Nonlinear Analysis and Estimation of Dynamic Stochastic General Equilibrium Models

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Introduction

Linearization of DSGE models (first-order perturbation method) has become a widespread numerical technique to solve "true" nonlinear Dynamic Stochastic General Equilibrium Models (DSGE). Since the linearization method makes it possible to obtain of a linear approximation of policy functions, which can be solved by matrix algebra techniques, it has become a standard tool for solving and estimating the model (Blanchard and Kahn (1985), Sims (2000)). Moreover, many DSGE models exhibit behavior that is close to linear, especially in the neighborhood of the steady state, which makes the linearization method a reasonable approximation. As supported by a number of studies, this method is widely used in evaluating likelihood-based DSGE models because it allows for the transformation of these models into state-space representations compatible with Kalman filtering (An (2005)).

However, the linearization of these models can omit interesting aspects provided by nonlinearity, especially in the conduct of welfare analysis, examination of asset pricing, and analysis of the changes in uncertainty over time in an economy within DSGE models. In this regard, the second-order approximation of solutions of DSGE can provide a more accurate solution to the DSGE model. To estimate the model resulting from the second and higher-order perturbation technique is challenging and requires a large computational effort. There are different methods of computing the likelihood function of DSGE models developed in Bayesian econometrics, including Sequential Monte Carlo methods (Fernandez-Villaverde and Rubio-Ramirez (2007)), and Exact Likelihood computation techniques (Amisano and Tristani (2011)). While the former method involves the simulation of a large number of particles, which requires significant processing power, the latter applies to small models with quite restrictive assumptions.

Methodology

To fill the apparent gap, we propose a relatively simple and fast approach to estimation of nonlinear DSGE models, which is also a likelihood-based approach. The gist of our proposed technique is to compute the higher order approximation of the likelihood function of a nonlinear DSGE model using a Laplace approximation method and imposing invertibility of the policy function in the approximation of the likelihood. Specifically, we impose invertibility in the approximation by using an implicit nonlinear invertible approximation of the policy function. We note that in the literature it has been pointed out that in some DSGE models the shocks can be recovered uniquely from some of the control variables, which implies that the policy function is invertible with respect to the shocks (e.g. Stokey et al. (1989), Hopenhayn and Prescott (1992)). This allows us to construct the likelihood function using a standard change of variables theorem and a Lagrange inversion formula. This likelihood can then be used for Bayesian analysis or Maximum Likelihood. In contrast with Amisano and Tristani (2011), this method allows for unobserved non-stochastic state variables, and unlike Fernandez-Villaverde (2010) it requires neither the introduction of additional shocks nor simulation to evaluate the likelihood.

Results and Discussion

In the first study, using US data, we demonstrate the proposed approach in the case of the well-known neoclassical growth model of Fernandez-Villaverde (2010). In addition to the baseline model, we modify the model by incorporating an uncertainty, defined as an increase in the standard deviation of the shocks that affect the economy (which implies that the shocks are heteroskedastic). We specify uncertainty by Generalized Autoregressive Conditional Heteroskedasticity (GARCH) processes. We estimated four structural models: the log-linearized model with homoscedastic shocks; the model resulting from second-order approximation with homoscedastic shocks; the model resulting from second-order approximation with unrestricted GARCH in the shocks; and the model resulting from the second-order approximation with a common factor in the GARCH processes.

As a result of the Bayesian estimation of four models, we find that the nonlinear heteroscedastic empirically performed better model than linearized models in terms of their log marginal likelihood, log-likelihood and predictive likelihood. The posterior analyses indicate that the monetary policy shock in a nonlinear heteroscedastic model is the main driving force of uncertainty in an economy.

In the second study we develop a more general New Keynesian model with limited heterogeneity featuring two agent properties (referred to in the macroeconomic literature as the Two-Agent New Keynesian (TANK) DSGE model) and estimate the model using a likelihood-based approach via the perturbation method. Following existing studies on two-agent models, we introduce Ricardian (financially unconstrained) and non-Ricardian (financially constrained) households. The model features price and wage rigidity dynamic and capital adjustment cost, and innovation to technology, monetary and government spending rules, as well as labor income and consumption tax shocks. Also, we model real and nominal frictions nonlinearly, where the former includes investment costs, the latter price and wage frictions.

We used quarterly aggregate Korean data for 1999 Q4–2021 Q4. Through likelihood-based estimation, we find that the linear representation of the TANK model generates a better fit of the model to the data than the nonlinear one, as measured by log-likelihood at the posterior mode. However, the nonlinear representation of the TANK model has better log marginal and predictive likelihood than the linearized model. These results still support the idea of including nonlinearities when analyzing the behavior of the TANK model. The standard deviations of the parameters are estimated to be much higher for the second-order approximation model than for the first-order approximation, indicating that the nonlinear likelihood is more dispersed than the linear. The analyses of Bayesian impulse responses for both models indicate that, in general, they align with findings of existing studies, most of which have mostly involved linear estimation. However, the magnitude varies by model: we find that positive government expenditure shock has an expansionary effect on aggregate consumption and output in both linear and nonlinear models.

Conclusion

Our findings argue in favor of the nonlinear estimation of DSGE models to assess the role of monetary and fiscal policy rules on macroeconomic outcomes. Linear DSGE models cannot capture important features such as consumer risk aversion, time-varying variances, asset pricing, and welfare evaluations. Although nonlinear DSGE models help address these issues, they present challenges for estimation. The new method presented here facilitates the estimation of nonlinear DSGE models and makes it possible to evaluate some nonlinear DSGE models that cannot be estimated with existing methods. As a practical policy issue, for example, the design of optimal monetary or fiscal policies to improve the welfare of individuals in an economy. As mentioned above, there are already methods to do this in the literature; the method proposed in this dissertation widens the tools for empirical analysis of these issues.

In addition, our results provide evidence that nonlinear models can deliver a more accurate representation of the economy, allowing for nonlinear relationships between variables. These features can affect the transmission of monetary policy and the response of macroeconomic variables to aggregate shocks, which are important considerations for central banks in their policy-making decisions. The implications of the findings can be useful for central banks in their efforts to stabilize the economy and achieve their policy objectives.

However, the proposed estimation technique is a relatively new approach, and we recognize several limitations to our study. First, the method is not applicable in all cases, as it requires the policy function to be invertible concerning shocks. Furthermore, the analysis is restricted to second-order approximation and to the case in which the number of shocks equals the number of observed variables. Since these two assumptions are rather restrictive, future studies could usefully address these challenges by further exploring the estimation of nonlinear DSGE models. Therefore, subsequent research analysis can investigate this issue further in order to work out how to relax these two assumptions.

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