GRIPS Research Report Series I-2002-0002

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First version: May 15, 2002

Research supported by Grant-in-Aid for Scientific Research (C)
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FOR POLICY STUDIES

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Kaoru Tone*and Biresh K. Sahoo †

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Abstract

This paper, first, points out the shortcomings of the cost efficiency evaluations in DEA models, and then propose a new approach to circumvent this problem. Using this new approach, which is based on cost-production correspondence, we propose a new scheme for measuring cost elasticity of production and compare the results with those in the traditional model. Thus, this paper gives evidence on irrationality latent in the traditional approach, focusing on the cost vs. production elasticity issue. Finally, a case study on Life Insurance Corporation of India is presented to illustrate the potential application of this new scheme to elsewhere in the economy.

Keywords: DEA, cost (overall) efficiency, returns to cost, cost elasticity

1 Introduction

Data Envelopment Analysis (DEA), first introduced by Charnes *et al.* (1978), has been widely applied to the productivity measurement of many organizations in public/private sectors. Subjects related to cost efficiency in DEA have been studied by several authors (Färe *et al.* (1985, 1994) and Sueyoshi (1997, 1999)). In the course of theoretical developments, these authors implicitly assume that the unit costs of input items are common to all decision-making units (DMUs), even though, in real world applications they differ

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from one DMU to another. Recently, Tone (2001) has pointed out the shortcomings of this assumption of common unit cost by demonstrating 'strange phenomena' latent in the traditional model. Then, he has proposed, using a new cost-production correspondence, another cost efficiency model, which is free from such shortcomings. In this paper we first point out that the cost elasticity estimates obtained from the old cost-efficiency DEA model are only illusory, and then apply this new scheme for cost elasticity evaluations within the framework of DEA.

This paper is organized as follows. In Section 2, we introduce the new cost efficiency model after pointing out the shortcomings of the traditional one. New formulae for measuring cost elasticity are presented in Section 3. We compare our new approach with that of Sueyoshi (1999) by applying both methods to an illustrative example in Section 4. We employ the Nippon Telegraph & Telephone (NTT) data in Sueyoshi (1997) for analyzing their cost elasticity in Section 5. Section 6 presents a case study in which the performance trend of Life Insurance Corporation (LIC) of India is examined before and after economic liberalization is introduced based on our new cost elasticity estimates. We conclude this paper with some remarks in Section 7.

2 Cost Efficiency Model

In this section, we point out the shortcomings of the cost efficiency model proposed so far in the literature, and, in an effort to overcome such irrationalities, we introduce a new cost efficiency scheme. Throughout this paper, we deal with n DMUs, each having m inputs for producing s outputs. For each DMU$_o$ ($o = 1, \ldots, n$), we denote respectively the input/output vectors by $\boldsymbol{x}_o \in R^m$ and $\boldsymbol{y} \in R^s$. The input/output matrices are defined by $X = (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n) \in R^{m \times n}$ and $Y = (\boldsymbol{y}_1, \ldots, \boldsymbol{y}_n) \in R^{s \times n}$. We assume that $X > O$ and $Y > O$. The technical efficiency of DMU$_o$ is evaluated as the optimal solution θ^* of the following input-oriented BCC model (BCC-I) (Banker *et al.* (1984)):

$$\begin{aligned} \text{[BCC-I]} \quad & \min \ \theta & (1) \\ \text{subject to} \quad & \theta \boldsymbol{x}_o \geq X\boldsymbol{\lambda} \\ & \boldsymbol{y}_o \leq Y\boldsymbol{\lambda} \\ & \boldsymbol{e}\boldsymbol{\lambda} = 1 \end{aligned}$$

$$\boldsymbol{\lambda} \geq \mathbf{0},$$

where $\boldsymbol{e} \in R^n$ is a row vector with all elements being equal to one.

2.1 Shortcomings of the Traditional Cost Efficiency Model

Given the unit cost vector $\boldsymbol{c}_o \in R^m(> \mathbf{0})$ for the input $\boldsymbol{x}_o$ of DMU$_o$, the traditional cost (overall) efficiency is defined as

$$\gamma^* = \boldsymbol{c}_o \boldsymbol{x}_o^* / \boldsymbol{c}_o \boldsymbol{x}_o, \tag{2}$$

where $\boldsymbol{x}_o^*$ is an optimal solution of the following linear programming problem:

$$\begin{aligned} \text{[Cost]} \quad & \min \quad \boldsymbol{c}_o \boldsymbol{x} && (3) \\ \text{subject to} \quad & \boldsymbol{x} \geq X\boldsymbol{\lambda} \\ & \boldsymbol{y}_o \leq Y\boldsymbol{\lambda} && (4) \\ & \boldsymbol{e}\boldsymbol{\lambda} = 1 \\ & \boldsymbol{\lambda} \geq 0. \end{aligned}$$

On this definition of cost (overall) efficiency, see Färe *et al.*(1994), Coelli *et al.*(1998), Byrnes and Valdmans (1994), Cooper *et al.*(1999) and Sueyoshi (1999). Let us now demonstrate several 'strange' cases latent in this model.

2.1.1 Single Input Case

Let us first consider the case where the number of inputs is one. Then we have

Theorem 1 *For the single input case, the technical efficiency θ^* is equal to the cost efficiency γ^*.*

Proof : Let us denote x as θx_o in [Cost] and change the variable from x to θx_o. Then, noting $x_o > 0$ and $c_o > 0$, [Cost] becomes

$$\begin{aligned} & \min \ c_o x_o \theta \\ \text{subject to} \quad & \theta x_o \geq \sum_{j=1}^{n} x_j \lambda_j \end{aligned}$$

$$y_{ro} \leq \sum_{j=1}^{n} y_{rj}\lambda_j \ (r = 1, \ldots, s)$$
$$\sum_{j=1}^{n} \lambda_j = 1$$
$$\lambda_j \geq 0. \ (\forall j)$$

This program is equivalent to the input-oriented BCC model [BCC-I], and its optimal objective value is $\theta^* c_o x_o$. Thus we have $\gamma^* = \theta^* c_o x_o / c_o x_o = \theta^*$ □

This sounds very strange, since, in this case the input cost has nothing to do with the cost efficiency.

2.1.2 General Case

Here we observe a more general case where we have m inputs $(x_1, \ldots, x_m)$. Suppose that DMUs A and B have the same amount of inputs and outputs, i.e., $\boldsymbol{x}_A = \boldsymbol{x}_B$ and $\boldsymbol{y}_A = \boldsymbol{y}_B$. Assume further that the unit cost of DMU A is twice that of DMU B for each input, i.e., $\boldsymbol{c}_A = 2\boldsymbol{c}_B$. Under these assumptions, we have the following theorem:

Theorem 2 *Both DMUs A and B have the same cost (overall) efficiency.*

Proof : Since DMUs A and B have the same inputs and outputs, they have the same technical efficiency, i.e., $\theta_A^* = \theta_B^*$. The cost efficiency of DMU A (or DMU B) can be obtained by solving the following LP:

$$\begin{aligned}
& \min \ \boldsymbol{c}_A \boldsymbol{x} (= 2\boldsymbol{c}_B \boldsymbol{x}) \\
\text{subject to} \quad & x_i \geq \sum_{j=1}^{n} x_{ij}\lambda_j \ (i = 1, \ldots, m) \\
& y_{rA} (= y_{rB}) \leq \sum_{j=1}^{n} y_{rj}\lambda_j \ (r = 1, \ldots, s) \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \geq 0. \ (\forall j)
\end{aligned}$$

Apparently, DMUs A and B have the same optimal solution (inputs) $\boldsymbol{x}_A^* = \boldsymbol{x}_B^*$, and hence the same cost efficiency, since we have

$$\gamma_A^* = \boldsymbol{c}_A \boldsymbol{x}_A^* / \boldsymbol{c}_A \boldsymbol{x}_A = 2\boldsymbol{c}_B \boldsymbol{x}_B^* / 2\boldsymbol{c}_B \boldsymbol{x}_B = \boldsymbol{c}_B \boldsymbol{x}_B^* / \boldsymbol{c}_B \boldsymbol{x}_B = \gamma_B^*.$$

□

This also sounds very strange, since DMUs A and B have same cost efficiency even though the cost of DMU B is half that of DMU A.

We should note that, if the unit cost is common to all DMUs in the data set, these 'strange' phenomena do not occur. However, in most real data sets, we observe that costs are different from one DMU to another.

2.2 New Cost Efficiency Model

The previous two examples reveal the shortcomings and irrationality of the cost efficiency DEA models. (We can similarly demonstrate that the allocative efficiency defined as the ratio cost efficiency to technical efficiency suffers from the same problems too.) These problems are due to the structure of the supposed production possibility set P as defined by

$$P = \{(\boldsymbol{x}, \boldsymbol{y}) | \boldsymbol{x} \geq X\boldsymbol{\lambda}, \boldsymbol{y} \leq Y\boldsymbol{\lambda}, \boldsymbol{e\lambda} = 1, \boldsymbol{\lambda} \geq 0\}. \tag{5}$$

P is defined only by using technical factors $X = (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n) \in R^{m\times n}$ and $Y = (\boldsymbol{y}_1, \ldots, \boldsymbol{y}_n) \in R^{s\times n}$, but has no concern with the unit input cost $C = (\boldsymbol{c}_1, \ldots, \boldsymbol{c}_n)$.

Let us define an another cost-based production possibility set P_c as

$$P_c = \left\{(\bar{\boldsymbol{x}}, \boldsymbol{y}) | \bar{\boldsymbol{x}} \geq \bar{X}\boldsymbol{\lambda}, \boldsymbol{y} \leq Y\boldsymbol{\lambda}, \boldsymbol{e\lambda} = 1, \boldsymbol{\lambda} \geq 0\right\}, \tag{6}$$

where $\bar{X} = (\bar{\boldsymbol{x}}_1, \ldots, \bar{\boldsymbol{x}}_n)$ with $\bar{\boldsymbol{x}}_j = (c_{1j}x_{1j}, \ldots, c_{mj}x_{mj})^T$.

Here, we assume that the matrices X, C and hence $\bar{X}$ are all positive. Also we assume that the elements of $\bar{x}_{ij} = (c_{ij}x_{ij})$ $(\forall(i, j))$ are denominated in homogeneous units, e.g., dollar, cent or pound so that adding up the elements of $\bar{x}_{ij}$ has a meaning. Though we only assume here the convexity for the sets P and P_c as defined respectively by (5) and (6), this convexity issue has been discussed at length in the case of constant returns to scale environment in Tone (2001).

