GRIPS Research Report Series I-2002-0006

INTEREST WITH APPRECIATION:

The Original Fisher Equation

James R. Rhodes

National Graduate Institute for Policy Studies
2-2 Wakamatsu-cho, Shinjuku-ku,
Tokyo 162-8677, Japan.

September 2002

GRIPS
NATIONAL GRADUATE INSTITUTE
FOR POLICY STUDIES

INTEREST WITH APPRECIATION:

The Original Fisher Equation

James R. Rhodes

National Graduate Institute for Policy Studies
2-2 Wakamatsu-cho, Shinjuku-ku
Tokyo 162-8677 Japan

September 2002

INTEREST WITH APPRECIATION:

The Original Fisher Equation

James R. Rhodes[*]

Abstract

In 1896, Irving Fisher published *Appreciation and Interest* and put into print the original form of the famous equation that links nominal and real rates of interest. In the past half century, a voluminous literature has misrepresented and misinterpreted Fisher's contribution. The conventional "Fisher equation" (CFE) is expressed in terms of expected inflation, whereas the original Fisher equation (OFE) uses the expected appreciation of money. Since the OFE is written in terms of the value of money (1/P), not the value of goods (P), criticisms of the conventional version based on Jensen's inequality do not apply. This paper derives the OFE and explicates its importance. It argues that Fisher's subtle, non-Patinkin, concept of "money illusion" provides an explanation for the departure from pure theory in his subsequent (1930) empirical work. The CFE, although using an inferior index for measuring expected appreciation, is more amenable to Fisher's psychological theory of monetary value. [JEL: E43, E40, G10]

Key Words: Fisher equation, Fisher hypothesis, Fisher effect, money illusion, nominal interest rate, purchasing power of money, value of money.

[*]Professor of Economics, National Graduate Institute for Policy Studies (GRIPS), 2-2 Wakamatsu-cho, Shinjuku-ku, Tokyo 162-8677 Japan TEL/FAX: +81-(0)3-3341-0332 E-mail: rhodes@grips.ac.jp

INTEREST WITH APPRECIATION:

The Original Fisher Equation*

At the outset the question arises, how can a merchant be said to foresee the appreciation of money? Appreciation is a subtle conception. Few business men have any clear ideas of it. Economists disagree as to its definition, and statisticians as to its measurement.

Irving Fisher (1896, p. 35)

I. Introduction

Over one hundred years after the publication of *Appreciation and Interest*, "appreciation of money" remains a subtle conception. The subtlety extends to Fisher's theory of the nominal interest rate which has been widely misrepresented and misunderstood in scores of textbooks, scientific papers, and popular writings. In recent years, the mistake has been compounded by the accusation that the "Fisher equation" is incorrect. The objective of this paper is to recover the original Fisher equation (OFE), interpret its meaning, and clarify its relationship to the bastardized version that appears in the modern literature. The major finding can be simply put: the master had it right, the scribes got it wrong.

The mistake involves the subtle concept of "appreciation." Appreciation to Fisher meant the rate of change in the value of money expressed in terms of (non-money) commodities. If the value of commodities is P, then the value of one unit of money (v) is 1/P.[1] Where Fisher used "expected appreciation of money," modern economists substitute "expected deflation." Confusion arises since the concepts are often used interchangeably in informal analysis. As a consequence of Jensen's inequality, however, the mathematical definitions are not equivalent.[2] This fact is the cornerstone of various arguments concerning the inappropriateness of the CFE. As it turns out, it is the profession at large, not Fisher, that has failed to use the proper index of monetary appreciation. For a given nominal interest rate, the OFE provides an unbiased forecast of the real rate of interest.

II. Fisher's Lost Equation

The Fisher equation describes the relationship between the nominal and real rates of interest.[3] The conventional Fisher equation (CFE) expresses a relationship between the nominal rate of interest (i) and expected inflation (π^e). A common linear representation is:

$$i = r + \pi^e + \rho\pi^e \qquad (1)$$

where r = real interest rate and π^e = expected change in the price level (P). The theory embodies the "Fisher hypothesis" (or "Fisher effect") of a one-to-one relationship between the nominal interest rate and expected inflation. The continuing popularity of this equation is somewhat puzzling, since the theory is now commonly accepted as wrong.[4] A lengthy empirical literature, starting with Fisher (1930), continues to search, with mixed success, for evidence supporting the "Fisher effect."[5]

Fisher's critics are barking up the wrong tree. The original Fisher equation (OFE) was expressed in terms of the *expected* appreciation of money (a) and written with the real interest rate (j), Fisher's "virtual interest in commodities," as the left-hand variable:

$$j = i + a + ia \qquad (2)$$

In this form, the original equation can easily be misinterpreted as the popular version.[6] That is unfortunate, for this equation properly accounts for the expected variation of money prices when expectations are formed rationally. Although Fisher did not use the mathematical expectation operator (E), a literal representation of the "expected value of money" would be: $v^* = E(1/P)$.

