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DEVOLUTION OF THE FISHER EQUATION:

Rational Appreciation to Money Illusion

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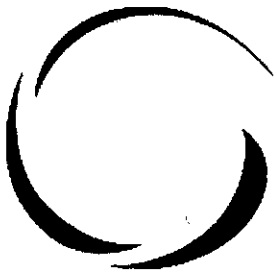

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Rational Appreciation to Money Illusion

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I first noted the discrepancy between the original and conventional versions of the Fisher equation some twenty years ago. At that time, the difficulty of finding empirical verification of the conventional Fisher equation (CFE) led a number of economists to the discovery that the CFE is inconsistent with rational behavior. As a Fisher buff, I found it difficult to accept that Fisher could have got the relationship wrong. My original hypothesis was that the conventional view was correct. In reading *Appreciation and Interest* (1896) I discovered that I had been wrong about the equation, but correct in my instincts about Fisher. The explanation for Fisher's switch in 1930 from the original Fisher equation (OFE) to the conventional specification eluded me until recently when "money illusion," another Fisher innovation, returned to academic respectability. The author is indebted to Levis Kochin for providing the original stimulus for this paper and to Milton Friedman for sustaining interest. Thanks are also due Joerg Bibow, Yang Ming Chang, Dana Ferrell, Thomas Humphrey, the late Mark Meador, Perry Mehrling, Francisco de A. Nadal-De Simone, Sumimaru Odano, Frank Packer, Kenichi Sakakibara, and Kenji Yamamoto for helpful comments on earlier versions of the paper.

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Abstract

In *Appreciation and Interest* Irving Fisher (1896) derived an equation connecting interest rates in any two standards of value. The original Fisher equation (OFE, 1896) was expressed in terms of the expected appreciation of money (the real return on money) whereas the conventional Fisher equation (CFE, 1930) uses expected inflation. Since the OFE is based on the value of money (1/P) it is not subject to standard criticisms of irrationality leveled against the CFE. Fisher's puzzling substitution of lagged inflation for money appreciation in 1930 is resolved by taking into account his theory of "money illusion." [JEL: B00, E40, B13, B22, B31]

Key Words: Fisher equation, Fisher hypothesis, Fisher effect, money illusion, nominal interest rate, purchasing power of money, value of money.

DEVOLUTION OF THE FISHER EQUATION:
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> *At the outset the question arises, how can a merchant be said to foresee the appreciation of money? Appreciation is a subtle conception. Few business men have any clear ideas of it. Economists disagree as to its definition, and statisticians as to its measurement.*
>
> Irving Fisher (1896, p. 35)

I. Introduction

Over one hundred years since the publication of *Appreciation and Interest* (1896), "appreciation of money" remains a subtle conception. The subtlety extends to Fisher's theory of the nominal interest rate which continues to be misrepresented and misunderstood in countless textbooks, scientific papers, and popular writings. The objective of this paper is to recover the original Fisher equation (OFE, 1896) and explain its relationship to and displacement by the conventional Fisher equation (CFE, 1930). The paper argues that the two equations are a consequence of different assumptions about expectation formation and variable measurement. The original specification reflects Fisher's early interest in rational behavior and measurement. The conventional specification is the product of Fisher's growing skepticism about the rationality of market expectations. The CFE has it's origins in the empirical work reported in *The Theory of Interest* (1930) which, in turn, is an outgrowth of Fisher's concept of money illusion.

The failure of the profession to distinguish the two Fisher equations involves the subtle concept of "money appreciation." Appreciation under a modern fiduciary standard refers to the rate of change in the value of (paper) money expressed in terms of commodities.[1] If the value of commodities is P, then the value of one unit of money (v) is 1/P. Where Fisher used "expected appreciation of money," modern economists substitute "expected deflation." Confusion arises since the concepts are often used interchangeably in informal analysis. As a consequence of

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[1] An alternative name for money appreciation is the real return on money (Eden, 1976).

