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ABSTRACT

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There is a wide consensus that transport infrastructure development stimulates economic competitiveness and growth. However, such kind of development is long-lusting in nature and requires huge budgetary allocation. Moreover, the contemporaneous public policy has to focus more on other socioeconomic problems such as health care, pension, education, etc. which puts the transport sector under strict financial constraint. The issue of optimal transport infrastructure development therefore, is a major concern among the transportation economists, policymakers, and academics of both the developed and developing countries. In recent years, the world has experienced a rapid growth both in the number and size of cities, and so this issue has become more crucial in urban contexts than before.

Most of the prior literature concerns with the specific type of transport infrastructure development, limited number of travel routes and monocentric city assumptions. There is no extent work on the optimal transport infrastructure in a city model under "general" settings that can explain any kind of transport structures, any number of modes and routes, and any number of city centers. Literature is also limited on the optimal circumferential highway within a city model and therefore,

the influence of a circumferential highway on the city characteristics remains unexplored. One "efficient" way to finance transport infrastructure is to utilize the toll revenue collected from the highway usages. The theory of transportation economics demonstrates that by following a "right" investment policy together with some specific characteristics of highway cost and trip production functions, it is possible to finance the cost of highways from the toll revenue. However, when a highway agency intends to maintain self-financing, while following a "wrong" or in other ward "naive" investment policy which is more likely in the real policymaking, it will result in a substantial amount of welfare loss; and no literature, so far, has seemed to concern with the recovery of such welfare loss.

These gaps in the existing literature clearly indicate that there is a need to address these issues. Thus, in general, the three important issues of transport development deserve careful and in-depth consideration: (a) the optimal urban transport development in a general settings by considering land-use model, (b) the optimal design of a circumferential highway within a city model, and (c) the welfare recovery when the highway investment is made according to "investment cost" policy and not according to "capital cost" policy (naïve policy with short-run marginal cost pricing) with a view to achieve long-run self-financing.

Throughout this dissertation we attempt to establish the optimality conditions of highway investment under "general" settings which can be applied to any kind of transport development, any number of transport route and mode, any number of city center, and any type of city shape. The study then determines the optimal design characteristics of a circumferential highway by using an urban land-use model. The final focus of the study is on the issue of self-financing of highway investment. We try to develop policy tools that can help restoring the welfare losses resulted from a wrongly conceived investment policy while targeting self-financing.

We applied various model setups to analyze these issues. First, to address the issue of the optimal transport infrastructure from a general perspective, we set up an urban land-use model in which households with identical preferences and endowments maximize their utility by choosing residential location, lot size, and travel mode. The transport system and the city characteristics are exogenous to them. The benevolent transport authority optimizes the uniform level of utility by controlling infrastructure characteristics. Second, to design the optimal circumferential highway, we used the same urban land-use model with three kinds of transport mode options: the city streets, the radial highways, and the circumferential highway. City streets are dense and slow. Radial highways are faster but need access travel. The third mode is the circumferential highway that reduces travel cost by providing quicker access to the radial highways; however, city dwellers have to pay equally for its construction. These three modes offer three different travel routes which divide the city into three market areas. By considering absentee land ownership, we numerically solve for the optimal design of the circumferential highway. Third, we consider a two-stage dynamic game model for addressing the issue of benefit restoration under naïve policy in highway investment. In the first stage, government fixes the toll for highway usages, and in the second stage, the transport authority collects the toll and invests the net of it into capacity expansion for the subsequent period. We consider the net present value of welfare and continuous time in this analysis.

From the analysis of the general optimal transport infrastructure in a closed city with residential land ownership, the key findings are that

- the infrastructure should be developed to an extent at which the marginal cost of development is equal to the marginal increase in aggregate differential land rent evaluated at the current level of land rent.

The study on optimal design of a circumferential highway in a monocentric city reveals that

- the optimal circumferential highway is partially used,

- the resulting city shape is such that there is a bulge of urbanized area around the circumferential highway,
- the cities with a larger number of radial highways require construction of the circumferential highway further outward from the CBD than the cities with a smaller number of radial highways.

From the analysis of welfare recovery with the second-best dynamic highway pricing under naïve investment policy, the major findings are that

- only when the interest rate is zero, naïve policy under short-run marginal cost pricing does not really cause any harm,
- a vertically disintegrated superior authority can partially recover the welfare loss by taking necessary adjustments in highway pricing,
- interest rate has a great effect on welfare recovery.

The research findings suggest that

- the post-project land price is not essential for determining the optimal level of transport investment,
- the cities of the developed countries need bigger circumferential highways than that of developing countries, and
- the developing countries with higher interest rates are more vulnerable to the welfare consequences resulted from the naïve policy and therefore needs to pay more attention while planning for self-financing in highway sector.

Conference and Seminar Presentations and Publication 2012-2014

Conferences and Seminars

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Discussion Paper

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Dedication

To my father, Tasir Uddin Ahmed; mother, Rahenara Begum; daughter, Raisa Amrin Ahmed; grandfather, the late Shahimuddin Sharkar (paternal), and the late Basir Uddin Ahmed (maternal); grandmother, the late Toifa Khatun (paternal) and the late Razia Begum (maternal); uncles, the late Shahibur Rahaman, and the late Mozammel Haque; and aunt, the late Mosammat Mojida Khatun.

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Nomenclature

SBNP	Second-best Naive Policy
SRMCP	Short-run Marginal Cost Pricing
TA	Transport Authority
CBD	Central Business District
MADLRc	Marginal Aggregate Differential Land Rent-Evaluated at Current Land Rent

1. INTRODUCTION

1.1. Significance of the Issue

Due to promotion of trade flow, emergence of new markets and expansion of supply chain, the mobility of passengers and goods has increased greatly and hence the necessity of transport infrastructure development has turned into a crucial issue. The key objective of the majority of transport policies is to facilitate efficient and sustainable mobility of passengers and goods that promotes economic activity and growth of the country. However, transport development is a long-run issue and it has effects on the location choice and land-use pattern and, most importantly, it requires huge capital investment. Furthermore, the recent increased necessity of government expenditures on other socioeconomic issues such as health care, pension, education etc. has imposed tighter budgetary conditions on the transport sector than ever before. The situation is more severe, especially in the urban context. Therefore, the questions arise as to what the socially efficient urban transport infrastructure requirements are, how these requirements are influenced by the land-use and locational choice behavior and, what the alternative efficient financial source of transport development should be.

There is plenty of literature that addresses the issue of optimum transport infrastructure development and its financing from different perspectives. Yet, literature is limited on issues of (a) the optimal urban transport development under "general" settings where the level of development and city characteristics are endogenously determined; (b) the efficient design of a circumferential highway (ring road) under endogenous city framework; and (c) the social consequences and remedial measures in the context of highway self-financing where the transport agency naïvely invests all of the toll revenue in capacity expansion. Since transport development involves huge capital investment, the above issues may have considerable economic ramifications.

First, the interdependence between the urban transport system and the city characteristics has been an issue of study by urban transport planners for the decades. Primary monocentric city models mainly focus on the residential pattern where city dwellers commute to the CBD by paying transport costs proportional to the transport distance (e.g., Alonso, 1964; Mills, 1967; Muth, 1969). Later, some other studies developed these basic models in various dimensions (e.g., Brueckner, 1987; Solow, 1973). Yet, most of the above studies are based on the equilibrium analysis.

Subsequently, a few works have focused on the optimal analysis of urban transport system by using land-use models (e.g., Anas & Moses, 1979; Kanemoto, 1984; Mills & De Ferranti, 1971). However, the main drawback of these models is that the number of available transport modes is limited to one or two and therefore cannot be applicable where, like in reality, commuters have more than two mode options. In addition, modern cities are likely to have more than one business center and so have employment center. Since the existing literature mostly considers monocentric city models with limited mode options, it is of academic interest to formulate and analyze a generalized city model that can be used to explain multi-mode and multi-city center issues while obtaining insight into the optimal requirements of transport development.

The second issue focuses on a particular type of transport development, namely, the provision of a new circumferential highway within a city. The existence of one or more circumferential highways is a common feature in most large cities. However, most of the cities of the developing world are suffering from huge traffic volume and are yet to construct circumferential highways. In fact, the circumferential highway helps to connect the city periphery with the radial highways which eventually facilitates quicker travel to the CBD for the residents living close to it. Thus, it diverts traffic from the dense city streets and influences the internal structure as well as the expansion and growth of the city.

Most existing works on circumferential highway largely depend upon the minimization of travel time or travel distance in exogenous city framework (e.g., Blumenfeld & Weiss, 1970; Pearce, 1974). This branch of literature fails to incorporate the two-way interaction between the geometric features of the circumferential highway and the city characteristics. Therefore, the design features obtained by using these models show incorrect signals to the market and distort the market mechanism and resource mobilization. Like other transport development projects, another important feature of this type of highway is that once built it is almost impossible to replace. This means that improper design of such highways will result in severe negative consequences. Therefore, it is important to find an economically optimal circumferential highway by using land-use and locational choice model.

Lastly, we focus on the welfare issue of the self-financing of highways. The root idea of self-financing comes from the user-pay principle. The toll charged to reduce congestion externality through short-run marginal cost pricing (SRMCP) can be used for construction financing and other related costs (e.g., operating, maintenance and depreciation costs) of the highway. For an efficient and sustained transport system, highway self-financing is a prospective issue. One of the ten goals towards an efficient transport system by the European Commission is "Move towards full application of “user pays” and “polluter pays” principles and private sector engagement to eliminate distortions, including harmful subsidies, generate revenues and ensure financing for future transport investments" (EC, 2011, p. 10).

The basic concept of highway self-financing was pioneered by Mohring & Harwitz (1962) and later analyzed in numerous works (e.g., De Palma & Lindsey, 2007; Hau, 1998; Newbery, 1989; Small, 1992; Small, 1999; Small & Verhoef, 2007; Strotz, 1964; Yang & Meng, 2002). However, most of these works address the issue in a static framework and therefore are incapable of handling real world dynamics. Notably, a few studies later extend the concept into intertemporal context and establish analytically the socially optimal relationship between revenue and investment (e.g., Arnott & Kraus, 1998). For maintaining self-financing in highway, investment should be made in such a way that the toll revenues just cover the capital cost (interest cost) of capacity stock at every moment in time.

However, in reality there may be a conceptual misunderstanding between "capital cost" and "investment cost". As noted by Verhoef and Mohring (2009), "This mistake may seem terribly naïve to the trained economist, but may in fact not be so far-fetched in the practice of policymaking, where investments are financed from public funds that are raised through taxation, and no interest is paid (at least directly) on the capital invested in public infrastructure" (p. 303). What is the welfare consequence, if the highway agency makes such a mistake and spends the toll revenue directly for capacity expansion in the next period? Verhoef and Mohring (2009) answered the question and showed numerically that there are indeed negative welfare consequences. The next question then arises as to what policy measure can recover or at least minimize this loss of social welfare when the transport agent, aiming at maintaining self-financing, behaves naïvely by investing all of the net toll revenue into capacity addition? To date, no literature has been found to answer this question.

Thus, the above mentioned issues in the area of transport planning, and investment namely, the requirements of the general optimal urban transport development, the optimal design of a circumferential highway, and the remedial measure against naïve investment in highway development deserve keen consideration.

1.2. Research Objective

Due to the perceived significance of the contemporary transport investment issues, this dissertation attempts to make a modest contribution to the urban and transportation economic literature, and to assist the academics, researchers and policymakers by investigating the three important issues of transport development.

The first objective of the dissertation is to determine the necessary and sufficient conditions for the optimality of urban transport development under general settings. For this, we consider locational choice and land-use effects. This analytical work attempts to contribute to urban transport theory and policymakers by deriving general optimality conditions which primarily emphasize that we do not need to consider the post-project land price in deciding optimal transport development; by introducing the isomorphism technique to the locational choice problem; by allowing provisions for multimodal (more than two modes) transport options to the commuters' route selection problem, and by relaxing the so called single-CBD assumption of a city model.

The second objective is to provide an optimal design (e.g., location and capacity) of an urban circumferential highway and to determine the resulting size and shape of the city. Thus, it implicitly aims at making a bridge between the city models and the pure transport models that considered the circumferential highway. This work also seeks to extend the model of Anas and Moses (1979).

The final objective of this research is to determine the policy measures that can recover the social optimum when the transport agency, aiming at achieving a long-run balanced budget through self-financing, mistakenly invests all of the toll revenue into capacity addition. Since such naïve behavior by real world policymakers is not unlikely, the findings of this study may be used as a guideline to recover the resulting welfare losses under such circumstances. This study is expected to contribute to the literature of transport financing by providing long-run policy options for the decision variables (e.g., toll) against naïve behavior in highway investment.

1.3. Dissertation Outline and Research Framework

There are five chapters in this dissertation. These chapters have been arranged as follows. Chapter 1 discusses the research significance, objectives and the basic framework of this study.

Chapter 2 determines the social optimal conditions for transport development in general settings. In doing so, we use an urban land-use model with a residential land ownership assumption. Households face a three-stage decision problem that include locational choice, travel mode choice and consumption choice, and they try to maximize their utility given their budget and the transport infrastructure. We analyze the household decision problem backwards. The transport planner chooses the level of transport development such that it maximizes the global equilibrium level of utility. By using isomorphism we transformed the two-dimensional locational choice problem into a one-dimensional general transport cost problem. One important feature of this study is that we do not limit the number of CBDs or the number of travel modes, and thus the analysis has general implications. We then discuss the sufficiency of the optimality conditions. The key finding of this study reveals that we should continue to develop the transport infrastructure until the marginal construction cost equals the aggregate differential land rent evaluated at the current level of land rent. Though our findings support the Henry George theorem and Samuelson condition, we have shown that they are different from that of the well-known capitalization hypothesis.

In chapter 3, we introduce a design procedure of a utility-maximizing circumferential highway for a city. We use a monocentric city model where landowners are absentees. Apart from the circumferential highway, the households have two other mode options: the radial highways and the direct city streets. With different combination of these three modes, households then have three route choices which divide the total city into three market areas. Perfect mobility together with homogeneous preference ensures same level of utility at the spatial equilibrium. The location and capacity of the circumferential highway are determined in such a way that they maximize the total utility. We solve the optimization problem numerically. The intersection of the best response functions for location and capacity yields the best design geometry of the circumferential highway. We also discuss the resulting city shapes for various locations of the circumferential highway. An interesting finding of this study is that an optimal circumferential highway is partially used. We also characterize the threshold number of radial highways, beyond which construction of any circumferential highway is not efficient. Finally, we explore that bigger cities require larger circumferential highways.

Chapter 4 addresses the self-financing issue of highways when the highway agency follows the naïve policy and invests all of its toll revenue in the future capacity addition. In the model, we consider a two-stage dynamic game in highway administration that includes the government and the transport agency, which are not vertically integrated. The main focus is to minimize the welfare losses resulted from the naïve policy. We use the optimal control theorem to solve the intertemporal problem. Finally, by using specific functional forms and numerical simulation, we construct the optimal transition paths for highway toll and price. We also provide a comparison of highway capacities and welfare measure between the first-best policy, naïve policy under SRMCP and the second-best naïve policy. We find that the welfare loss due to the naïve behavior of the transport agency cannot be fully recovered. Nevertheless, a remarkable portion of the losses can be recovered by a pre-designed optimal toll policy by the superior authority.

We conclude in Chapter 5. This chapter includes a brief discussion of the findings of the dissertation and draws some policy implications that might be helpful to the policymakers, researchers and academics. In the latter part of this chapter, we discuss the limitations of the studies and finally, propose some relevant future research ideas that might be academically and practically interesting.

2. Optimal Transport Infrastructure in a Closed City with Residential Land Ownership

2.1. Introduction

More than half of the world's population is living in cities, either big or small. The number of mega-cities, with a population of more than ten million, has reached 27. Moreover, the number of cities with a population over one million has increased from 74 to 442 since 1950 (National Geographic, 2011). According to Gwilliam (2002), ".... cities are the engines of economic growth in most developing countries, and that urban transport is the oil that prevents the engine from seizing up" (p. 22). Thus, urban transport plays a vital role in city characteristics.

The issue of investment in urban transport is definitely old, yet it is equally important among economists and city planners. Underinvestment in transport infrastructure hampers economic activities and slows down growth. Conversely, overinvestment in the transport sector not only incurs unnecessarily huge costs but also encourages dwellers to make more noncommercial trips, which may in turn slow down traffic movements and act against the essence of infrastructural development. Thus, suboptimal transport infrastructure is undesirable from society's perspective. This reality encourages researchers to search for economically-efficient transport infrastructure development.

There is a large body of literature addressing this issue from various viewpoints. We can broadly classify this literature into two categories. The first category focuses solely on the transport infrastructure and does not consider its impact on city characteristics although city characteristics are likely to influence such kind of public project (e.g., Lind, 1973). Thus, considering transport development alone fails to capture the overall scenario and thereby results in inefficient resource allocation, especially in the long run.

The second category of the literature considers the transport infrastructure development within the city framework and therefore deserves more attention than the previous one. Due to a certain complicacy in modeling the characteristics of real world cities, most second category of literature is based on the assumption of monocentric cities with direct city streets or radial highway networks. These studies deal with city shape, size, location, and mode choice as well as transport infrastructure and land-use patterns under special settings. To the best of our knowledge, literature is limited on the analysis of transport infrastructure development within a city model under a more "general" setting that can be applicable for a wide variety of transport infrastructure. The term “general” has been used in a sense that the model must not be limited to: (a) the number and type of available transport modes, (b) the number and type of transport routes, (c) the type of infrastructural development, (d) the number of CBD(s), (e) the shape of the city, (f) the directional restriction of travel, and (g) the functional form of the utility and construction cost. Given this background, in this study we try to find the social optimal requirements for the development of transport infrastructure in a more general setup with residential land ownership.[1]

Alonso (1964) was the first to illustrate urban land-use theory by using monocentric city models. This line of research continued with later analyzes by Mills and De Ferranti (1971), Solow (1973), and Brueckner (1987). Mills and de Ferranti determined optimal capacity of city streets that varied across locations. Solow provided an equilibrium analysis of land-use pattern and locational choice. These models

[1]By residential land ownership, we mean that land is owned by the residents living within the city boundary. This is opposite to the concept of absentee land ownership where landowners live out of the city.

mainly considered dense city streets as the means of travel to the central business district (CBD). Interestingly, Anas and Moses (1979) explicitly included the radial highways together with the dense city streets in a urban land-use model and described the shape of a city under different generalized transport cost structures. By considering dense city streets and radial highway systems, their work makes an real progress in the literature of city models. However, they considered only two travel modes and performed mainly equilibrium analysis. While they ultimately formulated an optimization problem, they did not go for a solution mainly due to the complexity of the problem. Since then, unfortunately, there has not been seen any further development of their model. As a matter of fact, the above models considered only one or two travel modes. Another limitation of the existing literature is that they mostly addressed particular type of transport infrastructure development (e.g., width and strength of road by Newbery, 1989; length of railway by Kanemoto, 1984; subway system by Kim, 1978 among others). Thus, we face a paucity of literature that analyzes transport development within a city model applicable for any type of transport infrastructure, any number of transport modes, and number of city centers, and any kind of city shapes.

Considering the above limitations of the existing literature, this study attempts to formulate a general theoretical model of a city served by a transport infrastructure that can be of any form (e.g., dense city streets, radial or circumferential highway, railways or subways). The purpose of this study is to determine the optimal investment principles of the urban transport development under general settings with residential land ownership and closed city assumptions. Infrastructural geometry is the choice variable in our model. The key findings of this study is that for the residential land ownership case, the transport development is optimum when the marginal cost of development and the marginal increase in the aggregated differential land rent evaluated at the current level of land rent are equal.

Although we construct a general urban transport model which has not been addressed so far, the findings of the present study may seemingly resemble to some existing works. For example, Kanemoto (1984) looked into the optimal pricing and investment policy of transport network in a circular, monocentric city where the city is served by a radial railway network. Using dynamic analysis, he found that in first best environment, the optimal length of the railway requires the extension cost and total differential land rent to be equal. However, in our case, the optimal requirement is that the marginal construction cost is equal to the marginal aggregate differential land rent evaluated at the current level of land rent. Thus, in our case we need to consider only a portion of the marginal aggregate differential land rent that is affected by the change in city size. His another finding is that, for utility-taker competitive railway companies, the gain from railway investment is completely capitalized into the land value. When a railway company increases fairs, the net income of the household decreases and eventually the land price decreases. The gain by raising fares is capitalized by lowering land rent.

In fact, the well-known capitalization hypothesis states that the benefit of a public projects is capitalized into land value (e.g., Starrett, 1981). Thus, capitalization hypothesis relates benefit and land value of a development project at the margin (Kuroda, 1994). Our findings are different from capitalization hypothesis. The assumption that is associated with the capitalization hypothesis is that the utility of households is fixed. This means that this hypothesis applies for a "small" and "open" city, or for a small-scale investment, where the equilibrium level of utility remains the same before and after the development. For a closed city, the value of land is affected both directly by the transport investment and indirectly by the change in utility and therefore requires general equilibrium analysis (see Polinsky & Shavell, 1975).

Our findings support Samuelson condition. Samuelson condition establishes a relationship between the investment cost and the benefit of a public project and states that when the level of investment is optimum, marginal value of these two are equal. We have the similar results. On the other hand, the Henry George theorem relates the total project cost and land rent when the size of population is optimum. Even though we did not optimize population, we have the similar result as long as Samuelson condition holds.

Kanemoto (2011) examined the benefits of a transport investment project under general equilibrium settings. Measuring benefits by the change in utility before and after the project, he explained that the benefits of an investment project can be determined by the increase in real national income for a small change in transport capacity. Using quasi-linear utility function with constant marginal utility of income, he showed that for a downward-sloping general equilibrium demand curve the real national income determined by using pre-investment prices gives the upper bound of the benefit and the real national income determined by using post-investment prices gives the lower bound of the benefit.

He also illustrated that under general equilibrium settings with no market failure, the benefits of a public investment project can be determined by the cost reduction in transport sector only. Thus, it is not necessary to know the changes in benefits in all other sectors. At the general equilibrium, the first best price equals the marginal cost and thus the benefits and costs become equal for every other sector and cancel out each other. The change in social surplus is the change in consumer surplus over the project cost.

To the great extent, the above mentioned works attempted to measure the benefits of a public projects under market equilibrium. By contrast, we consider optimal provision of the transport infrastructure, and therefore, our objectives and findings are indeed different from those. We examine the conditions of transport development that ensures optimal benefit to the society. By considering residential land ownership, we find that for the social optimum, we need to invest to the extent at which the marginal cost of infrastructural development equals the marginal increase in the aggregated differential land rent evaluated at the pre-project land price. To the best of our knowledge this optimality condition is a modest finding in urban transport literature under a general setting. What makes it different from existing findings is that we do not need to consider the post project land price and therefore do not need to depend on the forecasted value of land during the planning process.

We organize this chapter as follows. In Section 2.2, we present the general model framework, formulate the household and transport authority's problems, fix spatial equilibrium properties, and solve for the optimality conditions with residential land ownership assumption. We also discuss the relationships and applicability of the Samuelson condition, Henry George theorem and capitalization hypothesis using our model in this section. Section 2.3 concludes this study; we summarize the limitations and provide some future research directions. The mathematical proofs of the optimization problem and relevant derivations are presented in the appendix.

2.2. Optimal Transport Infrastructure

In this section we construct a city model with an endogenous transport system. We begin by assuming that the city is closed and inhabited by a fixed number of identical households. The closed-city assumption implies that migration to other cities is infinitely costly, whereas migration within the city is costless. The number of population is exogenously given, which readily implies that the general level of utility is endogenously determined. This is opposite to an open city where the population is endogenous (i.e., the population can migrate from city to city and the utility level is exogenous). We assume that the city land is owned by the residential land owners. Production takes place only in the CBD and therefore CBD is the only source of employment. The location of the CBD is exogenous as well. Unlike other city models, the number of CBD does not have to be one or be located at the center of the city. Moreover, the shape of the city does not have to be circular nor symmetric, and it can be as large as a country.

We assume that the demand for transport service is inelastic and thus does not depend on the transport cost. One member from each household works in CBD and makes a trip to CBD every day. Land is directly consumed by the household for their residence. Households maximize their utility by selecting the size of the lot, other goods, leisure time, as well as the residential location and travel route. Agriculture is the only alternative to urban land use. Thus, if a land is not used for residential purposes, it will be used for agriculture. The agricultural land market is perfectly competitive and thus needs to pay only the agricultural land rent, which is uniform over space. The wage rate w is exogenously given and is the same for every household. There exists a benevolent transport authority which runs the transport sector efficiently, collects taxes from the city residents, and uses this tax revenue to finance its development. Development of transport infrastructure facilitates reduces transport costs and therefore increases the utility of households. We ignore the land allocated to the transport infrastructure.

2.2.1. Generalized Transport Costs

Unlike the model of Anas and Moses (1979) and other city models in the literature, the number of travel modes in our model is not limited to a specific number. In fact, it can be of any value. For example, modes can be highway, railway, subway, city streets, etc. and commuters can choose any combination of these to reach CBD.

We assume that the size of CBD is zero and thus the travel cost within CBD is negligible. Any location within the city is represented by a polar coordinate system (r,θ), where r is the radial distance from CBD and θ is the angular displacement from the nearest radial highway. A travel route from a given location to the CBD is a combination of one or more travel modes. The number of travel routes can be any positive integer. We consider that there are m routes in the city and route h is such that $h = 1, ..., m$; m can be any positive integer. We do not consider congestion and therefore no toll is imposed for the network usage. The generalized transport cost is thus the user cost of travel that includes both time and monetary cost. Let $T_h(r,\theta)$

and $C_h(r,\theta)$ be the round-trip time and monetary costs of commuting from (r,θ) to the CBD per individual via route h,[2] respectively. The generalized transport cost, G_h for a round trip travel from (r,θ) to the CBD via route h is then

$$G_h = wT_h(r,\theta) + C_h(r,\theta). \tag{2.1}$$

The unit time cost of travel is considered to be equal to the wage rate.

Depending on the type of route chosen, the transport cost at each location varies. However, among these routes, one particular route costs minimum. Since households are utility maximizer, they choose the route that yields the minimum generalized transport cost to the CBD. We define $G(r,\theta)$ as the minimum generalized transport cost among all the routes at location (r,θ), that is

$$G(r,\theta) = \min_h G_h(r,\theta). \tag{2.2}$$

For convenience, we will hereafter refer to this minimum generalized transport cost as the "generalized transport cost".

2.2.2. Household Utility Maximization Problem

Households gain utility by consuming three kinds of goods: residential land Q, all other non-residential goods termed as composite good Z, and leisure time L. Regarding residential choice, households only care about the size of the lot they live and we ignore all other characteristics of residential demand. We assume that the price of the composite good does not depend on location and therefore, same everywhere. The cost of leisure is same as wage. Each household has an equal and fixed endowment of time, H, that it allocates for leisure, working, and commuting time. Households living further away from the city center will have a longer commuting time and less leisure time.

The household's decision-making process can be divided into three stages. In the

[2] Just as the CBD is not necessarily unique, its location does not have to be at $(0,0)$ to derive the main results. Similarly, the dimension of the city can be more, or less than two to derive the main results. However in what follows, our exposition postulates a classic two-dimensional monocentric city where CBD is at its center.

first stage, the household chooses residential location (r,θ) within the city (recall that the cost of migration within the city is zero). In the second stage, it chooses the route h given the residential location, while in the third stage it chooses the amount of consumption of composite good Z, land lot Q, and the leisure time L given the residential location and travel route . The problem is then solved by backward induction.

In the third stage, the household maximizes utility given the location (r,θ) and route h, as well as the budget and the time constraints. The aggregate differential land rent, Φ, is defined as the total land rent for residential use less the total land rent for agricultural use. In addition to wage income, each household receives an equal share of aggregated differential land rent.[3] Therefore, even though the wage rate is fixed, the total income of the household is endogenous (see Pines & Sadka, 1986). Let N be the total number of households in the city; each household pays a lump-sum tax D to the transport authority. This tax will be used to finance the development of infrastructure. Capital is the only input for transport development. Solving the third-stage problem yields an indirect utility function conditional on the location (r,θ) and the route h. In the second stage, the household chooses route h to maximize this conditional indirect utility.

We formulate the dual of the above problem. Denoting the land rent at location (r,θ) by $\widetilde{R}(r,\theta)$, the household's expenditure minimization problem becomes

$$\begin{aligned} E\left(\widetilde{R}(r,\theta),U\right) &= \min_{Z,Q,L} Z+\widetilde{R}(r,\theta)\,Q+wL \\ & \quad subject\ to\ U=U(Z,Q,L), \end{aligned}$$

where E is the expenditure function, $U(Z,Q,L)$ is the utility function, and U is the utility level. Here the price of the composite good Z is normalized to one. The solution of the above expenditure problem gives the compensated (Hicksian)

[3]This assumption is made by Pines and Sadka (1986), who labeled such a city structure as "fully-closed city". It differs from a "semiclosed city" in which the income level is exogenous (i.e., households do not receive any share from the aggregate differential land rent).

demand functions which are

$$\begin{aligned} Q^c &= Q^c\left(\widetilde{R}(r,\theta),U\right) \\ Z^c &= Z^c\left(\widetilde{R}(r,\theta),U\right) \\ L^c &= L^c\left(\widetilde{R}(r,\theta),U\right) \end{aligned}$$

where the superscript C denotes compensated demand.

Now by using Envelop Theorem and Shephard's Lemma, we get

$$\begin{aligned} \frac{\partial E}{\partial \widetilde{R}} &= Z^c_R + Q^c + \widetilde{R}(r,\theta)\,Q^c_R + wL^c_R \\ &= Q^c \end{aligned}$$

which simplifies to

$$Z^c_R + \widetilde{R}(r,\theta)\,Q^c_R + wL^c_R = 0 \tag{2.3}$$

Here subscript R indicates partial derivatives with respect to the land rent[4].

Because we do not consider savings, the household experiences a balanced budget. Under this assumption, the bid rent function $\widetilde{R}_h(r,\theta)$ gives the maximum amount that a household, commuting via route h, is willing to pay for the land lot provided that it achieves a given level of utility:

$$\widetilde{R}_h(r,\theta) = \frac{1}{Q^c}\left[wH - D + \frac{\Phi}{N} - G_h(r,\theta) - Z^c - wL^c\right]. \tag{2.4}$$

The landowners rent the land to the highest bidder. Therefore, the land rent is the maximum of the bid rents[5]:

$$\widetilde{R}(r,\theta) = \max_h \widetilde{R}_h(r,\theta).$$

The argument of the maximum here coincides with that of the minimum of generalized transport cost given in Equation (2.2). This is obvious because under no saving constraint, the minimum generalized transport cost yields the maximum amount of

[4]For example, see Kanemoto (1977).