Based on this new production possibility set P_c, a new technical efficiency $\bar{\theta}^*$ is obtained as the optimal solution of the following LP problem:

$$\text{[NTech]} \quad \bar{\theta}^* = \min\ \bar{\theta} \tag{7}$$

$$\text{subject to} \quad \bar{\theta}\bar{\boldsymbol{x}}_o \geq \bar{X}\boldsymbol{\lambda} \tag{8}$$

$$\boldsymbol{y}_o \leq Y\boldsymbol{\lambda} \tag{9}$$

$$e\boldsymbol{\lambda} = 1 \quad (10)$$
$$\boldsymbol{\lambda} \geq \mathbf{0}. \quad (11)$$

The new cost efficiency $\bar{\gamma}^*$ is defined as

$$\bar{\gamma}^* = e\bar{\boldsymbol{x}}_o^* / e\bar{\boldsymbol{x}}_o, \quad (12)$$

where $\bar{\boldsymbol{x}}_o^*$ is the optimal solution of the LP given below:

$$\begin{aligned} \text{[NCost]} \quad & e\bar{\boldsymbol{x}}^* = \min \ e\bar{\boldsymbol{x}} && (13) \\ \text{subject to} \quad & \bar{\boldsymbol{x}} \geq \bar{X}\boldsymbol{\lambda} && (14) \\ & \boldsymbol{y}_o \leq Y\boldsymbol{\lambda} && (15) \\ & e\boldsymbol{\lambda} = 1 && (16) \\ & \boldsymbol{\lambda} \geq \mathbf{0}. && (17) \end{aligned}$$

Theorem 3 *The new cost efficiency $\bar{\gamma}^*$ is not greater than the new technical efficiency $\bar{\theta}^*$.*

Proof : Let an optimal solution for (7)-(11) be $(\bar{\theta}^*, \boldsymbol{\lambda}^*)$. Then, $(\bar{\theta}^*\bar{\boldsymbol{x}}_o, \boldsymbol{\lambda}^*)$ is feasible for (14)-(17). Hence, it holds that $e\bar{\theta}^*\bar{\boldsymbol{x}}_o \geq e\bar{\boldsymbol{x}}_o^*$. This leads to $\bar{\theta}^* \geq e\bar{\boldsymbol{x}}_o^*/e\bar{\boldsymbol{x}}_o = \bar{\gamma}^*$. □

The new *allocative* efficiency $\bar{\alpha}^*$ is then defined as the ratio of $\bar{\gamma}^*$ to $\bar{\theta}^*$, i.e.,

$$\bar{\alpha}^* = \bar{\gamma}^* / \bar{\theta}^*. \quad (18)$$

We note here that the new efficiency measures $\bar{\theta}^*$, $\bar{\gamma}^*$ and $\bar{\alpha}^*$ are all units invariant so long as $\bar{X}$ has a common unit cost, e.g., dollar, cent or pound.

2.3 Rationale of the New Scheme

Concerning the above cost efficiency, we have the following theorems:

Theorem 4 *The new cost efficiency is strictly monotonic decreasing with the increase in unit cost.*

Corollary 1
(i) If $\boldsymbol{x}_A = \boldsymbol{x}_B, \boldsymbol{y}_A = \boldsymbol{y}_B$ and $\boldsymbol{c}_A = k\boldsymbol{c}_B \ (k > 0)$, then we have $\bar{\gamma}_A^* = \bar{\gamma}_B^*/k$.
(ii) If $\boldsymbol{x}_A = k\boldsymbol{x}_B \ (k > 0), \boldsymbol{y}_A = \boldsymbol{y}_B$ and $\boldsymbol{c}_A = \boldsymbol{c}_B$, then we have $\bar{\gamma}_A^* = \bar{\gamma}_B^*/k$.

Theorem 5 *The new cost efficiency is monotonic decreasing with the decrease in outputs.*

These theorems serve to demonstrate the rationality of the new cost efficiency scheme. See Appendix A for a proof of these statements. Also refer to Tone (2001) for further details on these and related subjects.

2.4 Another Formulation of the New Model

The new cost efficiency is evaluated by the program [NCost](13)-(17). The constraint (14) includes m inequalities, since $\bar{x}$ is an m-vector. Considering the objective function form $e\bar{x}$ and the intput constraints in [NCost], the aggregation of these m constraints into one constraint yields a new program [NCost-1] as follows:

$$\begin{aligned} &[\text{NCost-1}] \quad && \min \ e\bar{x} && (19)\\ &\text{subject to} && e\bar{x} \geq e\bar{X}\lambda && (20)\\ &&& y_o \leq Y\lambda && (21)\\ &&& e\lambda = 1 && (22)\\ &&& \lambda \geq 0. && (23) \end{aligned}$$

This program is simpler than the former in that it has only one aggregated constraint (20) in the input part. Now we prove the following theorem:

Theorem 6 *The optimal objective values of* [NCost] *and* [NCost-1] *are the same.*

Proof: Let an optimal solution of [NCost-1] be $(\tilde{x}^*, \tilde{\lambda}^*)$. Using $\tilde{\lambda}^*$, we define $\widehat{x}$ by

$$\widehat{x} = \bar{X}\tilde{\lambda}^*. \tag{24}$$

The vector $(\widehat{x}, \tilde{\lambda}^*)$ is feasible for [NCost], and it satisfies the following equalities:

$$e\widehat{x} = e\bar{X}\tilde{\lambda}^* = e\tilde{x}^*. \tag{25}$$

The second equality holds since $e\tilde{x}^*$ is optimal for [NCost-1]. The feasible region of [NCost] is a subset of that of [NCost-1] and hence (25) implies the optimality of $\widehat{x}$ for [NCost]. □

This aggregated model presents a correspondence between cost (input) and production (outputs).

Let us denote $e\bar{x}_j$ by $\bar{c}_j$, i.e.,

$$\bar{c}_j = \sum_{i=1}^{m} x_{ij}c_{ij}. \quad (j = 1, \ldots, n) \tag{26}$$

$\bar{c}_j$ is the input cost for the DMU_j for producing the output vector $\boldsymbol{y}_j$. Using this notation and notifying the expressions in [NCost-1], the new aggregated scheme reduces to the following LP:

$$\begin{aligned} &\text{[NCost-2]} \quad && \min \sum_{j=1}^{n} \bar{c}_j \lambda_j && (27)\\ &\text{subject to} && \boldsymbol{y}_o \leq Y\boldsymbol{\lambda} && (28)\\ & && \boldsymbol{e\lambda} = 1 && (29)\\ & && \boldsymbol{\lambda} \geq \mathbf{0}. && (30) \end{aligned}$$

Now we consider the cost-production correspondence set as defined by

$$P_{cp} = \left\{ (\bar{c}, \boldsymbol{y}) | \bar{c} \geq \sum_{j=1}^{n} \bar{c}_j \lambda_j, \boldsymbol{y} \leq Y\boldsymbol{\lambda}, \boldsymbol{e\lambda} = 1, \boldsymbol{\lambda} \geq \mathbf{0} \right\}. \tag{31}$$

P_{cp} is a convex polyhedron composed of every convex combination of $(\bar{c}_1, \boldsymbol{y}_1)$, $(\bar{c}_2, \boldsymbol{y}_2), \ldots, (\bar{c}_n, \boldsymbol{y}_n)$, plus activity $(\bar{c}, \boldsymbol{y})$ with an excess in cost $(\bar{c} \geq \sum_{j=1}^{n} \bar{c}_j \lambda_j)$ and shortfalls in production $(\boldsymbol{y} \leq Y\boldsymbol{\lambda})$. P_{cp} can be construed as a set representing a possible correspondence between the input (cost) and the output (production). Thus, we can determine every Pareto-Koopmans efficient point by employing the variable returns to scale DEA models such as the input/output oriented BCC models. Since we discuss the cost elasticity issue in production in the next section, we introduce here the output-oriented BCC model in P_{cp}. We solve the following LP for each DMU_o $(o = 1, \ldots, n)$.

$$\begin{aligned} &\text{[BCC-O]} \quad && \max \xi + \varepsilon(s^- + \boldsymbol{es}^+) && (32)\\ &\text{subject to} && \bar{c}_o = \sum_{j=1}^{n} \bar{c}_j \lambda_j + s^- && (33)\\ & && \xi \boldsymbol{y}_o = Y\boldsymbol{\lambda} - \boldsymbol{s}^+ && (34)\\ & && \boldsymbol{e\lambda} = 1 && (35)\\ & && \boldsymbol{\lambda} \geq \mathbf{0},\ s^- \geq 0, \boldsymbol{s}^+ \geq \mathbf{0}, && (36) \end{aligned}$$

where ε is a non-Archimedean infinitesimal. Actually we solve this LP using the two-stage process (see Cooper *et al.* (1999) for details). Let an optimal solution of the LP be $(\xi^*, \boldsymbol{\lambda}^*, s^{-*}, \boldsymbol{s}^{+*})$. The DMU$_o$ is Pareto-Koopmans (BCC) efficient, if and only if it has $\xi^* = 1$, and $s^{-*} = 0$ and $\boldsymbol{s}^{+*} = \mathbf{0}$ hold for every optimal solution. If the DMU$_o$ is not BCC-efficient, the projection onto the efficient frontier is given by the following formulae:

$$\bar{c}_o^* \leftarrow \bar{c}_o - s^{-*} \tag{37}$$

$$\boldsymbol{y}_o^* \leftarrow \xi^* \boldsymbol{y}_o + \boldsymbol{s}^{+*}. \tag{38}$$

The returns to scale issue can be identified in the BCC-O model (Banker and Thrall (1992)).

3 Cost Elasticity as Measured by New Cost-Production Correspondence

In a single output case, assume that the cost-production correspondence on the efficient frontier can be described by a function $c = f(y)$. If $f(y)$ is differentiable, the cost elasticity ρ is defined as the ratio of average cost (AC) to marginal cost (MC), i.e.,

$$\rho = \frac{\text{AC}}{\text{MC}} = \frac{c}{y} \Big/ \frac{dc}{dy}. \tag{39}$$

Returns to scale is said to be increasing, constant and decreasing if $\rho > 1$, $\rho = 1$ and $\rho < 1$ respectively. (See Baumol *et al.* (1982).) Figure 1 exhibits such a sample curve $c = f(y)$ to demonstrate cost elasticity. Cost elasticity is well-defined at a point on the efficient portion of the cost-production correspondence, e.g., the point A. For an inefficient DMU operating on point such as B, the cost elasticity is defined on its upward projected point B′. In the multiple output case, this projection is realized by means of the output-oriented BCC model (BCC-O), taking the production cost as input and the multiple production as output.

