_p , η_w and χ_w . In particular, the range $\eta_p = [0.6, 0.65, 0.7, 0.75]$ and $\chi_p = [0.00, 0.33, 0.67, 1.00]$ are respectively considered to compare with the baseline model. Similarly, we let Calvo wage parameters change. For comparison purpose, we consider a range of values: $\eta_w = [0.6, 0.65, 0.7, 0.75]$ and $\chi_w = [0.00, 0.33, 0.67, 1.00]$. Additionally, we also set the diverse values for the steady state level of trend inflation, persistence and standard deviation of trend inflation process to investigate the roles of this shock on the economy. Specifically, $\bar{\pi}^*$, $\rho_{\bar{\pi}}$ and $\sigma_{\bar{\pi}}$ are set to $[1.02^{0.25} \dots 1.06^{0.25}]$; $[0.9900 \dots 0.9999]$, $[0.100; 0.075; 0.05; 0.025; 0]$ ³², respectively.

4.5 Results

The present study focuses on quantifying the welfare cost of shifting trend inflation by developing the model with Calvo staggered price and wage contracts. We firstly present the results of welfare costs of the constant positive trend inflation before discussing the vital results on the welfare cost of shifting trend inflation³³. Theoretically, the shifting trend inflation distorts allocations of output due to stag-

³²These parameters are taken from the literature. Regarding the standard of shocks to trend inflation, the first one comes from Ireland (2007), while the others are taken from Cogley et al. (2009) for 1960-1979 subsample, 1982-2007 subsample, respectively.

³³We use Matlab code with the toolbox originally written by Schmitt-Grohe and Uribe for a second approximation to the policy rules. The codes performing these exercises are available from the author upon request.

gered price contract, and the relative allocation of labor across households due to staggered wage contract. In particular, the Calvo mechanism creates price and wage dispersion terms, which are sensitive to trend inflation. The shifting trend inflation drives the price and wage dispersion up by inducing a gap between the average price level and the price set by re-setting firms, and a gap between the average wage level and the wage set by re-setting households. As a result, these gaps generate efficiency loss in aggregate production and labor allocation. Clearly, shifting trend inflation distorts the economy through these two channels: staggered prices and staggered wages.

4.5.1 Welfare Costs of Constant Positive Trend Inflation

The Model with Staggered Prices and Wages

Table 4.2 compares the welfare of an economy in which central banks set constant trend inflation at 0 annualized percent and an economy in which central banks set constant trend inflation at 4 annualized percent. We firstly discuss the model featuring both staggered price and staggered wage contracts. Table 4.2 shows that the greater trend inflation results in large welfare costs (nearly 4.5%). This result is aligned line with Ascari et al. (2016) who also discuss the costs of trend inflation in the model with both staggered price and wage contracts. These welfare costs come mainly from reduction of the mean and volatility component, especially the volatility component. Conversely, the changes in the deterministic component play a small role in explaining the welfare difference. When central banks raise their inflation targets from 0 percent to 4 percent, there are changes in the mean as well as the variance of consumption and working hours. It is worth noticing about a significant increase in variance of consumption and working hours.

Table 4.2: Welfare Costs of Constant Trend Inflation: Model with Staggered Price and Wage Contracts

| | $\bar{\pi}^* = 1.00^{0.25}$ | $\bar{\pi}^* = 1.04^{0.25}$ |
|---------------|-----------------------------|-----------------------------|
| Welfare Cost | | 4.46% |
| Welfare | -1242.2 | -1281.4 |
| U_d | -1219.5 | -1219.6 |
| U_l | -0.05 | -0.24 |
| U_v | -22.6 | -61.5 |
| E(C)(*) | -0.015 | -0.041 |
| E(H)(*) | -0.018 | -0.050 |
| $100\sigma_C$ | 1.33 | 2.17 |
| $100\sigma_H$ | 2.50 | 3.22 |

Note: (*) expressed as percentage deviation from the deterministic steady-state. U_d , U_l and U_v are the deterministic steady-state, level and volatility component, respectively.

Two special cases: Staggered Prices and Staggered Wages

In the next exercise, we compare welfare and welfare costs of constant trend inflation in two special cases: the model with staggered price contract and completely flexible wage ($\eta_p > 0, \eta_w = 0$) and the model with completely flexible price and staggered wage ($\eta_p = 0, \eta_w > 0$). We call them the staggered price model and the staggered wage model, respectively. Table 4.3 reports results of these two special cases. Some main findings could be listed here. First, welfare costs due to a constant and positive trend inflation level in the staggered price model is trivial (0.22%) as compared to the staggered wage model (3.62%). The modest cost of constant trend inflation in the first special case is consistent to those of Ascari (2004), Amano et al. (2009), and Nakata (2014) that discuss trend inflation distorting the economy solely by the staggered price contracts. The higher costs in the staggered wage model when trend inflation increases from 0 percent to 4 percent suggest that a staggered wage mechanism plays a vital role in transmitting adverse impacts of varying trend inflation levels into the economy. This conclusion is similar to those discussed by

Ascari et al. (2016). The changes in the property of economy due to constant trend inflation are also reported in Table 4.3. In the staggered price model, changes in the mean and variance of consumption and working hours are very small, whereas there are significant movements in the staggered wage model. The higher dynamics of economy due to the staggered wage contracts in part explain high welfare costs due to a constant positive trend inflation through this channel.

Table 4.3: Welfare Costs of Constant Trend Inflation: Two Special Cases

| | Staggered Prices | | Staggered Wages | |
|---------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| | $\bar{\pi}^* = 1.00^{0.25}$ | $\bar{\pi}^* = 1.04^{0.25}$ | $\bar{\pi}^* = 1.00^{0.25}$ | $\bar{\pi}^* = 1.04^{0.25}$ |
| Welfare Cost | | 0.22% | | 3.62% |
| Welfare | -1232.6 | -1234.3 | -1240.2 | -1271.7 |
| E(C)(*) | -0.0046 | -0.0069 | -0.014 | -0.044 |
| E(H)(*) | -0.0001 | -0.0001 | -0.018 | -0.057 |
| $100\sigma_C$ | 0.92 | 0.93 | 1.28 | 2.02 |
| $100\sigma_H$ | 2.14 | 2.23 | 2.44 | 3.07 |

Note: (*) expressed as percentage deviation from the deterministic steady-state. Staggered prices and wages correspond to $\eta_p > 0, \eta_w = 0$ and $\eta_p = 0, \eta_w > 0$, respectively.