[5]In the model, mode "h" is discrete and so the land rent function generally varies across market areas (a market area is a set of locations where one particular route dominates). However, the assumption of an interior solution of the above problem provides a justification of taking the derivative of the land rent function.

income available to the household for its bidding lot size.

2.2.3. Spatial Equilibrium

The spatial equilibrium is such that households can't attain higher utility by changing their location and mode. Since the households are homogenous in their preference and earn same income (wage income plus income from the aggregate differential land rent), they achieve the same level of utility regardless of their location. For a closed city with a fixed population, the equilibrium utility and the city boundary are endogenously determined. Let the utility level that each household achieves in the spatial equilibrium be $\overline{U}$. To satisfy the same level of utility for all households, the land rent varies across locations, and the residential land rent at the city boundary is equal to the agricultural land rent.

Let $\mathfrak{R}$ represents the set of all locations within the city so that

$$\mathfrak{R} \equiv \left\{ (r,\theta) \,| \widetilde{R}(r,\theta) \geq R_A \right\}$$

where R_A is agricultural rent. The city area is such that at the spatial equilibrium it must accommodate all households. Thus, we have the following land constraint

$$N = \iint_{(r,\theta)\in\mathfrak{R}} \frac{r dr d\theta}{Q^c\left(\widetilde{R}(r,\theta),\overline{U}\right)}. \tag{2.5}$$

The term $1/Q^c$ represents the population density at location (r,θ) for a given $\overline{U}$. The city boundary is defined as locations beyond which land is used for agricultural purpose and inside of which land is used for residential purpose. Therefore, at the city boundary, there is a competition for land for residential and agricultural use which under spatial equilibrium implies that the residential bid rent must be equal to the agricultural land rent:

$$\widetilde{R}\left(\overline{r},\overline{\theta}\right) = R_A \tag{2.6}$$

where $\left(\overline{r},\overline{\theta}\right)$ represent the location at the city boundary. The aggregate differential land rent Φ is the sum of land rent over the agricultural land rent such that

$$\Phi = \iint_{(r,\theta)\in\mathfrak{R}} \left[\widetilde{R}(r,\theta) - R_A\right] r dr d\theta. \tag{2.7}$$

Solving Equations (2.5), (2.6) and (2.7) simultaneously, we get $\overline{U}$, $\left(\bar{r},\overline{\theta}\right)$, and Φ in terms of all exogenous variables as well as the lump-sum tax D.

We, now, use isomorphism to convert the two-dimensional space problem into a one-dimensional transport cost problem. Each physical location in the city is represented by the generalized transport cost for a round-trip from that location to the CBD. For this, let us define the land rent R as a function of G such that

$$R(G(r,\theta)) = \widetilde{R}(r,\theta).$$

Now using the above definition, we rewrite the consumption of lot, composite good, and leisure in terms of G:

$$Q^*(G) = Q^c\left(R(G),\overline{U}\right) \tag{2.8}$$

$$Z^*(G) = Z^c\left(R(G),\overline{U}\right) \tag{2.9}$$

$$L^*(G) = L^c\left(R(G),\overline{U}\right). \tag{2.10}$$

Using these three equations, $R(G)$ satisfies the following:

$$R(G) = \frac{1}{Q^*(G)}\left[y^d(G) - Z^*(G) - wL^*(G)\right] \tag{2.11}$$

where y^d is the disposable income such that

$$y^d(G) = wH - D + \frac{\Phi}{N} - G \tag{2.12}$$

and total household income, denoted by $y(G)$ is

$$y(G) = wH + \frac{\Phi}{N}. \tag{2.13}$$

Next let us define $\xi(G)\,dG$ as the land area where the generalized transport cost is equal to G. That is, by letting $\Xi(G)$ be the area of the city where the generalized transport cost is less than or equal to G, we have

$$\xi(G) = \frac{d\Xi(G)}{dG} \tag{2.14}$$

for $G \geq 0$. The spatial equilibrium can now be rewritten in terms of G:

$$N = \int_0^{\overline{G}} \frac{\xi(G)\,dG}{Q^*(G)} \tag{2.15}$$

$$R\left(\overline{G}\right) = \frac{wH - D + \frac{\Phi}{N} - \overline{G} - Z^*\left(\overline{G}\right) - wL^*\left(\overline{G}\right)}{Q^*\left(\overline{G}\right)} = R_A \tag{2.16}$$

$$\Phi = \int_0^{\overline{G}} \left(R(G) - R_A\right)\xi(G)\,dG \tag{2.17}$$

where $\overline{G}$ is the generalized transport cost for any household living at the city boundary such that

$$\overline{G} = G\left(\bar{r}, \overline{\theta}\right).$$

The equilibrium utility $\overline{U}$, the generalized transport cost at the city boundary $\overline{G}$, and aggregated differential land rent Φ satisfy these three equations:

$$\overline{U} = \overline{U}\left(D, \Xi(G), w, H, N, R_A\right) \tag{2.18}$$

$$\overline{G} = \overline{G}\left(D, \Xi(G), w, H, N, R_A\right) \tag{2.19}$$

$$\Phi = \Phi\left(D, \Xi(G), w, H, N, R_A\right). \tag{2.20}$$

Thus, the endogenous variables are expressed in terms of the parameters and the lump sum tax, D.

2.2.4. Optimization of Transport Infrastructure

We will now look into the other fold of the problem, namely, the optimization problem of transport authority[6]. Depending on the nature and level of development, the construction cost varies. Construction cost also depends on the economies of scale that the construction industry exhibits. However, we use a general construction cost function. Moreover, we do not specify the type of infrastructural development; it can be a completely new highway link, a new circumferential highway, or an extension of length or capacity of an existing highway. We denote the construction cost

[6]The term "transport authority" need not necessarily be limited to a government institution controlling transport sector, it can be a private transport company having the right to impose a head tax to finance transport investment.

and the characteristics of the transport infrastructure development by K and a vector η, respectively, so that

$$K = K(\eta). \tag{2.21}$$

The transport authority maintains a balanced budget; which means that the construction cost K must be financed by a lump-sum tax D collected equally from all households. Therefore, the budget constraint is given by

$$K = DN. \tag{2.22}$$

The transport authority optimizes households' benefits by choosing the network geometry and thus its problem is

$$\max_{(\eta)} \overline{U} = \overline{U}\left(D, \Xi(G), w, H, N, R_A\right)$$

subject to Equations and (2.21) and (2.22).

Let us suppose that the above optimization problem has an interior solution. The total differentiation of Equations (2.8) through (2.10) as well as Equation (2.11) yields that, for a given G,

$$\begin{aligned} dQ^* &= \frac{Q_R^c}{Q^*(G)}\left(-dD + \frac{d\Phi}{N}\right) + \left[Q_U^c - \frac{\lambda(G) Q_R^c}{Q^*(G)}\right] d\overline{U} \\ dZ^* &= \frac{Z_R^c}{Q^*(G)}\left(-dD + \frac{d\Phi}{N}\right) + \left[Z_U^c - \frac{\lambda(G) Z_R^c}{Q^*(G)}\right] d\overline{U} \\ dL^* &= \frac{L_R^c}{Q^*(G)}\left(-dD + \frac{d\Phi}{N}\right) + \left[L_U^c - \frac{\lambda(G) L_R^c}{Q^*(G)}\right] d\overline{U} \end{aligned} \tag{2.23}$$

where subscripts R and U represent partial derivatives and $\lambda(G) \equiv \partial E/\partial U = Z_U^c + R(G) Q_U^c + wL_U^c$.[7]

After taking total differentiation of Equations (2.15) through (2.17) as well as Equations (2.21) and (2.22) with respect to η and then using Equation (2.23) we get[8]

$$0 = \int_0^{\overline{G}} \frac{1}{Q^*(G)}\left[-\frac{\xi(G)}{Q^*(G)} dQ^* + \frac{\partial \xi(G)}{\partial \eta} d\eta\right] dG + \frac{\xi(\overline{G})}{Q^*(\overline{G})} d\overline{G}$$

[7]See Appendix A.1.1 for detailed derivation.

[8]See Appendix A.1.1 and Appendix A.1.2 for derivatives.

$$
\begin{aligned}
0 &= -dD+\frac{d\Phi}{N}-d\overline{G}-dZ^{*}|_{\overline{G}}-w\,dL^{*}|_{\overline{G}}-R_A\,dQ^{*}|_{\overline{G}} \qquad (2.24)\\
d\Phi &= \int_0^{\overline{G}}\left[\left(\frac{-dD+\frac{d\Phi}{N}-\lambda(G)\,d\overline{U}}{Q^{*}(G)}\right)\xi(G)+(R(G)-R_A)\frac{\partial\xi(G)}{\partial\eta}d\eta\right]dG\\
dK &= \frac{\partial K}{\partial\eta}d\eta\\
dD &= \frac{dK}{N}.
\end{aligned}
$$

These ultimately reduce to[9]

$$
\begin{aligned}
&\left[\int_0^{\overline{G}}\frac{\xi(G)}{(Q^{*}(G))^{2}}\left(Q_U^{c}-\frac{\lambda(G)-\lambda(\overline{G})}{Q^{*}(G)}Q_R^{c}\right)dG\right]d\overline{U}\\
&-\left[\frac{\xi(\overline{G})}{Q^{*}(\overline{G})}-\int_0^{\overline{G}}\frac{\xi(G)}{(Q^{*}(G))^{3}}Q_R^{c}dG\right]d\overline{G}\\
=&\left[\int_0^{\overline{G}}\frac{1}{Q^{*}(G)}\frac{\partial\xi(G)}{\partial\eta}dG\right]d\eta \qquad (2.25)
\end{aligned}
$$

where

$$
\begin{aligned}
d\overline{U} &= \left[\int_0^{\overline{G}}\lambda(G)\frac{\xi(G)}{Q^{*}(G)}dG\right]^{-1}\\
&\quad\left[\int_0^{\overline{G}}(R(G)-R_A)\frac{\partial\xi(G)}{\partial\eta}dG-\frac{\partial K}{\partial\eta}\right]d\eta. \qquad (2.26)
\end{aligned}
$$

Thus we arrived at the following propositions and corollary, of which the respective proofs follow.

Proposition 1. *At the optimum, marginal increase in the construction cost and the marginal change in aggregate differential land rent, evaluated at the current level of land rent, due to the transport infrastructure development must be equal.*

Proof: At the optimum, it is not possible to increase the household utility by altering the network design. Therefore, setting $\frac{d\overline{U}}{d\eta}=0$ in Equation (2.26) we get

$$
\frac{\partial K}{\partial\eta} = \int_0^{\overline{G}}(R(G)-R_A)\frac{\partial\xi(G)}{\partial\eta}dG. \qquad (2.27)
$$

The optimization requirement in Proposition 1 is quite interesting. It says that when city land is owned by the households, we do not need to consider the post-project

[9]See Appendix A.1.3 for detailed derivation of the optimality conditions.

land price to determine the optimum level of investment. By equating the marginal construction cost of the transport development with the marginal aggregate differential land rent evaluated in terms of the pre-project land price guarantees optimal investment.

Proposition 2. *At the optimum, the marginal change in construction cost due to a change in transport infrastructure design is equal to the marginal change in benefit derived from the change in land rent and reduction in transport cost measured in terms of the household living at the city boundary.*

Proof: At the optimum, we have from Equation (2.25),

$$\frac{d\overline{G}}{d\eta} = -\frac{\int_0^{\overline{G}} \frac{1}{Q^*(G)} \frac{\partial \xi(G)}{\partial \eta} dG}{\frac{\xi(\overline{G})}{Q^*(\overline{G})} - \int_0^{\overline{G}} \frac{\xi(G)}{(Q^*(G))^3} Q_R^c dG}. \tag{2.28}$$

We also have

$$d\overline{G} = -dD + \frac{d\Phi}{N} - \lambda\left(\overline{G}\right) d\overline{U}$$

which after applying optimality condition becomes

$$\frac{\partial K}{\partial \eta} = \frac{d\Phi}{d\eta} - N\frac{d\overline{G}}{d\eta}. \tag{2.29}$$

This simply says that marginal costs and benefits are equal at the optimum.

Corollary 1. *The optimal transport development requires that the disposable income of a household living at the city boundary does not change.*

Proof: Using Equation (2.12), the disposable income for a households living at the city boundary can be expressed by

$$y^d\left(\overline{G}\right) = wH - \overline{G} + \frac{\Phi}{N} - D.$$

Differentiating with respect to η and then using Proposition 2 yields

$$\frac{dy^d\left(\overline{G}\right)}{d\eta} = -\frac{d\overline{G}}{d\eta} + \frac{1}{N}\frac{d\Phi}{d\eta} - \frac{dD}{d\eta} = 0.$$

When lot size is a normal good, the global equilibrium utility for any household living in the city, depends on the land rent (in fact all prices) and disposable income.

However, the land rent for the people living at the city boundary is fixed which means that $\overline{U}$ depends only on his disposable income. Maximizing this person's utility is, thus, equivalent to maximizing his disposable income. This reveals that at the optimum, the disposable income of the households living at the city boundary is the maximum.

Proposition 3. *At the optimum, the marginal change in generalized transport cost at the city boundary due to a marginal change in infrastructural design is equal to the per capita marginal change in land value caused by the change in land rent.*

Proof: By taking total differentiation of Equation (2.17), we get

$$\frac{d\Phi}{d\eta} = \int_0^{\overline{G}} (R(G) - R_A) \frac{\partial \xi(G)}{\partial \eta} dG + \int_0^{\overline{G}} \xi(G) \frac{\partial R(G)}{\partial \eta} dG$$

which after using Proposition 1 becomes

$$\frac{d\Phi}{d\eta} = \frac{\partial K}{\partial \eta} + \int_0^{\overline{G}} \xi(G) \frac{\partial R(G)}{\partial \eta} dG.$$

Comparing this equation with Proposition 2 yields

$$\frac{d\overline{G}}{d\eta} = \frac{1}{N} \int_0^{\overline{G}} \xi(G) \frac{\partial R(G)}{\partial \eta} dG. \tag{2.30}$$

The marginal value of aggregate differential land rent has two components: one is due to change in city area, keeping land rent fixed and the other is due to change in land rent, keeping city area fixed. Proposition 1 is related to the former component and Proposition 3 is related to the later component. Figure 2.1 depicts the decomposition of marginal aggregate differential land rent and their relationships with the propositions.

It is to be noted that for residential land ownership,[10] Proposition 1 is independent

[10] We also derived the optimality conditions with absentee land ownership assumption. For details refer to Appendix A.1.5. The optimality conditions for Absentee landwonership case are as follows:

$$\frac{d\Phi}{d\eta} = -\frac{\partial K}{\partial \eta} + \int_0^{\overline{G}} (R(G) - R_A) \frac{\partial \xi(G)}{\partial \eta} dG, \tag{2.31}$$

$$\frac{d\overline{G}}{d\eta} = -\frac{1}{N} \frac{\partial K}{\partial \eta} = \frac{1}{N} \int_0^{\overline{G}} \xi(G) \frac{\partial R(G)}{\partial \eta} dG. \tag{2.32}$$

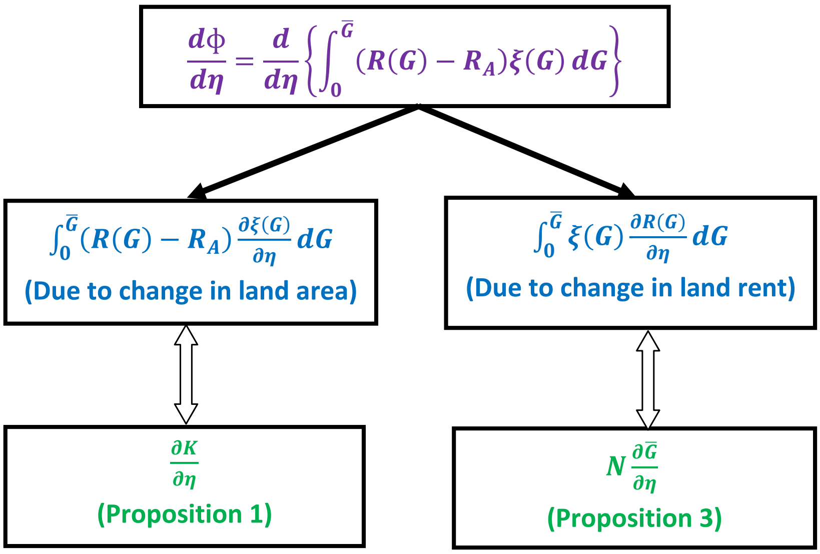

Figure 2.1. Decomposition of Marginal Aggregate Differential Land Rent.

of Propositions 2 and 3. Proposition 2 is independent of Propositions 1 and 3; however, Proposition 3 depends on Propositions 1 and 2. Thus, even if Proposition 3 does not hold, we have Propositions 1 and 2.

2.2.5. Relationships Between Samuelson Condition, Capitalization Hypothesis and Henry George Theorem

As shown in Kuroda (1994), in this section we will try to find the relationships between Samuelson condition, capitalization hypothesis and Henry George theorem. For residential land ownership case, the equilibrium condition of the household living at the city boundary is

$$\frac{NdV}{U_Z d\eta} = -N\frac{d\bar{G}}{d\eta} + \frac{d\Phi}{d\eta} - \frac{\partial K}{\partial \eta} + \frac{K}{N}\frac{dN}{d\eta} - \frac{\Phi}{N}\frac{dN}{d\eta}. \tag{2.33}$$

where V is the indirect utility function.[11] The above equation gives a monetary measure of the net marginal benefit derived from the infrastructural development. The first and second terms of the right side of the equation gives the marginal benefit derived from reduction in transport cost and increase in land rent respectively. The third term is the marginal construction cost. The forth term is the reduction in head tax and fifth term is the cost of having smaller lot size due to increase in population. Thus, the first three terms of the right hand side gives the net benefit due to marginal network development and last two terms gives the net benefit due to change in population. Therefore, the first three terms in the right side of the above equation are the net benefits for a infrastructural development and the last two terms are the net benefit due to change in population. The total net benefit is the left side of this equation.

Samuelson Condition

Samuelson condition states that for optimal public investment, the marginal benefit is equal to the marginal cost. From Equation (2.33), the condition for optimal transport investment is

$$-N\frac{d\overline{G}}{d\eta} + \alpha\frac{d\Phi}{d\eta} = \frac{\partial K}{\partial \eta}.$$

This is nothing but Proposition 2. Thus, our findings support Samuelson condition.

Henry George Theorem

Henry George theorem applies when population is optimum. In our case, population is fixed and need not necessarily be optimum. However, as long as Samuelson condition holds (which is always true for our case) we have from Equation (2.33),

$$\frac{dV}{dN} = \left(\frac{K}{N} - \frac{\Phi}{N}\right)\frac{U_Z}{N} = 0$$

[11] See Appendix A.1.4 for details.

which becomes

$$K = \Phi. \tag{2.34}$$

The above equation says that, the total investment should be equal to the aggregate differential land rent. Therefore, as long as Samuelson condition holds, we have $K = \Phi$. Thus, Henry George theorem holds in our case.

Capitalization Hypothesis

To hold the capitalization hypothesis, the project benefit needs to be equal to the land rent which means

$$-N\frac{d\overline{G}}{d\eta} + \frac{d\Phi}{d\eta} = \frac{d\Phi}{d\eta}$$

or

$$-N\frac{d\overline{G}}{d\eta} = 0.$$

This implies that capitalization hypothesis will hold, if the project does not affect the transport cost. Therefore, in general, (full) capitalization does not hold in our case. In other words, when considered simultaneously, the capitalization hypothesis does not bridge Henry George theorem and Samuelson condition in our model. Figure 2.2 represents the relationships between Henry George theorem, capitalization hypothesis and Samuelson condition. The dotted line mean that they capitalization hypothesis does not bridge the other two theorems.

Sufficiency Condition

So far we have derived the necessary conditions for optimality. However, these necessary conditions do not guarantee that the solution optimum. We will now focus on the condition that ensures sufficiency of the necessary conditions.

We can rewrite Equation (2.26) as

$$\frac{d\overline{U}}{d\eta} = \frac{-\frac{\partial K}{\partial \eta} + \int_0^{\overline{G}} (R(G) - R_A) \frac{\partial \xi(G)}{\partial \eta} dG}{\int_0^{\overline{G}} \lambda(G) \frac{\xi(G)}{Q^*(G)} dG}. \tag{2.35}$$

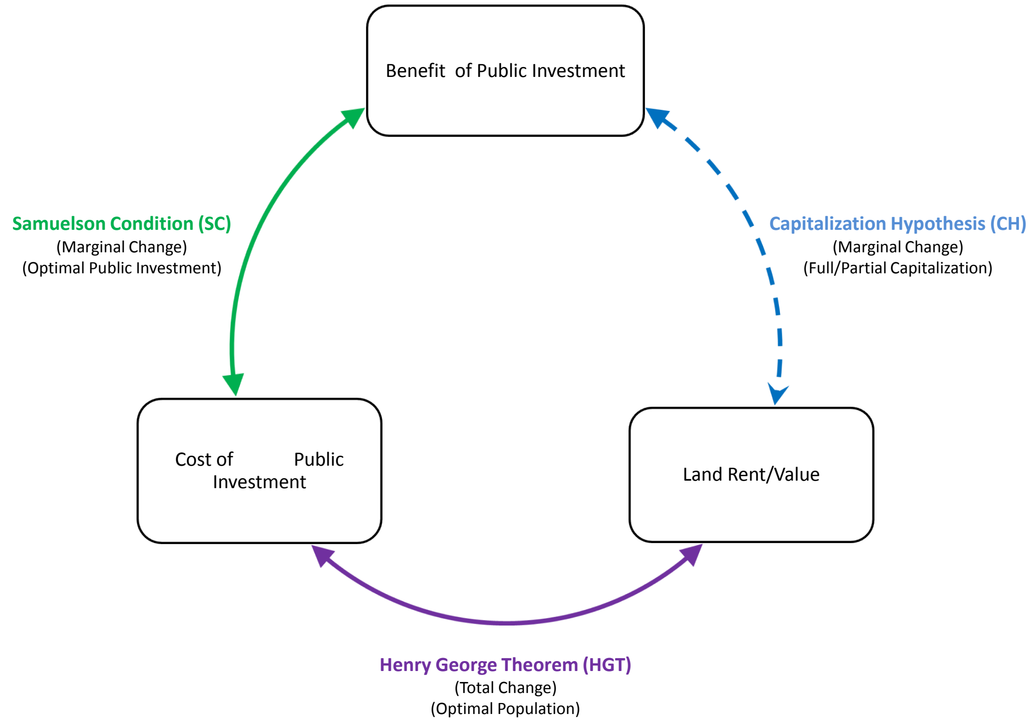

Figure 2.2. Relationships Between Henry George Theorem, Capitalization Hypothesis and Samuelson Condition.

Differentiating Equation (2.35) with respect to network geometry, η we get

$$\begin{aligned}\frac{d^2\overline{U}}{d\eta^2} &= \frac{\left[\int_0^{\overline{G}} \lambda(G)\frac{\xi(G)}{Q^*(G)}dG\right]}{\left[\int_0^{\overline{G}} \lambda(G)\frac{\xi(G)}{Q^*(G)}dG\right]^2}\left[-\frac{\partial^2 K}{\partial\eta^2}+\frac{d}{d\eta}\left\{\int_0^{\overline{G}}(R(G)-R_A)\frac{\partial\xi(G)}{\partial\eta}dG\right\}\right. \\ &- \left.\left\{-\frac{\partial K}{\partial\eta}+\int_0^{\overline{G}}(R(G)-R_A)\frac{\partial\xi(G)}{\partial\eta}dG\right\}\frac{d}{d\eta}\left\{\int_0^{\overline{G}}\lambda(G)\frac{\xi(G)}{Q^*(G)}dG\right\}\right].\end{aligned}$$

After using Proposition 1, the second term of the last square bracket of the R.H.S becomes zero. Therefore, we have

$$\frac{d^2\overline{U}}{d\eta^2} = \frac{-\frac{\partial^2 K}{\partial\eta^2}+\frac{d}{d\eta}\left\{\int_0^{\overline{G}}(R(G)-R_A)\frac{\partial\xi(G)}{\partial\eta}dG\right\}}{\int_0^{\overline{G}}\lambda(G)\frac{\xi(G)}{Q^*(G)}dG}.$$

Note that $\lambda(G)$ and $\xi(G)$ are always positive, which means the denominator must also be positive.

For sufficiency, we require $\frac{d^2\overline{U}}{d\eta^2} < 0$ which eventually implies

$$-\frac{\partial^2 K}{\partial\eta^2}+\frac{d}{d\eta}\left\{\int_0^{\overline{G}}(R(G)-R_A)\frac{\partial\xi(G)}{\partial\eta}dG\right\} < 0$$

or equivalently

$$\frac{\partial^2 K}{\partial \eta^2} > \frac{d}{d\eta}\left\{\int_0^{\overline{G}} \left(R\left(G\right) - R_A\right) \frac{\partial \xi\left(G\right)}{\partial \eta} dG\right\}. \tag{2.36}$$

Thus, the sufficient condition for optimality requires that the gradient of the marginal change in construction cost at the optimum should be strictly greater than the gradient of the marginal change in aggregate differential land rent $MADLR_C$ resulted from the change in city area.

We also have $\frac{\partial K}{\partial \eta} > MADLR_C$ for $\frac{d\overline{U}}{d\eta} < 0$; and $\frac{\partial K}{\partial \eta} < MADLR_C$ for $\frac{d\overline{U}}{d\eta} > 0$.

2.2.6. Some Possible Extensions

Issue of Polycentic City

We solved the optimization problem by converting a locational choice problem into a transport cost minimization problem. The location of a household can be identified by the generalized transport cost it pays. If there are multiple CBDs, and if we redefine the generalized transport cost such that it depends not only on the choice of routes but also on the choice of the CBDs. Thus we have

$$G\left(r,\theta\right) = \min_{h,j} G_{h,j}\left(r,\theta\right)$$

where j is such that $j = 1, ..., J$; J denotes the number of CBDs. Using this newly defined $G\left(r,\theta\right)$ in the previous problem, we can solve for the multiple CBD problem. However, in this case the underlying assumptions are that wage is same for every CBD and households can frequently change their employment location (CBD) in response to the minimum generalized transport cost.

Transport Infrastructure with Congestion

Now we discuss briefly how congestion can addressed within the model. We consider that the portion of transport infrastructure to be improved is prone to congestion. Let us denote $c_h\left(r,\theta\right)$ and $\tau_h\left(r,\theta\right)$ be the average user cost and per user toll respectively in route h. We also assume non-negativity of toll, i.e., $\tau_h\left(r,\theta\right) \geq 0$, and the $c_h(r,\theta) = c_h(N_h(r,\theta), \eta)$ where $N_h(r,\theta)$ is the number of travelers at location (r,θ) using route h. Then the generalized travel cost for route h is given by

$$\begin{aligned} G_h(r,\theta) &= c_h(N_h(r,\theta), \eta) + \tau_h(r,\theta) \\ &= G_h(r,\theta, N_h, \eta, \tau_h). \end{aligned}$$

At the equilibrium, the number of travelers at a given location will select the route that yields minimum generalized transport cost. Therefore, we have

$$G(r,\theta, N_h, \eta, \tau_h) = \min_h G_h(r,\theta, N_h, \eta, \tau_h)$$

for any h such that

$$\sum_{h=1}^{h} N_h = N.$$

We assume that transport authority will utilize toll to finance for its construction. However, the toll is not sufficient for its full finance. Therefore, it needs to impose an additional head tax, D as before. The transport authorities budget constraint then

$$K = DN + \tau N.$$

Other expressions in the previous problem need to be modified accordingly. Then, by maximizing the equilibrium utility of the household with respect to the toll and network geometry, we can determine their optimal values.

2.3. Conclusion

transport infrastructure development is a long-run supply-side management issue. Investments in such projects are generally lumpy and irreversible in nature. Therefore, careful consideration must be given while designing such facilities so that they can extract the maximum well-beings of the society. In this study we presented the optimality conditions of transport development that are applicable for multi-mode and multi-city-center options. We termed these as the "general" optimality conditions. Our general optimality conditions are applicable regardless of the number and type of travel modes or routes, the nature of development, the number of city-centers and the functional forms of the preference and construction technology. Thus, we considered a broader urban transport and land-use model compared to those in the existing literature. The transport infrastructure we presented in the model is an integral part of the city. Therefore, both transport and the city characteristics are endogenous to each other. We solved the model by converting the locational choice problem into a transport cost minimizing problem. In doing so we expressed all endogenous variables in terms of generalized transport cost. We re-identified the physical location of a household in the city by the minimum generalized transport cost needed to commute to the CBD (to the CBD that yields lowest transport cost for the case of polycentric city).

The general optimality condition (Proposition 1) for residential land ownership implies that, at the optimum, the marginal change in construction costs due to a change in network design is equal to the marginal change in aggregate differential land rent when current land prices are considered. Thus, for the optimality condition under residential land ownership case, we have to equate the marginal construction cost with the marginal aggregate differential land rent due to an increase in city area (while keeping land prices fixed).

According to Kanemoto (1984), the optimal length of railways should be such that the cost of building them is equal to the aggregate differential land rent. One might consider our findings in Proposition 1 similar to his findings especially, when one considers the length of railway track as the network geometry to be optimized. However, his findings distinctly differ from our findings in several capacities; first, our optimality condition equates the marginal change in construction cost with the marginal change in aggregate differential land rent measured at the current level of land price and thus we do not require considering the total marginal change in aggregate differential land rent. Second, our proposition demonstrates that we do not need to predict the change in land rent after the improvement; it is sufficient to calculate the change in aggregate differential land rent with respect to the current level of land rent which is a partial change. Thus, partial differentiation of the aggregate differential land rent with respect to network design is enough to find the optimum. Third, Kanemoto (1984) assumes a monocentric city where the shape of the city is given and that the railway network is radial, but our study derives the general optimality condition irrespective of the city shape, number of city centers, type of transport infrastructure, and number of transport modes available to the commuters.

The findings of this chapter differ from the capitalization hypothesis of public investment. For example, the capitalization hypothesis relates benefit of a public project and the land rent and says that for full capitalization both are equal. On the other hand, Propositions 1 particularly says that at that optimum, the marginal capital loss from investing in a project is equal to the marginal gain of increased total differential land rent evaluated at the current level of land rent. It shows us the extent to which we have to invest in order to ensure maximum social benefit.