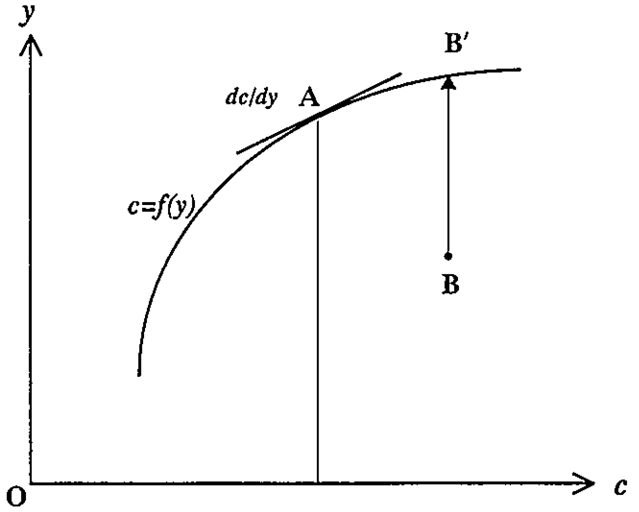

Figure 1: Cost Elasticity

In the multiple production environment we have to deal with the cost-production correspondence in an extended manner. In this section, we first discuss the cost elasticity of production when the cost is dealt with in an aggregated manner, as indicated in Section 2.4. Then, cost elasticity of individual input factor costs, *viz.*, labor, material and capital, is presented. Finally, we extend our analysis to the non-aggregated case where all the individual input factor costs are distinctly dealt with.

3.1 Aggregated Cost Case

Here we are concerned with the aggregated cost as expressed by (26) and the production possibility set P_{cp} in (31). Let us assume that DMU$_o$ is on the efficient frontier of P_{cp}. (If it is inefficient, we deal with the projected DMU as defined by (37) and (38).) In order to evaluate the marginal cost (dc/dy), we need to find a supporting hyperplane to P_{cp} at the point where DMU$_o$ is operating. The dual LP for [NCost-2] offers the basis to serve the purpose.

$$\text{[Dual]} \qquad \max \sum_{r=1}^{s} u_r y_{ro} + w \tag{40}$$

$$\text{subject to} \qquad \sum_{r=1}^{s} u_r y_{rj} + w \leq \bar{c}_j \quad (j = 1, \ldots, n) \tag{41}$$

$$u_r \geq 0 \ (\forall j), \ \ w : \text{free}. \tag{42}$$

Let an optimal solution for [NCost-2] and [Dual] be $\boldsymbol{\lambda}^*$ and $(\boldsymbol{u}^*, w^*)$ respectively. Then it holds that, since DMU$_o$ is efficient,

$$\bar{c}_o = \sum_{j=1}^{n} \bar{c}_j \lambda_j^* \quad \text{and} \quad \boldsymbol{y}_o = Y\boldsymbol{\lambda}^*. \tag{43}$$

By the duality theorem of linear programming, we have

$$\sum_{r=1}^{s} u_r^* y_{ro} + w^* = \bar{c}_o. \tag{44}$$

Now, we observe the hyperplane in $(\bar{c}, \boldsymbol{y})$ given by

$$\bar{c} = \sum_{r=1}^{s} u_r^* y_r + w^*. \tag{45}$$

This plane touches P_{cp} at $(\bar{c}_o, \boldsymbol{y}_o)$, i.e.,

$$\bar{c}_o = \sum_{r=1}^{s} u_r^* y_{ro} + w^*, \tag{46}$$

and satisfies the inequality

$$\bar{c}_j \geq \sum_{r=1}^{s} u_r^* y_{rj} + w^* \text{ for } j = 1, \ldots, n. \tag{47}$$

Thus, the hyperplane (45) is a supporting hyperplane to P_{cp} at $(\bar{c}_o, \boldsymbol{y}_o)$.

In order to unify the multiple outputs, let us define a scalar production y by a weighted sum of y_r as

$$y = \sum_{r=1}^{s} u_r^* y_r. \tag{48}$$

Then, we have a cost (c) to production (y) relationship at $(\bar{c}_o, \boldsymbol{y}_o)$ as

$$c = y + w^*. \tag{49}$$

From this equation we have

$$\frac{dc}{dy} = 1. \tag{50}$$

Now we focus on the average cost term c/y. From (46), we have

$$\frac{c}{y} \to \frac{\bar{c}_o}{\sum_{r=1}^{s} u_r^* y_{ro}} = \frac{1}{1 - w^*/\bar{c}_o}. \tag{51}$$

Summing up, we have the cost elasticity at $(\bar{c}_o, \boldsymbol{y}_o)$ as

$$\rho = \frac{1}{1 - w^*/\bar{c}_o}. \tag{52}$$

Theorem 7 *Returns to cost is characterized as* increasing *(IRS) if* $w^* > 0$, constant *(CRS) if* $w^* = 0$, *and* decreasing *(DRS) if* $w^* < 0$.

Theorem 8 *If there are multiple optima in* w^*, *then let its sup (inf) be* $\bar{w}^*$ ($\underline{w}^*$). *Then returns to scale is characterized as* increasing *(IRS) if* $\underline{w}^* > 0$, constant *(CRS) if* $\bar{w}^* \geq 0 \geq \underline{w}^*$, *and* decreasing *(DRS) if* $\bar{w}^* < 0$.

We summarize the computational process as follows:

[Cost Elasticity]

For each DMU_o $(o = 1, \ldots, n)$

- Solve the output-oriented BCC (BCC-O) model using single input $\bar{c}$ and multiple output Y.

$$\begin{aligned} &[\text{BCC-O}] && \max \xi + \varepsilon(s^- + \boldsymbol{e}\boldsymbol{s}^+) && (53)\\ &\text{subject to} && \bar{c}_o = \sum_{j=1}^{n} \bar{c}_j \lambda_j + s^- \\ &&& \xi \boldsymbol{y}_o = Y\boldsymbol{\lambda} - \boldsymbol{s}^+ \\ &&& \boldsymbol{e}\boldsymbol{\lambda} = 1 \\ &&& \boldsymbol{\lambda} \geq \boldsymbol{0},\ s^- \geq 0, \boldsymbol{s}^+ \geq \boldsymbol{0}, \end{aligned}$$

- If DMU_o is BCC-O efficient, then solve the following LP:

$$\begin{aligned} &[\text{RTS}] && \bar{w}^*(\underline{w}^*) = \max(\min) w && (54)\\ &\text{subject to} && \sum_{r=1}^{s} u_r y_{rj} + w \leq \bar{c}_j \quad (j = 1, \ldots, n) && (55)\\ &&& \sum_{r=1}^{s} u_r y_{ro} + w = \bar{c}_o && (56)\\ &&& u_r \geq 0\ (\forall r),\ w : \text{free}. && (57) \end{aligned}$$

Compute the upper and lower bounds of cost elasticity ρ respectively from the following formulae:

$$\bar{\rho} = \frac{1}{1 - \bar{w}^*/\bar{c}_o} \quad \text{and} \quad \underline{\rho} = \frac{1}{1 - \underline{w}^*/\bar{c}_o}.$$

- If $\underline{\rho} > 1$, then DMU$_o$ exhibits IRS.
- If $\underline{\rho} \leq 1 \leq \bar{\rho}$, then DMU$_o$ exhibits CRS.
- If $\bar{\rho} < 1$, then DMU$_o$ exhibits DRS.

- If DMU$_o$ is BCC-O inefficient, then project DMU$_o$ onto the efficient frontier by

$$\bar{c}_o^* \leftarrow \bar{c}_o - s^{-*} \tag{58}$$

$$y_o^* \leftarrow \xi^* y_o + s^{+*}. \tag{59}$$

Solve LP [RTS] above for the projected DMU.

3.2 Evaluating Individual Input Cost Elasticity

We evaluate here each input cost elasticity separately. The computational process of evaluation is just the same as is discussed at the end of our previous section. For instance, for the computation of labor cost elasticity one needs to take only labor cost (not total input cost) for $\bar{c}_j$ into consideration. This process will continue m number of times (as we have m number of inputs) for the computation of all the input cost elasticities.

3.3 Modification of [Cost Elasticity] for Dealing with Non-Aggregated Cost Factors

We have discussed the cost elasticity issue under the assumption that we can unify all cost items into a single aggregated cost as represented by $\bar{c}_j$ in (26). However, if all the individual input cost factors (e.g., labor, material and capital costs) need to be handled distinctly, we can modify our method as follows:

In the evaluation of [Cost Elasticity] procedure, we modify the program [BCC-O] into

$$\text{[BCC-O-1]} \quad \max \xi + \varepsilon(\boldsymbol{e}\boldsymbol{s}^- + \boldsymbol{e}\boldsymbol{s}^+) \tag{60}$$

$$\text{subject to} \quad \bar{x}_{io} = \sum_{j=1}^{n} \bar{x}_{ij}\lambda_j + s_i^- \ (i = 1, \ldots, m)$$

$$\xi \boldsymbol{y}_o = Y\boldsymbol{\lambda} - \boldsymbol{s}^+$$

$$\boldsymbol{e}\boldsymbol{\lambda} = 1$$

$$\boldsymbol{\lambda} \geq \mathbf{0}, \ \boldsymbol{s}^- \geq \mathbf{0}, \boldsymbol{s}^+ \geq \mathbf{0},$$

If DMU$_o$ is [BCC-O-1] efficient, then we modify the program [RTS] into

$$\text{[RTS-1]} \quad \bar{w}^*(\underline{w}^*) = \max(\min) w \tag{61}$$

$$\text{subject to} \quad -\sum_{i=1}^{m} v_i \bar{x}_{ij} + \sum_{r=1}^{s} u_r y_{rj} + w \leq 0 \ \ (j = 1, \ldots, n)$$

$$-\sum_{i=1}^{m} v_i \bar{x}_{io} + \sum_{r=1}^{s} u_r y_{ro} + w = 0$$

$$\sum_{r=1}^{s} u_r y_{ro} = 1$$

$$v_i \geq 0 \ (\forall i), \ u_r \geq 0 \ (\forall r), \ w : \text{free}.$$

If DMU$_o$ is [BCC-O-1] inefficient, then we project $(\bar{\boldsymbol{x}}_o, \boldsymbol{y}_o)$ onto

$$\bar{\boldsymbol{x}}_o^* \leftarrow \bar{\boldsymbol{x}}_o - \boldsymbol{s}^{-*} \tag{62}$$

$$\boldsymbol{y}_o^* \leftarrow \xi^* \boldsymbol{y}_o + \boldsymbol{s}^{+*}.. \tag{63}$$

We now apply [RTS-1] to the projected DMU.

The upper/lower bounds of cost elasticity $\bar{\rho}$ ($\underline{\rho}$) are respectively computed from the following formulae:

$$\bar{\rho} = 1 + \bar{w}^* \quad \text{and} \quad \underline{\rho} = 1 + \underline{w}^*. \tag{64}$$

See Appendix B for the derivation of (64).

It is to be noted that, in a single input case, i.e., $m = 1$, [BCC-O-1] and [RTS-1] reduce to [BCC-O] and [RTS], respectively. Hence, this model has broader applications in that it can deal with multiple input in a unified manner. Also, if we apply this model to the original data set, i.e., (X, Y), then we can evaluate the 'production' elasticity that examines elasticity between technological inputs and products in the output-orientation.

4 Comparison with the Traditional Model

We compare our new scheme with the traditional one using an illustrative example. By 'traditional' we mean method for evaluating the cost elasticity based on the cost efficiency model as described in [Cost] ((3)). We will refer to Sueyoshi (1997, 1999) as the representative and call it 'Sueyoshi method.'