Historians of thought, although employing Fisher's terminology, have inadvertently contributed to the misunderstanding of Fisher's theory. Tobin (1997, p. 374) describes the Fisher equation using the conventional specification (1). Dimand (1999, p. 746) accurately reproduces the original Fisher equation (2), but does so using the relationship between interest rates in a two commodity standard (gold and wheat). Later, he points out that Fisher's money and commodities model used the "(expected) purchasing power of money," but he fails to explain the connection with the two commodity equation (2) or note the difference with the conventional equation (1). In defining "expected inflation as the difference between real and nominal interest rates," Dimand (1997, p. 442; 1999, p. 748) implicitly assumes that there is no difference between the OFE (2) and the CFE (1). Humphrey (1983), in an otherwise illuminating discussion, uses the conventional, rather than the true, Fisher equation in describing Fisher's contribution to the history of thought. The two equations are *not* the same since, in general, $r \neq j$.

Did Fisher have an understanding, intuitive or otherwise, of Jensen's inequality?[7] One cannot say with absolute certainty, but a reading of his work on index numbers suggests he did. Fisher's choice of terminology also supports such an interpretation. When explaining the theoretical connection between nominal and real interest rates, Fisher consistently used terms such as "expected change in the value of money," "expected appreciation of money," or "expected change in the purchasing power of money."[8] Fisher (1896, 1905, 1930) used the same terminology and equation in all of his major works on the theory of interest. In his extensive empirical investigations, reported in detail in 1896, appreciation was consistently calculated as the percent change in the reciprocal of the price level. Fisher bemoaned, over and over again, the apparent inability of people to grasp the concept of money value. He did not take it to be a matter of inconsequence that people found it easier to calculate in terms of prices than money values. Indeed, Fisher insisted that newspapers publish his weekly Index Number of Wholesale Prices as the inverse of the original price series.[9]

Ubiquitous assertions to the contrary, one will not find the Fisher equation in *The Theory of Interest*. In that work the primary concern is with the *real* theory of interest.[10] To uncover the relationship between *ex ante* real and nominal interest rates, we must take Fisher's (1930, p. 39) advice and consult *Appreciation and Interest*. Here, careful reading and patience are required for an accurate understanding of the theory. It must be remembered that Fisher was writing in response to the bimetallic controversy, the most important economic issue of the period. His purpose was to show the relationship between interest rates expressed in different standards (e.g. gold and wheat).[11] In part I of his monograph, Fisher stresses that there are as many interest rates as there are commodity standards. It is not until part II that he introduces the modern convention of using fiduciary money and (aggregate) commodities as the two standards.

Part II of *Appreciation and Interest* uses bond market and price data from seven countries to examine the extent to which market interest rates adjust to the "appreciation of money in commodities." Fisher's examples reflect the period of investigation: money is the (relatively) appreciating standard and commodities is the (relatively) depreciating standard. Money appreciates when commodity prices (P) go down and depreciates when prices go up. In part II, Fisher actually used the equation derived in part I to obtain a measure of "expected appreciation." He achieved this remarkable feat by exploiting the difference in the yields of commodity (gold coin) bonds and paper (currency) bonds. Knowing the paper yield (i) and the commodity yield (j), Fisher solved for the expected appreciation of money (a); that is, "that rate of appreciation which would have made the two interest rates equally profitable" (Fisher, 1930, p. 42-43, n. 4).[12] He compared this expected appreciation with the realized *(ex post)* appreciation of money and discovered that expected appreciation consistently under predicted actual appreciation. This discovery, reinforced by subsequent research, called into question the rationality assumption implicit in his theoretical derivation.

III. Derivation of the Original Fisher Equation

Although a skilled mathematician, Fisher took great pains to make his works accessible and relevant to sophisticated laymen. *Appreciation and Interest* is no exception, but the nature of the subject matter imposes considerable demands on the reader. Modern readers, apparently, do not have the required patience. Fisher explicates the relationship between appreciation and interest by guiding the reader through a series of progressively more complicated calculations supported by numerous illustrations drawn from everyday business experience under a bimetallic standard. The essence of Fisher's approach can be captured by a simple present value model.

Consider a bond in a representative bond market. In discrete time, the formula for the nominal price of the bond is:

$$P_{B,0} = \sum_{t=1}^{n} \frac{C}{(1+i)^t} + \frac{F_n}{(1+i)^n} \qquad (3)$$

where $P_{B,0}$ ≡ bond price in the reference period, C ≡ coupon value, F_n ≡ future value at end of holding period, and i ≡ pre-tax nominal yield (for holding period).