In short, welfare consequences of constant positive trend inflation are severe in the model with staggered price and wage contracts. The staggered wage channel plays a vital role in transmitting adverse impacts of constant trend inflation into the economy.

4.5.2 Welfare Costs of Shifting Trend Inflation

The Model with Staggered Price and Wage Contracts

Table 4.4 reports welfare costs of shifting trend inflation when central banks set the inflation target level to 2 annualized percent and 4 annualized percent, respectively. Trend inflation participates in the model as a shock and the presence of this shock creates welfare costs. This part discusses how this shock affects the economy in terms of welfare costs. First, we focus on the 2-percent-trend-inflation

economy. The persistent trend inflation shock creates welfare costs, which is 0.6%. A reduction in welfare mainly comes from a decline of volatility component, while the deterministic steady-state component remains the same. A positive shock to trend inflation, therefore, drives the average levels of consumption and working hours down, while pushing their volatility up. In our exercise, the volatility component accounts for the largest proportion of welfare reduction, which shapes business cycle dynamics. Specifically, the standard deviation of working hours increases from 1.33 to 1.47, while an increase in those of consumption is from 2.52 to 2.62.

Table 4.4: Welfare Costs of Shifting Trend Inflation:
Model with Staggered Price and Wage
Contracts

| | $\bar{\pi}^* = 1.02^{0.25}$ | | $\bar{\pi}^* = 1.04^{0.25}$ | |
|---------------|-----------------------------|--------------------------|-----------------------------|--------------------------|
| | $\sigma_{\bar{\pi}} = 0$ | $\sigma_{\bar{\pi}} > 0$ | $\sigma_{\bar{\pi}} = 0$ | $\sigma_{\bar{\pi}} > 0$ |
| Welfare Cost | | 0.60% | | 4.27% |
| Welfare | -1242.4 | -1246.7 | -1243.9 | -1281.4 |
| U_d | -1219.6 | -1219.6 | -1219.6 | -1219.6 |
| U_l | -0.01 | -0.03 | -0.21 | -0.23 |
| U_v | -22.8 | -27.8 | -24.0 | -61.5 |
| E(C)(*) | -0.016 | -0.023 | -0.031 | -0.041 |
| E(H)(*) | -0.019 | -0.028 | -0.037 | -0.050 |
| $100\sigma_C$ | 1.33 | 1.47 | 1.35 | 2.17 |
| $100\sigma_H$ | 2.52 | 2.62 | 2.51 | 3.22 |

Note: (*) expressed as percentage deviation from the deterministic steady-state. U_d , U_l and U_v are the deterministic steady-state, level and volatility component, respectively.

When trend inflation is 4 annualized percent, the costs of shock to trend inflation become more severe. This shock generates larger welfare costs (4.27%). The welfare differences still result from reduction of volatility component. However, a fall in volatility component is more considerable when trend inflation is higher. The economy also becomes more volatile, which is reflected by a substantial rise in the standard deviation of consumption (1.35 to 2.17) and working hours (2.51 to

3.22). The changes in properties of economy suggest that the volatility component plays an increasingly important role in explaining a fall in welfare due to exogenous variations in trend inflation when trend inflation is high.

Two special cases: Staggered Prices and Staggered Wages

Subsequently, we discuss welfare costs of shifting trend inflation in two special cases: completely flexible price and staggered wage contract ($\eta_p = 0$ and $\eta_w > 0$), and staggered price contract and completely flexible wage ($\eta_p > 0$ and $\eta_w = 0$). Table 4.5 reports results of these two special cases. The most striking feature is that welfare costs of shifting trend inflation in the model with staggered wage contract (0.48%) are significantly greater than one with staggered price contract (0.04%). The modest costs of exogenous variation in trend inflation in the sticky price model is consistent with the results of Nakata (2014). Welfare differences in these two cases can be explained by changes in mean and variance of consumption and working hours, but through the staggered wage channel, any change is greater. The significant costs of shifting trend inflation in the model with sticky wage suggest that an existence of staggered wage contract is a more vital channel in transmitting adverse impacts of shifting trend inflation into the economy as compared to the staggered price.

Table 4.5: Welfare Costs of Shifting Trend Inflation:
Two Special Cases

| | Staggered Prices | | Staggered Wages | |
|---------------|--------------------------|--------------------------|--------------------------|--------------------------|
| | $\sigma_{\bar{\pi}} = 0$ | $\sigma_{\bar{\pi}} > 0$ | $\sigma_{\bar{\pi}} = 0$ | $\sigma_{\bar{\pi}} > 0$ |
| Welfare Cost | | 0.04% | | 0.48% |
| Welfare | -1232.9 | -1233.2 | -1240.3 | -1244.3 |
| E(C)(*) | -0.0051 | -0.0057 | -0.0159 | -0.0228 |
| E(H)(*) | -0.0001 | -0.0001 | -0.0205 | -0.0294 |
| $100\sigma_C$ | 0.92 | 0.92 | 1.29 | 1.41 |
| $100\sigma_H$ | 2.17 | 2.17 | 2.41 | 2.52 |

Note: (*) expressed as percentage deviation from the deterministic steady-state. Staggered prices and wages correspond to $\eta_p > 0, \eta_w = 0$ and $\eta_p = 0, \eta_w > 0$, respectively.