4.1 Sueyoshi Method

In his pioneering work, Sueyoshi evaluates cost elasticity, i.e., cost-based scale elasticity or degree of scale economies (DSE) in his terminology, using the following process. For each cost efficient DMU_o, Sueyoshi first solves the following LP:

$$\text{[Sueyoshi]} \quad \bar{w}^*(\underline{w}^*) = \max(\min) w \tag{65}$$

$$\text{subject to} \quad \sum_{r=1}^{s} u_r y_{rj} + w \leq \bar{c}_j \quad (j = 1, \ldots, n)$$

$$\sum_{r=1}^{s} u_r y_{ro} + w = c_o^* \tag{66}$$

$$u_r \geq 0 \ (\forall j), \ \ w : \text{free}.$$

Using $\bar{w}^*$ and $\underline{w}^*$, Sueyoshi determines the characteristics of cost elasticity by the same formulae as ours. The difference exists in the identification of 'efficient' DMUs and hence in the usage of c_o^* in (66). This value is obtained as the optimal cost value of [Cost] (2), which is usually different from $\bar{c}_o$ in (56) of our model. For an inefficient DMU, Sueyoshi projects the output $\boldsymbol{y}_o$ onto the efficient frontier of P_c by $\boldsymbol{y}_o \leftarrow \boldsymbol{y}_o + \boldsymbol{s}^{+*}$, where $\boldsymbol{s}^{+*}$ is obtained as the slacks to $\boldsymbol{y}_o$ in (4).

4.2 An Illustrative Example

Table 1 exhibits a simple example with 10 DMUs, each having a single input x and a single output y, along with the unit cost c and the cost cx of the inputs.

Table 1: A Simple Example

DMU	Input x	Output y	Unit cost c	Cost cx
A	1.5	1	20	30
B	1.5	1	40	60
C	2	3	10	20
D	2	3	20	40
E	4	6	5	20
F	4	6	10	40
G	6	7	1	6
H	6	7	2	12
I	8	7.5	0.775	6.2
J	10	8	1	10

4.2.1 Results obtained in Sueyoshi Method

Table 2 reports results obtained in Sueyoshi method. First, we evaluated the cost-efficiency γ^* and the reference set by using the procedure [Cost] in (2). As is demonstrated in Theorem 1, the unit cost has no role in explaining the cost efficiency in the single input case. We plot in Figure 2 input x on the horizontal and output y on the vertical axis. From this figure, we see that all the DMUs are on the efficient frontier of the variable returns to scale (BCC) model, and hence they are all cost efficient in this single input case. Applying the [Sueyoshi] procedure, we obtained the results related to cost elasticity, which are reported in this table. DMUs A and B are said to operate under IRS, DMUs C, D, E and F under CRS, and DMUs G, H, I and J under DRS. This can be observed from Figure 2 as well.

Table 2: Results by Sueyoshi Method

	Cost efficincy		Cost elasticity				RTS
DMU	γ^*	Reference	$\bar{w}^*$	$\underline{w}^*$	$\bar{\rho}$	$\underline{\rho}$	
A	1	A	30	25	∞	6	IRS
B	1	B	60	50	∞	6	IRS
C	1	C	12.5	0	2.667	1	CRS
D	1	D	25	0	2.667	1	CRS
E	1	E	0	-40	1	0.333	CRS
F	1	F	0	-80	1	0.333	CRS
G	1	G	-8	-22	0.429	0.214	DRS
H	1	H	-16	-44	0.429	0.214	DRS
I	1	I	17.05	-17.05	0.267	0.267	DRS
J	1	J	-22	$-\infty$	0.313	0	DRS

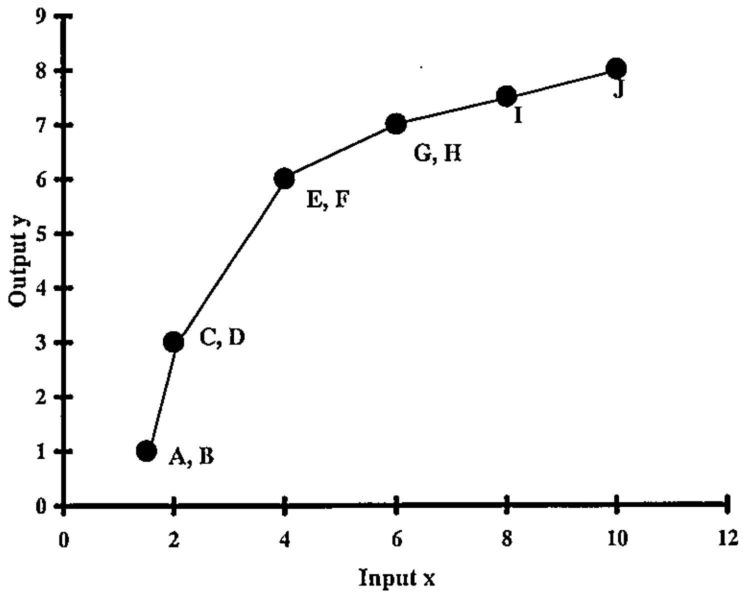

Figure 2: Production data

4.2.2 Results obtained in the New Scheme

Table 3 reports results obtained in our new scheme. We, first, evaluated the cost-efficiency $\bar{\gamma}^*$ and the reference set by using the procedure [NCost], which is equivalent to the input-oriented BCC model in this single input case. As can be observed, DMUs A and B have different cost efficiency scores: $\bar{\gamma}_A^* = 0.2$ and $\bar{\gamma}_B^* = 0.1$ reflecting the differences in their unit costs: $c_A = 20$ and $c_B = 40$. Similarly, we can discriminate the pairs (C and D), (E and F) and (G and H). All these pairs have the same cost efficiency scores in the traditional model. We plot in Figure 3 cost cx on the horizontal axis and the output y on the vertical axis. It is seen that DMUs G, I and J are on the efficient frontier of the variable returns-to-scale production possibility set P_{cp}. We then applied the procedure [Cost Elasticity] to the data set and obtained the results in Table 3. Here we applied the output-oriented BCC model (BCC-O), and the projected points (references) are not always the same with those in the input-oriented case. For example, it is seen from Figure 3 that in the input-oriented case, DMU A is projected to G: first move horizontally to the left to the boundary (reducing the cost) and then go up to G (enlarging product), whereas in the output-oriented case, first go up vertically to the boundary and then move to the left up to J. In the evaluation of cost elasticity, we employ the above principle as being demonstrated here with the help of point B in Figure 1: first enlarge the output and then reduce the cost.

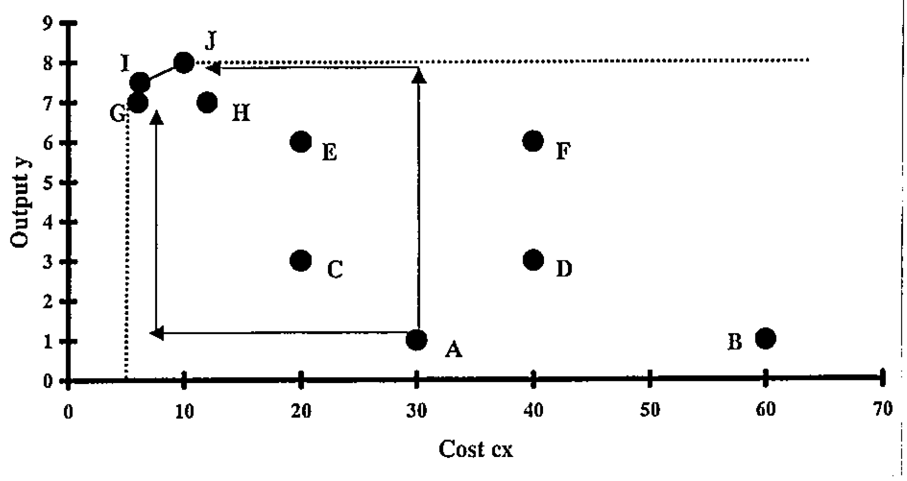

Figure 3: Cost-based production data

Table 3: Results by New Scheme

	Cost efficiency		Cost elasticity					RTS
DMU	$\bar{\gamma}^*$	Reference	$\bar{w}^*$	$\underline{w}^*$	$\bar{\rho}$	$\underline{\rho}$	Reference	
A	0.2	G	-50.8	$-\infty$	0.164	0	J	DRS
B	0.1	G	-50.8	$-\infty$	0.164	0	J	DRS
C	0.3	G	-50.8	$-\infty$	0.164	0	J	DRS
D	0.15	G	-50.8	$-\infty$	0.164	0	J	DRS
E	0.3	G	-50.8	$-\infty$	0.164	0	J	DRS
F	0.15	G	-50.8	$-\infty$	0.164	0	J	DRS
G	1	G	6	3.2	∞	2.142	G	IRS
H	0.5	G	-50.8	$-\infty$	0.164	0	J	DRS
I	1	I	3.2	-50.8	2.067	0.109	I	CRS
J	1	J	-50.8	$-\infty$	0.164	0	J	DRS

4.2.3 Observations

We now compare both sets of results obtained putting emphasis on the cost elasticity issue.

DMU A has $\bar{\rho}_A = \infty$ and $\underline{\rho}_A = 6$, and hence it is judged to be operating under *increasing* returns to cost in the Sueyoshi method. This means that the average cost of production is *at least* six times higher than the marginal cost of production at A and hence increasing the production is promising. On the contrary, in our scheme DMU A has $\bar{\rho}_A = 0.164$ and $\underline{\rho}_A = 0$, and hence it is judged to be under *decreasing* returns to cost. This means that the marginal cost of production is *at least* 6.1 (=1/0.164) times higher than the average cost of production at A, and hence decreasing the production is recommended! These two quite contrasting conclusions are due to the differences in the cost efficiency evaluation schemes.

Now we turn to DMU G which is operating under *decreasing* returns to cost (the marginal cost is high relative to the average cost) in [Sueyoshi], whereas our scheme identifies it under *increasing* RTS status (expanding production is recommended). Here we have too the opposite conclusion once again.

Turning to the original data in Table 1, we see that the unit cost of A is twenty times higher than that of G ($c_A = 20$, $c_G = 1$). It might be possible to liken A to Japan where the labor cost is high and G to China where there is lower labor cost. The traditional model might recommend to increase production in Japan and to decrease production in China. However, the actual movement should be in the opposite direction. Hence, the latter recommendation based on the traditional measurement is misleading.

5 The NTT Case Revisited

Sueyoshi (1997, 1999) has studied the cost elasticity of Nippon Telegraph and Telephone (NTT) using the total operating expenses as input (cost), and the toll, the local and other revenues as outputs (production) over the time interval of 39 years: 1953-1992. In this section the estimates of cost elasticity in our new scheme are first compared with those in Sueyoshi (1999) to see if there is any broad agreement with these two corresponding sets of estimates of elasticity. The upshot of this comparison is that the cost elasticity estimates of NTT mostly exhibit decreasing returns to scale throughout as opposed to increasing returns to scale in the Sueyoshi's measure. We then trace the sources of the decreasing cost elasticity by decomposing the total expenses into assets (capital), employees (labor) and access lines (material).