Jensen's inequality, however, the mathematical definitions are not equivalent.[2] This fact is the cornerstone of various arguments concerning the inappropriateness of the CFE. As it turns out, it is the profession at large, not Fisher (1896), which has failed to take proper account of expected appreciation.

Fisher, as he readily acknowledged, was not the first person to advance the theory that the nominal interest rate adjusts to changes in the value of money. Humphrey (1983) traces the lengthy development of this idea. Fisher's contribution, apparently, was being the first to write an equation for the relationship (Humphrey, 1983). Fisher was also the first to clearly show the derivation of the equation. Fisher's original equation, however, is *not* the one which is commonly attributed to him. This is fortuitous in the sense that the conventional representation of the "Fisher equation" is a misspecification of the relationship between nominal and real yields when market participants form rational expectations over uncertain future prices (Eden, 1975, 1976; Blejer and Eden, 1979; Kochin, 1980; and Benninga and Protopapadakis, 1983). But what if market expectations are based on money illusion rather than rational behavior? Ironically, it was Fisher's (1930) rejection of the rationality postulate that led to the conventional specification.

II. Fisher's Lost Equation

The Fisher equation, in its most common representation, describes the relationship between the nominal and real rates of interest.[3] This conventional Fisher equation (CFE)

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[2] The value of money is a convex function of the price level. The mean value of the secant connecting any two points on this function is greater than the average of the two points on the curve. For a non-degenerate random variable (P), Jensen's inequality implies: $E(1/P) \exists 1/EP$.

[3] Fisher (1896, p. 88) was familiar with the concepts of "real" and "nominal" interest rates, but he preferred not to use this Marshallian terminology (see chapter XII). Fisher's theory was more general in the sense that it connected any two interest rates in any two standards. The Fisher equation was the "law" that connected any two interest rates expressed in different standards of value. There are as many interest rates as there are standards.

expresses a relationship between the nominal rate of interest (i) and expected inflation (π). A common linear representation is:

$$i = r + \pi + r\pi \tag{1}$$

where r = *ex ante* real interest rate and π = expected rate of change in the price level (P). The theory embodies the "Fisher hypothesis" (or "Fisher effect") of a one-to-one relationship between the nominal interest rate and expected inflation. The theory remains popular in spite important limitations raised in the theoretical literature.[4] A lengthy empirical literature, starting with Fisher (1896, 1930), continues to search, with mixed success, for evidence supporting the "Fisher effect."[5]

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[4] In the theoretical literature, the Fisher effect is said to apply to only one or more special cases where inflationary expectations are held with certainty, where expected inflation is uncorrelated with the real interest rate, where income tax rates are zero, or where international arbitrage operates costlessly for both commodities and financial capital. The perfect certainty interpretation is advanced by Benniga and Protopapadakis (1983), Blejer and Eden (1979), and Kochin (1980). Mundell (1963), Tobin (1965), and Carmichael and Stebbing (1983) provide theories that highlight the neutrality proposition underlying the Fisher effect. The tax argument is expounded by Darby (1975), Feldstein (1976), and Tanzi (1976). The costless international arbitrage view is employed in Hansson and Stuart (1986)