Our optimality conditions support Samuelson condition and Henry George theorem. Samuelson condition considers optimal public investment and relates project benefit and cost at the marginal level. The findings of Propositions 2 is exactly similar to Samuelson condition in a sense that it exhibits a relationship between marginal project benefit and costs. On the other hand, Henry George theorem is applicable where population is optimum. In our closed city assumption, population is given and therefore need not necessarily be optimum. Yet, as long as Samuelson condition holds, the total marginal benefit is maximized when total construction cost is equal to the aggregate differential land rent. This is nothing but the Henry George theorem with residential land ownership.

Although the findings of this study are interesting, a more extensive work on this issue is necessary. We did not include the production sector (composite good or housing) of the economy. Moreover, we did not consider the growth of population over time. Even in our closed city assumption where migration is impossible, city population is likely to increase due to the natural growth rate of population. If the population grows at an exogenous rate, we can still incorporate this in the model by considering the discounted present values of the objective function and the constraints. In such a case, it is conjectured that discounted present value of the marginal construction cost would be equal to the discounted present value of the marginal aggregate differential land rent due to a change in city area.

Our model can be extended in various dimensions. Congestion externality can be considered in the model. The analysis can be extended for open city models too. One can also formulate the model under a general equilibrium framework, which may be tedious but practically useful for efficient allocation of funds in transport development. We hope future study will consider the above issues.

3. Optimal Capacity and Location of a Circumferential Highway in a Monocentric City

3.1. Introduction

The circumferential highway is a special type of transport infrastructure in the urban context. Most large cities have one or more circumferential highways that form a ring road around the city. One of the main purposes of the circumferential highway is to provide access to radial highways. Thus, the circumferential highway reduces extra traffic load on inner city streets and serves as a quicker and cheaper transport mode. In this way, circumferential highway affects the overall commuting cost, land rent, population density, and consumption pattern, which in turn influence the size and shape of the city. Looking from the other side, the city characteristics have considerable influence on the design of such a highway. This otherwise means that land-use and locational choice are needed to consider while planning for such a highway.

However, literature addressing such issues is limited. Most of the existing literature on monocentric city models deals with the locational and modal choices, transport network, land-use pattern, and the shape and size of a city only under the existence of direct city streets, radial highway or train transport. The purpose of this study is to find the optimal capacity and location of a circumferential highway and its influence on the shape and size of the city. Since most cities without a circumferential highway or without properly-designed circumferential highway face huge traffic problems - which is common in most developing cities - it is quite reasonable to search for an economic solution of a circumferential highway from land-use and locational theory. The difficulty arises from the fact that the analytical solution of the optimization problem of such a highway is not explicitly tractable. We, therefore, try to solve it by adopting a numerical simulation which helps us to look more deeply into the issue.

Among the city models in the existing literature, Alonso (1964) was the first who described the spatial characteristics of a monocentric city. Mills and de Ferranti (1971) illustrated the optimal distribution of land between residential and transport use by minimizing total transport cost (commuting cost plus land cost in transport) in a congested city. Solow (1973) presented an equilibrium analysis of urban land-use and residential redistribution and explained various internal features of the city. He provided a detailed comparative static analysis of city characteristics. The remarkable similarity among these models is that they consider direct routing from the residential location to the CBD. These direct routes are dense city streets; and there is no alternative mode of transport (e.g., highways) that provides faster travel. However, Anas and Moses (1979) considered radial highways together with the city streets and showed the impact of transport on land-use patterns in the city. They illustrated various types of city forms for various transport costs. Yet, all of these city models overlooked the existence of any circumferential highway in the urban transport network and its possible effects on the shape of the city.

Other than monocentric city models, there is some literature that discusses the design of circumferential highways. Among them Blumenfeld and Weiss (1970a) analyzed a circular city framework with two routes of travel, circumferential (ring) and direct routing. By locating the circumferential route at the edge of the city, they searched for the route with the minimum total travel time. In their another paper, Blumenfeld and Weiss (1970b) analyzed the optimal location of a second circumferential route while the first one was fixed at the city boundary. By minimizing the average travel distance, Pearce (1974) determined the optimal location of a ring road. This study also considered two types of travel route: direct routes, and a combination of direct and circumferential (ring) routes. However, this category of literature considered pure transport models and did not taken into account the locational choice and land value induced by the transport infrastructure development. The prime focused was on minimizing average travel distance or total travel time while considering city size and shape as exogenous; furthermore, they did not consider radial highways. But, a public project is likely to have influence on the existing pattern of city which will create a new equilibrium. This new equilibrium will be associated with new land rent and new locational pattern (Lind, 1973).

Given these limitations of the existing literature, we mainly focus on the economic design of the circumferential highway by considering land-use effects and locational redistribution. More specifically, we will try to answer the following questions: (a) what would be the optimal capacity and location of a circumferential highway; and (b) how does it affect the size and shape of the city?

In doing so, we include a circumferential highway in a model of a two-dimensional monocentric closed city with dense city streets and a radial highway system. By a combination of these three travel modes, the commuters now have three alternative routes to reach the CBD. The study then investigates the special equilibrium and optimal conditions of a circumferential highway with fixed lot size and absentee land ownership assumptions. Finally, we provide a numerical simulation to solve the optimization problem.

It is true that the monocentric city models are criticized as they do not replicate real world cities, yet there are many cities that have only one employment center. Moreover, understanding the features of monocentric city models helps us to analyze the characteristics of the more complex polycentric cities. Analyzing circumferential highway in an endogenous city feature provides interesting insight into the investment in such an urban transport project. Through this analysis we elucidate the optimal location and capacity of a circumferential highway, the resulting shape and size of the city as well as market boundaries and rent and density gradients.

This chapter includes four sections. In Section 3.2, we develop an urban transport model and describe the size and shape of the city and the optimality conditions of transport development under an assumption of equal lot size. Section 3.3 presents a numerical simulation for the optimal design features of a circumferential highway. We conclude in Section 3.4 by providing some interesting dimensions toward which the model can be extended in the future. The proof and necessary derivations have been summarized in the appendix.

3.2. The Model

In this section we consider a specific model of a two-dimensional monocentric closed city on a featureless plain. The city has three types of travel modes: (a) dense city streets on which commuters travel either radially or circumferentially (hereafter, Mode 1), (b) radial highways extending from the CBD (Mode 2), and (c) a circumferential highway (Mode 3).[1] We assume that the land allocated for highways and roads is negligible, yet this does not necessarily mean that highways have no width. In fact, the width of each highway is assumed to be uniform throughout the city and is exogenous for radial highways but is endogenous for the circumferential highway. That is, the widths of circumferential and radial highways differ. In reality, geographical limitations may preclude construction of a complete circular highway. However we assume that the circumferential highway is completely circular in shape and its capacity and the radius from the CBD, which we termed "location", is endogenously determined. As in most city models, commuters can only travel radially or circumferentially along the highways and streets. For analytical simplicity, traveling in any other direction is ignored.

Traveling on city streets is slower than highways while highways cannot be reached

[1] The model in this section closely follows that of Anas and Moses (1979) with the additional feature of a circumferential highway.

without city streets. By using these three modes, the commuters now have three alternative choices of routes to commute to the CBD. These three routes include (a) driving directly to the CBD on city streets (hereafter, Route 1), (b) first driving on city streets circumferentially to the nearest radial highway and then on the radial highway to the CBD (Route 2), and (c) first driving on the city streets in a radial direction to reach the circumferential highway and then using the radial highway to the CBD (Route 3). This implies that Route 1 uses only Mode 1, Route 2 uses Modes 1 and 2, and Route 3 uses all three modes. We illustrate this in Figure 3.1.

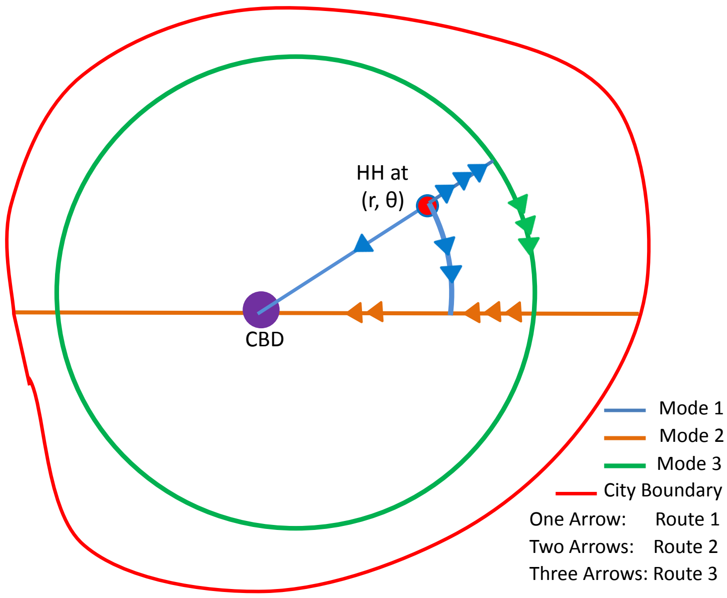

Figure 3.1. Modes and Routes.

We assume that radial highways are already constructed and their construction cost is sunk; and thereafter we focus on the optimal design of the circumferential highway. That is, in our model the number, say n and width w_r of the radial highways are exogenously given, and they extend sufficiently beyond the city boundary.[2] The implication of this assumption is that for any location of the circumferential highway, it can be connected with the radial highways. We further assume that all radial highways are evenly spaced in terms of angle, meaning they are separated by

[2]For a closed city without inter-city traffic, the optimal length of the radial highway may well be shorter than the distance between CBD and the city boundary; however, we do not consider such a case in this work.

an angle of $2\pi/n$. However, the model can easily be extended when radial highways are not symmetrically placed. Let us denote any location within the city by (r,θ) in polar coordinates. The city is therefore symmetric with its center being the CBD at location $(r,\theta) = (0,0)$. The circumferential highway with radius r_c and width[3] w_c has its center at the CBD . The width of the circumferential highway is assumed to be the same everywhere. Throughout the study the terms “width” and "capacity" are used interchangeably.

3.2.1. Structure of Generalized Transport Costs

We assume that the generalized transport cost at any location (r,θ) denoted by $G(r,\theta)$, includes round-trip time and monetary cost. To avoid complicacy in calculation, we ignore the fixed cost of travel. Therefore, we can express the generalized transport costs for the three routes in terms of the mode specific unit costs as follows:

$$G_1(r,\theta) = g_s r \tag{3.1}$$

$$G_2(r,\theta) = g_s r\theta + g_r r \tag{3.2}$$

$$G_3(r,\theta) = g_s |r_c - r| + g_c r_c \theta + g_r r_c. \tag{3.3}$$

where G_1, G_2, and G_3 are the generalized transport cost along Routes 1, 2, and 3, and g_s, g_r, and g_c are the round-trip transport costs (time and monetary cost) per unit distance along Modes 1, 2, and 3 respectively, with $g_s > g_r \geq g_c$ for an optimal circumferential highway. We assume that commuters travel either radially or circumferentially using either highway or city street, and that the transport cost increases linearly with distance traveled.

[3]As we do not consider congestion in our model, one might think the term “width” is virtually meaningless, especially for the case of highways. However, even in the absence of congestion width does has influence on the free flow speed. The reason can be explained by the fact that when there are many vehicles in the road, drivers require extra care to drive which increases effects the travel time and travel cost.

3.2.2. Shape of the City

In this subsection we establish the city boundary and the market boundary in order to obtain the city area $\Xi(G)$ and $\xi(G)$ where

$$\xi(G) = \frac{d\Xi(G)}{dG}. \tag{3.4}$$

We define market area of Route i as the area in the city from where traveling to the CBD through Route i yields the lowest generalized transport cost. The market boundary between, for example, Routes 1 and Route 2 is the location where a household is equally likely to use either of two different routes. Due to the inclusion of the circumferential highway the shape of the city is likely to be different from that in Anas and Moses (1979). We determine the shape of the city by fixing the city boundary and the market boundaries for all routes.

City Boundaries

A city boundary is a location where the maximum of the residential bid rents for the three routes are equal to the agricultural land rent R_A. The agricultural land rent is fixed and equal for every location outside the city. A household living at the city boundary pays agricultural land rent for residential use. This is the minimum possible land rent that a household pays. This otherwise means that the households at the city boundary pay the highest generalized transport cost, denoted by $\overline{G}$, such that $\overline{G} = G\left(\overline{r}, \overline{\theta}\right)$ where $\left(\overline{r}, \overline{\theta}\right)$ is the polar representation of locations on the city boundary. $\overline{G}$ is the same for every location along the city boundary. In this model the city boundary is endogenously determined.

City Boundary with Market Area 1: Let us denote the city boundary with Market Area 1 by $\overline{r}_1$. At the boundary, we have $\overline{G} = \overline{G}_1 = g_s\overline{r}_1$,which yields

$$\overline{r}_1 = \frac{\overline{G}}{g_s}. \tag{3.5}$$

Thus the city boundary of Market Area 1 is an arc of a circle with radius $\overline{G}/g_s$.

City Boundary with Market Area 2: The city boundary with Market Area 2, denoted by $\overline{\theta}_2(r)$, is such that $\overline{G} = \overline{G}_2 = g_s\overline{\theta}_2 r + g_r r$, and the corresponding $\overline{r}_2(\theta)$ is

$$\overline{r}_2(\theta) = \frac{\overline{G}}{g_s\theta + g_r} \tag{3.6}$$

for $r \in \left[\overline{G}/g_s, \overline{G}/g_r\right]$.

City Boundary with Market Area 3: Depending on the value of $\overline{G}$, the city boundary of Market Area 3 can exist either outside or inside of the circumferential highway.

City Boundary with Market Area 3 Outside of the Circumferential Highway: The city boundary of Market Area 3 outside of the circumferential highway, denoted by $\overline{\theta}_3^{out}$, is such that $\overline{G} = \overline{G}_3 = g_s(r - r_c) + g_c r_c \overline{\theta}_3^{out} + g_r r_c$ and the corresponding $\overline{r}_3^{out}(\theta)$ is

$$\overline{r}_3^{out}(\theta) = \frac{\overline{G} + g_s r_c - g_r r_c - g_c r_c \theta}{g_s} \tag{3.7}$$

for $r \in \left[r_c, r_{23A}^{out}\right]$ where r_{23A}^{out} is a point such that[4]

$$r_{23A}^{out} \equiv \overline{r}_2\left(\theta_{23A}^{out}\right) = \overline{r}_3^{out}\left(\theta_{23A}^{out}\right).$$

City Boundary with Market Area 3 Inside of the Circumferential Highway: The presence of the city boundary with Market Area 3 inside of the circumferential highway implies that optimal circumferential highway is partially utilized. Let us denote the city boundary inside of the circumferential by $\overline{\theta}_3^{in}$. Therefore, we have, $\overline{G} = \overline{G}_3 = g_s(r_c - r) + g_c r_c \overline{\theta}_3^{in} + g_r r_c$ and the corresponding $\overline{r}_3^{in}(\theta)$ is

$$\overline{r}_3^{in}(\theta) = \frac{-\overline{G} + g_s r_c + g_r r_c + g_c r_c \theta}{g_s} \tag{3.8}$$

for $r \in \left[r_c, r_{23A}^{in}\right]$ where r_{23A}^{in} is a point such that

$$r_{23A}^{in} \equiv \overline{r}_2\left(\theta_{23A}^{in}\right) = \overline{r}_3^{in}\left(\theta_{23A}^{in}\right).$$

[4]See Appendix A.2.1.

Intersection of the Inner and Outer City Boundaries of Market Area 3: When Market Area 3 has a city boundary both inside and outside of the circumferential highway, there is a point location that intersects these two boundaries and locates on the circumferential highway. Let $\left(r_c, \theta_{3A}^{out-in}\right)$ be the intersection of $\overline{\theta}_3^{out}$ and $\overline{\theta}_3^{in}$ which can be determined by replacing r_3^{out} with r_c in Equation (3.7).[5] Thus we have

$$\theta_{3A}^{out-in} = \frac{1}{g_c r_c}\left[\overline{G} - g_s\left(r_c - r_c\right) - g_r r_c\right] = \frac{1}{g_c r_c}\left[\overline{G} - g_r r_c\right].$$

Market Boundaries

The market boundary between two adjacent routes is defined as the location where the generalized transport costs are same for any route. Thus, the city dwellers living at the market boundary are equally likely to choose either of the routes adjacent to a market boundary.

Market Boundary between Market Areas 1 and 2: The market boundary between Market Areas 1 and 2 is the set of locations, denoted by $\left(r_{12}, \theta_{12}\right)$, where $G_1 = G_2$. This reveals that θ_{12} is constant over r such that

$$\theta_{12} = \frac{g_s - g_r}{g_s}. \tag{3.9}$$

Market Boundary between Market Areas 1 and 3: Note that if the value of θ is greater than a certain threshold value, households will use the city street directly to CBD as it is the cheapest travel mode to them. We denote such a threshold value of θ by θ_{13}, which is obtained by equating $G_1 = G_3$ or equivalently

$$\theta > \theta_{13} = \frac{g_s - g_r}{g_c}. \tag{3.10}$$

For $\theta \in [\theta_{12}, \theta_{13}]$, the market boundary between Maket Areas 1 and 3 that exists only inside of the circumferential highway. This is denoted by $r_{13}^{in}\left(\theta\right)$ and is ob-

[5] The subscript "A" is used to denote a point location.

tained by equating $G_1 = G_3$ such that

$$r_{13}^{in}(\theta) = \frac{g_s + g_c\theta + g_r}{2g_s} r_c, \tag{3.11}$$

and corresponding θ_{13}^{in} is

$$\theta_{13}^{in} = \frac{2g_s r - g_s r_c - g_r r_c}{g_c r_c}.$$

It is easy to see that we have $\theta_{13}^{in} \leq \theta_{13}$.[6]

By replacing $g_s r$ with $\overline{G}$ we get $\theta_{13A}^{in}\left(\overline{G}\right)$ at the city boundary which is

$$\theta_{13A}^{in} = \frac{2\overline{G} - g_s r_c - g_r r_c}{g_c r_c}. \tag{3.12}$$

Market Boundary between Market Areas 2 and 3: The market boundary between Market Areas 2 and 3 has two separate segments, one of which is inside of the circumferential highway and another of which is outside of it. We express each of these boundaries in terms of r, which is the function of θ with domain $\theta \in [0, \theta_{12}]$.

Market Boundary between Market Areas 2 and 3 Inside of the Circumferential Highway: The market boundary between Market Areas 2 and 3 inside of the circumferential highway, denoted by $\theta_{23}^{in}(r)$, is such that $G_2 = G_3$ with $r < r_c$. Therefore, we have

$$\theta_{23}^{in}(r) = \frac{(g_s + g_r)(r_c - r)}{g_s r - g_c r_c}$$

and corresponding

$$r_{23}^{in}(\theta) = \frac{\theta g_c + g_s + g_r}{\theta g_s + g_s + g_r} r_c. \tag{3.13}$$

Market Boundary between Market Areas 2 and 3 Outside of the Circumferential Highway: The market boundary between Market Areas 2 and 3 outside of the circumferential highway, denoted by θ_{23}^{out}, is such that $G_2 = G_3$ with $r > r_c$. This

[6]From the above definition, $\theta_{13}^{in}(G) = \frac{2G - g_s r_c - g_r r_c}{g_c r_c} = \frac{2}{g_c}\left(\frac{G}{r_c} - g_s\right) + \theta_{13}$. For $r \leq r_c$, we have $\frac{G}{r_c} - g_s \leq 0$ and therefore $\theta_{13}^{in}(G) \leq \theta_{13}$.

gives

$$\theta_{23}^{out}(r) = \frac{(g_s-g_r)(r-r_c)}{g_s r - g_c r_c}$$

with corresponding

$$r_{23}^{out}(\theta) = \frac{\theta g_c - g_s + g_r}{\theta g_s - g_s + g_r} r_c. \tag{3.14}$$

3.2.3. Area of the City

Let $\Xi\left(\overline{G}\right)$ be the total area of the city, namely from the CBD (which is a point in our case) to the city boundary where $G = \overline{G}$.

Depending on the value of r_c relative to the generalized transport cost $\overline{G}$, we have four different kinds of city shapes, each of which we label a "regime". Regime 1 is defined as the city shape where $r_c > \frac{\overline{G}}{g_r}$. In this regime, Route 3 is totally out-bidden by Routes 1 and 2. The shape of the city for Regime 1 is similar to that described by Anas and Moses (1979) for the case of equal fixed costs. Regime 2 corresponds to the city shape such that[7] $\frac{2g_s\overline{G}}{g_s(g_s+g_r)+g_c(g_s-g_r)} < r_c \leq \frac{\overline{G}}{g_r}$. Regime 3 is defined where $\frac{\overline{G}}{g_s} < r_c \leq \frac{2g_s\overline{G}}{g_s(g_s+g_r)+g_c(g_s-g_r)}$. Regime 4 is the area such that $r_c \leq \frac{\overline{G}}{g_s}$. For $\frac{\pi}{n} \leq \theta_{13}$, Market Area 1 does not have any city boundary.

The land area of the city for each of the above mentioned regimes is derived below. It is important to note that the determination of the area of the city mostly depends on which regime it belongs to.

Area of City for Regime 1

In this regime, we do not have Market Area 3. The radius of the circumferential highway is so large that Route 3 costs more than Routes 1 and 2, meaning that households have no incentive to use the circumferential highway. Thus, the city only has Market Areas 1 and 2.

[7] The lower limit of r_c in Regime 2 can be determined by equating $\theta_{13A}^{in}|_{\overline{G}=G} = \overline{\theta}_{13}^{in}$ and θ_{12} from Equations (3.12) and (3.9), respectively. This gives $\frac{2G-g_s r_c - g_r r_c}{g_c r_c} = \frac{g_s-g_r}{g_s}$, from which we get $r_c = \frac{2g_s\overline{G}}{g_s(g_s+g_r)+g_c(g_s-g_r)}$.

(Half of the market area shown)

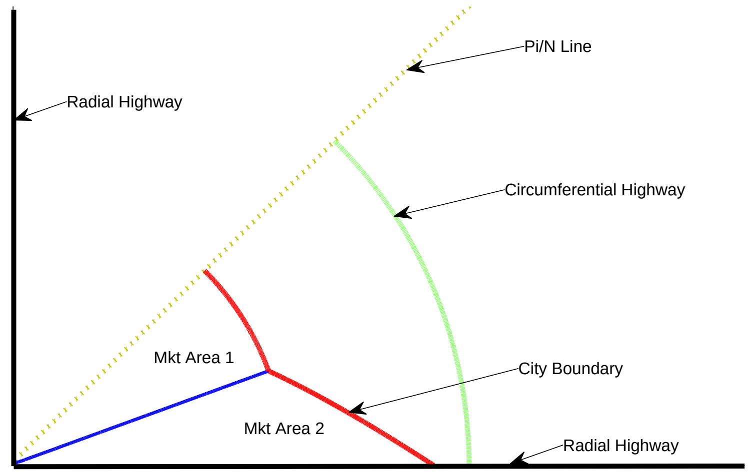

Figure 3.2. City Shape: Regime 1.

Area of Market 1: Market Area 1 is just an arc with radius $\overline{G}/g_s$, and the range of angle is $[min(\theta_{12,}\pi/n),\pi/n]$. Let us define r_1 such that, $r_1 = \bar{r}_1|_{\overline{G}=G}$. This area, denoted by $\Xi_1(\overline{G})$, is therefore

$$\Xi_1(\overline{G}) = 2n\left[\frac{1}{2}\int_{\theta_{12}}^{\pi/n}(r_1)^2 d\theta\right] = n\int_{\theta_{12}}^{\pi/n}(r_1)^2 d\theta.$$

Area of Market 2: Let the area of Market 2 in Regime 1 be $\Xi_2(\overline{G})$ and r_2 such that, $r_2 = \bar{r}_2|_{\overline{G}=G}$. This area is then

$$\Xi_2(\overline{G}) = 2n\left[\tfrac{1}{2}\int_0^{\theta_{12}}(r_2)^2 d\theta\right] = n\int_0^{\theta_{12}}(r_2)^2 d\theta.$$

The total city area for Regime 1, $\Xi(\overline{G})$, which is the sum of $\Xi_1(\overline{G})$ and $\Xi_2(\overline{G})$, is thus

$$\Xi(\overline{G}) = n\int_{\theta_{12}}^{\frac{\pi}{n}}(r_1)^2 d\theta + n\int_0^{\theta_{12}}(r_2)^2 d\theta. \tag{3.15}$$

Area of City for Regime 2

If we gradually decrease the radius of the circumferential highway from that in Regime 1, a new area will emerge around the circumferential highway, indicating that some of the households now use Route 3. This new area is Market Area 3.

Thus, in Regime 2, the city have three market areas namely, Market Areas 1, 2 and 3. Market Area 3 becomes attractive to some of the households previously using Market Area 2. Another important feature is that when the number of radial highways is less,[8] some portion of the circumferential highway are not fully utilized.

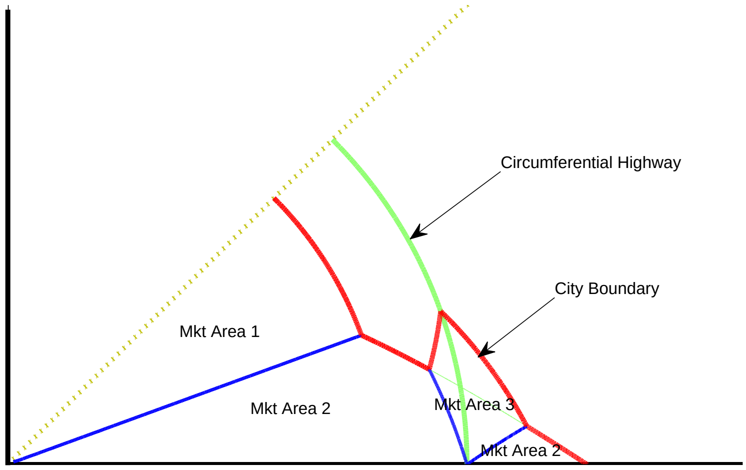

Figure 3.3. City Shape: Regime 2.

The total area of the city for Regime 2 is[9]

$$\begin{aligned}\Xi\left(\overline{G}\right) &= n\int_{\theta_{12}}^{\frac{\pi}{n}}(r_1)^2\,d\theta + n\int_0^{\theta_{12}}(r_2)^2\,d\theta + n\int_{\theta_{23A}^{out}}^{\theta_{3A}^{out-in}}\left(r_3^{out}\right)^2 d\theta \\ &\quad -n\int_{\theta_{23A}^{out}}^{\theta_{23A}^{in}}(r_2)^2\,d\theta - n\int_{\theta_{23A}^{in}}^{\theta_{3A}^{out-in}}\left(r_3^{in}\right)^2 d\theta.\end{aligned} \tag{3.16}$$

Area of City for Regime 3

In Regime 3, the circumferential highway is relatively closer to the CBD and therefore, smaller than that in Regime 2. In this regime the circumferential highway attracts some of the households from both Market Areas 1 and 2. Under the same condition as noted in Regime 2, the circumferential highway away from the radial highways remains unused.

[8] When $\frac{\pi}{n} > \theta_{3A}^{out-in}$, we have $n < \frac{\pi}{\theta_{3A}^{out-in}}$ and under such condition, there is an unused portion of circumferential highway.

[9] See Appendix A.2.2 for details.

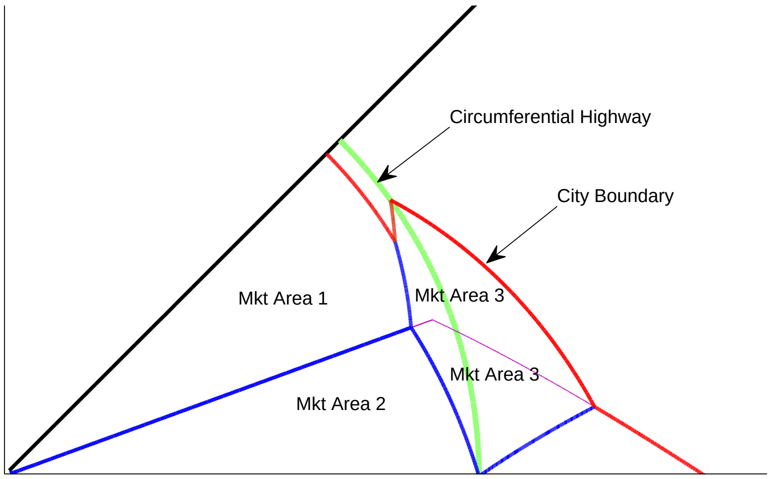

Figure 3.4. City Shape: Regime 3.

We have the total city area for Regime 3

$$\begin{aligned} \Xi\left(\overline{G}\right) \quad = \quad & n\int_{\theta_{13A}^{in}}^{\frac{\pi}{n}} \left(r_1\right)^2 d\theta + n\int_{0}^{\theta_{23A}^{out}} \left(r_2\right)^2 d\theta \\ & + n\int_{\theta_{23A}^{out}}^{\theta_{3A}^{out-in}} \left(r_3^{out}\right)^2 d\theta - n\int_{\theta_{13A}^{in}}^{\theta_{3A}^{out-in}} \left(r_3^{in}\right)^2 d\theta \end{aligned} \tag{3.17}$$

where

$$\overline{\theta}_{13A}^{in} = \frac{2\overline{G} - g_s r_c - g_r r_c}{g_c r_c} \tag{3.18}$$

with $(\overline{r}_{13A}^{in}, \overline{\theta}_{13A}^{in}(r))$ be the location where the inner city boundary of Market Area 3 intersects Market Area 1 such that $\overline{r}_3^{in} = \overline{r}_1$.[10]

Area of City for Regime 4

In this regime the radius of the circumferential highway is located closer to the CBD than in Regimes 1, 2 and 3. One important feature of this regime is that as long as $\frac{\pi}{n} > \theta_{13}$, Market Area 1 does have city boundary and therefore people living near to this boundary, use city streets directly, even though they cross the circumferential highway on their way. On the other hand, for $\frac{\pi}{n} \leq \theta_{13}$, the circumferential highway is fully utilized and Market Area 1 does not have any city boundary. Market Area 3

[10] See Appendix A.2.2 for details.

grabs some households from Market Areas 1 and 2.