5.1 Measurement of NTT Cost Elasticity

Sueyoshi (1997, 1999) analyzed NTT data and found that NTT had maintained high scale economies (increasing returns to scale) in the operations of 39 annual periods. We reexamine this data set comprising three inputs (total assets, employees and access lines) and three outputs (toll revenues, local revenues and other). Refer to Appendix 3 for the NTT data set, and its detailed explanations can be found in Sueyoshi (1997). Table 4 reports the estimates of cost elasticity (lower bound, upper bound and their average) obtained in both the measures. Keeping in mind whether both sets of estimates of cost elasticity are in broad agreement, we compare them in Figure 4 by taking their average. As can be observed, both sets of estimates yield the opposite conclusions. Our scheme finds 26 years of operations under DRS, 12 years under CRS and one year under IRS in a sample of 39 years, whereas

Sueyoshi finds 35 years of operations under IRS, four years under CRS, and zero year under DRS. This is due to the differences in the evaluation of their respective cost efficiency schemes.

Table 4: Comparisons of Results on NTT Data

	Cost Elasticity in New Scheme				Cost Elasticity in [Sueyoshi]			
DMU	Upper	Lower	Average	RTS	Upper	Lower	Average	RTS
1953/54	∞	0.86	∞	C	499012.1	2.57	249507.33	I
54/55	3.01	0.86	1.93	C	3.05	3.05	3.05	I
55/56	1.76	0.9	1.33	C	2.98	2.98	2.98	I
56/57	1.11	1	1.05	C	2.54	2.54	2.54	I
57/58	1.1	1	1.05	C	2.41	2.41	2.41	I
58/59	1.09	1	1.04	C	2.37	2.37	2.37	I
59/60	1.33	1.13	1.23	I	2.21	2.21	2.21	I
60/61	1.23	0.88	1.06	C	2.11	2.11	2.11	I
61/62	0.96	0.78	0.87	D	1.99	1.99	1.99	I
62/63	0.93	0.81	0.87	D	1.93	1.93	1.93	I
63/64	0.94	0.83	0.89	D	1.81	1.81	1.81	I
64/65	0.95	0.86	0.9	D	1.72	1.72	1.72	I
65/66	0.96	0.88	0.92	D	1.64	1.64	1.64	I
66/67	0.96	0.9	0.93	D	1.53	1.53	1.53	I
67/68	0.97	0.92	0.94	D	1.48	1.48	1.48	I
68/69	0.97	0.93	0.95	D	1.42	1.42	1.42	I
69/70	0.98	0.94	0.96	D	1.39	1.39	1.39	I
70/71	0.98	0.95	0.96	D	1.38	1.38	1.38	I
71/72	0.98	0.95	0.97	D	1.37	1.37	1.37	I
72/73	0.99	0.96	0.97	D	1.37	1.37	1.37	I
73/74	0.98	0.96	0.97	D	1.35	1.35	1.35	I
74/75	0.99	0.97	0.98	D	1.38	1.38	1.38	I
75/76	0.99	0.98	0.98	D	1.36	1.36	1.36	I
76/77	0.99	0.98	0.98	D	1.31	1.31	1.31	I
77/78	1	0.78	0.89	C	1.25	1.25	1.25	I
78/79	0.82	0.82	0.82	D	1.25	1.25	1.25	I
79/80	0.83	0.83	0.83	D	1.25	1.25	1.25	I
80/81	0.84	0.84	0.84	D	1.25	1.25	1.25	I
81/82	0.87	0.87	0.87	D	1.26	1.26	1.26	I
82/83	0.88	0.88	0.88	D	1.27	1.27	1.27	I
83/84	0.88	0.79	0.84	D	1.27	1.27	1.27	I
84/85	1	0.83	0.91	C	1.29	1.29	1.29	I
85/86	0.87	0.87	0.87	D	1.3	1.3	1.3	I
86/87	0.87	0.84	0.86	D	1.3	1.3	1.3	I
87/88	1	0	0.5	C	1.29	0.11	0.7	C
88/89	1	0	0.5	C	1.31	0.05	0.68	C
89/90	0.83	0.07	0.45	D	1.32	1.32	1.32	I
90/91	0.67	0	0.34	D	1.32	0	0.66	C
91/92	1	0	0.5	C	1.33	0	0.66	C

Note: I: IRS, C: CRS, D: DRS

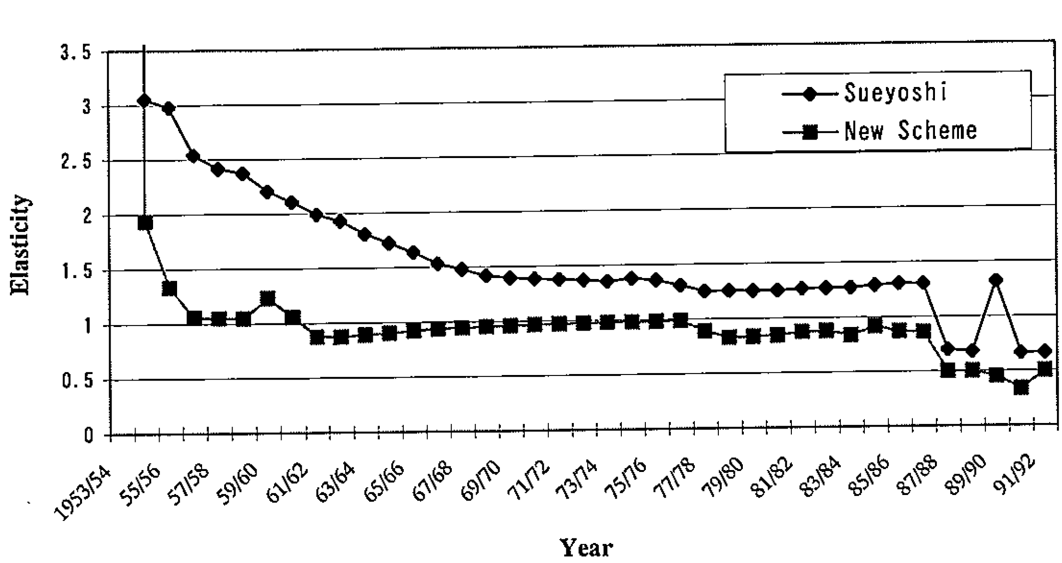

Figure 4: Comparisons of Averages

We have made an effort here to intuitively demonstrate the rationality of our approach by applying our scheme to a simplified case of a single input (Cost) and a single output (Revenue). The Revenue is meant here the sum of 'Toll Revenues,' 'Local Revenues' and 'Other.' In the NTT case these three outputs have a common unit of measurement, i.e., billion yen, and therefore summing up these three revenue figures has a meaning. However, notice that in our previous analysis, we treat them as independent factors. Although we do not report the results in detail, the DMU 88/89 is inefficient with its reference set composed of DMUs 87/88 and 89/90. We can depict the input-output relation of this simplified case on a two-dimensional graph in Figure 5.

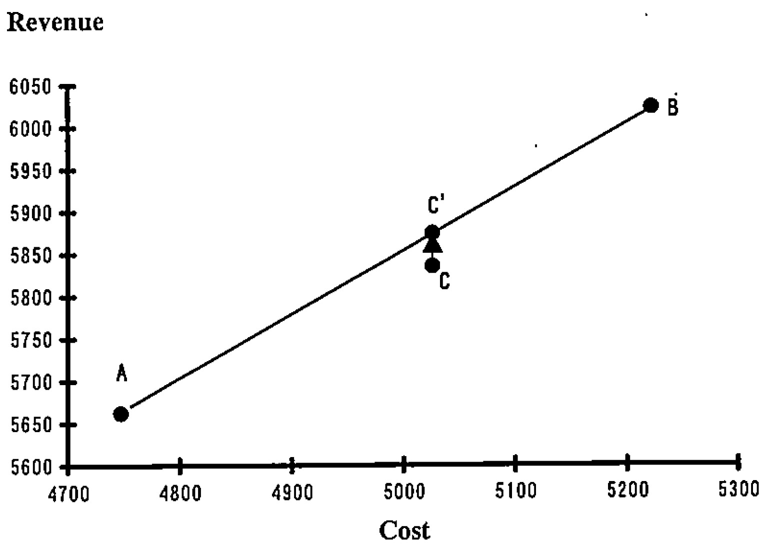

Figure 5: Cost Elasticity in a Simplified Case

The points A and B in this figure correspond to the efficient referent DMUs 87/88 (Cost = 4,747.7, Revenue = 5,662) and 89/90 (Cost = 5,222.4, Revenue = 6,022.4), respectively. The DMU 88/89 (Cost = 5,026.5, Revenue = 5,841.9) and its projected DMU (Cost=5,026.5, Revenue=5,873.8) are denoted by C and C$'$, respectively. The point C$'$ is on the efficient frontier spanned by A and B. Thus, we can calculate each component of C$'$'s cost elasticity in (39) as follows:

$$\text{AC} = (c/y) = 5026.5/5873.8 = 0.856$$

$$\text{MC} = (dc/dy) = (5222.4 - 4747.7)/(6022.4 - 5662) = 1.3171.$$

Thus, we have

$$\text{Cost elasticity of C} = 0.856/1.3171 = 0.65.$$

In the single input/output case our scheme boils down to the above simple calculations, and hence offers the basis in treating its cost elasticity index as rational. In the case of multiple input/output, we deal with the same thing by means of the virtual input/output as is usually being developed in the spirit of DEA.

5.2 The Sources of the Decreasing Cost Elasticity

We now turn to examine the sources of the decreasing cost elasticity as found in the NTT case. For this purpose, we applied our scheme to the NTT data set comprising the same three outputs (revenues) and a single input concentrating on each individual input factors, i.e., capital, labor and material cost. So it can safely be said that the corresponding elasticity indicates productivity of each input factor. Table 5 reports the results on lower bound, upper bound and their average of each input cost elasticity. We compare the average cost elasticity of each input cost with the total cost elasticity (measured in the previous subsection) in Figure 6.