In short, a shock to trend inflation produces large welfare costs, which result from reductions in volatility component of welfare. When trend inflation is higher, the volatility component plays an increasingly vital role in explaining the fall in welfare. Comparing two special cases, the staggered wage contract is the crucial channel, through which exogenous variations in trend inflation create a high welfare cost.

4.5.3 Sensitivity Analysis of Welfare Cost

In the following task, we conduct some exercises to investigate how welfare costs vary corresponding to changes in values of relevant parameters: the level of trend inflation ($\bar{\pi}^*$), the property of shock to trend inflation ($\sigma_{\bar{\pi}}$ and $\rho_{\bar{\pi}}$), Calvo price parameters (a combination of χ_p and η_p) and Calvo wage parameters (a combination of χ_w and η_w). Firstly, Table 4.6 reports the sensitivity of welfare costs to distinct levels of trend inflation. Two messages can be emphasized here. First, a higher trend inflation produces a great welfare cost. Second, the relationship between the trend inflation and welfare costs are non-linear. In other words, a given percentage point of rise in the steady-state level of trend inflation produces a larger proportion increase in welfare costs when trend inflation reaches a higher point. For example, welfare costs increase by nearly 4 percent (from 0.60 percent to 4.27 percent) when trend inflation increases from 2 percent to 4 percent. This cost, however, augments by nearly 10 percent (from 4.27 percent to 14.37 percent) when trend inflation rises from 4 percent to 6 percent.

Table 4.6: Welfare Costs of Shifting Trend Inflation: Various Trend Inflation Levels

| | $\bar{\pi}^* = 1.02^{0.25}$ | $\bar{\pi}^* = 1.04^{0.25}$ | $\bar{\pi}^* = 1.06^{0.25}$ |
|--------------|-----------------------------|-----------------------------|-----------------------------|
| Welfare Cost | 0.60% | 4.27% | 14.37% |

Further, changes in welfare costs due to growing values of standard deviation

level and persistence level of shock to trend inflation are illustrated in Table 4.7. When either the shock to trend inflation becomes more volatile or more persistent, welfare costs increase. Additionally, if these changes happen when the shock to trend inflation is either highly volatile or highly persistent, welfare costs skyrocket by a higher proportion. Specifically, welfare costs only rise by 0.18% (from 0.06 to 0.24 percent) when the variance of shock increases from 0.025 to 0.050. However, if shock's variance increases from 0.075 to 0.100, welfare costs of shifting trend inflation rise to nearly 1%. The same evidence is observed for the shock persistence level. Welfare costs rise only by 0.18 percent when the shock persistence level changes from 0.9 to 0.99, whereas these costs are 4% when the shock persistence level rises to 0.999. The results suggest that parameters governing the persistence and volatility level of shock to trend inflation play a vital role in explaining the welfare cost differences.

Table 4.7: Welfare Costs of Shifting Trend Inflation: Shifting Trend Inflation Shock

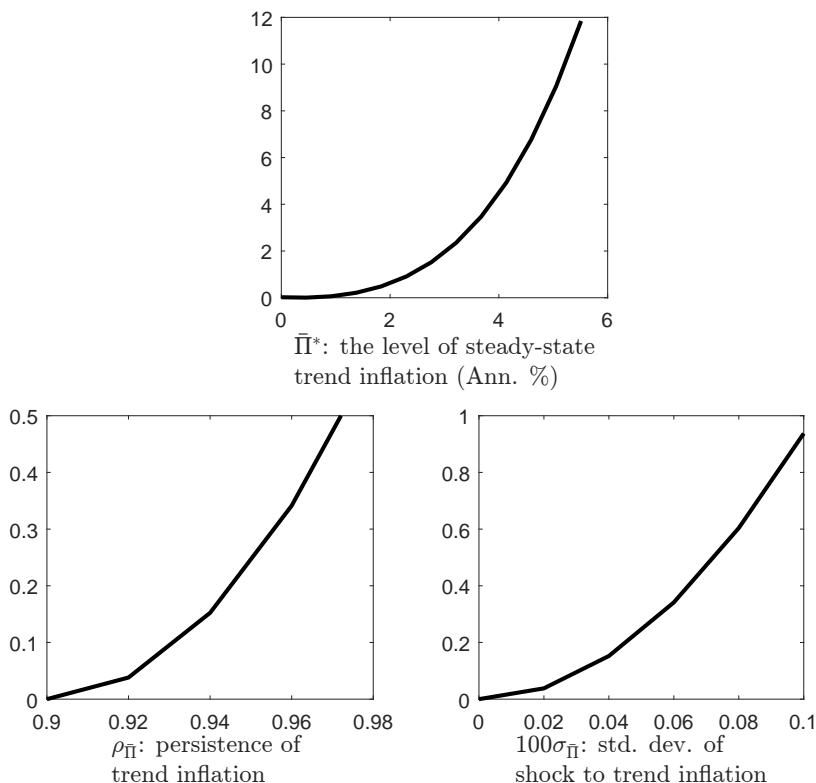
| | | Shock Volatility | | | | | Shock Persistence | | | | |
|----------------------|------|------------------|---------|-------|-------|--------------|-------------------|---------|---------|-------|-------|
| $100 * \sigma_{\pi}$ | wc | U | U_d | U_l | U_v | ρ_{π} | wc | U | U_d | U_l | U_v |
| 0.000 | | -1242.4 | -1219.6 | -0.01 | -22.8 | 0.900 | 0.01 | -1242.5 | -1219.6 | -0.01 | -22.9 |
| 0.025 | 0.06 | -1242.9 | -1219.6 | -0.01 | -23.3 | 0.990 | 0.19 | -1243.9 | -1219.6 | -0.02 | -24.3 |
| 0.050 | 0.24 | -1244.4 | -1219.6 | -0.02 | -24.8 | 0.995 | 0.60 | -1246.7 | -1219.6 | -0.03 | -27.8 |
| 0.075 | 0.53 | -1246.8 | -1219.6 | -0.03 | -27.2 | 0.999 | 4.07 | -1278.3 | -1219.6 | -0.10 | -58.6 |
| 0.100 | 0.94 | -1250.2 | -1219.6 | -0.05 | -30.6 | | | | | | |

Note: The welfare costs (wc) are measured in percent. U , U_d , U_l and U_v are the welfare, the deterministic steady-state, level and volatility component of welfare, respectively.