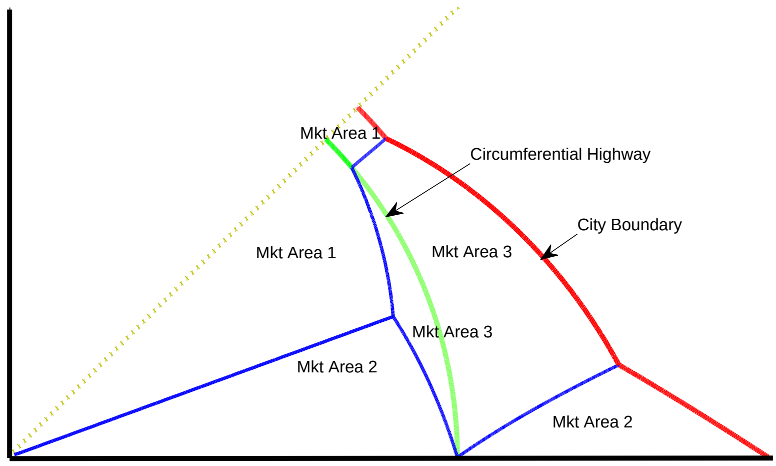

Figure 3.5. City Shape: Regime 4.

The total city area for Regime 4 is[11]

$$\Xi\left(\overline{G}\right) = n\int_{\theta_{13}}^{\frac{\pi}{n}} (r_1)^2\, d\theta + n\int_0^{\theta_{23A}^{out}} (r_2)^2\, d\theta + n\int_{\theta_{23A}^{out}}^{\theta_{13}} \left(r_3^{out}\right)^2 d\theta. \tag{3.19}$$

The area of the city varies with the location of the circumferential highway (for a given width). Essentially, these areas are from the supply side of the land which we summarize in Table 3.1. By differentiating, the expressions for the total city

Table 3.1. Formula for Land Area Supply for Different Regimes.

Regime	$\Xi(G)$
1	$n\int_{\theta_{12}}^{\frac{\pi}{n}} (r_1)^2\, d\theta + n\int_0^{\theta_{12}} (r_2)^2\, d\theta$
2	$n\int_{\theta_{12}}^{\frac{\pi}{n}} (r_1)^2\, d\theta + n\int_0^{\theta_{12}} (r_2)^2\, d\theta + n\int_{\theta_{23A}^{out}}^{\theta_{3A}^{out-in}} \left(r_3^{out}\right)^2 d\theta$ $-n\int_{\theta_{23A}^{out}}^{\theta_{23A}^{in}} (r_2)^2\, d\theta - n\int_{\theta_{23A}^{in}}^{\theta_{3A}^{out-in}} \left(r_3^{in}\right)^2 d\theta$
3	$n\int_{\theta_{13A}^{in}}^{\frac{\pi}{n}} (r_1)^2\, d\theta + n\int_0^{\theta_{23A}^{out}} (r_2)^2\, d\theta$ $+n\int_{\theta_{23A}^{out}}^{\theta_{3A}^{out-in}} \left(r_3^{out}\right)^2 d\theta - n\int_{\theta_{13A}^{in}}^{\theta_{3A}^{out-in}} \left(r_3^{in}\right)^2 d\theta$
4	$n\int_{\theta_{13}}^{\frac{\pi}{n}} (r_1)^2\, d\theta + n\int_0^{\theta_{23A}^{out}} (r_2)^2\, d\theta + n\int_{\theta_{23A}^{out}}^{\theta_{13}} \left(r_3^{out}\right)^2 d\theta$

areas for different regimes stated in Equations (3.15), (3.16), (3.17) and (3.19) with respect to G, we can get the expressions of $\xi(G)$.

[11] See Appendix A.2.2 for details.

3.2.4. Household's Problem and the Spatial Equilibrium

In this section, we briefly discuss the household utility maximization problem and the spatial equilibrium. We assume that land is owned by the absentee landowners. Household's preference is such that it can be represented by the Leontief utility function. One special characteristics of the Leontief utility function is that the price elasticity of demand is zero. Thus, the composite good Z, residential land Q and leisure L are consumed in a fixed proportion. This further implies that lot size is uniform for every household at every location and lot size is independent of land rent (See, for example, Kanemoto, 1980). Therefore, finding primary results does not depend on the existence of substitution effects among goods.

The utility of the household therefore

$$U = \min\{\alpha Z, \beta Q, \gamma L\}$$

where, (α, β, γ) is the fixed consumption vector with $\alpha \geq 0$, $\beta \geq 0$, and $\gamma \geq 0$. Compensated demand functions are therefore at the equilibrium

$$Z^c = \frac{U}{\alpha}, Q^c = \frac{U}{\beta}, L^c = \frac{U}{\gamma}. \tag{3.20}$$

Due to the absentee land ownership assumption, the aggregate differential land rent goes to outside of the city. Therefore, unlike the residential land ownership case as mentioned in Chapter 2, city dwellers do not receive any share of aggregate differential land rent. At the spatial equilibrium, all households achieve the same level of utility $\overline{U}$, meaning that they consume the same bundle of goods. From Equation (2.11) of Chapter 2, the equilibrium land rent function expressed in terms of generalized transport cost is

$$R(G) = \frac{1}{Q^*(G)}\left[y^d(G) - Z^*(G) - wL^*(G)\right], \tag{3.21}$$

with disposable income $y^d(G)$ such that $y^d(G) = wH - D - \frac{\Phi}{N} - G$. Here w is the wage rate, H is the total time endowment to each household, D is the head tax to finance the circumferential highway, Φ is the aggregate differential land rent, N is the total number of households, and the superscript * denotes the equilibrium value. By substituting compensated demand functions from Equation (3.20) into Equation (3.21), the land rent function becomes

$$R(G) \quad = \quad \beta\frac{wH - D - G}{\overline{U}} - \frac{\beta}{\alpha} - \frac{w\beta}{\gamma}.$$

The household at the city boundary paying generalized transport cost $\overline{G}$ faces the agricultural land rent for its residential use, namely $R\left(\overline{G}\right) = R_A$. Using these we have

$$\overline{U} = \frac{wH - D - \overline{G}}{\frac{1}{\alpha} + \frac{R_A}{\beta} + \frac{w}{\gamma}}, \tag{3.22}$$

$$R(G) = \frac{\overline{G} - G}{wH - D - \overline{G}}\left(\frac{\beta}{\alpha} + R_A + \frac{w\beta}{\gamma}\right) + R_A, \tag{3.23}$$

and

$$Q^* = \frac{wH - D - \overline{G}}{\frac{\beta}{\alpha} + R_A + \frac{w\beta}{\gamma}}. \tag{3.24}$$

At the spatial equilibrium, the residential land demand should be equal to the area of the city. Therefore,

$$\Xi\left(\overline{G}\right) \quad = NQ^* = \quad N\frac{wH - D - \overline{G}}{\frac{\beta}{\alpha} + R_A + \frac{w\beta}{\gamma}} \tag{3.25}$$

which implicitly defines $\overline{G}$. Note here that the aggregate differential land rent Φ can be expressed as[12]

$$\begin{aligned} \Phi \quad &= \quad \int_0^{\overline{G}} \left[R(G) - R_A\right]\xi(G)\,dG \\ &= \quad N\left[\overline{G} - \int_0^{\overline{G}} \frac{\xi(G)}{\Xi\left(\overline{G}\right)} G\,dG\right] \end{aligned} \tag{3.26}$$

where the terms in the square brackets is the difference between the maximum and the average transport costs. This, in line with the Henry George theorem, implies that the aggregate differential land rent and the aggregate differential transport cost are equal (See Arnott, 2004). If all households were living out of the city, then the

[12]See Appendix A.2.3.

daily aggregate transport cost for commuting to the city center would be $N\overline{G}$. However, since they live in the city, the actual aggregate transport cost is $N\int_0^{\overline{G}} \frac{\xi(G)}{\Xi(\overline{G})} GdG$. Thus the total savings in transport cost is $N\left[\overline{G} - \int_0^{\overline{G}} \frac{\xi(G)}{\Xi(\overline{G})} GdG\right]$ which is reflected in the (higher) land rent.

Although it is not possible to express $\overline{U}$,$\overline{G}$, or Φ explicitly in terms of parameters and D, we can solve them numerically by using Equations (3.22), (3.25), and (3.26).

3.2.5. Planners Problem and the Optimality Conditions

Let us define η to be the vector of the characteristics of the circumferential highway such that $\eta = \{r_c, w_c\}$. The construction cost of the circumferential highway K is therefore $K = K(\eta)$ and the budget constraint of the transport authority is

$$K = DN.$$

Thus the TA's problem is to maximize $\overline{U}$ with respect to η subject to the budget constraint mentioned above.

For the absentee land ownership case under general settings, we have shown the optimality conditions in Equations (2.31) and (2.32) of Chapter 2 which are

$$\frac{d\Phi}{d\eta} = -\frac{\partial K}{\partial \eta} + \int_0^{\overline{G}} (R(G) - R_A) \frac{\partial \xi(G)}{\partial \eta} dG$$

and

$$\frac{\partial K}{\partial \eta} = -N\frac{d\overline{G}}{d\eta}.$$

These conditions are applicable for any type of infrastructural development and are therefore applicable for the circumferential highway as well. However, in this study we will solve the optimization problem numerically at the later part of this section.

3.2.6. Strict Increase in Utility

To this point we have focused on the spatial equilibrium and optimal conditions for a new circumferential highway. Nonetheless, it may be the case that the above solution yields less utility than without having a circumferential highway. This may happen when we consider only local maxima. This situation is illustrated in Figure 3.6, in which the equilibrium level of utility without a circumferential highway is greater than with an (optimal) circumferential highway. Therefore, it is clearly suboptimal. In this subsection we pin down the condition that ensures an absolute higher level of utility after having a circumferential highway.

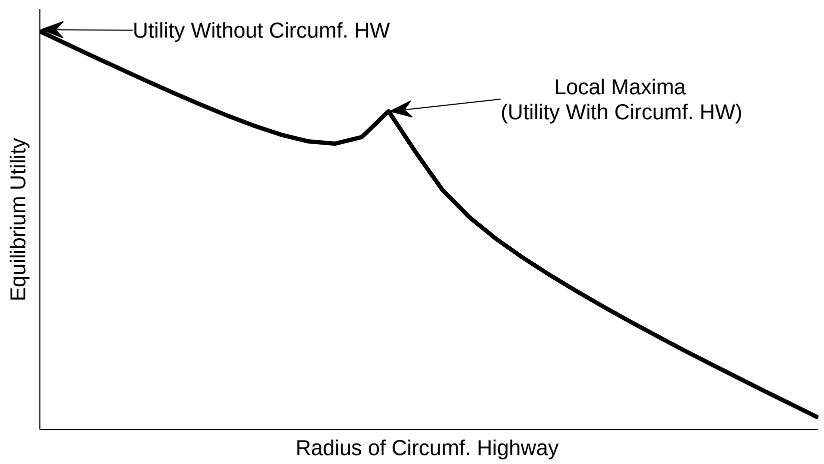

Figure 3.6. Local Maxima.

We have the equilibrium utility with a circumferential highway in Equation (3.22). Let Ξ_{wo}, and $\overline{G}_{wo}$ be the spatial equilibrium values of the land area, and generalized transport cost, respectively, prior to the construction of circumferential highway. In such a case the only means of transport are city streets and radial highways. Households do not need to pay head tax D. Therefore

$$\overline{U}_{wo} = \frac{wH - \overline{G}_{wo}}{\frac{\beta}{\alpha} + R_A + \frac{w\beta}{\gamma}}, \tag{3.27}$$

and

$$\Xi_{wo}\left(\overline{G}_{wo}\right) = N \frac{wH - \overline{G}_{wo}}{\frac{\beta}{\alpha} + R_A + \frac{w\beta}{\gamma}}. \tag{3.28}$$

The condition that the level of utility with an optimal circumferential highway will be strictly higher than without it is $\overline{U} > \overline{U}_{wo}$, or equivalently

$$\frac{wH - D - \overline{G}}{\frac{\beta}{\alpha} + R_A + \frac{w\beta}{\gamma}} > \frac{wH - \overline{G}_{wo}}{\frac{\beta}{\alpha} + R_A + \frac{w\beta}{\gamma}}$$

which requires that

$$\overline{G}_{wo} - \overline{G} > D.$$

Since we assume a balanced budget condition for transport authority, D requires is positive for any amount of positive investment in the infrastructure. This implies that construction of a circumferential highway will increase utility if and only if the benefit obtained from reduction in generalized transport cost by the household living at the city boundary is higher than the cost (head tax) that he pays for it. This condition has been derived by using equilibrium property. However, this must also hold at the optimum.

3.2.7. Threshold Number of Radial Highways

We have shown later in numerical analysis that the optimal radius of a circumferential highway increases with the number of radial highways. Keeping all other things fixed, the gain from reduction in transport cost decreases and per capita construction cost increases with the number of radial highways. The reason is that when there is a smaller number of radial highways, households living further away from it have to travel a longer distance to avail themselves of it. On the contrary, when the number of radial highways is larger, this distance becomes shorter.

Therefore, the reduction in transport cost with a circumferential highway for the households living far away from the radial highways is relatively less than its cost owing to larger number of radial highways. In other words, as the number of radial highways increases, the utility gain from having a circumferential highway becomes less than that without one. This readily implies that there exists a particular number of radial highways for which the utility with and without a circumferential highway is equal and beyond which households may experience a reduction in utility. In this subsection we derive the condition for such a threshold number of radial highways.

From the above discussion, the threshold number of radial highways n_t is such that at the optimum

$$\overline{U} = \overline{U}_{wo}$$

and for the Leontief utility case it is equivalent to

$$\Xi = \Xi_{wo}. \tag{3.29}$$

With Circumferential Highway

By replacing the value of $\Xi\left(\overline{G}\right)$ from Table 3.1 into Equation (3.25), we can solve for $\overline{G}$ and $\Xi\left(\overline{G}\right)$ in terms of n and other parameters. Let us express the city are in terms of n by the following function

$$\Xi = \sigma_1\left(n\right). \tag{3.30}$$

Without Circumferential Highway

The equilibrium land area without a circumferential highway $\Xi_{wo}\left(\overline{G}_{wo}\right)$ can also be expressed as a function of n. Nonetheless, in this case we have only two modes of travel, dense city streets and radial highways.

Therefore, without a circumferential highway the total land area supply is

$$\begin{aligned}\Xi_{wo}\left(\overline{G}_{wo}\right) &= 2n\left[\frac{1}{2}\int_{\psi}^{\frac{\pi}{n}}\left(r_1\right)^2 d\theta + \frac{1}{2}\int_{0}^{\psi}\left(r_2\right)^2 d\theta\right] \\ &= n\int_{\psi}^{\frac{\pi}{n}}\left(\frac{\overline{G}_{wo}}{g_s}\right)^2 d\theta + n\int_{0}^{\psi}\left(\frac{\overline{G}_{wo}}{g_s\theta+g_r}\right)^2 d\theta\end{aligned}$$

where the first and the second terms of the right side are the market areas for dense city streets and radial highways, respectively, with $\psi = min\left(\theta_{12,}\,\pi/n\right)$. Simplifying this we obtain

$$\Xi_{wo}\left(\overline{G}_{wo}\right) = \frac{n\overline{G}_{wo}^2}{g_s^2}\left[\frac{\pi}{n} - \psi + \frac{g_s}{g_r} - \frac{g_s}{g_s\psi+g_r}\right]. \tag{3.31}$$

Equations (3.28) and (3.31) simultaneously solves for $\overline{G}_{wo}$ and Ξ_{wo} in terms of n

and other parameters. Let us express this city area without circumferential highway in terms of n as

$$\Xi_{wo} = \sigma_2(n). \quad (3.32)$$

By replacing the values of Ξ from Equation (3.30) and Ξ_{wo} from Equation (3.32) into Equation (3.29), we can determine the threshold number of radial highways n_t.

3.3. Numerical Simulation

Since the policy variables are complex in forms, it is quite difficult to solve for them analytically. To get the solution of the optimization problem and to understand its economic implications, in this section we provide a numerical solution using simulation techniques.

3.3.1. Simulation

In order to perform a simulation analysis we consider a specific cost function and a specific value for each exogenous variable and parameter. We begin by assuming that the city is served by four radial highways ($n = 4$) that extend beyond the city boundary. The number of the households N is 1,819,927[13]; the total time endowment to each household H is equal to 24 hours a day; and the hourly wage rate w equals 60 taka[14]. In addition, we assume that agricultural land rent R_A equals 1000 taka per hector per day and the transport cost per unit distance for city streets g_s (which includes both time cost and monetary cost) equals 40 taka while that for radial highways g_r equals 20 taka. The average working time is eight hours per day and the average spending on housing is one-third of the wage income[15]. The coefficients of the Leontief utility function are calculated as $\alpha = 2/9$, $\beta = 1/9$ and

[13]This is the number of household in Dhaka City.

[14]Taka is the local currency of Bangladesh.

[15]In Bangladesh, the range of housing allowance in the government sectors varies between 45%-55% of the basic salary. Thus, it is reasonable to assume the average spending on housing in the capital city of Dhaka is one-third of the total wage income inclusive of any housing allowance.

$\gamma = 2/3$. Since we assume inelastic transport demand, one person from each household commutes every day to the CBD.

Specification of the Highway Construction Cost Function Let us assume that the highway construction cost exhibits neutral economics of scale with respect to width and length, and does not vary with location. Thus,

$$K = 2\pi r_c \kappa(w_c) \tag{3.33}$$

where $\kappa(w_c) = c_0 + c_1 w_c^{\rho}$ is the cost of capacity construction for a unit length of the circumferential highway (See Yoshida, 2011). However, in a departure from Yoshida (2011), we consider width as a measure of highway capacity.

The construction cost (excluding land acquisition cost) per unit length of a two-lane highway is on average 150.0 million taka. Using a 4% rental rate of capital, the construction cost per unit length per year becomes 6.0 million taka. Taking the width per lane of the highway as 3.66 meters, the width of two-lane of highway is 7.32 meters. Now, to determine the coefficient of c_0 and c_1 of the capacity cost function, let us consider the fixed cost c_0 is equal to 10% of the capacity cost $\kappa(w_c)$ and $\rho = 2.5$. Measuring w_c in meters, this gives $c_0 = 1643.84$ and $c_1 = 101.35$. Therefore, Equation (3.33) becomes

$$K = 2\pi r_c \left(1643.84 + 101.35 w_c^{2.5}\right). \tag{3.34}$$

Relationship between Capacity and Unit Cost of Travel We represent capacity by the width of the highway. As width increases the capacity increases and vice versa. The unit cost of travel g_c along circumferential highway decreases with width and therefore is determined endogenously as well. The relationship between g_c and w_c can be represented by the following function

$$g_c = \frac{165}{w_c}. \tag{3.35}$$

3.3.2. Simulation Results

For the case of Leontief utility with no substitution effect, the lot size Q^* is the same for every household. As household utility is proportional to the equilibrium lot size and the number of city population is given, the spatial equilibrium level of utility $\overline{U}$ increases with city area. Figure 3.7 is a typical representation of supply and demand of residential land against the generalized transport cost G for every value of r_c and w_c.[16]

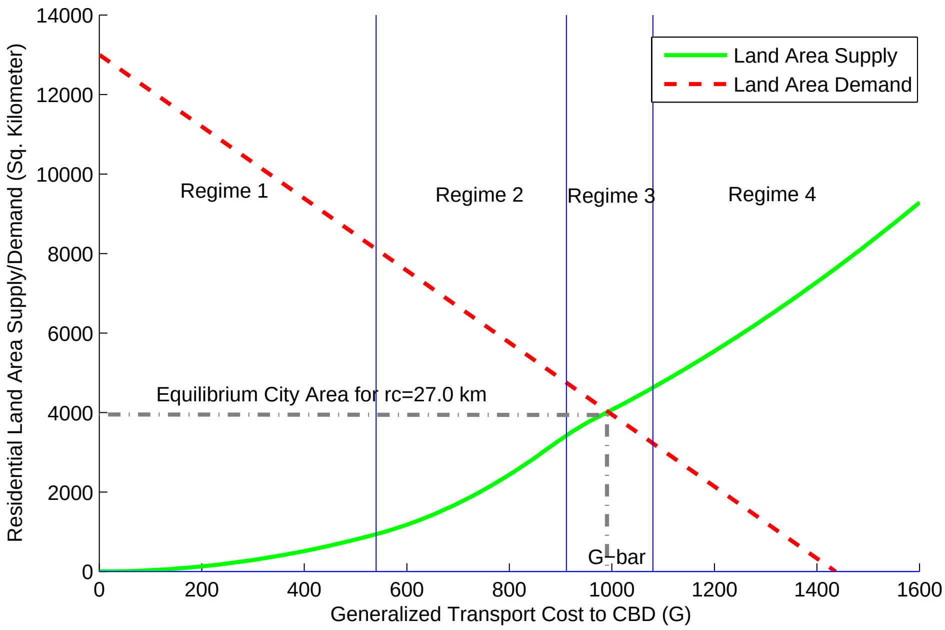

Figure 3.7. Residential Land vs. Generalized Transport Cost.

The point of intersection of the demand and the supply curve represents the equilibrium city area and $\overline{G}$. In this particular figure, the equilibrium occurs in Regime 3. However, depending on the parameter values the equilibrium may occur in other regimes. Obviously, not all of these equilibria are optimal. In the numerical simulation our objective is to determine an equilibrium which yields the highest uniform level of utility.

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Figure 3.7 have been drawn for $r_c = 27.0$, $n = 4$, $g_s = 40$, $g_r = 20$, $g_c = 15$, $w = 60$, $R_A = 1000$, $w_c = 11$, $H = 24$, $N = 1819927$, $\alpha = 0.22$, $\beta = 0.11$, $\delta = 0.66$, $c_0 = 1643.835$, $c_1 = 101.35$.

As we mentioned, the households receives no contribution from aggregate differential land rent as part of their income. This together with the fact that the head tax is equal for every household causes the disposable income to vary linearly with the generalized transport cost G. We have a downward slopping demand curve which is due to the fact that demand of residential land is proportional to disposable income. The land supply curve is upward slopping and increases with G, which is intuitive. If the households can afford to pay more transport costs, while the unit cost of travel remains constant, some of them will live farther away from CBD, which causes the overall city area to increase.

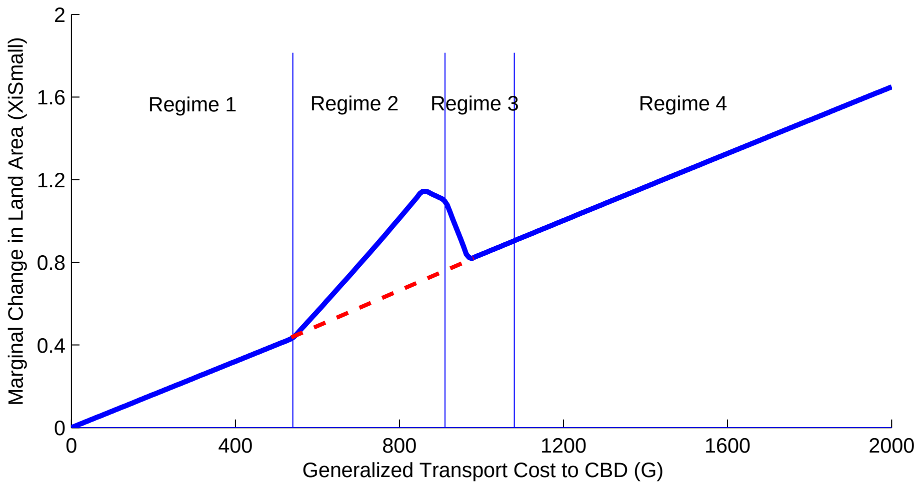

Figure 3.8. Marginal Change in Land Area.

Figure 3.8 illustrates that land supply increases more rapidly with G in Regime 2 than in Regimes 1 and 4; whereas it decrease sharply in Regime 3. Interestingly, the rate of change in land supply in Regime 4 is converging to that of Regime 1. This indicates that when the circumferential highway is too small (Regime 4) or too big (Regime 1), the change in $\xi(G)$ is almost the same. The reason is that for cases when the circumferential highway is too small (i.e., its radius is close to zero) or too big (i.e., no one will use it), it has negligible influence on the commuters travel cost reduction, and thus have negligible or no market share. In such a case, most of the city is served by Routes 1 and 2 and thus $\xi(G)$ changes almost the same way. In the case of the Leontief utility function, the equilibrium utility is proportional to the land area. In Regime 1, an increase in r_c will reduce utility, because it will increase cost without increasing city area or utility. In Regimes 4, an increase in the radius will always increase the equilibrium level of utility. So there will be no trade-off between r_c and $\overline{U}$ in Regime 1 and 4 and so we can safely rule out the existence of optimum in these two regimes.

The optimal r_c can be in Regime 3 or even in Regime 2. For smaller number of radial highways, $\frac{\pi}{n}$ is sufficiently large and therefore does not binding, so it is most likely that the equilibrium will be in Regime 3. Nonetheless, for a larger number of radial highways, $\frac{\pi}{n}$ is binding and equilibrium can be in Regime 2. In both cases, as long as $n < \frac{\pi}{\theta_{3A}^{out-in}}$, the optimal circumferential highway is partially utilized.[17] In fact, the portion of the circumferential highway located away from the radial highway is not used. This can be explained by the fact that households living away from the radial highway need to travel longer distance along the circumferential highway to reach the nearest radial highway and therefore incur higher travel costs than other routes, which are not economically viable to them. Thus, there is always an unused segment of the optimal circumferential highway.

To determine the optimal value of r_c, first we keep w_c fixed and allow r_c to vary. We then determine the corresponding $\overline{U}$ for each value of r_c. Figure 3.9 is a plot showing the variation of the equilibrium utility with r_c for a chosen value of w_c.

Obviously the peak in Figure 3.9 gives the highest equilibrium utility and the corresponding r_c is the optimal radius of the circumferential highway for a given w_c. The simulation result shows that the optimal r_c belongs to Regime 3. However, the width corresponding to this r_c need not necessarily be optimum. Therefore, we next change the value of w_c and repeat the same procedure. Thus, for each value of w_c we obtain one optimal value of r_c. In other words, each of these r_c is the best response of the corresponding w_c which we represent by the function, $r_c = f_1(w_c)$. We summarize these sets of values in Table 3.2. The optimum is such a particular

[17] See Figures 3.3 and 3.4.

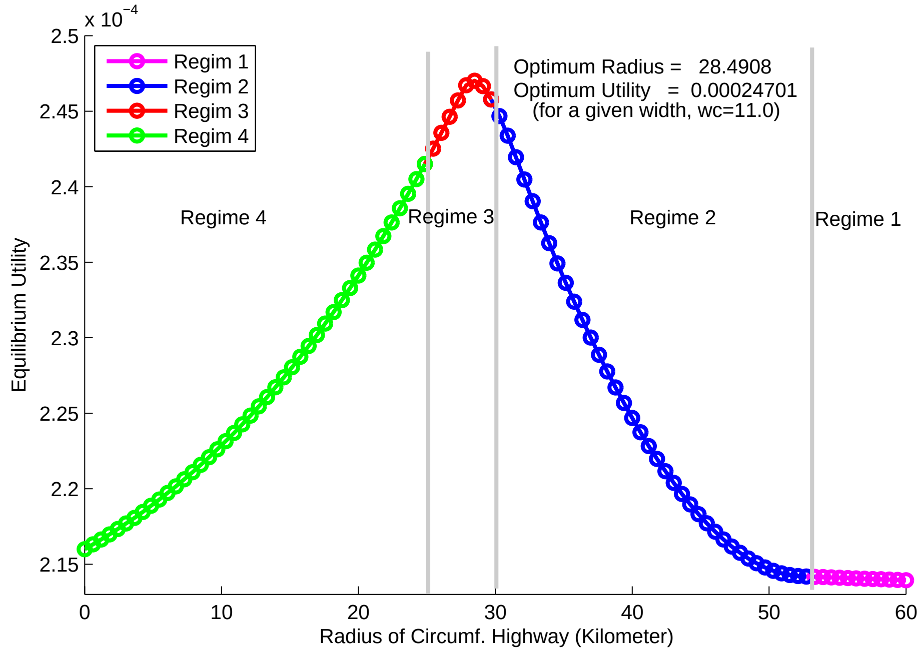

Figure 3.9. Utility vs. Radius of Circumferential Highway.

pair of r_c and w_c that yields the highest equilibrium level of utility $\overline{U}$.

Table 3.2. Simulation Results.

Sl.	g_c (Taka/Km)	Width, w_c (Meter)	City Area, Ξ^e (Sq. Km)	Radius, r_c (Kilometer)	Utility, $\overline{U}$ (x10^{-4})
1	4.10	40.25	4034.97	28.76	2.463
2	4.31	38.25	4081.37	29.00	2.492
3	4.55	36.25	4120.13	29.20	2.515
4	4.82	34.25	4153.62	29.37	2.536
5	5.12	32.25	4182.70	29.54	2.554
6	5.45	30.25	4207.06	29.68	2.569
7	5.84	28.25	4226.35	29.79	2.580
8	6.29	26.25	4240.17	29.87	2.589
9	**6.80**	**24.25**	**4247.97**	**29.90**	**2.593**
10	**7.42**	**22.25**	**4249.10**	**29.89**	**2.594**
11	8.15	20.25	4242.53	29.83	2.590
12	9.04	18.25	4226.30	29.71	2.580
13	10.15	16.25	4198.41	29.51	2.563
14	11.58	14.25	4151.60	29.21	2.535
15	13.47	12.25	4085.28	28.80	2.494
16	16.10	10.25	3992.39	27.99	2.437
17	20.00	8.25	3868.32	27.26	2.362
18	26.40	6.25	3695.70	26.45	2.256
19	38.82	4.25	3540.37	25.82	2.161

Figure 3.10 plots the values of column 6 against column 3 of Table 3.2 and shows how the equilibrium utility varies across r_c and w_c pairs.

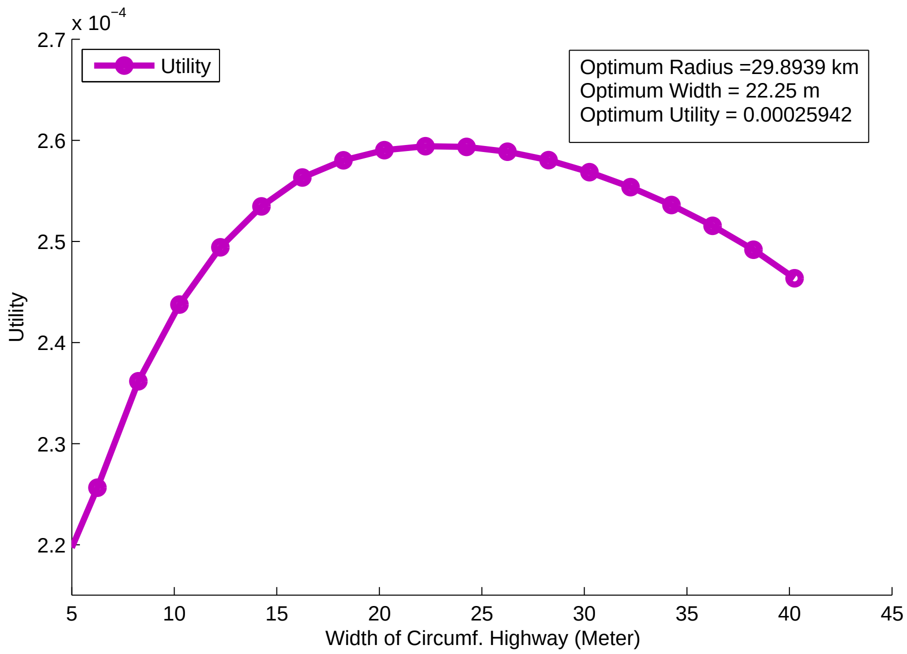

Figure 3.10. Utility vs. Width of Circumferential Highway.