Table 5: Cost Elasticity of Capital, Labor and Material

	Capital (Total assets)				Labor (Employees)				Material (Access lines)			
DMU	Upper	Lower	Av.	RTS	Upper	Lower	Av.	RTS	Upper	Lower	Av.	RTS
1953/54	0.69	0.67	0.68	D	∞	0.90	∞	C	∞	0.95	∞	C
54/55	∞	0.66	∞	C	1.15	1.15	1.15	I	4.44	1.00	2.72	C
55/56	3.07	0.63	1.85	C	1.52	1.14	1.33	I	1.50	1.49	1.49	I
56/57	0.80	0.71	0.75	D	1.51	1.26	1.39	I	1.42	1.41	1.42	I
57/58	0.81	0.71	0.76	D	1.23	1.23	1.23	I	1.34	1.34	1.34	I
58/59	0.81	0.72	0.77	D	1.28	1.21	1.24	I	1.32	1.32	1.32	I
59/60	1.04	1.03	1.03	I	1.37	1.30	1.33	I	1.37	1.24	1.30	I
60/61	1.03	0.67	0.85	C	1.35	0.97	1.16	C	1.26	1.11	1.19	I
61/62	0.77	0.62	0.70	D	1.06	0.86	0.96	C	1.17	1.09	1.13	I
62/63	0.71	0.68	0.70	D	1.00	1.00	1.00	C	1.15	1.08	1.11	I
63/64	0.75	0.72	0.74	D	1.06	1.00	1.03	C	1.12	1.06	1.09	I
64/65	0.79	0.76	0.78	D	1.06	1.00	1.03	C	1.10	1.05	1.08	I
65/66	0.84	0.81	0.82	D	1.00	1.00	1.00	C	1.08	1.04	1.06	I
66/67	0.87	0.85	0.86	D	0.96	0.93	0.94	D	1.06	1.03	1.05	I
67/68	0.89	0.87	0.88	D	1.02	1.00	1.01	C	1.05	1.03	1.04	I
68/69	0.91	0.89	0.90	D	1.03	0.85	0.94	C	1.04	1.02	1.03	I
69/70	0.92	0.91	0.91	D	1.02	0.86	0.94	C	1.04	1.02	1.03	I
70/71	0.93	0.92	0.93	D	0.91	0.91	0.91	D	1.03	1.02	1.03	I
71/72	0.94	0.93	0.93	D	1.02	0.89	0.95	C	1.03	1.02	1.02	I
72/73	0.95	0.94	0.94	D	0.93	0.91	0.92	D	1.02	1.01	1.02	I
73/74	0.95	0.95	0.95	D	0.94	0.92	0.93	D	1.02	1.01	1.02	I
74/75	0.96	0.95	0.96	D	0.95	0.94	0.94	D	1.02	1.01	1.01	I
75/76	0.97	0.96	0.96	D	0.96	0.95	0.95	D	1.02	1.01	1.01	I
76/77	0.97	0.96	0.97	D	0.96	0.95	0.95	D	1.01	1.01	1.01	I
77/78	0.97	0.97	0.97	D	1.01	0.63	0.82	D	1.01	1.00	1.01	C
78/79	0.97	0.97	0.97	D	0.71	0.70	0.70	D	1.01	1.00	1.01	C
79/80	0.98	0.97	0.97	D	0.99	0.57	0.78	D	1.01	1.00	1.01	C
80/81	0.98	0.97	0.98	D	0.71	0.70	0.70	D	0.52	0.27	0.39	D
81/82	0.98	0.98	0.98	D	0.75	0.75	0.75	D	0.53	0.28	0.40	D
82/83	0.98	0.98	0.98	D	0.73	0.72	0.73	D	0.53	0.29	0.41	D
83/84	0.98	0.98	0.98	D	0.74	0.56	0.65	D	0.54	0.29	0.41	D
84/85	0.99	0.99	0.99	D	0.79	0.62	0.71	D	0.54	0.29	0.42	D
85/86	0.99	0.00	0.50	D	0.71	0.60	0.65	D	1.01	0.27	0.64	C
86/87	0.98	0.00	0.49	D	0.72	0.61	0.67	D	0.89	0.35	0.62	D
87/88	0.98	0.00	0.49	D	0.99	0.00	0.50	D	0.73	0.00	0.37	D
88/89	0.99	0.00	0.50	D	0.74	0.00	0.37	D	0.60	0.00	0.30	D
89/90	0.98	0.12	0.55	D	0.44	0.00	0.22	D	0.86	0.01	0.44	D
90/91	1.00	0.00	0.50	C	0.45	0.00	0.23	D	0.32	0.00	0.16	D
91/92	1.00	0.00	0.50	C	1.00	0.00	0.50	C	0.98	0.00	0.49	D

Note: I: IRS, C: CRS, D: DRS

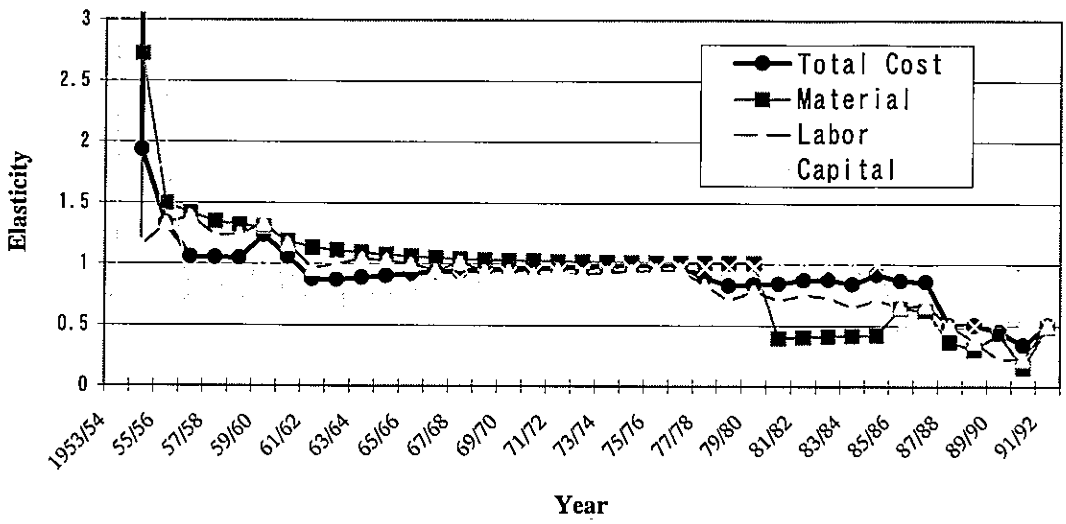

Figure 6: Comparison of Capital, Labor, Material and Total Cost Elasticity

Our observations are as follows:

1. The first period (1953-61) indicates the initial stage of NTT after World War II when NTT expanded its operation with the rapid economic development in Japan. This period can be seen as the cradle of NTT. Three indicators out of four show IRS.

2. The second period (62-79) is considered a stably expanding era. All indicators tend to unity. NTT expanded its operation consecutively as the monopolistic common carrier and a public entity under the control/regulation of the Japanese government.

3. The last period (80-92) endures rapid decreases in productivity, especially in material (access lines) and labor (employees), resulting in the low total cost elasticity after 87/88. In fact, NTT has been exposed in rapidly changing social situations, e.g., NTT privatization in 1985 and the declining monopolistic status due to liberalization of telecommunication systems. This deteriorating situation has been continuing until now. The mammoth is not yet free from the bureaucracy, and high cost structure that it enjoyed in the second period.

We do not pursuit further details of the relationship between the history of NTT management/achievements/technological innovations and the indicators obtained, since this needs an another paper to be adequately dealt with. However, it is interesting to point out that just a simple analysis of the NTT cost elasticity reveals the fundamental efficiency change over time for about 40 years.

6 A Case Study of Life Insurance Corporation of India

Life insurance corporation (LIC) is the only insurance company providing an insurance cover against various risks in life for the last 44 years. A monolith then, the corporation, enjoyed a monopoly status and became synonymoys with life insurance. Among its various products, endowment assurance (participating) and money back (participating) are most popular, comprising 80% of the life insurance business. For the year 1998-99 LIC had Rs.75606.26 crores [1 crore = 10 million] in assurances and Rs. 13.08 crores in annuities. The number of policies in assurance is 148.57 lakhs and in annuities is 0.06 lakhs. The annual premium in assurance is Rs.4880.52 crores and in annuities Rs.4.93 crores. Insurance penetration in 1997 is 1.39% compared to 9.42% in Japan according to Swiss Reinsurance Company (1999). Insurance density in 1997 is $5.4 compared to $3092 in Japan. Even though LIC in India enjoyed monopoly status, the above figures raise questions on the efficiency status of LIC.

Malhotra (1994) finds that LIC is a monolith and recommended liberalization in insurance. Banerjee (2000) found that effectiveness of insurance penetration and insurance density was low. The recent passage of the Insurance Regulatory and Development Authority (IRDA) Bill, 2000 along with expected amendments to the LIC and GIC Acts paves the way for the entry of private players, and possibly the privatization of the hitherto public monopolies LIC and GIC. See also the study of Ranade and Ahuja (2000) for detailed discussion on various issues in regulation of insurance. Since life insurance is as much about savings as about protection, it would increasingly compete with banks and mutual funds for people's savings. Recently, the Reserve Bank of India (200) issued guidelines for banks, which wish to enter into insurance business in India. In the light of above-mentioned studies, we here evaluate LIC's performance (in terms of cost elasticity/degree of scale economies)in this competitive environment. The study uses the aggregate time-series data of LIC over past 17 years from 1982-83 through 1998-99 covering both prior and post financial liberalization for the aforementioned purpose.

6.1 The Data

A modified version of the value added approach to measure life insurance output is adopted in our study. The value added approach counts as important outputs those that are significant value added, as judged using operating cost allocations (Berger and Humphrey (1992)). We follow the recent insurance efficiency literature in defining insurance output as the present value of real losses incurred (Berger *et al.* (1997), Cummins *et al.* (1999)). We have taken losses as the claims settled during the year including claims written back (y_1). The rationale for the use of losses to proxy for insurance output is that the primary function of insurance is risk pooling, i.e. the collection of funds from the policyholder pool and the redistribution of funds to those pool members who incur losses (Cummins *et al.* (1999)). Losses are deflated to the base 1995 using the Consumer Price Index (CPI). The CPI data are taken from International Financial Statistics Year Book, 1999.

Following the study of Brockett *et al.* (1998), the ratio of liquid assets to liabilities (y_2) is taken as the second output in our study. Liquid assets have been taken as the sum of outstanding premiums, interest, dividends, and rents outstanding; interest, dividends and rents accruing but not due; deposits with banks; cash and bank balance and remittances in transit. Liabilities are the probable future sacrifices of economic benefits stemming from present legal, equitable, or constructive obligations to transfer assets or to provide services to other entities in the future as a result of past events affecting the corporation. This ratio reflects a company's claims-paying ability; this is an important objective of an insurer firm, with improvement in claims-paying ability contributing to the likelihood of attracting and retaining customers.

Insurance inputs can be classified into four groups: business services (x_1), labor (x_2), debt capital (x_3) and equity capital (x_4). The business service is taken as commission to agents, which is material input, which is deflated by CPI. The input price index for business services (c_1) is calculated by dividing total deflated commission to agents with total active agents. The labor variable is taken as the total number of employees. The price per unit of labor (c_2) is calculated by dividing total deflated salary and other benefits to employees with total employees.