[5] Classical studies follow Fisher (1930) in regressing some measure of expected inflation on short-term nominal interest rates. Typically, these studies find that the coefficient on expected inflation is significantly less than one. This was also the finding of Summers (1983) in a study which attempted to extract a long-term relationship from 120 years of data. Fama (1975) reinterpreted the Fisher hypothesis as a test for market efficiency and found evidence that short term interest rates efficiently predict subsequent changes in the value of money. Nelson and Schwert (1977), however, found evidence contradicting Fama's joint hypothesis of market efficiency and real rate constancy. Kandel, Ofer, and Sarig (1996) found a negative correlation between the *ex ante* real interest rate and expected inflation. Ahmed and Rogers (2000), in finding Tobin-type effects of inflation on real variables, indirectly reject the Fisher hypothesis. Traditional estimates of the Fisher effect may be biased in that they fail to take into account the changing stochastic inflation process (Klein, 1975; Barsky, 1987; Hutchison and Keeley, 1989) and/or differences in the order of integration of the data (Rose, 1992). Some support for a long-run Fisher effect has been found when careful attention has been paid to the time series properties of the data. Studies finding support for a long-run Fisher hypothesis include Lucas (1980), Mishkin (1992), Wallace and Warner (1993), Evans and Lewis (1995), and Mishkin and Simon (1995). Mixed support was found by Lee, Clark, and Ahn (1998) and Carneiro, Divino, and Rocha (2002). Some recent support for a tax-adjusted Fisher equation was found

While criticism of the CFE has merit from an efficient markets perspective, it is inappropriate to lay the blame on Fisher. To understand why such criticism is misguided, we must go back to Fisher's original work on nominal interest rates. In *Interest and Appreciation* Fisher (1896) put forth a "multiple theory of interest." What does this mean? It does not mean that Fisher advanced a theory for the simultaneous determination of multiple interest rates. On the contrary, Fisher derived a single equation that showed the relationship between any two interest rates (i and j) in any *two* alternative standards of value. For any given transaction, there are as many interest rates as there are monetary standards.

The original Fisher equation is an arbitrage condition that makes the interest rate in one standard of value equivalent to the interest rate in another standard of value.[6] Fisher's point is that a loan contract that specifies payment in terms of money type X can be rewritten in equivalent form in terms of money type Y. If money X is expected to appreciate relative to money Y, then the interest rate (j) in the relatively depreciating standard (Y) should be greater than the interest rate (i) in the appreciating standard (X).

Although a skilled mathematician, Fisher took great pains to make his works accessible and relevant to sophisticated laymen. *Appreciation and Interest* is no exception, but the nature of the subject matter imposes considerable demands on the reader. Modern readers, apparently, do not have the required patience. Fisher explicates the relationship between appreciation and interest by guiding the reader through a series of progressively more complicated calculations

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by Crowder and Hoffman (1996) and Crowder and Wohar (1999). For surveys of the empirical literature see Choudhry, Placone, and Wallace (1991) and Friedman and Schwartz (1982).

[6] Don't be misled by advertisements of "true" Fisher equations. Fisher-like equations can be derived from a variety of models incorporating interest rate(s), the price level, and production. Only those models which impose the OFE/CFE as a constraint are consistent with Fisher's original analysis. As an arbitrage condition between two currencies, the "Fisher open equation" follows the spirit of Fisher. It is subject, however, to the same measurement issues posed by the "closed economy" CFE.

supported by numerous illustrations drawn from everyday business experience under a bimetallic standard. As Fisher (1896, chapter X) showed, the analysis can also be applied to a fiduciary standard in which paper money exchanges for commodity money.

The essence of Fisher's approach can be captured by a simple two period present value model. A modern touch is the introduction of the expectation operator (E). Following Fisher (1896), taxes and risk considerations are ignored. Fisher (1896, 1907) was aware that incorporating such considerations would lead to a modification in his analysis. Appendix A contains an n-period generalization of the two period Fisherian model.

Consider a contract in which future payment is to be made in paper money (dollars). The present value ($P_{B,t}$) of a future (dollar) benefit (D) sold at discount in period t and at a nominal (paper money) interest rate, i, is:

$$P_{B,t} = \frac{D_{t+1}}{1+i} \tag{2}$$

An alternative contract is one in which the future payment would be made in bushels (B) of commodity money (wheat in many of Fisher's examples). For market participants to be indifferent between the two contracts, the number of bushels (B) to be paid in the *initial* period must be equivalent in value to the number of dollars (D) required in the money contract. Future payments must take into account changes in the expected terms of trade between money and commodities. Under perfect risk-free arbitrage, the commodity (real) value of future money payments, $D_{t+i}E(v_{t+i})$, must equal the equivalent amount of future commodities, B_{t+i}.