The maximum point of this graph yields the optimal utility $\overline{U}$ and the corresponding w_c and r_c bundle represents the optimal width and radius of the circumferential highway.

However, so far we have discussed the radius which is the best response of width. We need to determine the width that is the best response of radius. Therefore, we next repeat the whole procedure by changing w_c first while keeping r_c fixed. We then alter the value of r_c and by repeating the same procedure, we get a set of values for w_c that are the best response for each r_c. We represent these values by the function $w_c = f_2(r_c)$. Thus, we have an optimal radius for each width and, conversely, an optimal width for each radius. Figure 3.11 plots the best response functions of radius and width.

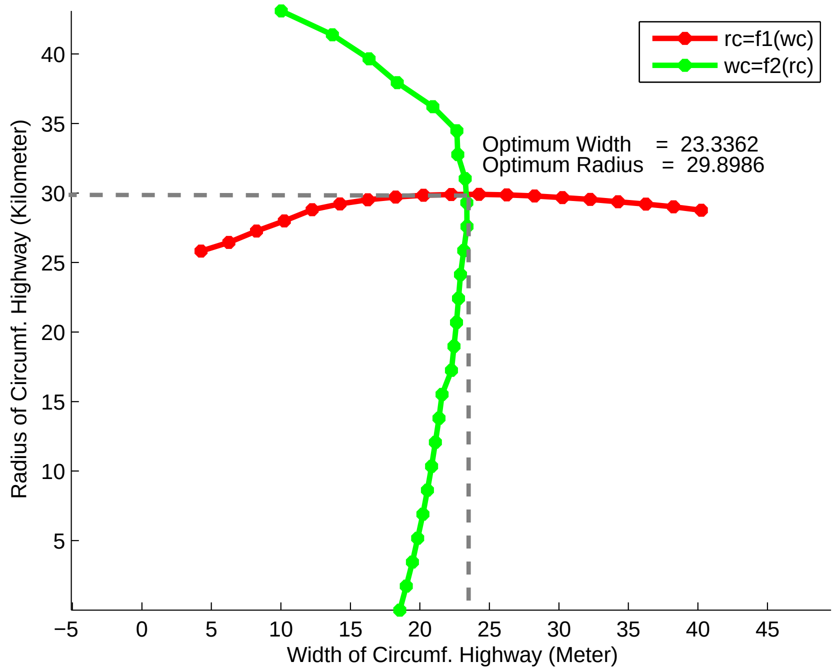

Figure 3.11. Best Response Functions of Radius and Width of Circumferential Highway.

The intersecting point of the two best response functions in Figure 3.11 gives the social optimal design features of the circumferential highway. For the specific parameter values as well as construction cost, the unit travel cost, and utility functions, the optimal design values for a circumferential highway are summarized in Table 3.3.

Table 3.3. Optimal Design Values.

Variables	Optimal values
Optimal Radius, r_c	29.90 kilometers
Optimal Width , w_c	23.34 meters
Utility (x10^{-4}), $\overline{U}$	2.5934 utils
Optimal City Area, Ξ	4248 square kilometers
City Shape	Regime 3.

The optimal radius is approximately 30.0 kilometers, the optimal width is approximated by six lanes and the resulting city size is 4248 square kilometers. The most important feature is that the optimum takes place in Regime 3 which implies that the circumferential highway is partially used.

3.3.3. Effect of the Number of Radial Highways on the Optimal Design

In this subsection, we focus on the effect of the number of radial highways on the optimal design of the circumferential highway. To do so, we keep width constant, and change the number of radial highways n in the above simulation. For each value of n we obtain one corresponding optimal value of r_c. We then plot such optimal values of r_c against n (for $w_c = 11.0$ meters) which is represented in Figure 3.12.

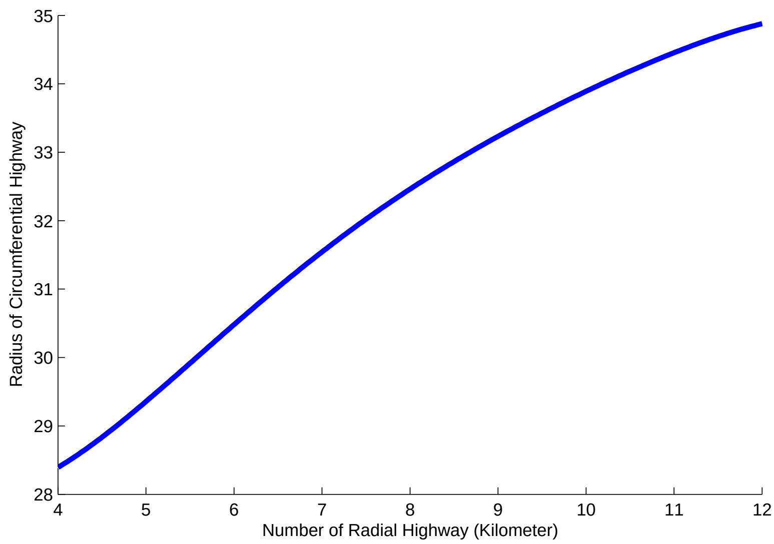

Figure 3.12. Optimal Radius of Circumferential Highway vs. Number of Radial Highways.

This figure illustrates that the optimal radius of the circumferential highway increases with the initial number of radial highways. This can be explained by the fact that the increase in the number of radial highways decreases the share of city area between the two consecutive radial highways for a given radius of the circumferential highway. In such a case, the transport cost savings to reach the nearest radial highway by traveling on the circumferential highway is less than the cost of investment. Therefore, there is an incentive to increase the radius of the circumferential highway.

In the real world, cities have different numbers of radial highways. It is reasonably assume that developed cities tend to have more radial highways than developing cities. Under this assumption, circumferential highway for developed cities should be constructed farther outward than for developing cities.

3.4. Conclusion

In this study, we demonstrated a procedure to solve for the optimal capacity and location of a circumferential highway using a city model. We also determined the resulting size and shape of the city. To the best of our knowledge, this is the first study that includes a circumferential highway in a city model and determines the socially optimal design characteristics of this highway. To do so, we assumed absentee land ownership. The city planner increases the utility level of the households by optimally designing a circumferential highway which, for no substitution case, results in a bigger city and lower transport costs.

One of the major findings of this study is that the optimal location of a circumferential highway is such that it is partially utilized. Although there is a bulge of urbanized area surrounding the circumferential highway, a portion of the circumferential highway away from the radial highways is not really used even though the cost along the circumferential highway is less than dense city streets.

Another finding is that cities with a larger number of radial highways require a larger circumferential highway than cities with smaller number of radial highways. The implication of this finding is that the developed cities with many radial highways need larger circumferential highways than do developing cities with a smaller number of radial highways.

Third, our study is an extension of the city model of Anas and Moses (1979). In our model households have three mode choices to reach the CBD. Thus, we have three different market areas. Depending on the parametric values and the location and capacity of the circumferential highway, the relative size and shape of the market and city areas can be determined endogenously.

Fourth, our method can be used for the cases of residential or public land ownership or for different utility functions after some minor adjustments.

However, one of the limitations is that we did not take into account the welfare of the absentee landowners. Kanemoto (1980) pointed out that for optimization with absentee land ownership, the welfare of the absentee landowner should be taken into consideration. In our study, we concentrated on the benefits of the city dwellers. In addition, our trip demand is not sensitive to transport costs. Another limitation of our analysis is that we considered only one circumferential highway; however, cities may require more than one circumferential highway. For example, a city like Tokyo has three circumferential highways. In fact, in the current version of our study, we limited our focus on the cities having no circumferential highway.

There is room for improvement of our analysis further by, for example, considering more than one circumferential highway in the model and investigating how it affects the city shape, city size, and the optimality conditions. On the other hand, the number of circumferential highways could be thought of as endogenous variable under intertemporal settings. The model can also be extended for elastic travel demand. The shape of the circumferential highway can be treated as endogenous, which would allow investigating whether the optimal shape of circumferential highway is truly circular throughout or something different. Since an inward turning of the circumferential highway near the intersection with the radial highway is supposed to reduce the commuting distance and hence the commuting cost, one can expect an inward bend of the circumferential highway near the intersection with the radial highway. Congestion externality creates a gap between social and private cost. Therefore, including congestion in the model would be a significant progress. In fact, it is possible to consider congestion in the circumferential highway in our numerical analysis by replacing Equation (3.35) with a standard user cost function that includes traffic flow and capacity together with some other concomitant adjustments. However, if we want to consider congestion in all three modes, the model would become quite complicated. This remains an issue to be addressed.

Circumferential highway plays an important role in the management of transport demand of a city. Therefore, careful supply side consideration is required for optimal provision of such facilities. In this study, we have sought to find a design procedure of a circumferential highway by taking into account its land-use effects. However, prior to applying such design features, we need to modify the model so that it best fits real world cities.

4. Welfare Recovery with the Second-best Dynamic Highway Pricing Under Naïve Investment Policy

4.1. Introduction

It goes without saying that highway construction involves huge capital investment. A substantial portion of this investment is funded from the general fiscal revenue in most countries. The theory of transportation economics, from decades long, advocates for highway pricing in order to achieve efficiency in this sector. Apart from enhancing efficiency in the short-run, a well-designed highway pricing generates revenue as a by-product. This revenue can be a potential source of funds to finance highway construction. “Appropriate patterns of congestion tolls are thus essential not only to the efficient utilization of existing facilities, but to the planning of future facilities” (Vickrey, 1969, p. 260). As the source and utilization of this revenue remains within the highway sector, it is generally well known as “self-financing”[1] in transportation literature. It is to be noted that self-financing does not necessarily mean “full” cost recovery. The main feature of full cost recovery or “exact self-financing” is that under some special conditions[2], the toll revenue will exactly cover the capital cost. If the term "capital cost" is misunderstood with "investment cost", then targeting exact self-financing will bring about substantial welfare loss (Verhoef & Mohring, 2009). To the best of our knowledge, no literature deals with the recovery issue of such welfare loss. Given this limitation of the present literature, this study seeks the policy tools that might be used to address the welfare loss

[1]This is also termed as "cost recovery" in the literature (e.g., De Palma & Fosgerau, 2011).
[2]The conditions are described latter.

resulted from naïve interpretation of the capacity investment.

Mohring and Harwitz (1962) were the first to address the issue of self-financing of highways. According to them, if: (a) highway capacity can be added continuously, (b) there exits neutral economies of scale in capacity production, and (c) the user cost function is homogenous of degree zero in usage and capacity, then optimal toll revenue will exactly cover the capital cost. In such a case, we do not need to outsource the costs of highway construction. "Moreover, the use of revenues for the financing of infrastructure appears to improve the acceptability of road pricing" (Verhoef, 2008, p. 22). However, when there exists economies or dis-economies of scale in the capacity cost function, the collected toll revenue from the road yields deficit or surplus respectively.

Since then, many other literature considers self-financing under various settings (see, for example, De Palma & Lindsey, 2007; Hau, 1998; Newbery, 1989; Small, 1992; Small, 1999; Small & Verhoef, 2007; Strotz, 1964; Yang & Meng, 2002). However, this bunch of literature did not consider self-financing under intertemporal settings until Arnott and Kraus (1998). Their work mainly examined the applicability of self-financing of highway in an intertemporal framework under growing demand. Another interesting feature of their work was that they considered the net social welfare in present value terms. They concluded that, with some exception, self-financing is indeed applicable for highways in a dynamic framework under the same technical assumptions as were made in a static framework.

It is observed that most literature of intertemporal self-financing, considered the first-best case where there is no distortion in the economy. Nonetheless, a study by De Palma, Proost, and Van Der Loo (2012) considered the second-best and even third best self-financing where borrowing is a constraint and the toll revenues at any time have to be utilized sorely for highway construction and maintenance for the next time. Using bottleneck congestion in a dynamic model, they mainly focused on the effect of inefficiency resulted from the initial capacity on the future transport infrastructure development. They found that, a “balanced budget” policy together with suboptimal initial capacity results in inefficient capacity in the long-run. They suggested that before imposition of any other financial restriction on the highway agency, it is important to take into account the extent of deviation of initial capacity from the optimum.

In fact, there is a clear distinction between “capital cost” and “investment cost” (Verhoef & Mohring, 2009). In the context of highway investment, “capital cost” implies the cost of interest of the highway capital stock. On the other hand, “investment cost” is the cost of extra capacity of the highway. In order to sustain exact self-financing, capital costs for a particular time should be balanced by the toll revenue generated at that time. A “naïve policy” with SRMCP is such that the highway agency invests all of its toll revenue into capacity addition at each point in time[3]. However, the price is charged equal to short-run marginal cost.

The following example clarifies the distinction between capital cost and investment cost. Let us assume that the interest rate, r is 10%; unit (constant) cost of capacity addition for a given length of highway, ρ is 10 million yen/ meter. Further we assume that at any given time t, capacity K_t is 9.50 meter, toll revenue, is 12 million yen, depreciation and maintenance cost is 2 million yen. Thus, the amount available for investment in the next period is 10 million yen. If we invest this amount in such a way that it exactly covers the amortized capital cost of the capacity stock in the next period, then there will be a total capacity of 10 meters. This means we increase capacity by 0.50 meter next period and the cost of new capacity will be 5 million yen. On the contrary, if we invest all the net revenues (revenue net over depreciation and maintenance costs, if considered) in the capacity expansion next period, then we can increase capacity by one additional unit and therefore, the total capacity and the end of next period would be 10.50 meter. This means that the cost of new capacity in this case is 10 million yen. We summarize the outcome of

[3]Though any kind of deviation from the basic concept of self-financing can be termed as " Naïve Policy", in general; however, we will continue to use this definition throughout this study.

the two policies in Table 4.1.

Table 4.1. Difference Between "Capital Cost" and "Investment Cost".

Items	Capital Cost Policy	Investment Cost (Naïve) Policy with SRMCP
Toll Revenue	12 million Yen	12 million Yen
Depreciation & Maintenance	2 million Yen	2 million Yen
Capital cost/ Investment cost	$r\rho K_{t+1} = 10$	$I_{t+1} = 10$
Capacity Addition, $\triangle K$	$+0.50$ meter $(= 10 - 9.50)$	$+1$ meter $(= 10/10)$
Capacity, K_{t+1}	10.00 meter	10.50 meter $(= 9.50 + 1)$.

The values in the table demonstrate that the two concepts of investment end up with different level of capacity. According to Verhoef and Mohring (2009), a naïve policy "would use revenues to expand capacity in the next period, so that capacity will grow over time as long as road use and toll are positive" (p. 303).

When we consider self-financing of highway, it is not unlikely to mix up the concept of “capital cost” and “investment cost”. A study by Nemoto, Misui and Kajiwara (2009), did not consider "capital cost" in capacity investment while focusing on long-run self-financing. In the real world, we cannot ignore the possibility that the decision-makers undertake naïve investment policy due to misunderstanding the concept of self-financing, or due to inadequate financial market, or due to an externally imposed borrowing constraint. The financial markets in most developing countries are not well-performed, and therefore, large amounts of borrowing, as generally required in highway construction, may be quite impossible. If we ignore foreign or any other kinds of borrowings or aids, then highway agency is more likely to finance the construction and maintenance costs of the highway sorely from the toll revenue which eventually implies highway agency will follow a policy similar to naïve policy, even though, in this case it is not due to misinterpretation of capital cost and investment cost. In many countries, highway sector experiences privatization either fully or partly. When government privatizes highway sector, there may have incentives to impose borrowing constraints on the private highway agency so that the possible risk of becoming another public burden can be avoided (De Palma et al., 2012).

Whatever is the reason; such a policy will result in a huge welfare loss. Therefore, it would be interesting to see whether it is possible to recover the welfare loss caused by naive policy, and if so then what policy we should follow. We also seek which parameter is crucial in welfare recovery. More particularly, we will try to find the welfare maximizing capacity, pricing and tolling policy and their long-run consequences. In doing so, we will consider continuous time and net present value of money. The use of continuous time is obvious because "...jurisdictions must balance present and future needs against costs when financing infrastructures" (Levinson, 2002, p. 96). It is well known that when one generation sorely pays for the infrastructure and another generation uses it without paying, there is a free-rider problem which will make distortion to the resource allocation over time.

The rest of this chapter is organized as follows: Section 4.2 discusses the theoretical background of the first-best self-financing under intertemporal context. It also characterizes and analyzes the basic features of naive policy with SRMCP. In Section 4.3, we formulate a model of the second-best naïve policy that can by used as an instrument to restore welfare losses. We also provide a numerical simulation at the end of this section and construct the trajectories of the key variables. In Section 4.4, we conclude this study.

4.2. Theoretical Background of Intertemporal Self-financing

In the first half of this section, we summarize the first-best capacity and pricing rules as well as the conditions for holding self-financing of highway in the intertemporal settings. We then discuss about the naïve policy with SRMCP.

4.2.1. Self-financing Under the First-best Policy

While reviewing the first-best conditions under self-financing, we will mostly follow Arnott and Kraus (1998). We assume that capacity can be adjusted continuously over time. The objective then is to choose toll (price), and investment so as to maximize the net social benefit over the entire planning horizon. The net social benefit is the difference between the gross social benefit and the social cost. For a downward sloping demand curve, the gross social benefit is the sum of the areas under the demand curve totaled over the planning period. The social cost has two components: (i) the highway user cost and (ii) the highway construction and depreciation costs. In our analysis, we ignore maintenance cost as well as the externality costs other than congestion. All of the components of costs and benefits are measured in terms of monetary units and are subjected to appropriate discount in order to reflect users' time preference.

We will use the following notations and assumptions for the rest of this chapter: t_0 is the initial time, t_1 is the final time, $\tau(t)$ is the toll at time t, $P(t)$ is (full or generalized) price at time t which includes user cost and toll , $K(t)$ is the one way capacity of highway at time t (e.g., vehicle/day), $I(t)$ is the investment (in monetary terms) at time t, $R(t)$ is the toll revenue at time t, r is the annual interest rate constant over time, $Q(.) > 0$ is the demand for trip at a given price and time which, otherwise, reflects the users' willingness to pay, and $C(Q(.), K(t))$ is the user cost function or trip production function at time t and depends on the demand $Q(.)$ and the highway capacity $K(t)$. Note that $C(.)$ includes only the time cost of the user. δ is the annual rate of depreciation of capacity which is constant over time. $F(K)$ is the cost function that the construction industry exhibits, and θ is the elasticity of capacity cost with respect to capacity. We also assume that $C(.)$, $K(.)$, $\tau(.)$, $Q(.)$, $P(.)$ and $F(.)$ are continuously differentiable with respect to their arguments. The initial capacity at time $t = t_0$ is K_0 which is given.

We will follow some well accepted properties of the user cost function, such as: $\frac{\partial C}{\partial Q} > 0$, $\frac{\partial C}{\partial K} < 0$, and for demand function we have $\frac{dQ}{dP} \leq 0$.

Now for the first-best case, we begin by assuming that the capacity cost function takes a general form. However, we will relax this assumption at the end of this

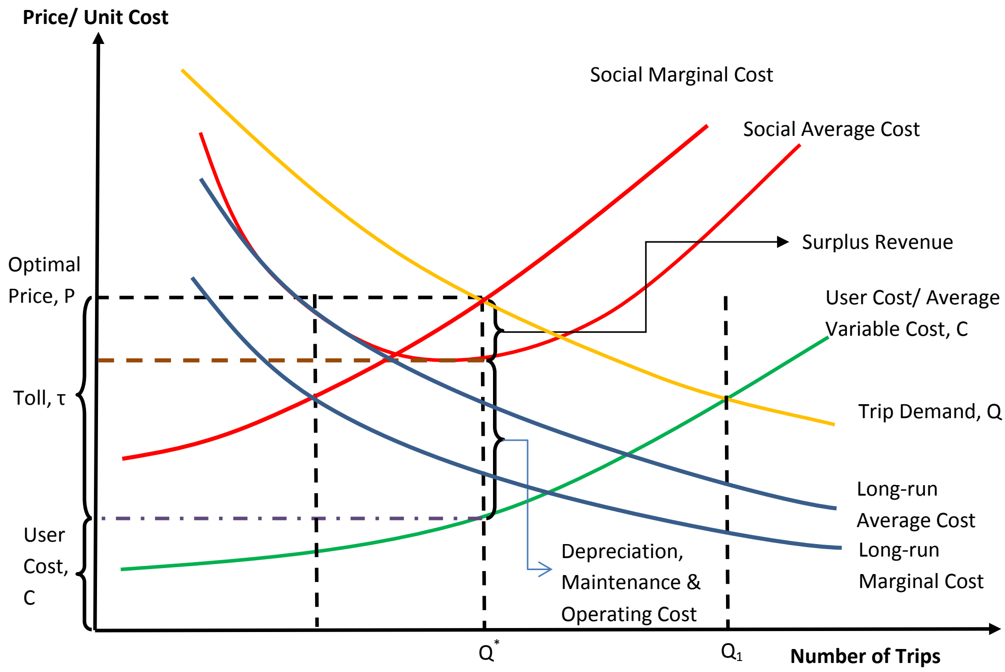

Figure 4.1. Internalization of the Congestion Externality.

subsection For an infinite horizon, the social benefit maximizing problem is

$$\max_{\tau(t),P(t),I(t)} \int_{t_0}^{t_1} e^{-rt} \left[\int_0^{Q(P(t))} Q^{-1}(q)\,dq - Q(P(t))\,C(Q(P(t)),K(t)) - I(t) \right] dt$$

subject to the law of evolution of the capacity over time

$$\dot{K}(t) = G(K(t),I(t),t) \tag{4.1}$$

and the price at each point in time

$$P(t) = C(Q(P(t)),K(t)) + \tau(t). \tag{4.2}$$

The capacity at the end of the planning horizon is free to choose. Therefore, we have the following initial and boundary conditions:

$$K(t_0) = K_0 \geq 0,\ t_0,\ t_1 \rightarrow fixed,\ K(t_1) \rightarrow free. \tag{4.3}$$

The cost of the initial capacity K_0 is not sunk and it has to be paid up during the planning period.

Since we consider the discounted value of welfare function, it is convenient to use current value Hamiltonian instead of the present value Hamiltonian. The current value Hamiltonian of the above problem is

$$
\begin{aligned}
H^c &= \int_0^{Q(P(t))} Q^{-1}(q)\,dq - Q(P(t))\,C(Q(P(t)),K(t)) - I(t) \\
&\quad + \mu(t)\left[G(K(t),I(t),t)\right]. \qquad (4.4)
\end{aligned}
$$

As the equality constraint in Equation (4.2) requires to be observed at every point along the planning horizon, the current value Augmented Hamiltonian or Lagrangian, L^c is

$$
\begin{aligned}
L^c &= H^c + \lambda(t)\left[P(t) - C(Q(P(t)),K(t)) - \tau(t)\right] \\
&= \int_0^{Q(P(t))} Q^{-1}(q)\,dq - Q(P(t))\,C(Q(P(t)),K(t)) - I(t) \\
&\quad + \mu(t)\,G(K(t),I(t),t) + \lambda(t)\left[P(t) - C(Q(P(t)),K(t)) - \tau(t)\right]. \qquad (4.5)
\end{aligned}
$$

where $\mu(t)$ is the costate variable of capacity and $\lambda(t)$ is the Lagrangian multiplier.

Maximum Principle Conditions

We get the maximum principle conditions by taking partial derivatives of the Augmented Hamiltonian with respect to τ, I, P, K, μ, and λ as follows:[4]

$$\lambda = 0. \qquad (4.6)$$

$$\mu = \frac{1}{G_I}. \qquad (4.7)$$

$$Q_P P - Q_P C - Q C_Q Q_P + \lambda(1 - C_Q Q_P) = 0. \qquad (4.8)$$

$$-QC_K - \mu G_K - \lambda C_K = -\dot{\mu} + r\mu. \qquad (4.9)$$

$$\dot{K} = G(K(t),I(t),t). \qquad (4.10)$$

$$P - C - \tau = 0. \qquad (4.11)$$

[4] The arguments are omitted for notational convenience.

The finite horizon transversality condition is

$$\mu\left(t_1\right)e^{-rt_1}=0. \tag{4.12}$$

Here a subscript means partial derivative with respect to that variable.

For finite horizon with final time t_1, the transversality condition implies that if there any unused capacity remains after the end of the planning horizon t_1, its discounted shadow price viewed from the time t_0 should be zero. In other words, when we have the flexibility to choose capacity in order to maximize the net social benefit, we must choose capacity to such an extent that its present discounted value becomes zero. If the present discounted value is positive, we have the incentive to increase net social benefit by increasing capacity and therefore not optimum.

By using Equations (4.6) and (4.7), Equations (4.8) and (4.9) become

$$P=C+QC_Q. \tag{4.13}$$

and

$$-QC_K=-\dot{\mu}+r\mu-\mu G_K. \tag{4.14}$$

From Equations (4.2) and (4.13) we have

$$\tau=QC_Q. \tag{4.15}$$

Equations (4.13), (4.14) and (4.15) give the first-best pricing, capacity and tolling rule respectively. The term QC_Q in the above equation is the marginal time cost (delay cost) of all travelers due to the addition of one extra traveler in the highway. It means that QC_Q is the congestion externality imposed by a traveler to the society. The term $C+QC_Q$ is the social marginal cost. Therefore, Equation (4.13) implies that at the social optimum, price should fully internalize the congestion externality (SRMCP). This means that toll should be charged equal to the congestion cost.

On the other hand, Equation (4.14) implies that capacity should be extended in such a way that the marginal cost of capacity expansion is equal to the marginal benefit of that extra capacity.

Self-financing Rule

We assume that capacity and investment follows

$$K(t) = \gamma I(t)^{\eta} \tag{4.16}$$

and the function $G(t)$ obeys

$$G(t) = -rK_0 - \delta K(t) + \gamma I(t)^{\eta} \tag{4.17}$$

where $\gamma > 0$ is a constant, and $\eta \geq 0$ is the elasticity of capacity with respect to capacity cost function. This means that $\eta = \frac{1}{\theta}$. We assume that the investment includes amortized cost of initial capacity and the depreciation cost. Thus, $G(.)$ depends on the depreciation of the highway, amortized cost of initial capacity and the investment cost. Then we have $G_I > 0$, $G_K < 0$ and $G_K < 0$.

The toll revenue $R(t)$ at time t is the product of toll and number of usages. Therefore, we have

$$R(t) \quad = \quad \tau Q = Q^2 C_Q.$$

If the user cost function $C(Q(P(t)), K(t))$ is homogeneous of degree zero in Q and K, then, from Euler's theorem we have, $QC_Q + KC_K = 0$. The elasticity of capacity cost with respect to capacity θ is given by

$$\theta = \frac{\frac{dF}{F}}{\frac{dK}{K}} = \frac{F_K}{\frac{F}{K}} = \frac{marginal\,cost}{average\,cost} = \frac{F_K K}{F}.$$

and thus

$$R = Q^2 C_Q = -KQC_K = (-\dot{\mu} + r\mu - \mu G_K) K$$

Now we assume the neutral economies of scale in capacity cost function such that $\mu = \frac{1}{G_I} = \rho = constant$ where ρ is the unit cost of capacity. This gives $\dot{\mu} = 0$. Thus, we have

$$R = (r + \delta) \rho K \tag{4.18}$$

Equation (4.18) is the self-financing rule for the intertemporal context. This implies that the toll revenue at any time should exactly cover the capital cost and the depreciation cost. This result is similar to that of the static self-financing case with $\theta = 1$ (see, for example, Verhoef & Mohring, 2009); however, the difference is that for the intertemporal case, Equation (4.18) must hold at every period in time.

To get the entire revenue in the long-run, we integrate both sides of Equation (4.18) with respect to time and consider the present value which gives

$$\begin{aligned} \int_0^\infty e^{-rt} R(t)\,dt &= \int_0^\infty e^{-rt}(r+\delta)K(t)\,dt \\ &= -e^{-rt}K(t)\big|_0^\infty + \int_0^\infty e^{-rt}\dot{K}(t)\,dt + \int_0^\infty e^{-rt}\delta K(t)\,dt \\ &= K_0 + \int_0^\infty e^{-rt}(I(t) - \delta K(t))\,dt + \int_0^\infty e^{-rt}\delta K(t)\,dt \\ &= K_0 + \int_0^\infty e^{-rt} I(t)\,dt. \end{aligned}$$

This is shown by Arnott and Kraus (1998). The above equation confirms that when toll revenue at any time is equal to amortized capital cost, the total discounted toll revenue can cover exactly the cost of initial capacity and the total discounted investment in the long-run. Thus, continuous short-run marginal cost pricing on congestion leads to long-run equilibrium when user cost and capacity cost functions exhibit neutral economies of scale.

Steady State

To derive a relatively simpler and tractable expression of the steady state values, we now set the specific functional forms. We assume the following constant elasticity trip demand function

$$Q(P(t)) = bP(t)^{-\varepsilon} \tag{4.19}$$

and a homogenous user cost function[5]

$$C(Q(P(t)), K(t)) = \alpha t_f + \alpha t_f \beta \left(\frac{Q(P(t))}{K(t)}\right)^m \tag{4.20}$$

[5]Here, we consider static congestion which mean that congestion does not vary within a unit period of time (e.g., within the day). However, in order to include the dynamic bottleneck congestion that considers time varying congestion (with endogenous trip timing), we can use the average user cost function e.g., $C(t) = \nu \frac{Q(t)}{K(t)}$ instead of Equation (4.20) where ν is the scheduled delay cost (see, for example, De Palma et al., 2012).

where ε is the (positive) price elasticity of trip demand such that, $\varepsilon = -\frac{dQ/Q}{dP/P}$; b is a non-negative parameter; α is the unit value of in-vehicle time; t_f is the free-flow time to travel a given length of road segment; β and m are parameters.