Figure 4.1 illustrates movements of welfare costs of shifting trend inflation with respect to changes in the trend inflation level, the persistence level and the standard deviation level of shock to trend inflation. It can be seen that either an increase in trend inflation (as in the top panel of Figure 4.1) or shock properties (as in the bottom panel of Figure 4.1) leads to an augment in welfare costs. The most striking feature is that all relationships are non-linear. It suggest that a given amount of rise in these parameters leads to a more sizeable rise in welfare costs when

these parameters are higher.

Figure 4.1: Welfare Costs of Shifting Trend Inflation

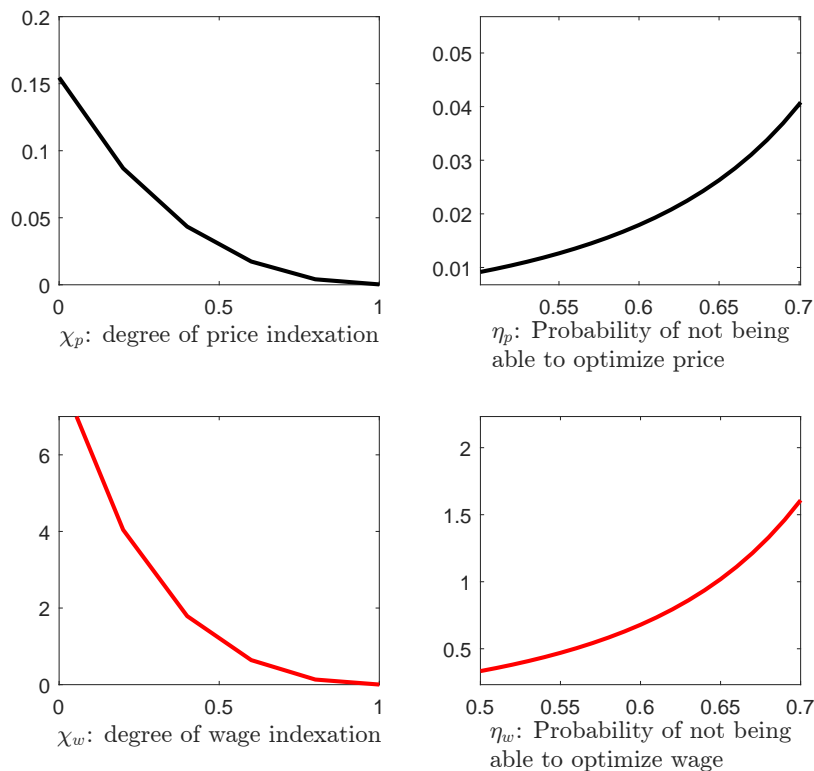


Calvo Price and Calvo Wage Setting Parameters

This section discusses how welfare costs of shifting trend inflation respond to changes in parameters governing the pricing environment and wage environment. The top panel of Figure 4.2 shows how welfare costs move with respect to an increase in the degree of price indexation and the frequency of price adjustment, while the bottom panel depicts responses of welfare costs corresponding to changes in the degree of wage indexation and the frequency of wage adjustment. Regarding Calvo price parameters, we observe the opposite trends. In particular, an increase in η_p and a decline in χ_p causes welfare costs to increase. To explain why, remember that all firms would produce the same amount if there is no friction in price-adjustment. A fall in aggregate output results from a larger cross-sectional price dispersion implying a higher cross sectional output dispersion, then a higher inefficiency production

allocation. When η_p increases, firms get an opportunity to adjust their price less frequently, thus cross sectional price dispersion expands. This leads to a lower output and welfare. A rise in χ_p , on the other hand, leads to a fall in welfare costs since a high χ_p implies that firms still update their prices to keep up with inflation despite of no opportunity to optimize the prices. Prices, hence, are less dispersed at any level of trend inflation rate when the price indexation is greater. Overall, the greater η_p creates the larger welfare costs, whereas the lower χ_p signifies welfare costs more considerably.

Figure 4.2: Welfare Costs: Calvo Price versus Calvo Wage Parameters



Similarly, the movements of welfare costs corresponding to variations of Calvo wage parameters are also consistent with the theory. Welfare costs increase with respect to an increase in η_w and a decrease in χ_w . Households adjust their wage less frequently due to a rise in η_w . As a result, cross sectional wage dispersion grows, and then lead to a higher welfare cost. By contrast, χ_w drives welfare cost down

because a high χ_w implies that household still have an opportunity to update their wage to keep up with optimizing wage of those who have wage bargaining power.

In summary, there are three main points worth emphasizing. First, the parameters governing the pricing and wage environment importantly determine welfare costs. Second, Figure 4.2 exhibits that the relationships between welfare costs of shifting trend inflation and Calvo price and wage parameters are non-linear. In other words, a given change in these parameters leads to a greater change in welfare costs when these parameters are high. Third, Calvo wage parameters play a vital role since their changes bring about significant changes in welfare costs. It provides more evidences on the discussion that the staggered wage contract is a crucial channel to transmit adverse impacts of shifting trend inflation on the economy.

In brief, shifting trend inflation produces large welfare costs especially in the high-trend-inflation economy. The welfare differences caused by shifting trend inflation can be mainly accounted by reduction in the volatility components. As trend inflation increases, the role of volatility component becomes increasingly important. Comparing two special cases, welfare costs created by shifting trend inflation through the staggered wage channel are more significant than those generated through the staggered price channel. The results suggest that the staggered wage is an important factor determining the welfare costs of shifting trend inflation³⁴.