Define the real (commodity) rate of interest as j and the terms of trade between money and commodities as v. Using these definitions, the real present value of the future (paper) money payment or receipt can be expressed:

$$P_{B,t}v_t = \frac{D_{t+1}Ev_{t+1}}{1+j} \tag{3}$$

where Ev_{t+k} is the expected value of money (1/P) in period t + k with k = 0 or 1. Equation (3) gives the present value in a commodity standard of a future money payment. Equating the price of the asset from equations (2) and (3), canceling, transposing, and rearranging terms yields:

$$\frac{1+i}{1+j} = \frac{1}{1+a} \tag{4}$$

where $a \equiv (Ev_{t+1} - v_t)/ v_t$.[7] Cross multiplying and collecting terms results in the OFE:

$$j = i + a + ia \tag{5}$$

As Fisher understood, the perfect certainty form of the OFE is a special case where expected appreciation (a) is equal to *ex post* appreciation (a*).

The original Fisher equation (OFE), when applied to a world of paper money ("money") and commodity money ("commodities"), is expressed in terms of the *expected* appreciation of money (a) and written with the real interest rate (j), Fisher's "virtual interest in commodities," as the left-hand variable. In this form, the original equation can easily be misinterpreted as the CFE. Hirshleifer (1970, pp. 135-36), in an influential work, reinforced the conventional view by representing anticipated inflation with the letter "a."

A more fundamental misunderstanding concerns the relationship between expected inflation (π) and expected appreciation of money (a). The OFE was derived by taking expectations over the value of money (v). The CFE can be derived from the same framework if expectations are taken over the value of goods (P) and expected inflation is defined with reference to the current price level: $\pi \equiv (EP_{t+1} - P_t)/ P_t$. It follows from Jensen's inequality, that $\pi \sqcup -a$. Fisher (1896, ch. II) was aware of this fact since he showed that the inequality holds even in the perfect foresight case where expected appreciation (a) is equal to *ex post*

[7] In his formal derivations, Fisher examined the special case where the expected value of money was equal to the future actual value of money, i.e. $Ev_{t+1} = v_{t+1}$. It is clear from his writings, however, that Fisher believed his formula also applied to the general case where foresight was less than perfect.

appreciation (a*).[8]

The two Fisher equations are not equivalent in the case where the appreciation of money is perfectly foreseen since they use different definitions (measurements) of the *ex post* appreciation of (paper) money and the *ex post* real return (j* versus r*). In the calculation above, the OFE used the current value of money (v_t) as the point of reference in the definition of the appreciation of money. The CFE uses the future value of money (v_{t+1}) as the implicit reference point (the denominator in calculations of discrete percentage changes). Under perfect certainty, the two definitions can be made compatible by using a common definition of the *ex post* real return.

The CFE and the OFE provide equivalent definitions of the *ex post* real return if *ex post* inflation (π^*) is calculated with respect to the future price level [i.e., $\pi^* \equiv (P_{t+1} - P_t)/ P_{t+1}$]. Under uncertainty, however, the difference in the OFE and CFE is not simply a matter of how the *ex post* real return is calculated. The two equations result from different assumptions about expectation formation. The OFE assumes that expectations are formed over the value of money (Ev); the CFE assumes that expectations are formed over the value of goods (EP). Even if comparable definitions are used for discrete calculations, the *expected* appreciation of money (a) will not equal the *expected* rate of deflation ($-\pi$) due to Jensen's inequality.[9]

Historians of thought, while occasionally employing Fisher's (1896) terminology, have inadvertently contributed to the misunderstanding of Fisher's theory. Tobin (1997, p. 374), Howitt (1992, 2, p. 123), and Dimand and Geanakoplos (2005), for example, describe the Fisher

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[8] Assuming perfect foresight, Fisher (1896, ch. II) showed that the interest rate relationship could be written in terms of the rate of depreciation (d) of money Y relative to money X: $j = i + d + jd$. Even in this special case where relative price changes are known, Fisher (p. 11) shows that the rate of appreciation (a) exceeds the rate of depreciation (d).