The real price of the bond can be expressed:

$$P_{B,0}v_0 = \sum_{t=1}^{n} \frac{Cv*_t}{(1+j)^t} + \frac{F_n v_n}{(1+j)^n} \qquad (4)$$

where $v*_t$ ≡ (expected) value of money in the commodity standard in period t and j ≡ pre-tax real (holding period) yield. The value of money in the current period (v_0) is simply the inverse of the price level ($1/P_0$).

Define period 0 as the base period such that $v_0 = 1$. Equate the right sides of equations 1 and 2. Divide both sides of the combined equation by C. Invert and expand the combined equation to obtain equation 5:

$$\frac{1}{v^*_1}(1+j) + \frac{1}{v^*_2}(1+j)^2 + \ldots + \frac{1}{v^*_n}(1+j)^n + \frac{1}{v^*_n}(1+j)^n(C/F_n) = (1+i) + (1+i)^2 + \ldots + (1+i)^n + (1+i)^n(C/F_n) \tag{5}$$

Assuming that expected value of money (v*) appreciates at a constant rate (a), we can write:

$$v^*_{t+1} = (1+a)v^*_t \tag{6}$$

for all t (t = 0,...,n). Define: I ≡(1+i), A ≡ 1/(1+a), and J ≡ (1+j). Using these definitions and equation 6, one can substitute progressively into equation 5 and, after simplification, obtain:

$$AJ + (AJ)^2 + \ldots + (AJ)^n + (AJ)^n(C/F_n) = I + I^2 + \ldots + I^n + I^n(C/F_n) \tag{7}$$

This implies

$$AJ = I \tag{8}$$

or

$$j = i + a + ia \tag{9}$$

This is the OFE expressed in terms of the expected appreciation of money (a).[13] Our understanding of Fisher depends on our interpretation of this "subtle conception."

Appreciation of money (a) is the percent change in the expected value of money (v*). How do we interpret v*? If v* = E(1/P), then the conventional interpretation of Fisher (1) is incorrect. In this interpretation, expected appreciation (a) measures the expected change in the *inverse* of the price level. To obtain the conventional specification, one would have to define $v^* = 1/EP$. This requires us to assert that expectations should be formed over the price level rather than the value of money. This does not ring true to Fisher. Fisher, an accomplished mathematician and statistician, used words with precision. In all of his theoretical work, Fisher consistently referred to the expected value of money, not the expected value of commodities.[14]

Why did Fisher insist on formulating the problem in terms of the expected appreciation of money (a) rather than expected deflation of commodities (π^e)? One cannot be sure. As "the greatest expert of all time on index numbers"(Tobin, 1987, p. 369), Fisher understood that care must be exercised in the method of calculating mean values.[15] He understood, for example, that the arithmetic mean (A) of a series would be greater than its harmonic mean (H).[16] Likewise, the *expected* rate of change in a variable (P) and its inverse (v) would not simply be mirror images.[17]

As it turns out, Fisher's preference for money appreciation (rather than goods depreciation) has an economic justification. The OFE gives an unbiased prediction of the *ex post* real rate of interest (ρ). The *ex post* real rate of interest is obtained by rewriting the formula for the real bond (4) in terms of actual (*ex post*) values of money (v) and the *ex post* real interest rate (ρ). Assuming a constant rate of appreciation of money (α), the equation system can be manipulated to obtain the *Fisher identity*:

$$\rho \equiv i + \alpha + i\alpha \tag{10}$$

where $\alpha \equiv (v_{t+1} - v_t)/v_t$. This equation is written in terms of the realized appreciation of money (α) which is *identically* equal to the realized rate of price deflation ($-\pi$).

The problem is to show that the *ex ante* real return (j) provided by the OFE gives an unbiased forecast of the expected *ex post* real return ($E\rho$). Assume that there is a finite probability distribution (γ) which associates a probability (γ_k) with each vector of possible future price levels and, hence, with each possible appreciation rate (α_k) and real return (ρ_k). Using the Fisher identity and the known probability distribution, calculate the *ex ante* real return:

$$E\rho = \gamma_1\rho_1 + \gamma_2\rho_2 + ... + \gamma_n\rho_n \quad (11)$$

The *ex post* real return for price vector k is:

$$\rho_k = i + \alpha_k + i\alpha_k \quad \text{with } k = 1, 2...., n. \quad (12)$$

Substituting equation (12) into equation (11) gives the expected real return:

$$E\rho = i + \sum_{k=1}^{n} \gamma_k \alpha_k + i\left(\sum_{k=1}^{n} \gamma_k \alpha_k\right) \quad (13)$$

Fisher's *ex ante* real return is:

$$j = i + a + ia \quad (14)$$

Using the known probability distribution, we calculate the expected depreciation of money:

$$a = \gamma_1\alpha_1 + \gamma_2\alpha_2 + ... + \gamma_n\alpha_n \quad (15)$$

Substituting equation (15) into equation (14) and consolidating terms establishes that $j = E\rho$. The OFE provides an unbiased prediction of the expected real return for a given nominal rate of interest.