4.6 Conclusions

The literature on shifting trend inflation has so far only considered the price rigidities. The present study, therefore, developed the model featuring a staggered price and wage contracts to fill the existing gap. Further, we assumed implicit inflation target to be positive and time-varying due to a lack of commitment in

³⁴To support these two conclusions in this chapter, we conduct additional analyses that use the asymmetric parameter values. The results of these exercises are reported in Appendix 3.

the policy implementations of central banks. Thus, trend inflation was modelled as a highly persistent AR(1) process. We then compared welfare of an economy with zero and the other with positive variance of innovation to the trend inflation process. To analyse the role of each channel importantly determining welfare costs of shifting trend inflation, we discussed two special cases: the model with staggered price contract and completely flexible wage, and the model with completely flexible price and staggered wage contract.

With these extensions, we showed that welfare consequences of constant positive trend inflation and shifting trend inflation are severe, especially when trend inflation is high. Furthermore, among the two channels, staggered wage contracts play a vital role in transmitting adverse impacts of constant and shifting trend inflation into economy. We also conducted exercises to examine the sensitivity of welfare costs to a wide range of plausible parameters. The sensitivity analysis results show that if the price and wage friction are sufficiently large, the price and wage indexation level are sufficiently small, or there is upward biased trend inflation process, welfare costs become more sizeable. Moreover, changes in parameters governing the wage environment cause more substantial movements of welfare costs as opposed to parameters controlling the pricing environment. The result provides compelling evidence that the staggered wage contracts are an important factor determining welfare costs of shifting trend inflation.

APPENDIX

Appendix 1 Variables and Parameters

Table B1.1: Description of Variables

| Variables | | Note |
|--------------------------|--|--------------------------------|
| $h_t(i)$ | Units of labor provided to an intermediate-goods firm i | |
| H_t | Labor supply | |
| $H_t(j)$ | Labor supply of household j | |
| $W_t(j)$ | Wage received from employment agencies by the supplier of type j | |
| W_t | Nominal wage rate | |
| D_t | Nominal dividend | |
| B_t | Units of bonds | |
| T_t | Lump-sum tax | |
| G_t | Government spending | $G_t = (1 - \frac{1}{g_t})Y_t$ |
| $Y_t(i)$ | Units of the intermediate goods i | |
| Y_t | Units of the final goods | |
| C_t | Units of the consumption goods | |
| λ_t | Non-negative Lagrange multiplier on the budget constrain | |
| π_t | Gross inflation rate | $\pi_t = P_t/P_{t-1}$ |
| $\bar{\pi}_t$ | Trend inflation | |
| P_t | Nominal price of final goods | |
| $P_t(i)$ | Nominal price of intermediate goods i | |
| Z_t | Aggregate technology | |
| R_t | Gross interest rate | |
| Calvo Price | | |
| P_t^* | An optimizing price of intermediate-goods producing firm | |
| $f_{1,t}$ | A numerator of equation of the optimizing price | |
| $f_{2,t}$ | A denominator of equation of the optimizing price | |
| ϕ_t^* | A relative optimal price | |
| $s_{p,t}$ | A price dispersion | |
| Calvo Wage | | |
| W_t^* | An optimizing wage of household | |
| $f_{3,t}$ | A numerator of equation of the optimizing wage | |
| $f_{4,t}$ | A denominator of equation of the optimizing wage | |
| $s_{w,t}$ | A wage dispersion | |
| Exogenous Shocks | | |
| ϵ_{R_t} | Monetary shocks | |
| ϵ_{g_t} | Government spending shocks | |
| ϵ_{z_t} | Technology shocks | |
| $\epsilon_{\bar{\pi}_t}$ | Trend inflation shocks | |

Table B1.2: Calibration

| Parameter | Description | Calibrated Value |
|---------------------------------|---|--|
| β | Discount factor | 0.9974 |
| γ | Consumption habit | 0.91 |
| ω | Labor supply disutility | 1.00 |
| ν | Inverse Frisch elasticity of labor supply | 1.00 |
| $1 - \bar{g}^{-1}$ | Steady state share of Government expenditure | 0.26 |
| ρ_Z | AR(1) coefficient for technology shock | 0.40 |
| ρ_g | AR(1) coefficient for government spending shock | 0.98 |
| $100\sigma_Z$ | Standard deviation of technology shock | 1.10 |
| $100\sigma_g$ | Standard deviation of government spending shock | 0.55 |
| Monetary Policy | | |
| ϕ_π | Taylor coefficient on the inflation gap | 1.92 |
| ϕ_y | Taylor coefficient on the output gap | 0.08 |
| ρ_R | AR(1) coefficient for monetary shock | 0.81 |
| $100\sigma_R$ | Standard deviation of monetary shock | 0.25 |
| Calvo Price Setting | | |
| θ_p | Price elasticity | 10.0 |
| η_p | Probability of not being able to optimize | [0.6,0.65,0.7,0.75] |
| χ_p | Degree of price indexation | [0.0,0.33,0.67,1.0] |
| μ_p | Weight on lagged inflation | 1.00 |
| Calvo Wage Setting | | |
| θ_w | Wage elasticity | 10.0 |
| η_w | Probability of not being able to optimize | [0.6,0.65,0.7,0.75] |
| χ_w | Degree of wage indexation | [0.0,0.33,0.67,1.0] |
| μ_w | Weight on lagged inflation | 1.00 |
| Shifting Trend Inflation | | |
| $\bar{\pi}^*$ | Steady-state level of trend inflation | [1.00 ^{0.25} ... 1.06 ^{0.25}] |
| $\rho_{\bar{\pi}}$ | Persistence level of shocks to trend inflation | [0.99 ... 0.995 ... 0.9999] |
| $100\sigma_{\bar{\pi}}$ | Standard deviation of shocks to trend inflation | [0.1,0.075,0.05,0.025,0] |