[9] $a = -[1 - P_t E(1/ P_{t+1})] \neq - \pi = [1 - P_t (1/E P_{t+1})]$

equation using the conventional specification (1). Dimand (1999a) accurately reproduces the original Fisher equation (5) and points out that Fisher's money and commodities model used the "(expected) purchasing power of money." In defining "expected inflation as the difference between real and nominal interest rates," however, Dimand (1997, p. 442; 1999a, p. 748; 1999b, p. 36) assumes that there is no difference between the OFE and the CFE. Humphrey (1983), in an otherwise illuminating discussion, uses the conventional, rather than the original, Fisher equation in describing Fisher's contribution to the history of thought. The mistake is in viewing the two equations as equivalent. With uncertainty, the CFE and the OFE are rival equations, not different forms of the same equation!

The continuing popularity of the CFE is puzzling in that it provides a biased estimate of the relationship between nominal and real bond yields when expectations are formed rationally over uncertain future prices (Benniga and Protopapadakis, 1983; Blejer and Eden, 1979; and Kochin, 1980). What is the extent of the bias? If the size of the bias is small, then the CFE is a reasonable approximation to the OFE. Theoretical models suggest that this is a risky assumption, particularly in cases where price level volatility is large (Eden, 1975, 1976; Sarte, 1998), expectation horizons are long (McCulloch and Kochin, 2000), or individual expectations are diffuse (Kochin, 1980). Fama (1975, 1976), in a shrewd attempt to avoid inflation-uncertainty bias, wrote the Fisher relationship in terms of the expected value of money [E(1/P)].[10] In doing so, he inadvertently rediscovered the OFE. Appendix B illustrates the effects of Jensen's inequality in a simple graphical model.

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[10] The issue of CFE bias has been engaged in the empirical literature. Since the source of the bias in the CFE is the failure to properly account for variability in the price level, regression equations for nominal interest rates sometimes include a measure of inflation variability. For the U.S. data, the sign, magnitude, and statistical significance of this coefficient have varied across studies. Recent studies include: Chan (1994), Ireland (1998), Sarte (1998), and Shome, Smith, and Pinkerton (1988).

III. Money Appreciation and the Original Equation

Why did Fisher (1896) insist on formulating the problem in terms of the expected appreciation of money (a) rather than expected deflation of commodities (π)? One cannot be sure, but one should not overlook a simple explanation. As a neoclassical economist, Fisher defined the value of money as the inverse of the price of goods.[11] It verges on the obvious that if you use the wrong definition of the relative price of money, you will end up with a biased measure of the appreciation of money. Fisher's preference for money appreciation (rather than goods depreciation) has, however, an economic justification. Although not established by Fisher, the OFE yields an unbiased prediction of the *ex post* real rate of interest (j*). A simple proof of this proposition is provided in Appendix C.

Fisher knew that the definition of money appreciation has consequences. As "the greatest expert of all time on index numbers" (Tobin, 1987, p. 369), Fisher understood that care must be exercised in the method of calculating mean values. He knew, for example, that the arithmetic mean (A) of a series would be greater than its harmonic mean (H).[12] Likewise, the rate of change in the price level (P) would be greater (in absolute value) than the rate of change of its inverse (v).[13] Schumpeter (1954, pp. 1091) points out that the work of Fisher and others

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[11] Alternative definitions of money value include the interest rate (the inter-temporal value of money) and the foreign exchange rate.

[12] Suppose there are n possible outcomes represented by the variable x. The arithmetic mean (A) is $A = \{3\ x\}/n$; the harmonic mean (H) is $H = n/\{3(1/x)\}$ with $H \# A$. These concepts appear repeatedly in Fisher's works.