It can now be verified that the CFE gives a biased prediction of the *ex post* real return. A counter-factual proof is offered. In order for the *ex ante* real return (r) of the CFE to be equal to the *ex ante* real return (j) of the OFE, it would have to be the case that $a = -\pi$. This is not true in the general case. According to Jensen's inequality, $E(1/P) \geq 1/EP$. This implies that $|a| \leq |\pi|$ and, therefore, that $j \geq r$. In general, the CFE under predicts the *ex post* real return. It fails to adequately compensate the bond holder for purchasing power risk.

What is the extent of bias? If the size of the bias is small, then the CFE may be a reasonable approximation to the OFE. Theoretical models suggest that this is a risky assumption, particularly in cases where price level volatility is large (Sarte, 1998), expectation horizons are long (McCulloch and Kochin, 2000), or individual expectations are diffuse (Kochin, 1980).[18]

IV. Rationalizing the Conventional Specification

Of Irving Fisher's works on interest rate behavior, the one which is most frequently cited is *The Theory of Interest.* Published in 1930, this book is often erroneously credited as the source of the Fisher equation and Fisher hypothesis. It is easy to see how a reader who consults only this work would find support for the conventional interpretation of Fisher's theory. Setting a pattern for subsequent research, Fisher (1930, ch. XIX) examined the correlation between the nominal interest rate and the price of commodities. As is well known, he found a positive but weak contemporaneous relationship between the rate of change of commodity prices and the nominal interest rate. Applying a distributed lag model, Fisher found that past inflation influenced the interest rate with a long and variable lag.

Fisher's *empirical* model is the source of confusion over the Fisher equation. Why did Fisher switch from expected appreciation to lagged inflation in his post-1896 empirical work? The answer can be found in Fisher's psychological theory of expectations. Fisher's early empirical work (1896) led him to question the rationality of market expectations. Conversations with businessmen and workers further convinced him that the value of money was too subtle a concept for the representative agent to comprehend. The popular view, according to Fisher (1930, p. 399) is that "money itself does not change." If this is the case, then bond market participants would not form expectations over the value of money and a viable empirical model of the interest rate would not be constrained by the OFE. To better explain the nominal interest rate, Fisher's (1930) statistical estimations embody the money illusion hypothesis.

Money illusion is traditionally defined as a situation where market participants make economic decisions based on money prices rather than theoretically correct relative prices and real wealth (Patinkin, 1965, pp. 22-23). Workers suffering from money illusion bargain in terms of money wages rather than real wages. Business managers, to the extent they suffer from the disease, fail to adequately take account of the general price level in making pricing and output decisions. Fisher used such notions throughout his collected works, especially in *The Money Illusion* and other business cycle writings where some type of fooling assumption is required to explain output and employment effects of monetary disturbances. In Fisher's theory of interest, however, money illusion is not limited to the case where people fail to calculate the relative prices of (non-money) goods and services. Money illusion can exist even in the unlikely event that people correctly perceive current changes in prices and accurately forecast future changes. Money illusion results when market participants fail to take proper account of the changing value of money over time.

To Fisher, money illusion is the notion that the value of money doesn't change, that "a dollar is a dollar" (Fisher, 1896, p. 35; 1930, p. 399).[19] In an uncertain world, the failure to form expectations over the value of money is the heart of the money illusion problem. Under money illusion, loan contracts fail to take proper account of the changing opportunity cost of money.

The presence of money illusion rules out the direct impact of expected appreciation on interest rates as presumed in the OFE. The possibility remains of a more round-about influence. In various writings, Fisher conjectured that changes in commodity inflation would have an *indirect* and *lagged* impact on the nominal interest rate. In *The Theory of Interest*, for example, Fisher (1930, p. 400) described the adjustment process of loan market participants:

> Yet it may be true that they do take account, to some extent at least, even if unconsciously, of a change in the buying power of money, under guise of a change in the level of prices in general. If the price level falls in such a way that they may expect for themselves a shrinking margin of profit, they will be cautious about borrowing unless interest falls, and this very unwillingness to borrow, lessening the demand in the money market, will tend to bring interest down. On the other hand, if inflation is going on, they will scent rising prices ahead and so rising money profits, and will be stimulated to borrow unless the rate of interest rises enough to discourage them, and their willingness to borrow will itself tend to raise interest.