Appendix 2 The Model

Appendix 2.1 A List of Nonlinear Equilibrium Conditions in Stationary Variables

$$\varrho_t^* = \frac{\theta_p}{\theta_p - 1} \frac{f_{1,t}}{f_{2,t}} \quad (\text{NL1})$$

$$f_{1,t} = \tilde{w}_t + \beta \eta_p (\bar{\pi}_t^{-\chi_p \theta_p})^{1-\mu_p} (\pi_t^{-\chi_p \theta_p})^{\mu_p} E_t[\pi_{t+1}^{\theta_p} f_{1,t+1}] \quad (\text{NL2})$$

$$f_{2,t} = 1 + \beta \eta_p (\bar{\pi}_t^{-\chi_p (1-\theta_p)})^{1-\mu_p} (\pi_t^{\chi_p (1-\theta_p)})^{\mu_p} E_t[\pi_{t+1}^{\theta_p-1} f_{2,t+1}] \quad (\text{NL3})$$

$$1 = \left[(1 - \eta_p) \varrho_{i,t}^*{}^{1-\theta_p} + \eta_p \pi_t^{\theta_p-1} \left(\frac{\bar{\pi}_t^{\chi_p (1-\mu_p)} \pi_{t-1}^{\chi_p \mu_p}}{\pi_t^{\chi_p (1-\mu_p)} \pi_{t-1}^{\chi_p \mu_p}} \right)^{1-\theta_p} \right]^{\frac{1}{1-\theta_p}} \quad (\text{NL4})$$

$$s_{p,t} = (1 - \eta_p) (\varrho_{i,t}^*)^{-\theta_p} + \eta_p (\bar{\pi}_t^{-\chi_p \theta_p})^{1-\mu_p} (\pi_{t-1}^{-\chi_p \theta_p})^{\mu_p} \pi_t^{\theta_p} s_{p,t-1} \quad (\text{NL5})$$

$$\tilde{\lambda}_t = \frac{1}{C_t - \gamma C_{t-1}} - \beta \gamma E_t \left(\frac{1}{C_{t+1} - \gamma C_t} \right) \quad (\text{NL6})$$

$$\tilde{\lambda}_t = \beta R_t E_t \left(\tilde{\lambda}_{t+1} \frac{1}{\pi_{t+1}} \right) \quad (\text{NL7})$$

$$(\tilde{w}_t^*)^{1+\theta_w v} = \frac{\theta_w}{\theta_w - 1} \frac{f_{3,t}}{f_{4,t}} \quad (\text{NL8})$$

$$f_{3,t} = \omega \left(\tilde{w}_t \right)^{\theta_w (1+v)} H_t^{1+v} + \beta \eta_w [\bar{\pi}_t^{-\chi_w \theta_w (1-\mu_w)(1+v)}] [\pi_t^{-\chi_w \theta_w \mu_w (1+v)}] [\pi_{t+1}^{\theta_w (1+v)}] f_{3,t+1} \quad (\text{NL9})$$

$$f_{4,t} = \lambda_t \left(\tilde{w}_t \right)^{\theta_w} H_t + \beta \eta_w [\bar{\pi}_t^{\chi_w (1-\theta_w)(1-\mu_w)}] [\pi_t^{\chi_w (1-\theta_w) \mu_w}] [\pi_{t+1}^{\theta_w-1}] f_{4,t+1} \quad (\text{NL10})$$

$$w_t = \left\{ (1 - \eta_w) (\tilde{w}_t^*)^{1-\theta_w} + \eta_w \left(\frac{\tilde{w}_{t-1}}{\pi_t} \right)^{1-\theta_w} [\bar{\pi}_{t-1}^{\chi_w (1-\mu_w)} \pi_{t-1}^{\chi_w \mu_w}]^{(1-\theta_w)} \right\}^{\frac{1}{1-\theta_w}} \quad (\text{NL11})$$

$$s_{w,t} = (1-\eta_w) \left(\frac{w_t^*}{w_t}\right)^{-\theta_w(1+v)} + \eta_w \left(\frac{w_{t-1}}{w_t}\right)^{-\theta_w(1+v)} (\bar{\pi}_t^{-\chi_w \theta_w(1+v)})^{1-\mu_w} (\pi_{t-1}^{-\chi_w \theta_w(1+v)})^{\mu_w} \pi_t^{\theta_w(1+v)} s_{w,t-1} \quad (\text{NL12})$$

$$H_t Z_t = Y_t s_{p,t} \quad (\text{NL13})$$

$$C_t = \frac{1}{g_t} Y_t \quad (\text{NL14})$$

$$\ln(Z_t) = \rho_Z \ln(Z_{t-1}) + \epsilon_{Z_t} \quad (\text{NL15})$$

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\bar{\pi}_t}\right)^{\phi_\pi} \left(\frac{y_t}{\bar{Y}}\right)^{\phi_y} \right]^{1-\rho_R} e^{\epsilon_{R_t}} \quad (\text{NL16})$$

$$\ln \bar{\pi}_t = (1 - \rho_\pi) \ln \bar{\pi}^* + \rho_\pi \ln \bar{\pi}_{t-1} + \epsilon_{\bar{\pi}_t} \quad (\text{NL17})$$

$$\ln(g_{t+1}) = (1 - \rho_g) \ln(\bar{g}) + \rho_g \ln(g_t) + \epsilon_{g_t} \quad (\text{NL18})$$