The debt capital of insurers consists of funds borrowed from policyholders. These funds are measured in real terms as the life insurance fund deflated using CPI. The cost of the policyholder supplied debt capital (c_3) is the rate of interest realized on the mean life insurance fund. Equity capital is an input for the risk-pooling function because it provides assurance that the company can pay claims even if there are larger than expected losses. The equity capital has been taken as sum of shareholders* paid up capital; general reserve; reserve for bad and doubtful debts, loans; reserve for house property and investment reserve. This value of equity capital deflated by CPI is considered an input category. Following the study of Gutfinger and Meyers (2000), the cost of equity capital (c_4) is taken as 9% + rate of inflation. To summarize we use four inputs: business services, labor, policyholder-supplied debt capital, and equity capital. The input-output data are reported in Table 6.

Table 6: The LIC Data on Input, Output and Input Prices

Year	y_1	y_2	x_1	x_2	x_3	x_4	c_1	c_2	c_3	c_4
1982/83	12.248	0.050	2.893	58595	243.147	2.444	0.000023478	0.000057550	0.0881	0.206
1983/84	13.221	0.051	2.655	60440	254.555	2.318	0.000019969	0.000067178	0.0919	0.175
1984/85	15.731	0.060	2.889	62977	275.643	2.274	0.000019547	0.000085656	0.0946	0.145
1985/86	16.489	0.051	3.332	66476	286.560	2.141	0.000019312	0.000083894	0.0987	0.179
1986/87	17.317	0.053	3.729	70207	302.129	2.008	0.000018765	0.000083699	0.1030	0.176
1987/88	18.913	0.059	4.524	72619	316.195	4.218	0.000018861	0.000081843	0.1050	0.186
1988/89	21.450	0.063	5.868	73283	350.695	4.040	0.000019775	0.000101529	0.1095	0.151
1989/90	23.508	0.076	7.087	88243	386.050	3.800	0.000020345	0.000089911	0.1113	0.180
1990/91	25.302	0.079	7.854	96289	410.419	3.406	0.000018933	0.000086409	0.1144	0.228
1991/92	27.733	0.080	8.509	104918	448.209	3.187	0.000018319	0.000082187	0.1195	0.208
1992/93	33.114	0.072	9.388	114927	498.157	3.121	0.000018937	0.000084563	0.1156	0.153
1993/94	36.980	0.070	10.318	123785	547.583	3.098	0.000019674	0.000079317	0.1243	0.192
1994/95	40.761	0.075	10.642	121410	599.789	2.866	0.000020486	0.000088333	0.1221	0.193
1995/96	41.580	0.075	11.019	125736	667.707	4.472	0.000021442	0.000101879	0.1229	0.180
1996/97	48.729	0.072	12.418	126620	751.369	4.017	0.000023293	0.000110113	0.1239	0.162
1997/98	50.507	0.079	12.909	125619	800.551	4.467	0.000023113	0.000104098	0.1237	0.222
1998/99	55.554	0.069	14.666	124385	933.253	4.770	0.000024517	0.000121662	0.1196	0.123

6.2 Measurement of LIC Cost Elasticity

Table 7 show the estimates of lower bound (infimum), upper bound (supremum), and their average of the cost elasticity of LIC for whether returns to scale (which are reported in the last column) is changing over years. We find here that LIC has been operating under CRS for the first three years of our study after which diminishing returns to scale completely sets in.

Table 7: Cost Elasticity and RTS in LIC

	Cost Elasticity			
Year	Lower	Upper	Average	RTS
1982/83	0.778	∞	0.937	CRS
1983/84	0.794	1.096	0.945	CRS
1984/85	0.429	1.083	0.756	CRS
1985/86	0.658	0.978	0.818	DRS
1986/87	0.677	0.980	0.829	DRS
1987/88	0.692	0.981	0.836	DRS
1988/89	0.723	0.723	0.723	DRS
1989/90	0.456	0.743	0.600	DRS
1990/91	0.104	0.728	0.416	DRS
1991/92	0	0.358	0.179	DRS
1992/93	0.653	0.989	0.821	DRS
1993/94	0.685	0.944	0.815	DRS
1994/95	0.397	0.948	0.673	DRS
1995/96	0.427	0.427	0.427	DRS
1996/97	0.740	0.740	0.740	DRS
1997/98	0	0.750	0.375	DRS
1998/99	0	0.792	0.396	DRS

6.3 Sources of Returns to Scale

Let us now turn to see the possible input cost factors determining the underlying behavior of such returns to scale. In Table 8 the lower and upper bounds of cost elasticity and RTS are provided for each individual input cost. The most striking feature of this table is that barring for the first three years all the input costs show decreasing returns over the years. Now coming to the first three years of LIC operations we find that debt capital cost shows constant returns whereas mix results (CRTS and DRTS) are found for the remaining input costs, which possibly explains why CRTS operates for LIC. It now remains to be seen whether physical inputs exhibit any returns. We find that barring equity capital all the remaining inputs exhibit increasing returns in the first two to three years (The results are not reported here). However, subsequently all these physical inputs mostly show diminishing returns.

Table 8: Source of RTS in LIC

	Business Services			Labor			Debt Capital			Equity Capital		
Year	Lower	Upper	RTS	Lower	Upper	RTS	Lower	Upper	RTS	Lower	Upper	RTS
1982/83	0.22	0.24	D	0.40	∞	C	0.92	∞	C	0.29	0.30	D
1983/84	2.71	∞	I	0.44	0.48	D	0.93	1.28	C	0.50	0.95	D
1984/85	0.19	2.61	D	0.51	0.55	D	0.41	1.24	C	0.45	∞	C
1985/86	0.33	0.59	D	0.67	0.67	D	0.65	0.92	D	0.48	0.95	D
1986/87	0.35	0.61	D	0.68	0.68	D	0.67	0.93	D	0.94	2.82	C
1987/88	0.39	0.39	D	0.68	0.68	D	0.68	0.93	D	0	0.36	D
1988/89	0.47	0.47	D	0.73	0.73	D	0.71	0.71	D	0.33	0.34	D
1989/90	0.38	0.40	D	0.61	0.64	D	0.43	0.74	D	0	0.36	D
1990/91	0.26	0.41	D	0.35	0.65	D	0.09	0.69	D	0	0.36	D
1991/92	0	0.54	D	0	0.76	D	0	0.29	D	0	0.36	D
1992/93	0.57	0.57	D	0.78	0.78	D	0.58	0.96	D	0.28	0.96	D
1993/94	0.61	0.82	D	0.89	0.89	D	0.62	0.87	D	0.10	0.10	D
1994/95	0.37	0.83	D	0.68	0.90	D	0.39	0.88	D	0.10	0.57	D
1995/96	0.39	0.39	D	0	0.70	D	0.41	0.41	D	0.13	0.13	D
1996/97	0.44	0.71	D	0	0.64	D	0.64	0.77	D	0.13	0.13	D
1997/98	0	0.72	D	0	1.02	C	0	0.68	D	0	0.19	D
1998/99	0	0.53	D	0	0.67	D	0	0.74	D	0	1.64	C

Note: RTS: Returns to scale, I: IRS, C: CRS, D: DRS

LIC's operations over the past 17 years can be broadly divided into two sub-periods (1982/83-1991/92 and 1992/93-1998/99) in order to see effect of financial liberalization. Figure 7 shows the plot of average cost elasticity (as a whole) as well cost elasticity of each of the individual input cost over time. It is observed here that in first sub-period LIC shows a monotonic decline trend of the cost elasticity, even to the extent of 0.179, which is lowest in our sample period. And the performance pattern then begins to show an upward-rising trend within the next two years after economic liberalization is introduced. Even though there is a performance spurt within these two years, it has only been to catch up with what was once attained in the beginning periods of our study. However, with the intense competition that LIC faced with financial intermediaries, it could not continue this process of recovery after 1993-94 as is seen from the downward trend in the cost elasticity score. During this time period the policy pressures, which seemed to revolving on LIC to behave less profligately, has resulted in decrease in cost efficiency because of increasing allocative inefficiency. This finding is also in line with the findings of our earlier study (Tone and Sahoo (2002)). This finding calls into question the deregulation of publicly held monopoly LIC as redundant because the profit motive and guarding its monopoly status may not be paramount objectives to a public sector firms, and there may be in-built procedures in its operations to deal with issues normally addressed by a regulator. So the widely held belief that competition bodes well for better performance remains to be verified yet!

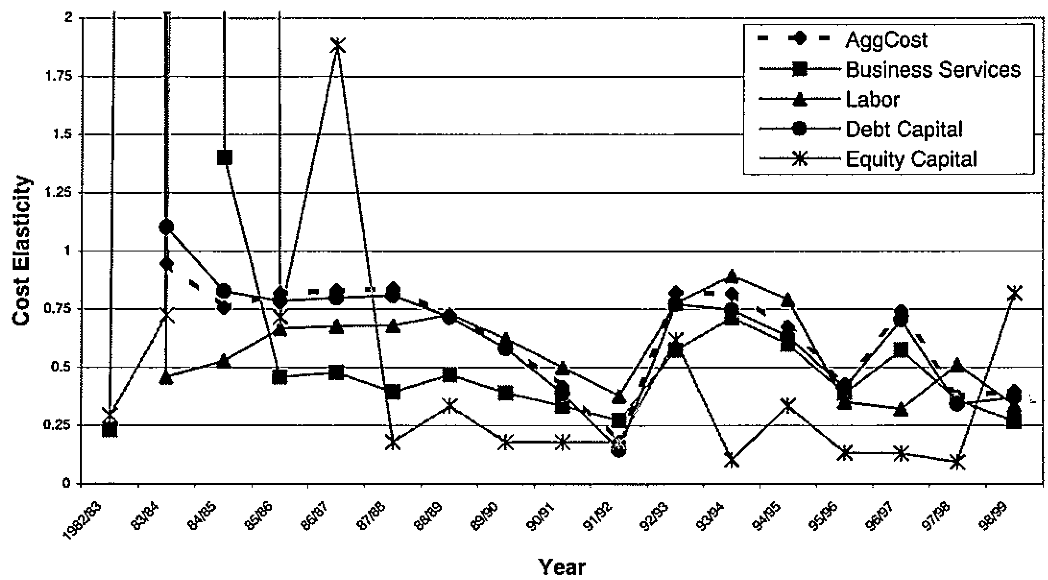

Figure 7: Cost Elasticity vs. Individual Input Factor Cost Elasticity

7 Concluding Remarks

The [Cost] DEA model is often used in the empirical literature to estimate cost/economic efficiency and cost elasticity. The estimates of economic efficiency are often utilized to inform decision-makers about potential savings deriving from the choice of correct input mix in the light of prevailing market prices. Similarly the estimates of cost elasticity are used to inform whether expansions/reductions in the scale of operation of DMUs will be profitable. So there is a need that these estimates must be examined with more prudence than previously thought for the DMUs' financial viability and success. The aim of this paper was to correctly assess the accuracy of the [Cost] model in estimating economic efficiency and cost elasticity. We have shown here that the [Cost] estimates of economic efficiency and cost elasticity discovered in individual DMUs are only illusory. We have then proposed a new scheme for measuring the cost elasticity of production, and demonstrated its superiority over the traditional [Cost] model. We hope that this article contributes to straightening the cost related researches in DEA.