In his post-1896 empirical work, Fisher used the CFE implicitly and the adaptive expectation hypothesis explicitly to capture his notions about money illusion and the lagged adjustment of interest rates to commodity inflation.

V. Conclusion

This paper argues that there are two Fisher equations describing the relationship between nominal and real interest rates. The original equation is written in terms of the expected appreciation of money (a). This is the product of Fisher's work as a theoretical economist. Under the assumption that expectations are formed rationally, the OFE provides an unbiased forecast of the *ex post* real return. The second equation, which is the conventional specification, is written in terms of expected inflation (π^e). Although the conventional form does not appear in Fisher's published works, the CFE reflects his views as an amateur psychologist and an applied statistician. If Fisher's theory of inflation psychology is correct, then empirical studies using the CFE should more accurately predict the behavior of the nominal interest rate than those based on the OFE.

Fisher, as he readily acknowledged, was not the first person to advance the theory that the nominal interest rate adjusts to changes in the value of money. Humphrey (1983) traces the lengthy development of this idea. Fisher's contribution, apparently, was being the first to write an equation for the relationship (Humphrey, 1983; Dimand, 1999). Fisher was also the first to clearly show the derivation of the equation. The equation Fisher wrote, however, is not the one which is commonly attributed to him. This is fortuitous in the sense that the conventional representation of the "Fisher equation" yields a biased estimate of the real return under the modern theory of efficient markets. Fisher's theoretical work anticipated the empirical studies of Fama (1975, 1976) by 79 years! Fama, in shrewdly avoiding the Jensen inequality problem, wrote the Fisher relationship in terms of the expected value of money. The literature, if not the profession, has subtly converged on Fisher.

NOTES

*I first noted the discrepancy between the original and conventional versions of the Fisher equation some twenty years ago. At that time, the difficulty of finding empirical verification of the conventional "Fisher equation" (CFE) led a number of economists to the discovery that the equation is mis-specified. As a Fisher buff, I found it difficult to accept that Fisher could have got the relationship wrong. My original hypothesis was that the conventional view was correct. In reading *Appreciation and Interest* I discovered that I had been wrong about the equation, but correct in my instincts about Fisher. The author is indebted to Levis Kochin for providing the original stimulus for this paper. Thanks are also due Joerg Bibow, Yang Ming Chang, Dana Ferrell, the late Mark Meador, Perry Mehrling, Francisco de A. Nadal-De Simone, Frank Packer, and Kenichi Sakakibara for comments on earlier versions of the paper.

1. "The index numbers for two dates, as 1826 and 1829, being given, their inverse ratio gives the relative value of money (in commodities) at those two dates." (Fisher, 1930, p. 59, n. 12)

2. The value of money is a convex function of the price level. The mean value of the secant connecting any two points on this function is greater than the average of the two points on the curve. For a non-degenerate random variable (P), Jensen's inequality implies: $E(1/P) \geq 1/EP$. The two expressions are equal only under the special cases where expectations are held with perfect certainty.

3. Fisher (1896, p. 88) was familiar with the concepts of "real" and "nominal" interest rates, but he preferred not to use this Marshallian terminology (see chapter XII). In *Appreciation and Interest* he constructed a theory of multiple interest rates. The Fisher equation was the "law" that connected any two interest rates expressed in different standards. There are as many real interest rates as there are commodity standards.

4. In the theoretical literature, the Fisher effect is said to apply to only one or more special cases where inflationary expectations are held with certainty, where expected inflation is uncorrelated with the real interest rate, where income tax rates are zero, or where international arbitrage operates costlessly for both commodities and financial capital. The perfect certainty (zero risk) interpretation is advanced by Benninga and Protopapadakis (1983), Blejer and Eden (1979), and Kochin (1980). Mundell (1963), Tobin (1965), and Carmichael and Stebbing (1983) provide theories that highlight the neutrality proposition underlying the Fisher effect. The tax argument is expounded by Darby (1975), Feldstein (1976), and Tanzi (1976). The costless international arbitrage view is employed in Hansson and Stuart (1986).

