博士論文審査結果報告
Report on Ph.D. / Doctoral Dissertation Defense

National Graduate Institute for Policy Studies (GRIPS)
政策研究大学院大学
Professor LEON-GONZALEZ Roberto
教授　LEON-GONZALEZ Roberto

審査委員会を代表し、以下のとおり博士論文審査に合格したことを報告します。
On behalf of the Doctoral Dissertation Review Committee, I would like to report the pass result of the Doctoral Dissertation Defense as follows.

プログラム名 Program	政策分析プログラム Policy Analysis Program	
学位申請者氏名（ID） Ph.D. Candidate (ID)	Majoni Blessings (PHD20303)	
Dissertation Title 論文タイトル (タイトル和訳)	Univariate and Multivariate Inverse Gamma Stochastic Volatility Models 単変量・多変量逆ガンマ分布確率的ボラティリティモデルに関する研究	
学位名 Degree Title	博士（国際経済学） Ph.D. in International Economics	
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論文最終版提出日/ Submission Date of the Final Dissertation	2024 年 2 月 21 日/ February 21, 2024	
審査委員会/ Doctoral Dissertation Review Committee	主査 Main referee	LEON-GONZALEZ Roberto
	審査委員 Referee	土谷　隆 TSUCHIYA Takashi
	審査委員 Referee	竹之内　高志 TAKENOUCHI Takashi
	審査委員 Referee	大森　裕浩 東京大学 OMORI Yasuhiro The University of Tokyo
	審査委員（博士課程委員会） Referee (Doctoral Programs Committee)	知花　武佳 CHIBANA Takeyoshi

※ タイトルが英文の場合、文部科学省に報告するため、和訳を付してください

Please add a Japanese title that will be reported to MEXT.

1. Summary of Defense and Evaluation

The work of Nobel prize winner Robert Engle showed that it is empirically very important to model the time-varying variance of an observed variable. He first proposed the Auto Regressive Conditional Heteroscedasticity (ARCH) models, and since then there has been a large literature which tries to improve such models. Stochastic volatility (SV) models were later proposed as a better alternative, but they require simulation for estimation, and for evaluating the likelihood. There are very few SV models whose likelihood function can be evaluated analytically without simulation. This is a necessary condition for Maximum Likelihood Estimation (MLE) to be possible, which is an efficient and popular estimation method. This thesis finds a new SV model for which the likelihood can be evaluated analytically, and therefore can be estimated with MLE. It obtains the analytical expression by obtaining a series with infinite terms, which can be truncated by discarding the negligible terms. It then carries out a large empirical exercise to compare the new model with previous existing models using inflation and exchange rate data from many countries. It finds that the new model often performs better empirically than the alternatives.

The second research chapter generalizes the method to the multivariate case. In the context of a large Vector Autoregressive (VAR) model it finds the analytical expression for the likelihood when using the newly developed SV process, in the context of a Common Stochastic Volatility (CSV) framework. It then uses large sets of macroeconomic variables from many countries to compare empirically the new method with previous alternatives in the literature. It finds that the new model often outperforms previous alternatives, while having the advantage of admitting an analytical expression for the likelihood.

The computer code to calculate the likelihood uses parallel threads for faster computations, and has been made freely available as a user-friendly R package through the CRAN repository.

2. Dissertation overview and summary of the presentation.

This dissertation makes several contributions to the stochastic volatility model literature. The key contribution is that it obtains a novel closed form expression of the likelihood for a stationary inverse gamma Stochastic Volatility (SV) model. As a result, using this expression of the log likelihood, it is possible to obtain the Maximum Likelihood Estimator (MLE) for this class of non linear non Gaussian state space models for the univariate model. The dissertation provides two empirical studies to demonstrate this approach.

First, chapter 3 proposes a novel method to explicitly calculate the likelihood for a stationary inverse gamma Stochastic Volatility model, which is conventionally approximated using sampling methods. The derived likelihood is expressed by infinite series of functions and its calculation is implemented by truncating higher order terms. This expression of the likelihood allows the estimation of the parameters and unobserved states for this model class by MLE. Further, the chapter provides the analytical expressions for both the filtering and smoothing distributions of the volatilities as mixture of gammas and therefore it is able to provide the smoothed estimates of the volatility. The chapter shows that by marginalising out the volatilities, the model that is obtained has the resemblance of a Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model in the sense that the formulas are similar, which simplifies computations significantly. Another significant contribution of this chapter is that, the computer code to perform the analysis has been publicly made available through an R package that is freely available from the Comprehensive R Archive Network (CRAN).