Steady state is defined when $t_1 \to \infty$. Since we do not have growth in the model, therefore, at steady state, every variable is constant. Thus, $\dot{\mu}(t) = 0$, $\dot{\tau}(t) = 0$, $\dot{P}(t) = 0$, and $\dot{K}(t) = 0$. After solving, we have the following implicit steady state values of investment: $b^{\frac{1}{\varepsilon}} d_1^{\frac{-1}{\varepsilon}} \overline{I}^{\frac{d_2}{\varepsilon}} \left(\frac{\gamma\overline{I}^{\eta} - rK_0}{\delta}\right)^{\frac{-1}{\varepsilon}} - \alpha t_f - \alpha t_f \beta (1+m) \left(d_1 \overline{I}^{-d_2}\right)^m = 0$. The other steady state values can be derived from the following expressions: $\overline{\tau} = \alpha t_f \beta m \left(d_1 \overline{I}^{-d_2}\right)^m$; $\overline{K} = \frac{\gamma\overline{I}^{\eta} - rK_0}{\delta}$; $\overline{Q} = d_1 \overline{I}^{-d_2} \overline{K}$; $\overline{P} = b^{\frac{1}{\varepsilon}} \overline{Q}^{\frac{-1}{\varepsilon}}$; $\overline{\mu} = \frac{1}{\gamma\eta\overline{I}^{\eta-1}}$; where $d_1 = \left[\frac{r+\delta}{\alpha t_f \beta m \gamma \eta}\right]^{\frac{1}{m+1}}$ and $d_2 = \frac{\eta-1}{m+1}$. Here the bar over a variable represents its steady state value.

4.2.2. Naïve Policy Under Short-run Marginal Cost Pricing

To obtain the first-best outcome, under the above model, the highway agency should follow an investment rule so that Equation (4.7) is observed. This implies that the toll revenue should be invested in capacity in such a manner that it exactly covers the capital cost of the capacity stock and the depreciation cost at that time.

A naïve policy is defined when the highway agency, instead of following the above self-financing rule, invests all of its instantaneous toll revenue directly into the capacity expansion. However, the pricing policy follows the short-run marginal cost pricing. We term this policy as "naïve policy with SRMCP". The investment strategy that naïve policy with SRMCP follows is

$$I(t) = Q(P(t))\, \tau(t). \tag{4.21}$$

In fact, all other rules are the same as in the first-best case. The only policy that makes it different from the first-best policy is the investment policy.

Since naive policy is not intentional, so in this case the highway agency does not have a different welfare function. It just thinks that it maximizes the first-best policy, but due to misunderstanding of the investment concept, it indeed follows the wrong path.

Thus we have the following rules for naïve policy with SRMCP:

pricing rule:

$$P = C + QC_Q; \tag{4.22}$$

capacity rule:

$$-QC_K = -\dot{\mu} + r\mu - \mu G_K; \tag{4.23}$$

$$\dot{K} = G; \tag{4.24}$$

and the investment rule:

$$I = \tau Q. \tag{4.25}$$

Steady State

By using the same functional form as mentioned in the first-best case, we have the following implicit steady state value of the costate variable of capacity $\overline{\mu}$: $\frac{b\delta}{d_3}\left[\alpha t_f + \alpha t_f \beta (1+m) d_3^m\right]^{-\varepsilon} - \gamma\left[b\alpha t_f \beta m d_3^m \left\{\alpha t_f + \alpha t_f \beta (1+m) d_3^m\right\}^{-\varepsilon}\right]^{\eta} + rK_0 = 0$. The other steady state values are: $\overline{\tau} = \alpha t_f \beta m d_3^m$; $\overline{K} = \frac{\gamma \overline{I}^{\eta} - rK_0}{\delta}$; $\overline{Q} = d_3 \overline{K}$; $\overline{P} = b^{\frac{1}{\varepsilon}} \overline{Q}^{\frac{-1}{\varepsilon}}$; and $I = b\alpha t_f \beta m d_3^m \left[\alpha t_f + \alpha t_f \beta (1+m) d_3^m\right]^{-\varepsilon}$; where $d_3 = \left[\frac{(r+\delta)\overline{\mu}}{\alpha t_f \beta m}\right]^{\frac{1}{1+m}}$.

When interest rate r is zero together with the fact that $\eta = 1$, it is straightforward to show that the steady state values of first-best and naive policy with SRMCP are the same. Thus, in a hypothetical world of zero interest rate, naive policy does not cause any harm. However, it does have negative welfare effect, when interest rate is greater than zero. We will illustrate this in the numerical analysis in the following section.

4.3. The Model

In this section, we formulate a models in order to minimize the welfare losses resulted from the naïve policy. We consider one origin-destination pair connected by a single highway. The capacity of the highway is limited and therefore, likely to be congested. We assume that the capacity is same at each direction. The road users are homogenous in their valuation of time. Capacity can be added continuously and the interest rate on capital is constant over time. We consider that roads are built with same technical standard over time and thus ignore the issue of durability. Toll can be adjusted instantaneously without any additional cost. The overall objective of the system is to maximize the net social benefit. There are two tiers of highway administration: the government itself and the highway agency[6]; however, they are vertically disintegrated. The optimization problem, therefore, is a two-stage dynamic game. In the second stage, considering toll as given, the highway agency collects the toll revenue from the highway users and invests the net amount of it directly to the capacity expansion. This creates a distortion to the basic self-financing concept. Government observes this distortionary policy and is keen aware of its negative consequences. However, since the highway agency is vertically disintegrated, government cannot administratively control the behavior of highway agency. Therefore, government utilizes economic tools to control the behavior of highway agency. In so doing, in the first stage, government adjusts toll (price). The highway agency runs under zero-profit condition.

4.3.1. Second-Best Naïve Policy

Now we will begin by illustrating the second-best case that minimizes the welfare losses when the highway agency follows the naïve investment policy. We name this corrective policy as the “second-best naïve policy”. In our model, we assumed

[6]Though we termed the lower tier of highway administration as "highway agency", it may be an autonomous body, or a leasing company, or a privately operated highway company. The main characteristic is that they are vertically disintegrated from the government in decision making.

that under this second-best policy, capacity-cost relationship is such that it can have any economies of scale. We also assume that the same economies of scale of the capacity cost function holds every time.

First Stage: Government

In first stage, government maximizes the discounted net social benefit by choosing toll (price) for highway usages. Contrary to the first-best case, in this case government does not decide investment. Thus, Government's problem becomes,

$$\max_{\tau(t),P(t)} \int_{t_0}^{t_1} e^{-rt} \left[\int_0^{Q(P(t))} Q^{-1}(q)\,dq - Q(P(t))\,C(Q(P(t)),K(t)) - I(t) \right] dt$$

subject to Equation (4.2).

Second Stage: Highway agency

At the second stage, the highway agency considers toll (since by Equation (4.2) toll is included within the price) as given and therefore the problem of the highway agency is not really an optimization problem. This is due to the special kind of behavior of the highway agency: it just collects the toll revenue and invests the net of it to increase the capacity. Therefore, the decision rule that the highway agency follows can be termed as "invest-as-you get". Thus, we have

$$I(t) = Q(P(t))\,\tau(t)\,. \tag{4.26}$$

Equation (4.26) is the budget constraint of the highway agency. As long as the road usage and toll are positive, the revenue is positive and so does the investment. The rate of depreciation δ is constant in this problem. It should be noted that in this case, a positive investment does not necessarily mean a positive capacity addition in general. The investment net of depreciation and amortized cost of initial capacity is utilized for capacity expansion. The boundary conditions are the same as in Equation (4.3).

The change in highway capacity can be characterized by the following dynamic equation,

$$\dot{K}(t) = G(K(t), I(t), t). \tag{4.27}$$

Solution of the Problem

We solve this two-stage game by backward induction. Combining the second stage conditions (Equations (4.26) and (4.27)) together, we have

$$\dot{K} = G(K(t), (Q(P(t))\,\tau(t)), t). \tag{4.28}$$

Now by using Equation (4.28), the whole problem can be re-written as:

$$\max_{\tau(t),P(t)} \int_{t_0}^{t_1} e^{-rt} \left[\int_0^{Q(P(t))} Q^{-1}(q)\,dq - Q(P(t))\,C(Q(P(t)), K(t)) - Q(P(t))\,\tau(t) \right] dt$$

subject to

$$\dot{K} = G(K(t), (Q(P(t))\,\tau(t)), t) \tag{4.29}$$

together with Equation (4.2) and Equation (4.3).

Current Value Hamiltonian

The current value Hamiltonian of the above problem is

$$\begin{aligned} H^c &= \int_0^{Q(P(t))} Q^{-1}(q)\,dq - Q(P(t))\,C(Q(P(t)), K(t)) - Q(P(t))\,\tau(t) \\ &\quad + \mu(t)\,G(K(t), (Q(P(t))\,\tau(t)), t) \end{aligned} \tag{4.30}$$

and the current value augmented Hamiltonian or Lagrangian, L^c is

$$\begin{aligned} L^c &= H^c + \lambda(t)\,[P(t) - C(Q(P(t)), K(t)) - \tau(t)] \\ &= \int_0^{Q(P(t))} Q^{-1}(q)\,dq - Q(P(t))\,C(Q(P(t)), K(t)) - Q(P(t))\,\tau(t) \\ &\quad + \mu(t)\,G(K(t), (Q(P(t))\,\tau(t)), t) \qquad (4.31) \\ &\quad + \lambda(t)\,[P(t) - C(Q(P(t)), K(t)) - \tau(t)] \end{aligned}$$

where $\mu(t)$ is the costate variable for highway capacity at time t, and can be interpreted as the shadow value of welfare gain for a unit increase in highway capacity. If there is a marginal change in capacity at any time period t, and we modify the problem optimally, then the change in the total value of the objective function would be at a rate $\mu(t)$ (See, for example, Dorfman, 1969). Thus, $\mu(t) = \frac{dW(K_0,t_0)}{dK}$, where $W(K_0,t_0)$ is the maximized value of the objective function (net social benefit). $\lambda(t)$ is the shadow value or welfare cost for marginal increase in highway toll.

Maximum Principle Conditions

The maximum principle conditions for τ, P, K, μ, and λ are respectively

$$\lambda = (\mu G_I - 1) Q. \tag{4.32}$$

$$Q_P P - Q_P C - Q C_Q Q_P - Q_P \tau + \mu G_I Q_P \tau + \lambda (1 - C_Q Q_P) = 0. \tag{4.33}$$

$$- Q C_K + \mu G_K - \lambda C_K = -\dot{\mu} + r\mu. \tag{4.34}$$

$$\dot{K} = G. \tag{4.35}$$

$$P - C - \tau = 0. \tag{4.36}$$

Transversality Condition

For the finite horizon, we have the following transversality condition[7]

$$\mu(t_1) e^{-rt_1} = 0. \tag{4.37}$$

For $G_I > 0$, λ and μ move in the same direction.

Optimal Pricing/Tolling Rule

Combining Equations (4.32) and (4.33) we have,

$$P = C + QC_Q + \left(\frac{\mu - \frac{1}{G_I}}{\mu} \right) \frac{P}{\varepsilon}. \tag{4.38}$$

Here, P/ε is the monopolistic markup such that $P/\varepsilon = -Q/Q_P$. Equation (4.38) is the "pricing rule" in our model. For any given amount of capacity, the above pricing rule maximizes the efficiency of the road use. For the first-best case, we have

[7]For the infinite horizon, we have the following transversality condition: $\lim_{t \to \infty} \mu(t) K(t) e^{-rt} = 0$.

seen that optimal toll is just sufficient to internalize the externality cost of highway congestion, QC_Q and therefore, price equals $C + QC_Q$. However, in the second-best case, we have an adjustment term on the right hand side of Equation (4.38) which is $\left(\frac{\mu - \frac{1}{G_I}}{\mu}\right)\frac{P}{\varepsilon}$. The above pricing policy is, in fact, the robust pricing policy which holds for both first-best with SRMCP and second-best naïve policy[8]. However, in the first-best case, investment in capacity is optimal which gives $\mu = \frac{1}{G_I}$ and the adjustment term disappears. On the other hand, the naive policy with SRMCP is a wrong policy and therefore, its pricing policy cannot be expressed like robust pricing policy.

In fact, the term $\frac{\mu - \frac{1}{G_I}}{\mu}$ is the surplus benefit ratio for capacity investment and the term $\frac{P}{\varepsilon}$ is the markup. The surplus benefit ratio becomes zero when the marginal benefit and cost of capacity investment are equal or when the trip demand is perfectly elastic. In either case, the second-best naïve policy will replicate the first-best price and toll. Since $\frac{P}{\varepsilon}$ and G_I are always positive, the sign of the third term $\left(\frac{\mu - \frac{1}{G_I}}{\mu}\right)\frac{P}{\varepsilon}$ depends on μ. When surplus benefit ratio is positive, negative or zero (together with the fact that the transport market is not perfectly elastic), the optimal pricing policy should be higher, lower or equal to the first-best pricing policy respectively.

We can see that the adjustment term depends on two factors: (1) the investment policy, and (2) the market power. Some extreme value of any of these factors can independently vanish the adjustment term. As we mentioned that in the first-best case, the adjustment term vanishes due to an ideal investment policy i.e., the "apital cost policy". However, in the second-best naïve policy, the adjustment term can vanish due to lack of market power. When $\varepsilon = \infty$, i.e., when demand is perfectly elastic, the superior authority does not have market power over the highway users. In such a case, even if we start with undercapacity, and the highway price is increased above the short-run social marginal cost, commuters are no longer willing to use the high-

[8]Equation (4.13) and Equation (4.7) together enable us to express the pricing policy of the first-best case exactly same as Equation (4.38).

way which will drastically reduce the number of users and thus will push the price back to the short-run social marginal cost. In reality, this situation may arise when commuters have perfectly substitutable routes or travel modes (e.g., subway, bus, car, monorail etc.) or when commuters can costlessly change their work-place.

Here, we briefly discuss some of the special cases of the second-best pricing policy:

(a) when $\mu(t) \neq \frac{1}{G_I}$ and $\varepsilon \neq \infty$, the pricing policy at that time should be something other than marginal cost pricing;

(b) when $\mu(t) \neq \frac{1}{G_I}$ but $\varepsilon \to \infty$ that is, for the case of perfect competition in the transport market, the monopoly markup multiplier, $m = \varepsilon/(\varepsilon - 1) \to 1$ and the pricing policy should be same as marginal cost pricing;

(c) when $\mu(t) > \frac{1}{G_I}$ then pricing policy should be such that price is higher than marginal cost;

(d) when $\mu(t) \to \infty$, i.e., $\mu(t)$ is very large then $\frac{\mu - \frac{1}{G_I}}{\mu} \to 1$ and $P \to C + QC_Q + \frac{P}{\varepsilon}$; this means that the pricing would be such that the congestion externality is internalized. In addition, there is an extra charge equal to the full monopolistic markup $\frac{P}{\varepsilon}$. Likewise, we can express price such that, $P \to \frac{\varepsilon}{\varepsilon - 1}(C + QC_Q)$ which means price should be equal to the social marginal cost multiplied by monopoly markup multiplier $\varepsilon/(\varepsilon - 1)$ with $\varepsilon/(\varepsilon - 1) \geq 1$.

Hence, we can think the factor $\frac{\mu - \frac{1}{G_I}}{\mu}$ as the degree of deviation from the first-best pricing. Since $\mu(t)$ represents a path over time, we may have one or more combination of cases (a) to (d) stated above.

Optimal Capacity Rule

From Equation (4.34)

$$-QC_K + \frac{\dot{\mu}}{\mu G_I} = \frac{r}{G_I} - \frac{G_K}{G_I}. \tag{4.39}$$

Since $G_I > 0$, and $G_K < 0$, the right hand side of Equation (4.39) is positive which is the marginal cost of capacity addition. Thus, the above equation implies that socially optimal capacity is such that the marginal cost of capacity should be equal to the marginal savings (benefit) in travel time plus an additional term $\frac{\dot{\mu}}{\mu G_I}$, the growth rate of marginal benefit. When $\mu = \frac{1}{G_I}$ $\forall t : t_0 \leq t \leq t_1$, then Equation (4.39) converges to the first-best capacity rule for the simple case (Equation (4.14)).

We provide a comparison of the rules for each policy in Appendix A.3.1.

Steady State

Variables are constant in the steady state, therefore, $\dot{\mu}(t) = 0$, $\dot{\tau}(t) = 0$, $\dot{P}(t) = 0$, and $\dot{K}(t) = 0$. For this second-best naïve policy case, we have the following steady state:

$\overline{I} - \left[b^{\frac{1}{\varepsilon}} d_1^{\frac{-1}{\varepsilon}} \overline{I}^{\frac{d_2}{\varepsilon}} \left(\frac{\gamma \overline{I}^{\eta} - rK_0}{\delta} \right)^{\frac{-1}{\varepsilon}} - \alpha t_f - \alpha t_f \beta \left(d_1 \overline{I}^{-d_2} \right)^m \right] d_1 \overline{I}^{-d_2} \left(\frac{\gamma \overline{I}^{\eta} - rK_0}{\delta} \right) = 0$; $\overline{\tau} = \frac{\overline{I}}{\overline{Q}}$; $\overline{P} = b^{\frac{1}{\varepsilon}} \overline{Q}^{\frac{-1}{\varepsilon}}$; $\overline{Q} = d_1 \overline{I}^{-d_2} \overline{K}$; $\overline{K} = \frac{\gamma \overline{I}^{\eta} - rK_0}{\delta}$; $\overline{\mu} = \frac{\frac{1}{\gamma \eta I^{\eta - 1}}}{1 - \frac{\varepsilon}{\overline{P}} \left[\overline{\tau} - \alpha t_f \beta m \left(d_1 \overline{I}^{-d_2} \right)^m \right]}$; where d_1 and d_2 are same as before. Note that in the first expression above, the steady state investment is given implicitly.

4.3.2. Numerical Simulation

In this subsection we provide a numerical simulation to explain our analytical findings. First, we look into how naïve policy with SRMCP distorts the optimum. Then we construct the transition paths of policy variables and show how much welfare the second-best naïve policy can restore. We also discuss the role of some important parameters on the welfare recovery.

The Welfare Index

In order to compare the welfare of the three different policies, a welfare index would be helpful. We particularly use the welfare index constructed by De Palma et al. (2012). This index considers the first-best welfare as the benchmark. Then, the welfare index in “current” term defined as follows:

$$WIC = \frac{CS^{opt}(t) - CS(t)}{CC^{opt}} + 1 \tag{4.40}$$

where WIC is the current welfare index, CS and CC are consumer surplus and construction cost of the concerned policy, and the superscript $'opt'$ represents the value at the optimum. Since net welfare is the sum of discounted instantaneous welfare over time, the “welfare index” is

$$WI = r\int_{t_0}^{t_1} (WIC - 1)\, e^{-rt} dt + 1$$

or,

$$WI = r\int_{t_0}^{t_1} \left[\frac{CS^{opt}(t) - CS(t)}{CC^{opt}}\right] e^{-rt} dt + 1. \tag{4.41}$$

The value of the index is one when there is no welfare loss. In such a case, naïve policy and the first-best optimum have the same welfare effects. For $WI > 1$, the concerned policy results in welfare loss as compared with the optimum policy. The greater the index, the greater is the welfare loss.

The Parameter Values

For simulation, we assume the following parameter values: the user value of in-vehicle time, $\alpha = 62.86$ yen/min; the free-flow time to travel a road segment of 1 km, $t_f = 1$ min/km (assuming 1 kilometer of a road segment with free-flow speed of 60 km/hr); $\beta = 0.15$ and $m = 4.0$ [see, for example, Yoshida (2011) and Nemoto et al. (2009)].

The capacity $K(t)$ and flow $Q(t)$ are expressed in terms of vehicles per day (veh/day); investment cost $I(t)$ is measured in yen/day/km; toll $\tau(t)$ and price $P(t)$ are measured in terms of yen/veh/km; and revenue $R(t)$ is measured in yen/day/km. The annual interest rate r and the depreciation rate of highway capacity δ have be taken as 5% and 4.5% respectively. We choose (positive) price elasticity of demand[9]

[9] It is to be noted that for a constant-elastic trip demand function with (positive) price elasticity $e < 1$, the welfare function is not tractable (e.g., see Yoshida (2011). Due to this reason, in our analysis we assumed elasticity of trip demand as 1.25 which is greater than 1.

$\varepsilon = 1.25$; and $b = 8571900$.

Since exact self-financing is associated with the neutral economies of scale of the capacity cost function, we assume $\eta = 1$ and he total cost of adding one extra unit of capacity (one veh/day) ρ is 23958 yen[10]. This means that $\gamma = \frac{1}{\rho^{\eta}} = 4.174X10^{-4}$. We use MATLAB software to solve for the optimal time paths for the variables. In particular, we follow the "relaxation algorithm" by Trimborn, Koch, and Steger (2008) in our simulation.

Simulation Results and Optimal Time Paths

By using the above parameter values, we have constructed the transition paths of capacity for various initial level of capacities K_0. Figure 4.2 depicts such transition paths for initial capacities equal to 25%, 50%, 75% and 100% of the optimal capacity for the case of naïve policy with SRMCP. We found that when initial capacity is below the optimum, naïve policy with SRMCP results in overcapacity in the long-run. The same was found by Verhoef and Mohring (2009). One notable feature is that when the initial capacity is equal to the optimal capacity, the transition path of capacity of naive policy coincides with the first-best policy. This is obvious as in such a case we do not have any distortion. This provides a robustness check of our simulation analysis. Another important feature is that the more the initial capacity differs from the optimum level, the more is the deviation of the long-run capacity. This supports the findings by De Palma et al. (2012).

The effects of second-best naïve policy have been shown in Figure 4.3. The first-best optimum capacity is 56573.53 veh/day, and for the naïve policy with SRMCP, the long-run capacity is 63039.43 veh/day. However, the second-best naïve policy results in a long-run capacity of 58859.29 veh/day. This implies that though naïve policy with SRMCP brings about a remarkable deviation of capacity from the

[10] According to Nemoto et al. (2009), the construction cost of one lane of highway is 287.5 million yen. Assuming a capacity of 12,000 vehicles/day/lane, the unit cost of capacity becomes 287.5*1000000/12000 or 23958 yen. It is to be noted that to get the true construction cost for one unit capacity we have to multiply this figure by $r/365$.

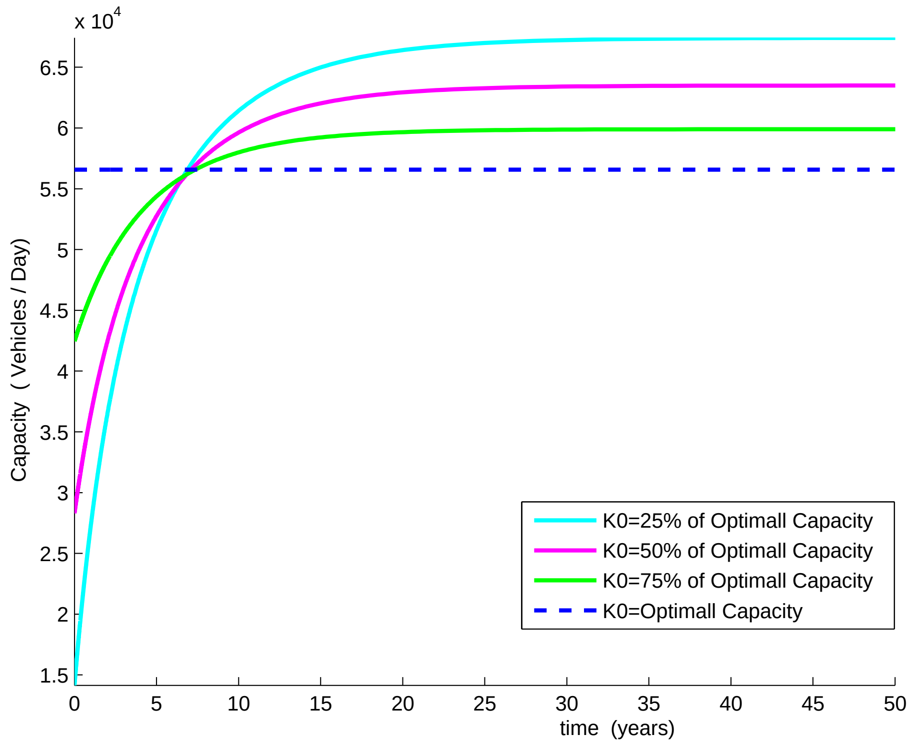

Figure 4.2. Capacity Comparison for Various Level of Initial Capacities for the Optimum, Naïve Policy with SRMCP and Second-best Naïve Policy.

optimum level, the second-best naïve policy minimizes the deviation significantly. However, it cannot completely restore the first-best optimum.

Next we determine the welfare index for the naïve policy with SRMCP and the second-best naïve policy as defined in Equation (4.41). While calculating this index, we assume that initial capacity is at 25% of the optimal capacity for both policies. The welfare index for naive policy with SRMCP is 1.1427 and for Second-best naive policy is 1.0838. This means that if the capacity cost of the first-best policy is increased by 14.27%, then it would have the same amount of welfare as in the case of naive policy with SRMCP. On the other hand, if the capacity cost of the first-best policy is increased by 8.38%, then first-best and the second-best naive policy would have the same welfare. Thus, the welfare of the second-best naïve policy is closer to first-best case than that of the naïve policy with SRMCP. This confirms that the second-best naïve policy indeed reduces the welfare loss.

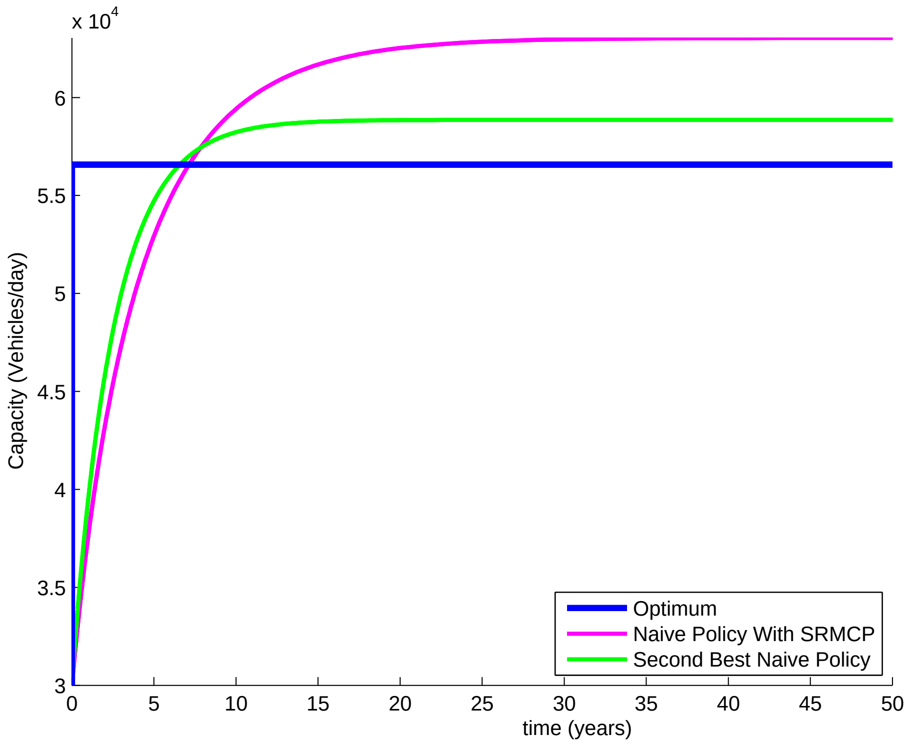

Figure 4.3. Transition Paths of Highway Capacity for the Optimum, Naïve Policy with SRMCP and Second-best Naïve Policy.

The Toll and Price Profile

Figure 4.4 and Figure 4.5 represent the toll and price profile of the three policies. The toll profile of the second-best naïve policy implies that in order to minimize welfare loss, initially we have to charge toll higher than optimum. Since in our simulation the initial capacity is suboptimal, charging higher initial toll will help to generate more revenue and therefore more capacity addition. This helps to reduce the capacity-gap faster. Since we consider the discounted net total welfare, the early recovery of capacity yields a higher value of the objective function. After a time period of about five years, we need to charge toll lower than optimum. This lower toll eventually prevents capacity to grow over time and keeps it as close to the optimum as possible.

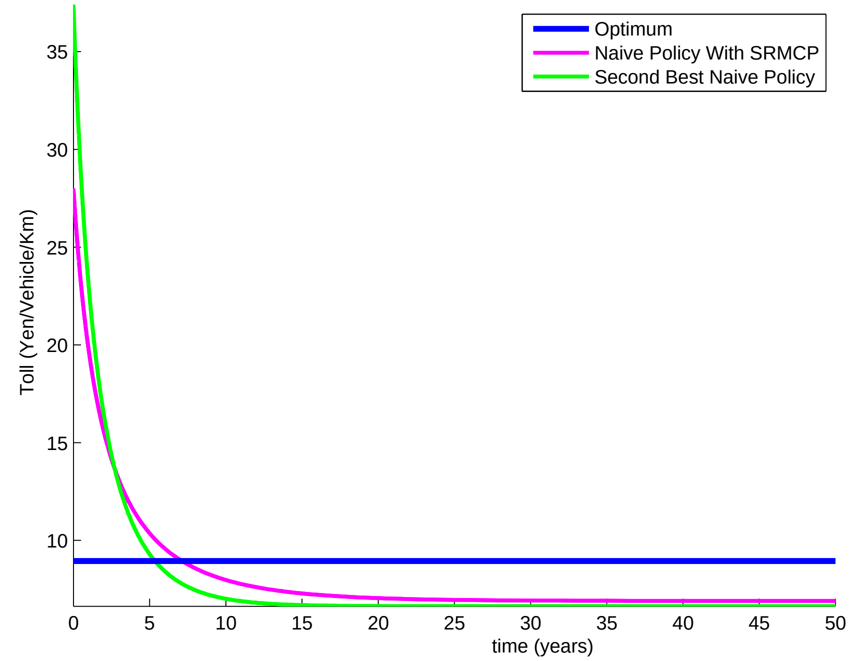

Figure 4.4. Transition Paths of Congestion Toll for the Optimum, Naïve Policy with SRMCP and Second-best Naïve Policy.

The pattern of the price profile is similar to that of the toll profile.