5. Classical studies follow Fisher (1930) in regressing some measure of expected inflation on short-term nominal interest rates. Typically, these studies find that the coefficient on expected inflation is significantly less than one. This was also the finding of Summers (1983) in a study which attempted to extract a long-term relationship from 120 years of data. Fama (1975) reinterpreted the Fisher hypothesis as a test for market efficiency and found evidence that short term interest rates efficiently predict subsequent changes in the value of money. Nelson and Schwert (1977), however, found evidence contradicting Fama's joint hypothesis of market efficiency and real rate constancy. Kandel, Ofer, and Sarig (1996)

found a negative correlation between the *ex ante* real interest rate and expected inflation. Ahmed and Rogers (2000), in finding Tobin-type effects of inflation on real variables, indirectly reject the Fisher hypothesis. Traditional estimates of the Fisher effect may be biased in that they fail to take into account the changing stochastic inflation process (Klein, 1975; Barsky, 1987; Hutchison and Keeley, 1989) and/or differences in the order of integration of the data (Rose, 1992). Some support for a long-run Fisher effect has been found when careful attention has been paid to the time series properties of the data. Studies finding support for a long-run Fisher hypothesis include Lucas (1980), Mishkin (1992), Wallace and Warner (1993), Evans and Lewis (1995), and Mishkin and Simon (1995). Mixed support was found by Lee, Clark, and Ahn (1998) and Carneiro, Divino, and Rocha (2002). Some recent support for a tax-adjusted Fisher equation was found by Crowder and Hoffman (1996) and Crowder and Wohar (1999). For a survey of the empirical literature see Choudhry, Placone, and Wallace (1991).

6. Hirshleifer (1970, pp. 135-36), in an influential work, reinforced the conventional view by representing anticipated inflation with the letter "a."

7. Jensen did not publish his formal proof concerning convex functions until 1906.

8. Elsewhere in his writings, Fisher was not so consistent. In his subsequent work on business cycles, for example, the analysis is in terms of changing price levels or inflation. Fisher's business cycle work, however, involves various concepts of "money illusion." Fisher's pure theory of interest assumed rational behavior.

9. "The chief purpose of this newspaper publication was to invert the ordinary index number representing the price level, thereby obtaining an index number representing the purchasing power of the dollar, the idea being to accustom the public to the thought that the dollar is not a constant but a variable." (Fisher, as quoted in I. N. Fisher, 1956, p. 191)

10. Fisher (1930, p. 45): "While the deviations of the money rate of interest from the real rate are of tremendous practical importance, they may be regarded as belonging more to the problem of money than to the problem of interest..."

11. According to Fisher (1896, p. 92), "[t]hese rates are mutually connected and our task has been merely to state the law of that connection. We have not attempted the bolder task of explaining the rates themselves." Fisher's initial attempt at "the bolder task" was *The Rate of Interest*, published in 1907. His definitive treatment of the subject is his 1930 work, *The Theory of Interest*.

12. $\hat{a} = (j - i)/(1 + i)$.

13. In the above analysis, the assumptions of constant periodic rates of interest and appreciation were made for convenience only. When interest rates and appreciation rates vary, the Fisher identity can be interpreted as a relationship between the yield to maturity (Fisher's rate of return over cost) and the *average* rate of appreciation of money. In this case, equations (3) and (4), which use appropriately weighted average yields, are used in place of the actual expressions for which they are equivalent in present value. Likewise, the rate of appreciation of money (a) may be interpreted as the average rate that generates a set of prices which, when substituted for the actual prices in equation (4), would yield the same present

value. This is precisely the argument made by Fisher (1906, pp. 392-93).

14. Fisher's later works on the real theory of interest (1907, 1930) reproduced, almost word for word, his original analysis (1896) of the theory of nominal interest rates.

15. Schumpeter (1974, pp. 1091) points out that the work of Fisher and others on index numbers was the "statistical complement" to the "theoretical discussion on the purchasing power of money."

16. Suppose there are n possible outcomes represented by the variable x. The arithmetic mean (A) is $A = \{\sum x\}/n$; the harmonic mean (H) is $H = n/\{\sum(1/x)\}$ with $H \leq A$. These concepts appear repeatedly in Fisher's works.

17. Coggeshall (1886-87), who Fisher (1927, p. 81n) cited, advocated use of the harmonic mean a full decade before the publication of *Appreciation and Interest*. In general, the *harmonic* mean, not the *arithmetic* mean, is the appropriate measure of central tendency when dealing with rates of change. The arithmetic mean overstates the true average rate of inflation since it fails to take account of the shorter length of time required to achieve a particular price level at a higher rate of inflation. A simple example illustrates: Suppose there are two possible rates, each of equal probability, at which the price level might rise from a level of 100 to 120: 10 percent p.a. and 20 percent p.a. The arithmetic average (A) of the two inflation rates is 15 percent p.a., but the true average time it would take to cover the "distance" would equal the harmonic mean: $H = 2/[1/10 + 1/20] = 13.33$ percent p.a. (total distance of 40 divided by total time of 3 years).

18. The issue of CFE bias has been engaged in the empirical literature. Since the source of the bias in the CFE is the failure to properly account for variability in the price level, regression equations for nominal interest rates sometimes include a measure of inflation variability. For the U.S. data, the sign, magnitude, and statistical significance of this coefficient have varied across studies. Some recent studies report a small risk premium, but the difficulty of measurement suggests caution is warranted. Studies that find a "small" risk premium include: Chan (1994), Ireland (1998), Sarte (1998), and Shome, Smith, and Pinkerton (1988).