We note that the new scheme developed in this paper can be applied not only for the cost elasticity evaluations but also for any combination of multi-inputs vs. multi-outputs elasticity analysis. Also, we can modify our algorithm in Section 3.3 so as to accommodate the evaluation of input-oriented elasticity as opposed to the current output-orientation. This study points to avenues for future research, which include further studies in this new scheme, applications to the revenue and profit efficiency evaluations, and the relaxation of the convexity assumption on the production possibility set.

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Appendix A: Proof of Propositions in Section 2.3

[**Theorem 4**]: Assume that two DMUs A and B in the data set have $\boldsymbol{x}_A = \boldsymbol{x}_B, \boldsymbol{y}_A = \boldsymbol{y}_B$ and $\boldsymbol{c}_A \leq \boldsymbol{c}_B$. This means that $\bar{\boldsymbol{x}}_A \leq \bar{\boldsymbol{x}}_B$. [NCost] has the same optimal solution $\bar{\boldsymbol{x}}^*$ for A and B. Hence, $\bar{\gamma}_A^* = \boldsymbol{e}\bar{\boldsymbol{x}}^*/\boldsymbol{e}\bar{\boldsymbol{x}}_A \geq \boldsymbol{e}\bar{\boldsymbol{x}}^*/\boldsymbol{e}\bar{\boldsymbol{x}}_B = \bar{\gamma}_B^*$. Strict inequality holds if we have $\bar{\boldsymbol{x}}_A \leq \bar{\boldsymbol{x}}_B$ and $\bar{\boldsymbol{x}}_A \neq \bar{\boldsymbol{x}}_B$.

[**Corollary 1**]: In both cases (i) and (ii), we have $\bar{\boldsymbol{x}}_A = k\bar{\boldsymbol{x}}_B$. The optimal solution of [NCost] for A and B is the same. Hence, it holds that $\bar{\gamma}_A^* = \bar{\gamma}_B^*/k$.

[**Theorem 5**]: The feasible region of [NCost] expands with the decrease in outputs, resulting in a decrease in the optimal objective value. Hence, the cost efficiency decreases with the decrease in outputs.

Appendix B: Derivation of (64)

The dual program for [BCC-O-1] is described as follows:

$$\text{[Dual-1]} \qquad \min \sum_{i=1}^{m} v_i \bar{x}_{io} - w \tag{67}$$

$$\text{subject to} \qquad -\sum_{i=1}^{m} v_i \bar{x}_{ij} + \sum_{r=1}^{s} u_r y_{rj} + w \leq 0 \quad (j = 1, \ldots, n) \tag{68}$$

$$\sum_{r=1}^{s} u_r y_{ro} = 1 \tag{69}$$

$$v_i \geq \varepsilon \ (\forall i), \ u_r \geq \varepsilon \ (\forall r), w : \text{free.} \tag{70}$$

If $(\boldsymbol{x}_o, \boldsymbol{y}_o)$ is [BCC-O-1] efficient, there exist optimal solutions $(\eta^* = 1, \boldsymbol{\lambda}^*, \boldsymbol{s}^{-*} = \boldsymbol{0}, \ \boldsymbol{s}^{+*} = \boldsymbol{0})$ for [BCC-O-1] and $(\boldsymbol{v}^*, \boldsymbol{u}^*, w^*)$ for [Dual-1], and the following relationships hold:

$$\sum_{i=1}^{m} v_i^* \bar{x}_{io} - w^* = 1 \tag{71}$$

$$\sum_{r=1}^{s} u_r^* y_{ro} = 1 \tag{72}$$

$$-\boldsymbol{v}^* X + \boldsymbol{u}^* Y + \boldsymbol{e} w^* \leq \boldsymbol{0}. \tag{73}$$

Thus, the following hyperplane is a supporting hyperplane to P_c at $(\boldsymbol{x}_o, \boldsymbol{y}_o)$.

$$-\sum_{i=1}^{m} v_i^* \bar{x}_i + \sum_{r=1}^{s} u_r^* y_r + w^* = 0. \tag{74}$$

Let us define a virtual cost (c) and a virtual production (y) by

$$c = \sum_{i=1}^{m} v_i^* \bar{x}_i \quad \text{and} \quad y = \sum_{r=1}^{s} u_r^* y_r. \tag{75}$$

Then, we have a cost (c) to production (p) relationship at $(\boldsymbol{x}_o, \boldsymbol{y}_o)$ as

$$c = y + w^*. \tag{76}$$

From this equation we have

$$\frac{dc}{dy} = 1. \tag{77}$$

Furthermore, at $(\boldsymbol{x}_o, \boldsymbol{y}_o)$, it holds that

$$\frac{c}{y} = \frac{y + w^*}{y} = 1 + \frac{w^*}{\sum_{r=1}^{s} u_r^* y_{ro}} = 1 + w^*. \tag{78}$$

Thus, the cost elasticity can be expressed as

$$\rho = 1 + w^*. \tag{79}$$

If there are multiple optima in w^*, then we evaluate its sup and inf by [RTS-1].

Appendix C: The NTT Data Set

	Outputs				Inputs			Input Prices			
DMU	y_1	y_2	y_3	TR	x_1	x_2	x_3	c_1	c_2	c_3	Cost
1953/54	49.7	24.5	28.0	102.2	282.0	1630	176.9	0.0677	0.0221	0.1962	89.822
54/55	59.1	29.1	28.7	116.9	313.8	1600	196.6	0.0590	0.0252	0.1831	94.832
55/56	67.4	33.1	29.3	129.8	358.5	1610	217.5	0.0538	0.0270	0.1802	101.951
56/57	81.5	36.8	30.0	148.3	400.6	1670	239.7	0.0694	0.0286	0.1840	119.668
57/58	94.1	40.8	32.1	167.0	445.2	1720	263.8	0.0649	0.0305	0.1895	131.344
58/59	105.7	44.6	31.5	181.8	508.4	1740	290.3	0.0598	0.0329	0.1839	141.034
59/60	126.2	50.2	36.0	212.4	586.2	1790	321.6	0.0597	0.0353	0.1831	157.068
60/61	149.0	57.4	41.1	247.5	728.7	1840	363.3	0.0564	0.0387	0.1844	179.299
61/62	178.5	65.8	49.1	293.4	895.6	1900	415.3	0.0585	0.0444	0.1917	216.366
62/63	197.4	75.0	48.9	321.3	1058.2	1990	478.1	0.0632	0.0488	0.1933	256.407
63/64	234.2	87.4	50.8	372.4	1262.7	2090	547.7	0.0658	0.0530	0.1990	302.848
64/65	275.2	101.2	59.4	435.8	1479.9	2190	633.9	0.0687	0.0589	0.2062	361.370
65/66	314.3	117.8	66.3	498.4	1724.7	2290	730.3	0.0797	0.0656	0.2151	444.770
66/67	378.1	139.0	79.0	596.1	1996.4	2370	846.6	0.0898	0.0730	0.2333	549.799
67/68	451.6	162.4	86.5	700.5	2295.5	2460	988.9	0.0932	0.0828	0.2401	655.063
68/69	522.8	189.6	97.0	809.4	2586.0	2550	1136.2	0.1040	0.0908	0.2439	777.603
69/70	610.7	237.5	108.6	956.8	2917.5	2640	1300.5	0.1069	0.1040	0.2422	901.422
70/71	692.3	294.0	124.5	1110.8	3307.6	2730	1517.3	0.1098	0.1195	0.2440	1059.631
71/72	779.7	337.0	136.2	1252.9	3800.5	2820	1731.3	0.1098	0.1356	0.2450	1223.855
72/73	909.0	391.9	161.6	1462.5	4466.5	2880	2098.5	0.1084	0.1551	0.2313	1416.240
73/74	1056.9	459.1	189.0	1705.0	5117.4	2980	2416.6	0.1101	0.1781	0.2287	1646.840
74/75	1147.3	521.9	212.8	1882.0	5784.4	3040	2744.4	0.1121	0.2266	0.2474	2016.260
75/76	1289.0	582.2	239.1	2110.3	6427.7	3120	3034.3	0.1167	0.2567	0.2628	2348.431
76/77	1531.1	706.2	280.9	2518.2	7013.3	3190	3242.7	0.1195	0.2733	0.2785	2613.008
77/78	1981.3	1085.4	336.9	3403.6	7722.8	3230	3394.5	0.1193	0.3052	0.2914	2896.283
78/79	2076.5	1146.0	399.9	3622.4	8314.6	3270	3549.4	0.1222	0.3302	0.2915	3130.448
79/80	2188.1	1206.9	460.6	3855.6	8874.1	3290	3704.6	0.1222	0.3489	0.2869	3295.146
80/81	2253.2	1272.5	480.6	4006.3	9459.1	3270	3849.0	0.1201	0.3802	0.2925	3505.124
81/82	2306.9	1332.9	527.3	4167.1	9900.7	3270	3933.1	0.1200	0.4022	0.3015	3689.108
82/83	2416.9	1390.0	537.4	4344.3	10248.8	3230	4110.4	0.1197	0.4315	0.2957	3835.972
83/84	2526.4	1457.6	568.4	4552.4	10521.9	3180	4245.5	0.1206	0.4651	0.2911	3983.824
84/85	2558.3	1553.4	644.5	4756.2	10791.7	3140	4401.9	0.1223	0.5073	0.2874	4177.853
85/86	2745.1	1488.6	857.8	5091.5	11368.6	3040	4486.1	0.1288	0.5753	0.2498	4333.815
86/87	2872.0	1496.9	984.7	5353.6	11377.5	2980	4672.5	0.1244	0.6166	0.2775	4549.448
87/88	3009.4	1544.4	1108.2	5662.0	11455.9	2911	4797.7	0.1209	0.6450	0.3095	4747.501
88/89	3010.9	1814.5	1016.5	5841.9	11559.7	2833	4990.4	0.1210	0.7166	0.3201	5026.279
89/90	2964.0	1763.2	1295.2	6022.4	12158.4	2729	5199.2	0.1160	0.7657	0.3312	5221.945
90/91	2937.4	1904.2	1410.0	6251.6	12225.2	2649	5408.4	0.1170	0.8227	0.3573	5542.102
91/92	2825.2	2023.0	1550.2	6398.4	12444.9	2577	5580.0	0.1199	0.8568	0.3679	5752.999

Note: y_1: Toll revenue, y_2: Local revenue, y_3: Other, TR: Total revenue, x_1: Total assets, x_2: Employees, x_3: Access lines, c_1, c_2, and c_3 are respectively the unit cost of total assets, employees and access lines. Source: Sueyoshi(1997, pp:788-789).