Appendix 2.2 A List of Steady-State Variables

$$R(\bar{\pi}) = \frac{\bar{\pi}}{\beta} \quad (\text{ss1})$$

$$\wp_t^*(\bar{\pi}) = \left[\frac{1 - \eta_p \bar{\pi}^{(1-\chi_p)(\theta_p-1)}}{1 - \eta_p} \right]^{\frac{1}{1-\theta_p}} \quad (\text{ss2})$$

$$s_p(\bar{\pi}) = \frac{1 - \eta_p}{1 - \eta_p \bar{\pi}^{(1-\chi_p)\theta_p}} \wp_t^*(\bar{\pi})^{-\theta_p} \quad (\text{ss3})$$

$$w(\bar{\pi}) = \frac{\theta_p - 1}{\theta_p} \frac{1 - \eta_p \beta \bar{\pi}^{(1-\chi_p)(\theta_p)}}{1 - \eta_p \beta (\bar{\pi})^{(1-\chi_p)(\theta_p-1)}} \quad (\text{ss5})$$

$$f1(\bar{\pi}) = \frac{w(\bar{\pi})}{1 - \eta_p \beta (\bar{\pi})^{(1-\chi_p)(\theta_p)}} \quad (\text{ss6})$$

$$f2(\bar{\pi}) = \frac{1}{1 - \eta_p \beta (\bar{\pi})^{(1-\chi_p)(\theta_p-1)}} \quad (\text{ss7})$$

$$w^*(\bar{\pi}) = \left\{ \frac{1 - \eta_w \bar{\pi}^{\chi_w(1-\theta_w)}}{1 - \eta_w} \right\}^{\frac{1}{1-\theta_w}} * w(\bar{\pi}) \quad (\text{ss8})$$

$$c(\bar{\pi}) = \left\{ \left(\frac{\theta_w - 1}{\theta_w} \right) \left(\frac{1}{\omega} \right) \left(\frac{(w(\bar{\pi})^*)^{1+\theta_w v} w(\bar{\pi})^{-\theta_w v}}{(s(\bar{\pi})\bar{g})^v} \right) \left(\frac{1 - \beta\gamma}{1 - \gamma} \right) \left(\frac{1 - \beta\eta_w \bar{\pi}^{\theta_w(1+v)(1-\chi_w)}}{1 - \beta\eta_w \bar{\pi}^{(1-\chi_w)(\theta_w-1)}} \right) \right\}^{\frac{1}{1+v}} \quad (\text{ss9})$$

$$y(\bar{\pi}) = \bar{g}c(\bar{\pi}) \quad (\text{ss10})$$

$$H(\bar{\pi}) = s(\bar{\pi})y(\bar{\pi}) \quad (\text{ss11})$$

$$\lambda(\bar{\pi}) = \frac{z - \beta\gamma}{c(\bar{\pi})(1 - \gamma)} \quad (\text{ss12})$$

$$f3(\bar{\pi}) = \frac{\omega(w(\bar{\pi}))^{\theta_w(1+v)} H(\bar{\pi})^{1+v}}{1 - \beta\eta_w \bar{\pi}^{\theta_w(1+v)(1-\chi_w)}} \quad (\text{ss13})$$

$$f4(\bar{\pi}) = \frac{\lambda(\bar{\pi})(w(\bar{\pi}))^{\theta_w} H(\bar{\pi})}{1 - \beta \eta_w \bar{\pi}^{(1-\chi_w)(\theta_w-1)}} \quad (\text{ss14})$$

$$s_w(\bar{\pi}) = \frac{1 - \eta_w}{1 - \eta_w \bar{\pi}^{(1-\chi_w)\theta_w(1+v)}} \left(\frac{w^*(\bar{\pi})}{w(\bar{\pi})} \right)^{-\theta_w(1+v)} \quad (\text{ss15})$$

Appendix 3 The Additional Analyses

To conduct a robust check of two conclusions in Chapter 4, we consider another case that there are asymmetric parameter values for staggered prices and wages (the asymmetric case). While the main exercises of Chapter 4 use symmetric parameter values for staggered prices and wages that is similar to those of Fernandez-Villaverde et al. (2010) and Ascari et al. (2016), we employ the parameters estimated by Justiniano and Primiceri (2008) for the asymmetric case. We select Justiniano and Primiceri (2008) for a comparison purpose since our model and theirs share many similar features. Details about parameter values are reported in B1.3.

Table B1.3: Calibration: An Asymmetric Case

| Parameter | Description | Calibrated Value |
|---------------------------------|---|------------------|
| β | Discount factor | 0.9974 |
| γ | Consumption habit | 0.81 |
| ω | Labor supply disutility | 1.00 |
| v | Inverse Frisch elasticity of labor supply | 1.59 |
| $1 - \bar{g}^{-1}$ | Steady state share of Government expenditure | 0.26 |
| ρ_Z | AR(1) coefficient for technology shock | 0.28 |
| ρ_g | AR(1) coefficient for government spending shock | 0.98 |
| $100\sigma_Z$ | Standard deviation of technology shock | 1.10 |
| $100\sigma_g$ | Standard deviation of government spending shock | 0.55 |
| Monetary Policy | | |
| ϕ_π | Taylor coefficient on the inflation gap | 1.92 |
| ϕ_y | Taylor coefficient on the output gap | 0.1 |
| ρ_R | AR(1) coefficient for monetary shock | 0.81 |
| $100\sigma_R$ | Standard deviation of monetary shock | 0.25 |
| Calvo Price Setting | | |
| θ_p | Price elasticity | 10.0 |
| η_p | Probability of not being able to optimize | 0.9 |
| χ_p | Degree of price indexation | 0.84 |
| μ_p | Weight on lagged inflation | 1 |
| Calvo Wage Setting | | |
| θ_w | Wage elasticity | 10.0 |
| η_w | Probability of not being able to optimize | 0.61 |
| χ_w | Degree of wage indexation | 0.09 |
| μ_w | Weight on lagged inflation | 1 |
| Shifting Trend Inflation | | |
| $\bar{\pi}^*$ | Steady-state level of trend inflation | $1.02^{0.25}$ |
| $\rho_{\bar{\pi}}$ | Persistence of trend inflation | 0.995 |
| $100\sigma_{\bar{\pi}}$ | Standard deviation of shocks to trend inflation | 0.08 |

The results for welfare cost computations are presented in Table B1.4. The results confirm two conclusions in Chapter 4. First, the consequences of constant positive trend inflation and shifting trend inflation are severe. Second, staggered wages more importantly determine costs of both constant and shifting trend inflation.