19. "Most people are subject to what may be called 'the money illusion,' and think instinctively of money as constant and incapable of appreciation or depreciation." (Fisher, 1930, pp. 399-400)

REFERENCES

Ahmed, Shaghil, and John H. Rogers. "Inflation and the great ratios: Long term evidence from the U.S." *Journal of Monetary Economics* 45 (2000): 3-35.

Barsky, Robert B. "The Fisher Hypothesis and the Forecastability and Persistence of Inflation." *Journal of Monetary Economics* 19 (1987): 3-24.

Benninga, Simon, and Protopapadakis, Aris. "Real and Nominal Interest Rates under Uncertainty: The Fisher Theorem and the Term Structure." *Journal of Political Economy* 91, no. 5 (October 1983): 856-67.

Blejer, Mario I, and Eden, Benjamin. "A Note on the Specification of the Fisher Equation Under Inflation Uncertainty." *Economic Letters* 3 (1979): 249-255.

Carmichael, Jeffrey and Stebbing, Peter W. "Fisher's Paradox and the Theory of Interest." *American Economic Review* 73, no. 4 (September 1983): 619-630.

Carneiro, Francisco G.; Divino, Jose Angelo C.A.; and Rocha, Carlos A. "Revisiting the Fisher hypothesis for the cases of Argentina, Brazil and Mexico." *Applied Economics Letters* 9 (2002): 95-98.

Chan, Louis K.C. "Consumption, Inflation Risk, and Real Interest Rates: An Empirical Analysis." *Journal of Business* 67, no. 1 (January 1994): 69-96.

Choudhry, Taufiq; Placone, Dennis; and Wallace, Myles. "Changes in the Fisher Effect in the 1980s: Evidence from Various Models." *Journal of Economics and Business* 43 (1991): 59-68.

Coggeshall, F. "The Arithmetic, Geometric, and Harmonic Means," *Quarterly Journal of Economics* 1 (1886-87): 83-86.

Crockett, Jr., John H. "Irving Fisher on the financial economics of uncertainty," *History of Political Economy*, 12, no. 1 (1980): 65-82.

Crowder, William J, and Hoffman, Dennis L. "The Long-Run Relationship between Nominal Interest Rates and Inflation: The Fisher Equation Revisited." *Journal of Money, Credit, and Banking* 28, no. 1 (February 1996): 102-118.

Crowder, William J. and Wohar, Mark E. "Are Tax Effects Important in the Long-Run Fisher Relationship? Evidence from the Municipal Bond Market." *Journal of Finance* 54, no. 1 (1999): 30717.

Darby, Michael R. "The Financial and Tax Effects of Monetary Policy on Interest Rates." *Economic Inquiry* 13 (June 1975): 266-76.

Dimand, Robert W. "Irving Fisher and the Fisher relation: setting the record straight," *Canadian Journal of Economics* 32, no. 3 (May 1999): 744-50.

______. "Irving Fisher and Modern Macroeconomics," American Economic Review, 87, no. 2 (May 1997): 442-44.

Evans, Martin D.D. and Lewis, Karen K. "Do Expected Shifts in Inflation Affect Estimates of the Long-Run Fisher Relation?" *Journal of Finance* 50, no. 1 (March 1995): 225-53.

Fama, Eugene F. "Inflation Uncertainty and Expected Returns on Treasury Bills." *Journal of Political Economy* 84, no. 3 (June 1976): 427-48.

_______. "Short-Term Interest Rates as Predictors of Inflation." *American Economic Review* 65, no. 3 (June 1975): 269-82.

Feldstein, Martin. "Inflation, Income Taxes, and the Rate of Interest: A Theoretical Analysis." *American Economic Review* 66 (December 1976): 809-820.

Fisher, Irving. *Appreciation and Interest.* New York: MacMillan, 1896. Reprinted in Fisher, 1997, vol. 1.

_______. *The Making of Index Numbers.* New York: Houghton Mifflin Co., 1922. Reprinted in Fisher, 1997, 7.

_______. *The Money Illusion.* New York: Adelphi, 1928. Reprinted in Fisher, 1997, p. 8.

_______. *The Nature of Capital and Income.* New York: MacMillan, 1906. Reprinted in Fisher, 1997, vol. 2.

_______. *The Purchasing Power of Money.* New York: MacMillan, 1910. Reprinted in Fisher, 1997, vol. 4.

_______. *The Rate of Interest.* New York: MacMillan, 1907. Reprinted in Fisher, 1997, vol. 3.

_______. *The Theory of Interest.* New York: MacMillan, 1930. Reprinted in Fisher,1997, vol.9.

_______. *The Works of Irving Fisher*, 14 vols, ed. W. J. Barber assisted by R.W. Dimand and K. Foster; consulting ed. J. Tobin. London: Pickering & Chatto, 1997.