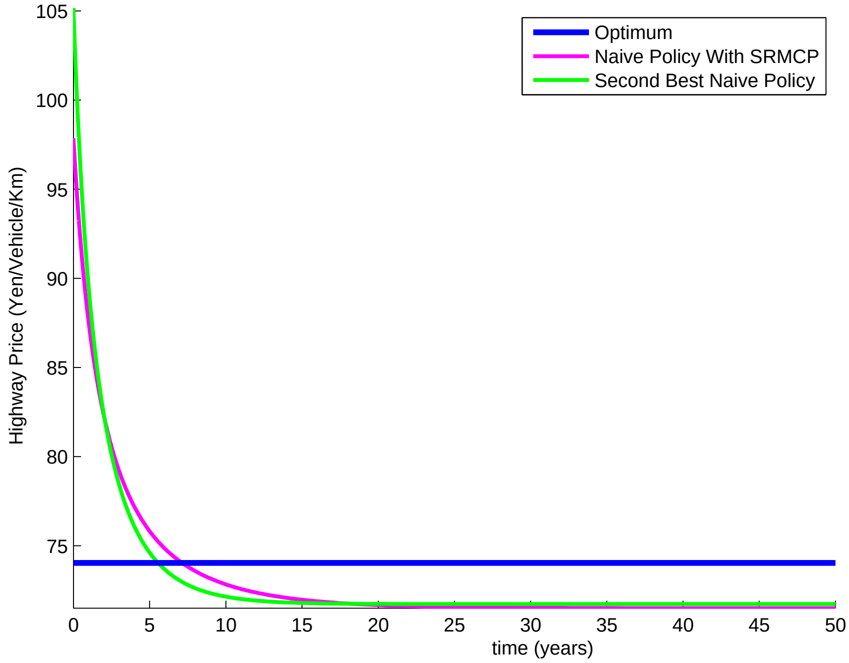

Figure 4.5. Transition Paths of Highway Price for the Optimum, Naïve Policy with SRMCP and Second-best Naïve Policy.

The Welfare-effect of Interest Rate

The cost of capital depends on the interest rate. Therefore, interest rate has a significant effect in our analysis. In Figure 4.6, we represent the steady state capacities for the three policies against interest rate. For the purpose of consistency, we assume that the initial capacity is 25% of the corresponding optimal capacity. The results govern that as the interest rate increases, the steady state level of capacity for each policy decreases. This can be explained by the fact that for higher interest rate, the highway agency has to pay more annuities for the initial capacity. So there remains less additional revenue to invest in future capacity expansion. That is why, the long-run capacity will be low. However, the interest rate has greater effect on the first-best optimal capacity. The reason is that for this case, capacity investment is made according to "capital cost" policy which directly depends on interest rate and affects the provision of future capacity.

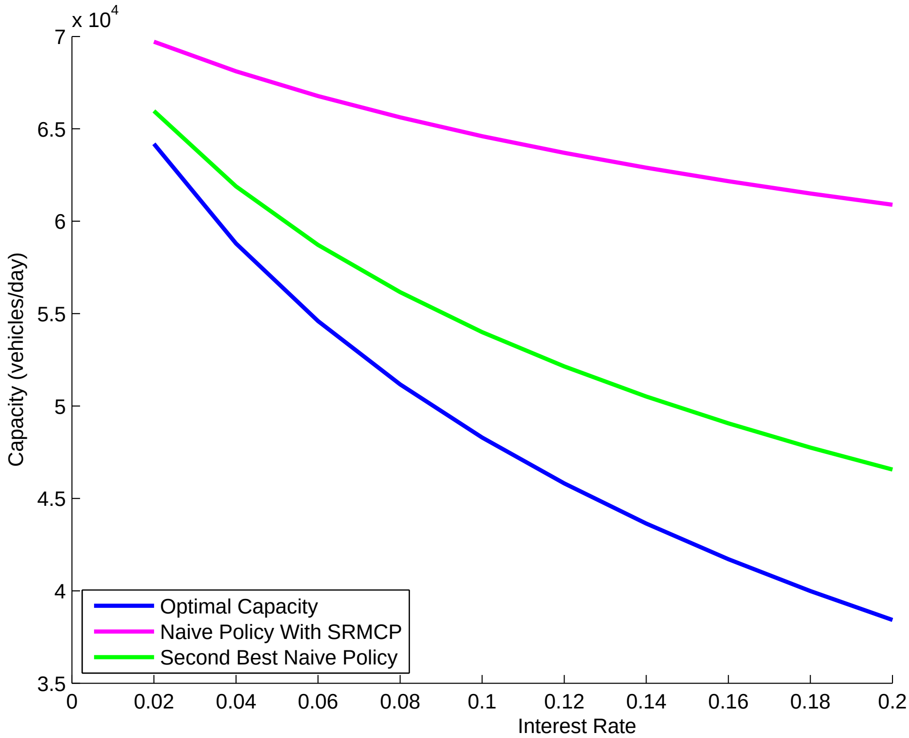

Figure 4.6. Variation of Capacity Against Interest Rate for Naïve Policy with SRMCP and Second-Best Naïve Policy as Compared with the Optimum.

The corresponding welfare indices imply that there is a remarkable loss in welfare for naïve policy with SRMCP. The second-best naïve policy, though reduces welfare loss to a significant amount, leaves a larger welfare-gap at higher interest rates. Thus, interest rate is a crucial variable that has a significant effect on the outcome of second-best naïve policy. This further implies that naïve policy has greater negative consequences to developing countries with higher interest rates. Therefore, careful consideration must be given while planning for self-financing in highway sectors to those countries.

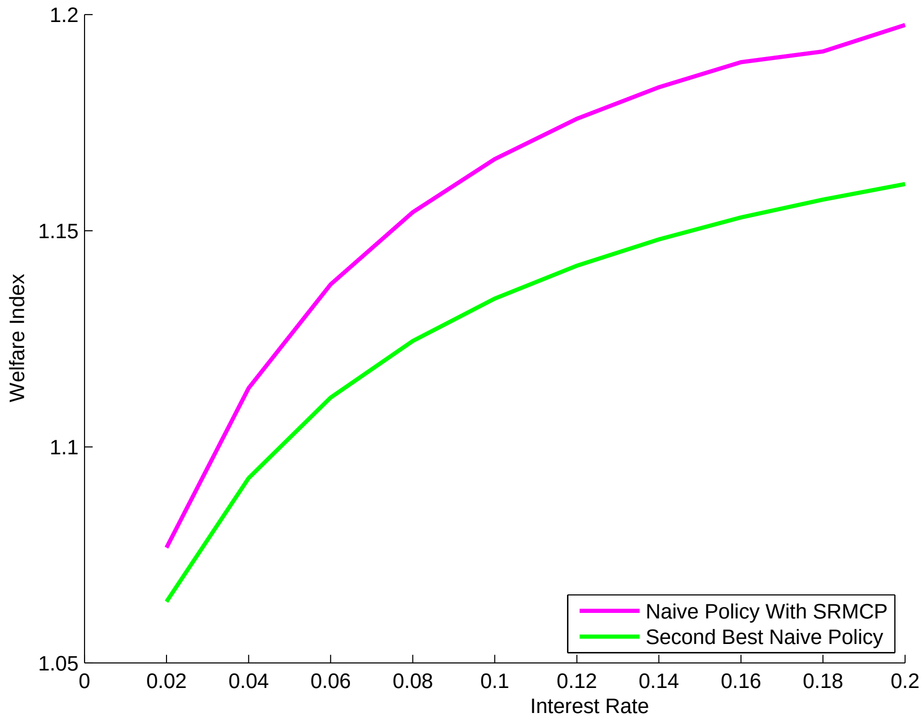

Figure 4.7. Variation of Welfare Index with Interest Rate for Naïve Policy with SRMCP and Second-best Naïve Policy as Compared with the Optimum.

The Welfare-effect of Price Elasticity of Demand

The simulation results show that price elasticity of demand has effect on the long-run capacity. We present these in Figure 4.8. As before, the initial capacity is kept at 25% of the corresponding optimal capacity. For higher (positive) elasticity, naïve policy yields lower capacity and vice versa. This is intuitive because for higher elasticity, people are more sensitive to price and are less willing to make trips when price is higher. Figures 4.8 and 4.9 shows the capacity profile for naïve policy with SRMCP and second-best naïve policy for (positive) price elasticities of demand elasticity 1.10, 1.25 and 1.50. The shapes of capacity profiles are similar for both cases; however, the magnitudes are different.

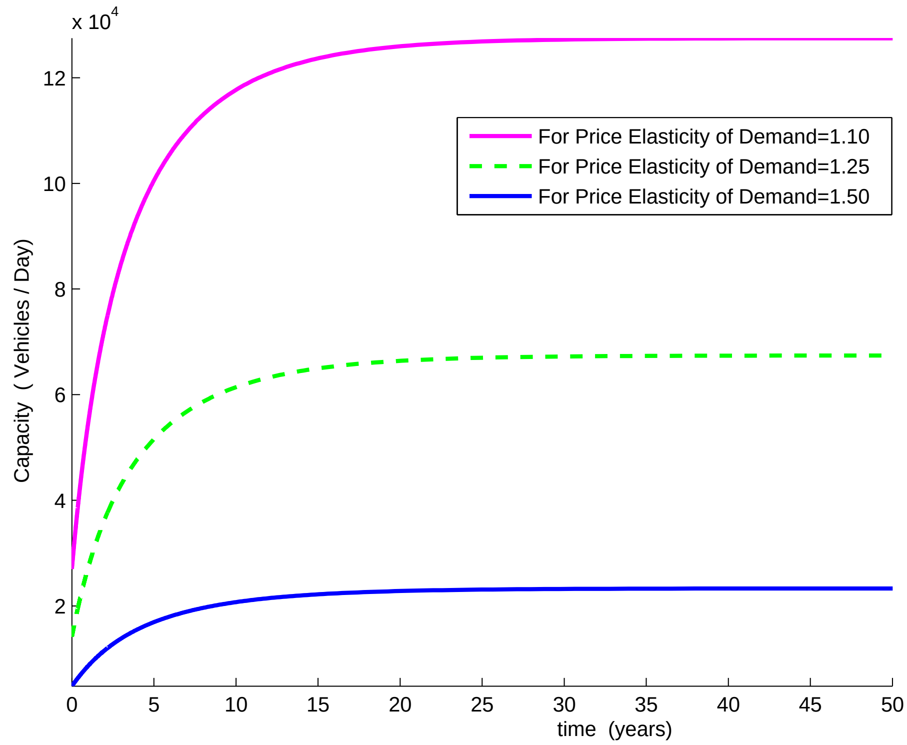

Figure 4.8. The Capacity Profile for Different Price Elasticity of Demand for Naïve Policy with SRMCP.

The welfare indices for naïve policy with SRMCP are 1.1254, 1.1267 and 1.1279 for demand elasticities 1.10, 1.25 and 1.50 respectively.

For the second-best naïve policy, the corresponding welfare indices are 1.1002, 1.1030 and 1.1063 respectively. These numbers reveal that even though the second-best naïve policy recovers welfare when demand elasticity is less, but for practical purpose, the amount of recovery is really insignificant. Thus, the effect of demand elasticity has turned to be negligible on welfare.

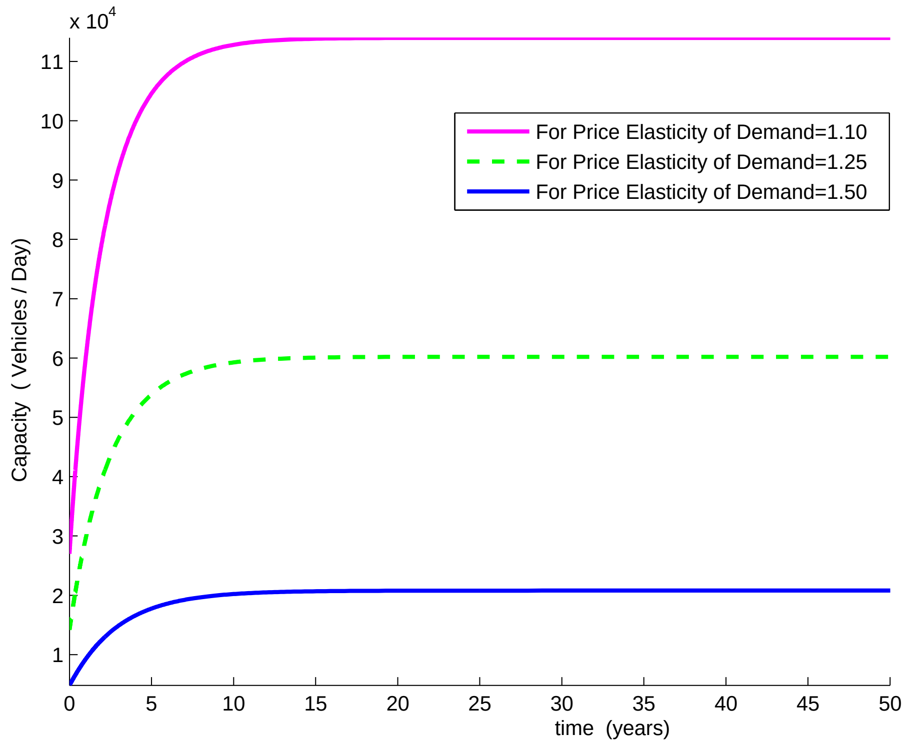

Figure 4.9. The Capacity Profile for Different Price Elasticity of Demand for Second-best Naïve Policy.

Parameter γ also have similar negligible welfare effects.

4.4. Conclusion

In this study, we addressed the welfare issue of naïve implementation of highway investment where the highway agency's prime target is to achieve self-financing. We proposed a welfare recovery policy measure which we term as "second-best naïve policy". We formulated the model in intertemporal settings and considered the net present value of money. The two tiers of highway management are vertically disintegrated. The benevolent superior authority decides highway toll (price) in advance, knowing that the highway agency will follow a wrong investment policy. We analytically derived the optimality conditions. To minimize the negative consequences of naïve policy, the superior authority needs to charge an well-designed second-best toll. As naïve policy with SRMCP results in over capacity in the long-run. Under the assumptions of homogenous user cost and depreciation functions with neutral economies of scale in capacity production, we numerically solved for the trajectories of the highway capacity, toll and price for the naïve policy with SRMCP and the second-best naïve policy. We then compared the resulting welfare of these two policies. We found that when the interest rate is zero, naïve policy does not have any adverse effect on the welfare. For other cases, naïve policy with SRMCP results in over capacity and higher welfare losses. Though the remedial measure against naïve policy cannot fully restore the first-best optimum in the long-run, yet it is possible to recover a significant portion of the welfare losses. Thus, our remedial measure deserves consideration for cost recovery policy formulation in highways. We also showed that interest rate plays a vital role in welfare recovery. The welfare recovery is less for higher interest rates. This readily implies that developing countries are more vulnerable to welfare consequences due to naïve investment policy and therefore careful consideration should be given while planning for self-financing in highway sectors especially.

However, the model of this study can be extended for time varying travel demand and interest rates. We considered only one highway agency running under zero profit margin. In reality, it may be the case that there are more than one agencies operating under oligopolistic manner. It might be interesting to extend the model in such a context. Also we considered single origin-destination pair. However, as stated by Verhoef & Mohring (2009), we need to charge optimal toll to the whole network for achieving self-financing. Therefore, the model needs to be extended for the entire network. In the numerical simulation, we mainly considered neutral-economies of scale in the construction industry. One might consider other types of economies of scale in the capacity cost function and seek the modified version of the welfare recovery policies.

5. CONCLUSION

5.1. Introduction

The issue of an efficient and sustainable transport infrastructure is the major concern for the transport policymakers in most countries. Many studies have been carried by researchers and academics to address this issue. However, most of their works are particular in nature, and there is no study that addresses the optimality requirements of transport infrastructural development under general settings. There is no city model that includes circumferential highway as a means of travel and there is no work on circumferential highway that considers land-use and locational choice. The underlying concept of highway self-financing is highly technical and therefore most likely to be perceived wrongly especially, by the policy makers or by the highway operators. In fact, three important agenda items deserves in-depth inquiry: the 'general' optimality conditions for urban transport infrastructure, the optimal design of a circumferential highway within an endogenous city framework, and the welfare recovery due to naïve policy in highway investment. These issues have not been addressed in the prior literature.

Keeping that in mind, this dissertation, we attempted to analyze the optimal transport development in terms of three aspects. The first is the optimal transport development under general settings with residential land ownership. For this, we formulated a general urban land-use model and determined general optimality conditions of transport investment where the city characteristics are interrelated with the level of development. We also tried to establish the relationships between the Samuelson condition, Henry George theorem and capitalization hypothesis and to verify their applicability within the model. The second aspect is the optimal circumferential highway in a monocentric city. In this study, we determined the capacity and location of the circumferential highway using a city model where the landowners are absentees. We then explored the resulting size and shape of the city. We also discussed about the threshold number of radial highways beyond which the construction of a new circumferential highway does not improve welfare. The last aspect is the determination of welfare recovery policy measures when highway agency follows naïve policy in capacity investment. For this, we formulated a two-stage dynamic game model between the government and the transport agency and tried to determine the intertemporal pricing (tolling) policy by maximizing the discounted net welfare function of the society. We also presented a comparison of welfare between the naïve policy with SRMCP and the second-best naïve policy.

5.2. Summary of the Findings

In Chapter 2 of this dissertation, we developed the optimality conditions for transport development with the assumption that households are landowners. The analysis revealed that for a socially optimal transport infrastructure, the marginal cost of infrastructural development should be equal to the marginal aggregate differential land rent evaluated at pre-development land rent. Another finding of this chapter is that at the optimum, the marginal change in transport cost at the city periphery due to a infrastructural development is equal to the per capita marginal change in land value evaluated at the post-development land rent. The marginal change in aggregate differential land rent, in the former finding, arises due to a change in land rent while keeping the city size fixed. However, in the later findings the marginal change in aggregate differential land rent arises due to a change in city area while keeping the land rend fixed. In fact, when there is an development in transport facility, the aggregate differential land rent changes by a two-way interaction: (a) the change in city area while keeping land rent fixed; and (b) the change in land rent while keeping city area fixed. Thus, at the optimum, the first component of aggregate differential land is required to be equal to the marginal development cost and the second component of aggregate differential land rend is required to be equal to the total (over population) marginal change in transport cost at the city boundary.

In Chapter 3, we provided an estimation procedure for the optimal capacity and location of a circumferential highway by using an urban land-use model with three mode options. Our numerical analysis reveals that an optimal circumferential highway is not required to be fully utilized. At first sight, this might seem contradictory to the fact, but it has economic reasoning. The households living near the circumferential highway but far away from the radial highways have no incentive to use the circumferential highway; because, in such a location, direct commute to CBD by using city streets provides them cheaper travel. Among other findings, this study demonstrates that there is a positive relationship between the radius of the circumferential highway and the number of radial highways. Therefore, if a city has more radial highways at the initial stage, the optimal circumferential highway should be larger. On the other hand, cities having radial highways more than a threshold number do not require circumferential highway at all.

In Chapter 4, we derived the welfare recovery conditions where the transport agency deviates from the basic principles of self-financing and invests the net toll revenue in future capacity expansion. We have shown that when the interest rate is zero, naïve policy replicates the social optimum in the long-run. In all other cases, naïve policy results in negative welfare consequences and over capacity. We analytically derived the remedial policy. Then by using numerical simulation, we constructed the transition paths of toll and price that might be considered as a counter measure against such misleading policy. We have provided a comparison between the outcomes of the naïve policy with SRMCP and the second-best naïve policy and have shown that second-best toll is higher initially and then lower as compared with the first-best toll. Since our numerical simulation considers suboptimal initial capacity, therefore, higher initial toll will generate higher revenue and so we can increase capacity more rapidly. On the other hand, as naïve policy with SRMCP results in overcapacity in the long-run, we need to charge lower toll subsequently in order to minimize the deviation in capacity from the optimum. Though second-best naïve policy cannot fully restore the first-best outcome, it indeed recovers substantial portion of the welfare losses. We also have shown that interest rate plays a crucial role in welfare recovery. The higher the interest rate, the less is the welfare recovery.

5.3. Academic Importance and Policy Implication

The main purpose of the transport policy is to establish a socially effective and economically viable transport infrastructure and thus to contribute positively to the socio-economic development and quality of living at the region. This underlines the fact that any kind of infrastructural improvement should be planned in such a way that by utilizing the minimal resource it can meet the transport demand of the society. The findings of the general optimality conditions for transport development have important implication to the literature on urban transport economics and policy formulation. It demonstrates that we do not need to predict post-project land price while determining the optimal amount of transport investment; a message that might be helpful in the planning process of highway improvement. The model can also be used to analyze the cities having more than one CBD.

The procedure illustrated in Chapter 3, can be used to determine the optimal circumferential highway when migration is not frequent between cities. The finding that an optimal circumferential highway is partially used, together with some other factors, could be helpful in explaining urban sprawl in the cities. Another policy implication is that the city planners of developing countries with a smaller number of radial highways need not choose an excessively large circumferential highway.

Long-run self-financing in highway sector means to break-even over the entire planning horizon. In order to sustain long-run self-financing by short-run marginal cost pricing, it is, therefore, not necessary to be break even at every point in time. In fact, investing the net toll revenue directly into the capacity addition for each point in time is detrimental from the welfare perspective. When the policy of the government is to ensure long-run self-financing, a poorly conceived investment policy by the highway agency can be corrected a priori through fixing optimal toll profile. The findings of Chapter 4 can be used as a policy guideline to recover welfare losses resulting from the naïve policy with SRMCP. The developing countries with higher interest rate need to pay careful considerations while planning for self-financing in highway sector.

5.4. Limitations and Direction for Future Research

Like other research works, this dissertation has some limitations. The major limitation of Chapter 2 is that we did not consider congestion, without which the model does not really capture the true travel and locational choice behavior. Another limitation is that we considered closed city which means the number of population is fixed. Nonetheless, the number of households of a city varies due to migration as well as natural growth of its population. Since transport development is a long-run issue, relaxing the closed city assumption has practical importance. However, when the population grows at an exogenous rate, we can accommodate this change in the model by considering discounted present value of the objective function. We did not include the production sector of the economy in the model and the transport demand was inelastic with the transport supply. It may be the case that households are not willing to work if the transport cost is too high; this may affect construction costs and wages. It can be easily understood that most of the limitations in Chapter 1 arose from the difficulty in handling analytical complexity. However, the above limitations might be considered in future research.

Chapter 3 can be regarded as an applied version of Chapter 2, and as such most limitations of Chapter 2 are also limitations of Chapter 3. Unlike Chapter 2, we assumed absentee land ownership and, therefore, the welfare of the land owners should be taken into account in the optimization problem. It is possible to incorporate the congestion and elastic travel demand issue in the numerical analysis of Chapter 3 by replacing the unit travel cost function (which depends only on capacity) with an appropriate trip demand function (which depends on both capacity and usages). Another limitation is that we considered only one circumferential highway. Therefore, the model cannot be generalized to cities that already have one or more circumferential highways and need to construct another. Lastly, we ignored the fixed cost of travel. However, fixed costs are not fixed across locations and therefore, they are likely to affect the results. We regard all of these as points to be addressed in future works.

In Chapter 4, we assumed that capacity can be added continuously. However, this is the theoretical necessity for self-financing to hold and therefore, not, in fact, a limitation of this paper. Moreover, while considering a large transport network, this assumption would seem to be not unreasonable. Another issue arises from the fact that we do not consider growing demand for transport services. We also overlooked the time-varying congestion. We ignored the operating cost of the highway agency; however, it is possible to accommodate this in the model. Time-varying interest rates can be incorporated in the model. Lastly, we limited our analysis to one origin-destination pair of the highway network, which needs to be extended to the full transport network. We hope that future research will enrich this study by incorporating and analyzing the limitations listed above and thus make it academically more interesting and practically more applicable.

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A. Appendix

A.1. Appendix 1

A.1.1. Derivatives

Derivatives of Consumption Goods

Here we briefly describe the derivation of $dQ*$, $dZ*$, and $dL*$. The main idea here is to eliminate dR from the total derivatives of Equations (2.8), (2.9), (2.10) and (2.11). The total derivative of Equations (2.8) through (2.11) yields

$$\begin{aligned} dQ^* &= Q_R^c dR + Q_U^c d\overline{U} \\ dZ^* &= Z_R^c dR + Z_U^c d\overline{U} \\ dL^* &= L_R^c dR + L_U^c d\overline{U} \end{aligned}$$

and

$$Q^* dR + R dQ^* = dy^d - dZ^* - w dL^*.$$

By using $Z_R^c + RQ_R^c + wL_R^c = 0$ we have

$$\begin{aligned} dR &= \frac{dy^d(G) - \left(Z_R^c dR + Z_U^c d\overline{U}\right) - R(G)\left(Q_R^c dR + Q_U^c d\overline{U}\right) - w\left(L_R^c dR + L_U^c d\overline{U}\right)}{Q^*(G)} \\ &= \frac{dy^d(G) - \lambda(G)\, d\overline{U}}{Q^*(G)} \end{aligned}$$

and

$$\begin{aligned} dQ^* &= Q_R^c \frac{dy^d(G) - \lambda(G)\, d\overline{U}}{Q^*(G)} + Q_U^c d\overline{U} \\ dZ^* &= Z_R^c \frac{dy^d(G) - \lambda(G)\, d\overline{U}}{Q^*(G)} + Z_U^c d\overline{U} \\ dL^* &= L_R^c \frac{dy^d(G) - \lambda(G)\, d\overline{U}}{Q^*(G)} + L_U^c d\overline{U} \end{aligned}$$

which ultimately gives

$$\begin{aligned} dQ^* &= \frac{Q_R^c}{Q^*(G)}\left(-dD+\frac{d\Phi}{N}\right)+\left[Q_U^c-\frac{\lambda(G)\,Q_R^c}{Q^*(G)}\right]d\overline{U} \\ dZ^* &= \frac{Z_R^c}{Q^*(G)}\left(-dD+\frac{d\Phi}{N}\right)+\left[Z_U^c-\frac{\lambda(G)\,Z_R^c}{Q^*(G)}\right]d\overline{U} \\ dL^* &= \frac{L_R^c}{Q^*(G)}\left(-dD+\frac{d\Phi}{N}\right)+\left[L_U^c-\frac{\lambda(G)\,L_R^c}{Q^*(G)}\right]d\overline{U} \end{aligned}$$

where $\lambda(G) \equiv \partial E/\partial U = Z_U^c + R(G)\,Q_U^c + wL_U^c$.

Derivative of The Land Constraint

Taking total differentiation of the Equation (2.15) we get

$$dN = \int_0^{\overline{G}}\left[\frac{1}{Q^*(G)}\frac{\partial\xi(G)}{\partial\eta}d\eta-\frac{\xi(G)\,dQ^*}{\{Q^*(G)\}^2}\right]dG+\frac{\xi(\overline{G})}{Q^*(\overline{G})}d\overline{G}.$$

Since number of population N is given so we have

$$0 = \int_0^{\overline{G}}\frac{1}{Q^*(G)}\left[-\frac{\xi(G)}{Q^*(G)}dQ^*+\frac{\partial\xi(G)}{\partial\eta}d\eta\right]dG+\frac{\xi(\overline{G})}{Q^*(\overline{G})}d\overline{G}. \quad \text{(A.1)}$$

A.1.2. Derivative of the Aggregated Land Rent Equation

Taking total differentiation of Equation (2.17) we get,

$$d\Phi = \int_0^{\overline{G}}\xi(G)\,dR(G)\,dG+\int_0^{\overline{G}}(R(G)-R_A)\frac{\partial\xi(G)}{\partial\eta}d\eta dG.$$

Replacing the value of $R(G)$ from Equation (2.11) and rearranging we get,

$$\begin{aligned} d\Phi &= \int_0^{\overline{G}}\xi(G)\left[\frac{-dD+\frac{d\Phi}{N}-R(G)\,dQ^*(G)-dZ^*(G)-wdL^*(G)}{Q^*(G)}\right]dG \\ &\quad +\int_0^{\overline{G}}(R(G)-R_A)\frac{\partial\xi(G)}{\partial\eta}d\eta dG \end{aligned}$$

$$\begin{aligned} d\Phi &= \int_0^{\overline{G}}\left[\frac{-dD+\frac{d\Phi}{N}}{Q^*(G)}\right]\xi(G)\,dG-\int_0^{\overline{G}}\left[\frac{R(G)\,dQ^*(G)}{Q^*(G)}\right]\xi(G)\,dG \\ &\quad -\int_0^{\overline{G}}\left[\frac{dZ^*(G)}{Q^*(G)}\right]\xi(G)\,dG-\int_0^{\overline{G}}\left[\frac{wdL^*(G)}{Q^*(G)}\right]\xi(G)\,dG \\ &\quad +\int_0^{\overline{G}}(R(G)-R_A)\frac{\partial\xi(G)}{\partial\eta}d\eta dG. \end{aligned}$$

Now replacing the value of dQ^*, dZ^* and dL^* from Equation (2.23) together with using $Z_R^c + RQ_R^c + wL_R^c = 0$ and $\lambda(G) = Z_U^c + R(G)Q_U^c + wL_U^c$, the above equation reduces to

$$d\Phi = \int_0^{\overline{G}} \left[\frac{-dD + \frac{d\Phi}{N} - \lambda(G)\,d\overline{U}}{Q^*(G)} \xi(G) + (R(G) - R_A) \frac{\partial \xi(G)}{\partial \eta} d\eta \right] dG \qquad \text{(A.2)}$$

A.1.3. Optimality Conditions with Residential Ownership

We can write Equation (2.24) as follows:

$$\begin{aligned}
0 &= - \left[\int_0^{\overline{G}} \frac{\xi(G)}{(Q^*(G))^3} Q_R^c dG \right] \left(-dD + \frac{d\Phi}{N} \right) + \left[\int_0^{\overline{G}} \frac{1}{Q^*(G)} \frac{\partial \xi(G)}{\partial \eta} dG \right] d\eta \\
&\quad - \left[\int_0^{\overline{G}} \frac{\xi(G)}{(Q^*(G))^2} \left(Q_U^c - \frac{\lambda(G) Q_R^c}{Q^*(G)} \right) dG \right] d\overline{U} + \frac{\xi(\overline{G})}{Q^*(\overline{G})} d\overline{G} \\
d\overline{G} &= -dD + \frac{d\Phi}{N} - \lambda(\overline{G})\, d\overline{U} \\
d\Phi &= N\left(-dD + \frac{d\Phi}{N} \right) - \left[\int_0^{\overline{G}} \lambda(G) \frac{\xi(G)}{Q^*(G)} dG \right] d\overline{U} \\
&\quad + \left[\int_0^{\overline{G}} (R(G) - R_A) \frac{\partial \xi(G)}{\partial \eta} dG \right] d\eta \\
dD &= \frac{1}{N} \frac{\partial K}{\partial \eta} d\eta
\end{aligned}$$

where the second equation uses $Z_R^c + RQ_R^c + wL_R^c = 0$ and the third equation uses $\int_0^{\overline{G}} \xi(G)/Q^*(G)\,dG = N$.