Table B1.4: Welfare Cost Results: An Asymmetric Case

| Welfare Costs (%) | Constant Trend Inflation | Shifting Trend Inflation |
|--------------------------|--------------------------|--------------------------|
| General Case | 4.91 | 0.56 |
| Staggered Prices | 0.22 | 0.05 |
| Staggered Wages | 4.13 | 0.49 |

Chapter 5

Conclusions

The empirical evidence indicates that the sustained rise in inflation and the time-varying volatility are important features of the U.S. data. Although previous studies have exploited implications of these two factors for various aspects of macroeconomics, there exist three important gaps. First, the literature on policy uncertainty has so far mostly concentrated on aggregate effects of the short-run fluctuations of volatility on macroeconomy. Little is investigated on its welfare effects. Second, these two features, a sustained rise in inflation and time-varying volatility, have been examined largely in isolation thus far. Third, previous studies on shifting trend inflation have mostly focused on the sticky price model.

To in part fill these gaps, this dissertation analysed the relationship between shifting trend inflation, policy uncertainty and economic welfare. We proposed three research questions: (i) how large are costs of shifting trend inflation? (ii) what are the roles of time-varying volatility in affecting the welfare costs of shifting trend inflation? and (iii) Do staggered price or wage contracts more importantly determine the welfare costs of shifting trend inflation?

To seek answers for each question, a range of studies on shifting trend inflation and policy uncertainty were conducted. Specifically, we designed the DSGE model simultaneously incorporating time-varying stochastic volatility shocks and shocks to trend inflation in Chapter 3 to solve the first two questions. Regarding policy uncertainty, the focus lies on the monetary policy (Policy Rate Risk) that is the volatility shock arising in the monetary policy shock. The stochastic volatility shocks were

assumed to follow an AR(1) process as Shephard (2008) and Fernandez-Villaverde et al. (2011). The main goal of the present chapter is to model an economy that is characterized by two features: a sustained high inflation (as expressed by shifting trend inflation) and high volatility (by using the time-varying volatility shock).

The results of this chapter emphasized interactions between shifting trend inflation and policy uncertainty. On the one hand, an increase in the variance of shocks to trend inflation decreases welfare not only by decreasing average levels of consumption and leisure, but also by increasing their volatilities. If there exists uncertainty about monetary policies, these changes become more sizeable, thus a higher welfare consequence. On the other hand, welfare costs as well as adverse impacts of monetary policy uncertainty on the economy are more substantial when trend inflation is higher.

In Chapter 4, we solved the last question by developing a New Keynesian model featured with Calvo price setting and Calvo wage setting to study welfare consequences of exogenous variations in trend inflation. In particular, allocations of output across firms and allocations of labor across households are simultaneously distorted by shifting trend inflation. We showed that the consequences of constant positive trend inflation and shocks to trend inflation are severe, especially when trend inflation is high. Among two channels, staggered wage contracts play a vital role in transmitting adverse impacts of constant and shifting trend inflation into economy.

The results of the dissertation have crucial implications for both policy makers and economists. Based on the preceding analyses, raising inflation targets would seem to be a bad policy prescription. On the policy front, the significant welfare costs of constant positive and shifting trend inflation present a warning against policy proposals requiring central banks to raise their inflation targets. This policy implication is aligned with those of Ascari and Sbordone (2014) and Ascari et al. (2016). However, these results might conflict with the fact that central banks target

a positive level of inflation, around 2 percent per year. The discussion against targeting a zero or a low rate of inflation is the Zero Lower Bound (ZLB, henceforth) constraint. In particular, Summers (1991) discusses that with a very low average inflation rate, the central banks might encounter limitations of conducting an effective stabilization policy since this policy requires non-negative real interest rates and nominal rates. The need for a negative real rate of interest when there exists the zero lower bound would be accommodated by an inflation target ranged from 1 to 3 percent as argued by Summers (1991) and Fischer (1996). Krugman (1998) raises a similar point when he argues that an inflation target of 4 percent for several years in Japan is necessary to generate the negative real rates and curb deflation.

In recent years, economists have emphasized on the likelihood and consequences of hitting the ZLB, for example, Reifschneider and Williams (1998), Schmitt-Grohe and Uribe (2011), and Coibion et al. (2012). They argue that raising the inflation targets could reduce the costs related to the ZLB as well as the probability of hitting the ZLB. Moreover, the literature has so far advocated a higher inflation target, which is realized by a temporary increase in expected inflation instead of a permanent increase in the long-run inflation objective. To generate the expectation of higher inflation, the key is the commitment to a history-dependent policy as argued by Eggertsson and Woodford (2003), Coibion et al. (2012), and Ascari and Sbordone (2014). The commitment to the future policy can decrease the current real interest rate and provide economic stimulus. Without the commitment to a history-dependent policy, it would be more likely to emphasize the costs of positive trend inflation. Further, our study and previous work like Kozicki and Tinsley (2001), Cogley and Sbordone (2008), Cogley et al. (2009), and Nakata (2014) suggest that enough attention has to be also paid to the costs of positive inflation.

Unexpected changes in inflation targets cause policy implementations to be riskier, and lead to changes in long-term expectation of households and firms. Therefore, the policy implementations in the future become inefficient. It is worth not-

ing that the consequences of changing inflation targets become more severe if the economy experiences policy uncertainty, especially uncertainty arising in the monetary policy. This uncertainty can produce adverse impacts on the economy. Under this circumstance, the consistency in policy implementation, especially inflation-targeting policy, is required to not signify the consequences of policy uncertainty.

Sometimes, changes in policy objectives are necessary to suitably adapt to each stage of development. The dissertation recommends that the proposal to raise inflation targets and perhaps other policy changes should be implemented in a stable economy. When central banks intend to raise inflation targets, accompanying policies to stabilize prices and wages issues are encouraged to minimize consequences of this proposal. We also recommend that the resolution of wage issues should be prioritized to create efficiency in mitigating adverse impacts of raising inflation targets.

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