Fisher, Irving N. *My Father: Irving Fisher.* New York: Comet Press Books, 1956.

Hirshleifer, J. *Investment, Interest, and Capital.* Englewood Cliffs, N.J.: Prentice-Hall, 1970.

Humphrey, Thomas M. "The Early History of the Real/Nominal Interest Rate Relationship." Federal Reserve Bank of Richmond, *Economic Review* 69 (May/June 1983): 2-10.

Hutchison, Michael M. and Keeley, Michael C. "Estimating the Fisher Effect and the Stochastic Money Growth Process." *Economic Inquiry* 27, no. 2 (April 1989): 219-37.

Ireland, Peter N. "Long-Term Interest Rates and Inflation: A Fisherian Approach." Federal Reserve Bank of Richmond *Economic Quarterly* 82, no. 1 (Winter 1996).

Kandel, Shmuel, Ofer, Aharon R., and Sarig, Oded. "Real Interest Rates and Inflation: An Ex-Ante Empirical Analysis." *Journal of Finance* 51, no. 1 (March 1996): 205-25.

Klein, Benjamin. "Our New Monetary Standard: The Measurement and Effects of Price Uncertainty, 1880-1973." *Economic Inquiry* (December 1975): 461-84.

Kochin, Levis. "Interest Rates and Uncertain Future Price Levels: Irving Fisher's Money Illusion," Unpublished manuscript, University of Washington, 1980.

Lee, Jeung-Lak; Clark, Carolyn; and Ahn, Sung K. "Long- and short-run Fisher effects: new tests and new results." *Applied Economics* 30 (1998): 113-24.

McCulloch, J. Huston and Kochin, Levis A. "The Inflation Premium Implicit in the US Real and Nominal Term Structures of Interest Rates." Ohio State University Working Paper, 2000.

Mishkin, Frederic S. "Is the Fisher Effect for Real?" *Journal of Monetary Economics* 30, no 2 (November 1992): 195-215.

Mishkin, Frederic S. and Simon, John. "An Empirical Examination of the Fisher Effect in Australia." *Economic Record* 71, no. 214 (September 1995): 217-29.

Mitchell, Douglas W. "Expected Inflation and Interest Rates in a Multi-asset Model: A Note." *Journal of Finance* 40, no. 2 (June 1985): 595-99.

Mundell, Robert A. "Inflation and Real Interest." *Journal of Political Economy* 71 (June 1963): 622-26.

William. "Short-Term Interest Rates as Predictors of Inflation: On Testing the Hypothesis that the Real Rate of Interest is Constant." *American Economic Review* 67, no3. (1977): 478-86.

Patinkin, Don. *Money, Interest, and Prices: An Integration of Monetary and Value Theory*, 2nd ed. New York: Harper and Row, 1965.

Rose, Andrew K. "Is the Real Interest Rate Stable?" *Journal of Finance* 43 no. 5 (December 1988): 1095-112.

Sarte, Pierre-Daniel G. "Fisher's Equation and the Inflation Risk Premium in a Simple Endowment Economy." Federal Reserve Bank of Richmond *Economic Quarterly* 84, no. 4 (Fall 1998): 53-72.

Schumpeter, Joseph A. *History of Economic Analysis*. New York: Oxford University Press, 8th ed., 1974.

Shome, Dilip K., Smith, Stephen D. and Pinkerton, John M. "The Purchasing Power of

Money and Nominal Interest Rates: A Re-Examination." *Journal of Finance* 43, no. 5 (December 1988): 1113-1125.

Summers, Lawrence H., "The Non-Adjustment of Nominal Interest Rates: A Study of the Fisher Effect." In Tobin, James, ed., *Macroeconomics, Prices, and Quantities*. Oxford: Basil Blackwell, 1983, pp 201-44.

Tanzi, Vito. "Inflationary Expectations, Economic Activity, Taxes, and Interest Rates," *American Economic Review* 70 (March 1980): 12-21.

Tobin, James. "Fisher, Irving." In *The New Palgrave Dictionary of Economics*. London: The Macmillan Press, 1987.

_______. "Money and Economic Growth." *Econometrica* 33 (October 1965): 671-84.

Wallace, Myles S. and Warner, John T. "The Fisher Effect and the Term Structure of Interest Rates: Tests of Cointegration." *Review of Economics and Statistics* 75, no. 2 (May 1993): 320-24.