Eliminating dD and $d\Phi$ using the third and the fourth equations, the above system of equations converts to

$$\begin{aligned}
0 &= - \left[\int_0^{\overline{G}} \frac{\xi(G)}{(Q^*(G))^3} Q_R^c dG \right] \left(d\overline{G} + \lambda(\overline{G})\, d\overline{U} \right) + \left[\int_0^{\overline{G}} \frac{1}{Q^*(G)} \frac{\partial \xi(G)}{\partial \eta} dG \right] d\eta \\
&\quad - \left[\int_0^{\overline{G}} \frac{\xi(G)}{(Q^*(G))^2} \left(Q_U^c - \frac{\lambda(G) Q_R^c}{Q^*(G)} \right) dG \right] d\overline{U} + \frac{\xi(\overline{G})}{Q^*(\overline{G})} d\overline{G}
\end{aligned}$$

and

$$\left[\int_0^{\overline{G}} \lambda(G) \frac{\xi(G)}{Q^*(G)} dG \right] d\overline{U} = - \left[\frac{\partial K}{\partial \eta} \right] d\eta + \left[\int_0^{\overline{G}} (R(G) - R_A) \frac{\partial \xi(G)}{\partial \eta} dG \right] d\eta$$

which can be expressed as (2.25) and (2.26).

A.1.4. Relationships Between Henry George Theorem, Samuelson Condition, and Capitalization Hypothesis

For the residential landownership, the utility maximization problem of the households residing at the city boundary is

$$\max_{Z,Q,L} \quad U = \quad U(Z,Q,:L) \tag{A.3}$$

$$s.t. \quad wH - D + \tfrac{\Phi}{N} - \overline{G} = \quad Z\left(\overline{G}\right) + R_A Q\left(\overline{G}\right) + wL\left(\overline{G}\right). \tag{A.4}$$

We have the Lagrangian as

$$\mathscr{L} = U(Z,Q,L) + \Pi\left(wH - D + \frac{\Phi}{N} - \overline{G} - Z - RQ - wL\right) \tag{A.5}$$

with Π is a Lagrangian multiplier. Solving this we get, $U_Z = \Pi$, $U_Q = \Pi R_A$, and $U_L = \Pi w$ and therefore, the prices are $R_A = U_Q/U_Z$, and $w = U_L/U_Z$.

The Marshallian Demands are

$$Z^* = Z\left(R_A, \overline{G}, D, \Phi, N, w, H\right),$$

$$Q^* = Q\left(R_A, \overline{G}, D, \Phi, N, w, H\right),$$

$$L^* = L\left(R_A, \overline{G}, D, \Phi, N, w, H\right),$$

from where we get the indirect utility function

$$V = V\left(R_A, \overline{G}, D, \Phi, N, w, H\right).$$

From Envelop Theorem

$$\frac{dV}{d\overline{G}} = \frac{\partial \mathscr{L}}{\partial \overline{G}} = \frac{\Pi}{N}\frac{\partial \Phi}{\partial \overline{G}} - \Pi = -U_Z$$

$$\frac{dV}{dD} = \frac{\partial \mathscr{L}}{\partial D} = -\Pi = -U_Z$$

$$\frac{dV}{d\Phi} = \frac{\partial \mathscr{L}}{\partial \Phi} = \frac{\Pi}{N} = \frac{U_Z}{N}$$

$$\frac{dV}{dN} = \frac{\partial \mathscr{L}}{\partial N} = -\frac{\Pi\Phi}{N^2} = -\frac{U_Z\Phi}{N^2}$$

$$\frac{dV}{d\eta} = \frac{\partial \mathscr{L}}{\partial \eta} = 0.$$

Now the problem of the Planner is

$$\max_{\eta} \quad V\left(R_A, \overline{G}, D, \Phi, N, w, H\right) \tag{A.6}$$

subject to Equations (2.21), (2.22), (2.15), (2.16) and (2.17).

After taking total differentiation, we have

$$\frac{\partial K}{\partial \eta} d\eta = DdN + NdD.$$

Also,

$$\begin{aligned} dV &= V_{R_A} dR_A + V_G d\overline{G} + V_D dD + V_\Phi d\Phi + V_N dN \\ &= 0 - U_Z d\overline{G} - U_Z dD + \frac{U_Z}{N} d\Phi - \frac{U_Z \Phi}{N^2} dN. \end{aligned}$$

Thus,

$$\begin{aligned} \frac{NdV}{U_Z d\eta} &= -N\frac{d\overline{G}}{d\eta} - N\frac{dD}{d\eta} + \frac{d\Phi}{d\eta} - \frac{\Phi}{N}\frac{dN}{d\eta} \\ &= -N\frac{d\overline{G}}{d\eta} - \frac{-DdN + \frac{\partial K}{\partial \eta} d\eta}{d\eta} + \frac{d\Phi}{d\eta} - \frac{\Phi}{N}\frac{dN}{d\eta} \\ &= -N\frac{d\overline{G}}{d\eta} + \frac{d\Phi}{d\eta} - \frac{\partial K}{\partial \eta} + \frac{K}{N}\frac{dN}{d\eta} - \frac{\Phi}{N}\frac{dN}{d\eta}. \end{aligned}$$

A.1.5. Optimality Condition with Absentee Land Ownership

In this case the modified expression for the disposable income of a household is

$$y^d(G) = wH - D - G$$

and the residential land rent is

$$R(G) = \frac{1}{Q^*(G)}\left[y^d(G) - Z^*(G) - wL^*(G)\right]. \tag{A.7}$$

Thus at spatial equilibrium we have

$$N = \int_0^{\overline{G}} \frac{\xi(G)\, dG}{Q^*(G)} \tag{A.8}$$

$$R\left(\overline{G}\right) = \frac{wH - D - \overline{G} - Z^*\left(\overline{G}\right) - wL^*\left(\overline{G}\right)}{Q^*\left(\overline{G}\right)} = R_A \tag{A.9}$$

$$\Phi = \int_0^{\overline{G}} \left(R(G) - R_A\right) \xi(G)\, dG \tag{A.10}$$

Derivative of Aggregate Differential Land Rent

Taking derivative of the aggregate differential land rent of Equation (A.10), we get

$$d\Phi = \int_0^{\overline{G}} \left[\left(\frac{-dD - \lambda(G)\, d\overline{U}}{Q^*(G)} \right) \xi(G) + (R(G) - R_A) \frac{\partial \xi(G)}{\partial \eta} d\eta \right] dG$$

which can be written this as

$$\frac{d\Phi}{d\eta} = \int_0^{\overline{G}} \left[\frac{\xi(G)}{Q^*(G)} \left(-\frac{dD}{d\eta} - \lambda(G) \frac{d\overline{U}}{d\eta} \right) + (R(G) - R_A) \frac{\partial \xi(G)}{\partial \eta} \right] dG \tag{A.11}$$

Optimization of transport Network

For absentee landownership case, Equation (2.23) becomeTaking total differentiation of the Equations (A.8), (A.9) as well as (2.21) and (2.22) with respect to network design η yields

$$\begin{aligned} 0 &= \int_0^{\overline{G}} \frac{1}{Q^*(G)} \left[-\frac{\xi(G)}{Q^*(G)} dQ^* + \frac{\partial \xi(G)}{\partial \eta} d\eta \right] dG + \frac{\xi(\overline{G})}{Q^*(\overline{G})} d\overline{G} \\ 0 &= -dD - d\overline{G} - dZ^*|_{\overline{G}} - w\, dL^*|_{\overline{G}} - R_A\, dQ^*|_{\overline{G}} \\ dK &= \frac{\partial K}{\partial \eta} d\eta \\ dD &= \frac{dK}{N}. \end{aligned}$$

Using Equation (**??**) along with $Z_R^c + RQ_R^c + wL_R^c = 0$ and then eliminating dD in the above set of equation we get

$$\begin{aligned} 0 = - & \left[\int_0^{\overline{G}} \frac{\xi(G)}{(Q^*(G))^3} Q_R^c dG \right] \left(d\overline{G} + \lambda(\overline{G})\, d\overline{U} \right) + \left[\int_0^{\overline{G}} \frac{1}{Q^*(G)} \frac{\partial \xi(G)}{\partial \eta} dG \right] d\eta \\ & - \left[\int_0^{\overline{G}} \frac{\xi(G)}{(Q^*(G))^2} \left(Q_U^c - \frac{\lambda(G) Q_R^c}{Q^*(G)} \right) dG \right] d\overline{U} + \frac{\xi(\overline{G})}{Q^*(\overline{G})} d\overline{G} \end{aligned}$$

and

$$d\overline{G} = -\frac{1}{N} \frac{\partial K}{\partial \eta} d\eta - \lambda(\overline{G})\, d\overline{U}$$

which eventually reduces to

$$\begin{aligned} 0 = - & \left[\int_0^{\overline{G}} \frac{\xi(G)}{(Q^*(G))^3} Q_R^c dG \right] \left(\frac{d\overline{G}}{d\eta} + \lambda(\overline{G}) \frac{d\overline{U}}{d\eta} \right) + \left[\int_0^{\overline{G}} \frac{1}{Q^*(G)} \frac{\partial \xi(G)}{\partial \eta} dG \right] \\ & - \left[\int_0^{\overline{G}} \frac{\xi(G)}{(Q^*(G))^2} \left(Q_U^c - \frac{\lambda(G) Q_R^c}{Q^*(G)} \right) dG \right] \frac{d\overline{U}}{d\eta} + \frac{\xi(\overline{G})}{Q^*(\overline{G})} \frac{d\overline{G}}{d\eta} \end{aligned}$$

and

$$\frac{d\overline{G}}{d\eta} = -\frac{1}{N}\frac{\partial K}{\partial \eta} - \lambda\left(\overline{G}\right)\frac{d\overline{U}}{d\eta}. \tag{A.12}$$

Combining these two equations, we get

$$\frac{d\overline{U}}{d\eta} = \frac{-\left[\frac{\xi(\overline{G})}{Q^*(\overline{G})} - \int_0^{\overline{G}} \frac{\xi(G)}{(Q^*(G))^3} Q_R^c dG\right] \frac{1}{N}\frac{\partial K}{\partial \eta} + \left[\int_0^{\overline{G}} \frac{1}{Q^*(G)} \frac{\partial \xi(G)}{\partial \eta} dG\right]}{\left[\int_0^{\overline{G}} \frac{\xi(G)}{(Q^*(G))^2} \left\{Q_U^c - \frac{\lambda(G) Q_R^c}{Q^*(G)}\right\} dG + \lambda\left(\overline{G}\right) \frac{\xi(\overline{G})}{Q^*(\overline{G})}\right]}$$

Therefore, at the optimum we have

$$\frac{\partial K}{\partial \eta} = N \frac{\int_0^{\overline{G}} \frac{1}{Q^*(G)} \frac{\partial \xi(G)}{\partial \eta} dG}{\frac{\xi(\overline{G})}{Q^*(\overline{G})} - \int_0^{\overline{G}} \frac{\xi(G)}{(Q^*(G))^3} Q_R^c dG}. \tag{A.13}$$

By using Equation (A.12) the above equation can be written as

$$\frac{\partial K}{\partial \eta} = -N\frac{d\overline{G}}{d\eta}.$$

This implies that in a closed city with absentee landowners, at the optimum the per capita increase in construction cost due to a marginal change in the network design is equal to the decrease in generalized transport cost for the individual living at city boundary.

From Equation (A.11), at the optimum, we have

$$\begin{aligned}\frac{d\Phi}{d\eta} &= -\frac{dD}{d\eta}\int_0^{\overline{G}} \frac{\xi(G)}{Q^*(G)} dG + \int_0^{\overline{G}} (R(G) - R_A) \frac{\partial \xi(G)}{\partial \eta} dG \\ &= -\frac{\partial K}{\partial \eta} + \int_0^{\overline{G}} (R(G) - R_A) \frac{\partial \xi(G)}{\partial \eta} dG.\end{aligned}$$

and by using $\frac{\partial K}{\partial \eta} = -N\frac{d\overline{G}}{d\eta}$, this can be written as

$$\frac{d\Phi}{d\eta} = N\frac{d\overline{G}}{d\eta} + \int_0^{\overline{G}} (R(G) - R_A) \frac{\partial \xi(G)}{\partial \eta} dG.$$

This implies that in a closed city with absentee land ownership, at the optimum the marginal change in aggregate differential land rent due to a change in network design is equal to the sum of the aggregate differential land rent, evaluated at the current level of land rent, and the marginal change in generalized transport cost at the city multiplied by the total number of households.

Finally we have

$$\frac{d\overline{G}}{d\eta} = \frac{1}{N}\int_{0}^{\overline{G}} \xi\left(G\right)\frac{\partial R\left(G\right)}{\partial \eta}dG.$$

In fact, the above optimality condition holds for any kind of land ownerships (see Equation 2.30 for residential land ownership case). It says that the marginal cost of network improvement is equal to the marginal increase in the aggregated differential land rent (caused by change in land prices) evaluated at post-project land price. Thus, regardless of who owns the land, the optimality conditions do not require considering the total change of the aggregate differential land rent; we just need to consider the partial change of it. However, the key difference between these two land ownership formations is that for the case of residential land ownership, we have to equate the marginal construction cost to the marginal aggregate differential land rent due to an increase in city area (keeping land prices fixed), and in the optimality condition for the case of absentee land ownership, we have to equate the marginal construction cost with the marginal aggregate differential land rent due to an increase in land prices (keeping land fixed).

A.2. Appendix 2

A.2.1. City Boundary with Market Area 3

Based on the values of number of radial highway n and the generalized transport cost $\overline{G}$, Market Area 3 may have boundary both side of the circumferential highway.

City boundary with Market Area 3 outside of the circumferential highway

This boundary can be established by using $\overline{G} = \overline{G}_3$ gives

$$g_s\left(r-r_c\right)+g_c r_c \overline{\theta}_3^{out}+g_r r_c = \overline{G}$$

or

$$\overline{\theta}_3^{out} = \frac{1}{g_c r_c}\left[\overline{G}-g_s\left(r-r_c\right)-g_r r_c\right]$$

and respective $\overline{r}_3^{in}(\theta)$ is

$$\overline{r}_3^{out}(\theta) = \frac{\overline{G} + g_s r_c - g_r r_c - g_c r_c \theta}{g_s} \qquad \text{(A.14)}$$

with $r \in \left[r_c, r_{23A}^{out}\right]$ where r_{23A}^{out} can be obtained from

$$r_{23A}^{out} \equiv \overline{r}_2\left(\theta_{23A}^{out}\right) = \overline{r}_3^{out}\left(\theta_{23A}^{out}\right).$$

We thus have

$$\begin{aligned} g_s g_c r_c \left(\theta_{23A}^{out}\right)^2 - \left(g_s \overline{G} + g_s^2 r_c - g_s g_r r_c - g_r g_c r_c\right) \theta_{23A}^{out} & \\ + \left(g_s \overline{G} - g_r \overline{G} - g_s g_r r_c + g_r^2 r_c\right) &= 0 \end{aligned}$$

which yields

$$\theta_{23A}^{out} = \frac{b_1 - \sqrt{b_1^2 - 4 g_s g_c r_c c_1}}{2 g_s g_c r_c} \qquad \text{(A.15)}$$

with $b_1 = g_s \overline{G} + g_s^2 r_c - g_s g_r r_c - g_r g_c r_c$ and $c_1 = g_s \overline{G} - g_r \overline{G} - g_s g_r r_c + g_r^2 r_c$. It is to be noted that we disregard the positive sign of the square root in the above expression, otherwise, it would result $r_{23A}^{out} < r_c$, which is unacceptable in this case.

City boundary with market area 3 inside of the circumferential highway

This boundary can be established by using $\overline{G} = \overline{G}_3$ which gives

$$g_s(r_c - r) + g_c r_c \overline{\theta}_3^{in} + g_r r_c = \overline{G}$$

or

$$\overline{\theta}_3^{in} = \frac{1}{g_c r_c}\left[\overline{G} - g_s(r_c - r) - g_r r_c\right]$$

and respective $\overline{r}_3^{in}(\theta)$ is

$$\overline{r}_3^{in}(\theta) = \frac{-\overline{G} + g_s r_c + g_r r_c + g_c r_c \theta}{g_s} \qquad \text{(A.16)}$$

with $r \in \left[r_c, r_{23A}^{in}\right]$ where r_{23A}^{in} can be obtained from

$$r_{23A}^{in} \equiv \overline{r}_2\left(\theta_{23A}^{in}\right) = \overline{r}_3^{in}\left(\theta_{23A}^{in}\right).$$

We thus have

$$g_s g_c r_c \left(\theta_{23A}^{in}\right)^2 - \left(g_s\overline{G} - g_s^2 r_c - g_s g_r r_c - g_r g_c r_c\right)\theta_{23A}^{in} + \left(-g_s\overline{G} - g_r\overline{G} + g_s g_r r_c + g_r^2 r_c\right) = 0$$

which yields

$$\theta_{23A}^{in} = \frac{b_2 + \sqrt{b_2^2 - 4 g_s g_c r_c c_2}}{2 g_s g_c r_c} \tag{A.17}$$

with $b_2 = g_s\overline{G} - g_s^2 r_c - g_s g_r r_c - g_r g_c r_c$ and $c_2 = -g_s\overline{G} - g_r\overline{G} + g_s g_r r_c + g_r^2 r_c$. It is to be noted that we disregard the negative sign of the square root in the above expression, otherwise, it would result negative θ_{23A}^{in} and r_{23A}^{in}, which is unacceptable in this case.

A.2.2. Determination of City Area

Area of City with Regime 2

Area of city with regime 2 consists of Market Areas 1, 2 and 3.

Area of Market 1: Like in Regime 1, in Regime 2, Market Area 1 is an arc with of radius $r_1 = G/g_s$ with angle between $[min(\theta_{12,}\pi/n), \pi/n]$. This area is

$$\Xi_1\left(\overline{G}\right) = 2n\left[\frac{1}{2}\int_{\theta_{12}}^{\frac{\pi}{n}} (r_1)^2 d\theta\right] = n\int_{\theta_{12}}^{\frac{\pi}{n}} (r_1)^2 d\theta.$$

Area of Market 2 with: In this regime, Market Area 3 grabs some portion of Market Area 2 and therefore the expression of Market Area 2 is something different from that in Regime 1. Let r_{23}^{in} and r_{23}^{out} be such that $r_{23}^{in} = \bar{r}_{23}^{in}|_{\overline{G}=G}$ and $r_{23}^{out} = \bar{r}_{23}^{out}|_{\overline{G}=G}$. Thus, Market Area 2 in this regime is given by

$$\Xi_2\left(\overline{G}\right) = n\int_0^{\theta_{12}} (r_2)^2 d\theta - n\int_0^{\theta_{23A}^{out}} \left(r_{23}^{out}\right)^2 d\theta - n\int_{\theta_{23A}^{out}}^{\theta_{23A}^{in}} (r_2)^2 d\theta + n\int_0^{\theta_{23A}^{in}} \left(r_{23}^{in}\right)^2 d\theta.$$

Area of Market 3 with: Here, without losing any generality, we assume that π/n is sufficiently large so that Market Area 1 exists; this further implies that $\pi/n > \theta_{23A}^{out}$

and $\pi/n > \theta_{23A}^{in}$. Let r_3^{in} and r_3^{out} be such that $r_3^{in} = \overline{r}_{23}^{in}|_{\overline{G}=G}$ and $r_3^{out} = \overline{r}_{23}^{out}|_{\overline{G}=G}$. The area of Market 3 denoted by $\Xi_3\left(\overline{G}\right)$, which is

$$\begin{aligned}\Xi_3\left(\overline{G}\right) &= n\int_0^{\theta_{23A}^{out}} \left(r_{23}^{out}\right)^2 d\theta \quad + \quad n\int_{\theta_{23A}^{out}}^{\theta_{3A}^{out-in}} \left(r_3^{out}\right)^2 d\theta \\ &- \quad n\int_0^{\theta_{23A}^{in}} \left(r_{23}^{in}\right)^2 d\theta \quad - \quad n\int_{\theta_{23A}^{in}}^{\theta_{3A}^{out-in}} \left(r_3^{in}\right)^2 d\theta.\end{aligned}$$

The total area for regime 2 is the sum of $\Xi_1\left(\overline{G}\right)$, $\Xi_2\left(\overline{G}\right)$ and $\Xi_3\left(\overline{G}\right)$, which is thus

$$\begin{aligned}\Xi\left(\overline{G}\right) &= n\int_{\theta_{12}}^{\frac{\pi}{n}} (r_1)^2 d\theta + n\int_0^{\theta_{12}} (r_2)^2 d\theta + n\int_{\theta_{23A}^{out}}^{\theta_{3A}^{out-in}} \left(r_3^{out}\right)^2 d\theta \\ &\quad - n\int_{\theta_{23A}^{out}}^{\theta_{23A}^{in}} (r_2)^2 d\theta - n\int_{\theta_{23A}^{in}}^{\theta_{3A}^{out-in}} \left(r_3^{in}\right)^2 d\theta.\end{aligned}$$

Area of City with Regime 3

Area of Market 1: Market Area 3 partly overlaps Market Area 1. Let the inner boundary of Market Area 3 and Market Area 1 intersects at point $(\overline{r}_{13A}^{in}, \overline{\theta}_{13A}^{in}(r))$. In this location, $\overline{r}_3^{in} = \overline{r}_1$, which gives

$$\overline{\theta}_{13A}^{in} = \frac{2\overline{G} - g_s r_c - g_r r_c}{g_c r_c}.$$

Thus, as r_c varies within this regime, the point $\overline{\theta}_{13A}^{in}$ moves along the $\overline{\theta}_{13}^{in}$ boundary. Let r_{13}^{in} and θ_{13A}^{in} be such that, $r_{13}^{in} = \overline{r}_{13}^{in}|_{\overline{G}=G}$ and $\theta_{13A}^{in} = \overline{\theta}_{13A}^{in}|_{\overline{G}=G}$. Therefore, Market Area 1 is an arc of radius $\overline{G}/g_s$ bounded between $\left[min\left(\theta_{13A}^{in}, \theta_{13}, \pi/n\right), \pi/n\right]$ plus an additional area with a non-circular arc having angles in the range between $min\left(\theta_{12}, \pi/n\right)$ and $min\left(\theta_{13A}^{in}, \theta_{13}, \pi/n\right)$. The area then

$$\Xi_1\left(\overline{G}\right) = n\int_{\theta_{13A}^{in}}^{\frac{\pi}{n}} (r_1)^2 d\theta + n\int_{\theta_{12}}^{\theta_{13A}^{in}} \left(r_{13}^{in}\right)^2 d\theta.$$

Area of Market 2: The area of Market 2 in this regime is

$$\begin{aligned}\Xi_2\left(\overline{G}\right) &= n\int_0^{\theta_{12}} (r_2)^2 d\theta - n\int_0^{\theta_{23A}^{out}} \left(r_{23}^{out}\right)^2 d\theta - n\int_{\theta_{23A}^{out}}^{\theta_{12}} (r_2)^2 d\theta \\ &\quad + n\int_0^{\theta_{12}} \left(r_{23}^{in}\right)^2 d\theta.\end{aligned}$$

Area of Market 3: The area of Market 3 is

$$\begin{aligned}\Xi_3\left(\overline{G}\right) &= n\int_0^{\theta_{23A}^{out}} \left(r_{23}^{out}\right)^2 d\theta + n\int_{\theta_{23A}^{out}}^{\theta_{3A}^{out-in}} \left(r_3^{out}\right)^2 d\theta - n\int_0^{\theta_{12}} \left(r_{23}^{in}\right)^2 d\theta \\ &\quad - n\int_{\theta_{12}}^{\theta_{13A}^{in}} \left(r_{13}^{in}\right)^2 d\theta - n\int_{\theta_{13A}^{in}}^{\theta_{3A}^{out-in}} \left(r_3^{in}\right)^2 d\theta.\end{aligned}$$

Thus the total area of the city is given by

$$\begin{aligned}\Xi\left(\overline{G}\right) &= n\int_{\theta_{13A}^{in}}^{\frac{\pi}{n}}(r_1)^2 d\theta + n\int_0^{\theta_{23A}^{out}}(r_2)^2 d\theta \\ &\quad + n\int_{\theta_{23A}^{out}}^{\theta_{3A}^{out-in}}\left(r_3^{out}\right)^2 d\theta - n\int_{\theta_{13A}^{in}}^{\theta_{3A}^{out-in}}\left(r_3^{in}\right)^2 d\theta.\end{aligned}$$

Area of City with Regime 4

Area of Market 1:Market Area 1 is an area of radius G/g_s which is bounded by $[min(\theta_{13},\pi/n),\pi/n]$ plus an additional area bounded by a non-circular arc with angle between $min(\theta_{12},\pi/n)$ and $min(\theta_{13},\pi/n)$. The area of Market 1 for Regime 4 is therefore

$$\Xi_1\left(\overline{G}\right) = n\int_{\theta_{13}}^{\frac{\pi}{n}}(r_1)^2 d\theta + n\int_{\theta_{12}}^{\theta_{13}}\left(r_{13}^{in}\right)^2 d\theta \ .$$

Area Market 2: The area of Market 2 for Regime 4 is

$$\Xi_2\left(\overline{G}\right) = n\int_0^{\theta_{23A}^{out}}(r_2)^2 d\theta - n\int_0^{\theta_{23A}^{out}}\left(r_{23}^{out}\right)^2 d\theta + n\int_0^{\theta_{12}}\left(r_{23}^{in}\right)^2 d\theta \ .$$

Area of the Market 3: In Regime 4, the area of Market 3 is

$$\Xi_3\left(\overline{G}\right) = n\int_0^{\theta_{23A}^{out}}\left(r_{23}^{out}\right)^2 d\theta + n\int_{\theta_{23A}^{out}}^{\theta_{13}}\left(r_3^{out}\right)^2 d\theta - n\int_0^{\theta_{12}}\left(r_{23}^{in}\right)^2 d\theta - n\int_{\theta_{12}}^{\theta_{13}}\left(r_{13}^{in}\right)^2 d\theta.$$

The total area of the city $\Xi\left(\overline{G}\right)$ in this regime is then

$$\Xi\left(\overline{G}\right) = n\int_{\theta_{13}}^{\frac{\pi}{n}}(r_1)^2 d\theta + n\int_0^{\theta_{23A}^{out}}(r_2)^2 d\theta + n\int_{\theta_{23A}^{out}}^{\theta_{13}}\left(r_3^{out}\right)^2 d\theta.$$

A.2.3. Total Differential Land Rent with Leontief Utility

As in equilibrium for Leontief utility function, $\overline{U} = \alpha Z^* = \beta Q^* = \gamma L^*$. So,

$$\begin{aligned}R(G) &= \frac{1}{Q^*}\left[\left\{wH - D + \frac{\Phi}{N} - G\right\} - \left(\frac{\beta}{\alpha} + \frac{\beta w}{\gamma}\right)Q^*\right] \\ &= \frac{1}{Q^*}\left\{wH - D + \frac{\Phi}{N} - G\right\} - \beta\left(\frac{1}{\alpha} + \frac{w}{\gamma}\right).\end{aligned}$$

Therefore,

$$\Phi = \int_0^{\overline{G}}\left[\frac{1}{Q*}\left\{wH - D + \frac{\Phi}{N} - G\right\} - \beta\left(\frac{1}{\alpha} + \frac{w}{\gamma}\right) - R_A\right]\xi(G)dG$$

$$= \Phi + \left[\frac{wH-D}{Q^*} - \beta\left(\frac{1}{\alpha}+\frac{w}{\gamma}\right) - R_A\right]\int_0^{\overline{G}} \xi(G)dG - \frac{1}{Q^*}\int_0^{\overline{G}} G\xi(G)dG.$$

This can be written as

$$\left[wH - D - Q^*\left\{\frac{\beta}{\alpha} + R_A + w\frac{\beta}{\gamma}\right\}\right]\int_0^{\overline{G}} \xi(G)dG = \int_0^{\overline{G}} G\xi(G)dG.$$

Replacing the value of $Q^*$from Equation (3.24) we get,

$$\left[wH - D - \left(wH - D + \frac{\Phi}{N} - \overline{G}\right)\right]\int_0^{\overline{G}} \xi(G)dG = \int_0^{\overline{G}} G\xi(G)dG$$

which ultimately reduces to

$$\Phi = N\left[\overline{G} - \int_0^{\overline{G}} \frac{\xi(G)}{\Xi(\overline{G})} GdG\right].$$

A.3. Appendix 3

A.3.1. Comparison Between the Optimality Conditions

Table A.1 provides a comparison between the optimality conditions of the First-best, Naïve and the Second-best Policies with neutral economies of scale in the capacity cost and user cost functions.

Table A.1. Comparison of the Optimality Conditions Between the First-best, Naïve Policy with SRMCP and the Second-best Naive Policy.

Rule	First-best Policy (Optimum)	Naïve Policy with SRMCP	Second-best Naive Policy
		Naïve Policy	
Pricing	$P = C + QC_Q + \left(\frac{\mu - \frac{1}{G_I}}{\mu}\right)\frac{P}{\varepsilon}$	$P = C + QC_Q$	$P = C + QC_Q + \left(\frac{\mu - \frac{1}{G_I}}{\mu}\right)\frac{P}{\varepsilon}$
Tolling	$\tau = QC_Q + \left(\frac{\mu - \frac{1}{G_I}}{\mu}\right)\frac{P}{\varepsilon}$	$\tau = QC_Q$	$\tau = QC_Q + \left(\frac{\mu - \frac{1}{G_I}}{\mu}\right)\frac{P}{\varepsilon}$
Capacity	$-QC_K + \dot{\mu} = r\mu - \mu G_K$	$-QC_K + \dot{\mu} = r\mu - \mu G_K$	$-QC_K + \frac{\dot{\mu}}{\mu G_I} = \frac{r}{G_I} - \frac{G_K}{G_I}$
Costate μ	$\dot{K} = G$	$\dot{K} = G$	$\dot{K} = G$
Investment	$\mu = \frac{1}{G_I}$	$I = \tau Q$	$I = \tau Q$

In all of the above mentioned cases, we have additional two equations which are: $P = C + \tau$ and $Q = bP^{-\varepsilon}$ that together with the conditions mentioned in Table A.1

describes the transition paths.

A.3.2. Summary of the Unknowns and Equations of the Problems

Table A.2 summarizes the number of unknowns and equations for different problems.

Table A.2. Summery of the Unknowns and Equations of the Problems.

Items	First-best Policy (Optimum)	Naïve	Policy
		Naïve Policy with SRMCP	Second-best Naive Policy
Unknowns	P, Q, K, τ, I, μ $Total = 6$	P, Q, K, τ, I, μ $Total = 6$	P, Q, K, τ, I, μ $Total = 6$
Equations	6 (2 diff. equations)	6 (1 diff. equation)	6 (2 diff. equations).