The performance of the proposed method was evaluated using several macroeconomic and financial data sets and its results show that the proposed method achieved accurate calculation of the likelihood with low computational cost. The macroeconomic application uses quarterly inflation data for four countries, that is, UK, USA, Japan and Brazil. A range of other models are also estimated to evaluate the empirical performance of the proposed model. The proposed

model performs better than all other models in 50% of the applications in terms of the Bayesian Information Criterion (BIC), with very large gains for the Brazil dataset. The second application uses exchange rates data for 7 currencies (GBP, EUR, JPY, CND, AUD, BRL, ZAR) and finds that the empirical fit of the proposed model is overall better than alternative models in 4 of the 7 currencies in terms of the BIC, being much superior in the case of currencies with turbulent episodes, such as the Brazilian Real.

Chapter 4 extends the univariate approach to estimating large Vector Autoregressions (VAR) in a multivariate common stochastic volatility (CSV) model. CSV models capture the commonality that is often observed in volatility patterns. However, they assume that all the time variation in volatility is driven by a single multiplicative factor. Given the empirical evidence on fat tailed distributions, and the commonality that is observed in volatility patterns, this model combines stochastic volatility, heavy tailed distributions and a common heteroscedastic latent factor. The volatility for this novel CSV model follows an inverse gamma process, which implies student-t type tails for the observed data.

An analytic expression for the likelihood is obtained for this CSV model, which facilitates model comparison. This model is estimated using 4 macroeconomic applications that use 20 variables each for Japan, Brazil, US, and the UK. A second application uses financial data of daily exchange rate returns for a small VAR of 4 currencies and a larger VAR of 8 currencies. The comparison method is based on marginal likelihoods and the one step ahead out of sample predictive likelihoods. The proposed model is compared to other CSV models proposed in the literature with encouraging results.

3. **Evaluation Notes from the Doctoral Dissertation Review Committee (including changes required to the dissertation by the referees)**

During the final defense the candidate presented her thesis for about 45 minutes and afterwards the examination committee gave comments and questions that after the defense were passed to the candidate in written form. The candidate worked on the revisions for 4 months and a half, and produced a revised version of the thesis as well as a document explaining how she incorporated the comments of the examiners. I attach the document, which deals with comments such as:

1) The literature that you cite seems to have started a long time ago. Why does it still continue up to this date, why was not a satisfactory solution given already?

2) The differences in the numbers given in the second research chapter seem to be small. If there were slight changes in the data, would the differences still hold?

3) Your method requires a truncation point. You illustrated the effect of the truncation point in the first research chapter, for the univariate model. What about in the case of the multivariate model in the second research chapter?

4) There needs to be more explanation of the econometric models that you use, specially in your second research chapter, so that a person not familiar with the econometric literature (for example VAR or stochastic volatility factor models) can more easily understand your chapter.

5) Please provide more details about the R package that you made in your thesis. It is an important contribution and it deserves to be highlighted.

6) Your model does not include a leverage effect. You don't use stock market data in your thesis, but if you did, the leverage effect becomes important. Can you modify your model to include a leverage effect? Can you analyze some stock market data using models with a leverage effect?

4. Confirmation by the Main Referee that changes have been done to the satisfaction of the referees and final recommendations

In the evaluation after the defense, the median evaluation by the examining committee was 5, so they left to the main adviser the responsibility for the final check of the revisions. Although the evaluation was 5, some of the revisions required a substantial amount of work. The candidate worked for 4 months and a half, and although was not able to do everything that was asked, because some of the comments required new technical derivations, the revised thesis incorporated most of the comments from the examiners and was substantially improved with respect to the first version. The main advisor shared the revised version of the thesis, and the document explaining the revisions with the examination committee. The plagiarism software did not find any problem, so the main advisor recommended the candidate to submit the final version through the program coordinator.

Therefore, the doctoral dissertation review committee recommends that GRIPS awards the degree of Ph.D. in International Economics to Ms. Blessings MAJONI.