[13] Coggeshall (1886-87), who Fisher (1927, p. 81n) cited, advocated use of the harmonic mean a full decade before the publication of *Appreciation and Interest.* In general, the *harmonic* mean, not the *arithmetic* mean, is the appropriate measure of central tendency when dealing with rates of change. The arithmetic mean overstates the true average rate of inflation since it fails to take account of the shorter length of time required to achieve a particular price level at a higher rate of inflation. A simple example illustrates: Suppose there are two possible rates, each of equal probability, at which the price level might rise from a level of 100 to 120: 10 percent p.a. and 20 percent p.a. The arithmetic average (A) of the two inflation rates is 15 percent p.a., but the true average time it would take to cover the "distance" would equal the harmonic mean:

on index numbers was the "statistical complement" to the "theoretical discussion on the purchasing power of money."

Jensen did not publish his formal proof concerning convex functions until 1906. Did Fisher (1896) have an understanding, intuitive or otherwise, of Jensen's inequality? One cannot say with absolute certainty, but a careful reading of his subsequent work on index numbers suggests he did. Fisher's choice of terminology also supports such an interpretation. When explaining the *theoretical* connection between nominal and real interest rates, Fisher consistently used terms such as "expected change in the value of money," "expected appreciation of money," or "expected change in the purchasing power of money." Fisher (1896, 1905, and 1930) used the same terminology in all of his major works on the theory of interest. In his extensive empirical investigations, reported in detail in 1896, appreciation was consistently calculated as the percent change in the *reciprocal* of the price level.

Fisher bemoaned, over and over again, the apparent inability of people to grasp the concept of money value. He did not take it to be a matter of inconsequence that people found it easier to calculate in terms of prices than money values. Indeed, Fisher insisted that his weekly Index Number of Wholesale Prices be published as the *inverse* of the original price series. According to his son, I. N. Fisher (1896, p. 35):

> The chief purpose of this newspaper publication was to invert the ordinary index number representing the price level, thereby obtaining an index number representing the purchasing power of the dollar, the idea being to accustom the public to the thought that the dollar is not a constant but a variable.

Taking a tip from Mehrling (2001), one may see in Fisher's dogged persistence a conviction that a proper measure of money value would enhance social welfare.

Despite frequent assertions to the contrary, the original source of the "Fisher equation" is not *The Theory of Interest.* To uncover the nature of the theoretical relationship between ex

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$H = 2/[1/10 + 1/20] = 13.33$ percent p.a. (total distance of 40 divided by total time of 3 years).

ante real and nominal interest rates, we must take Fisher's (1930, p. 39) advice and consult *Appreciation and Interest.* Here, careful reading and patience are required for an accurate understanding of the theory. It must be remembered that Fisher was writing in response to the bimetallic controversy, the most important economic issue of the period. His purpose was to show the relationship between interest rates expressed in different standards (e.g. gold and wheat). According to Fisher (1896, p. 92), "[t]hese rates are mutually connected and our task has been merely to state the law of that connection. We have not attempted the bolder task of explaining the rates themselves." Fisher's initial attempt at "the bolder task" was *The Rate of Interest*, published in 1907. His definitive treatment of the subject is his 1930 work, *The Theory of Interest.*

In part I of his 1896 monograph, Fisher stresses that there are as many interest rates as there are monetary standards. It is not until part II that he introduces the modern convention of using fiduciary money and (aggregate) commodities as the two standards. It is also in part II that Fisher drops the simplifying assumption of perfect foresight and makes clear that the OFE is in terms of *expected* appreciation.

The empirical question Fisher (1896) attempted to address was the extent to which *ex post* appreciation (a*) was captured by expected appreciation (a). In testing his theory, Fisher (1896) used a variety of alternative definitions of money X and money Y: gold and wheat (ch. II), gold and paper (ch. VIII), gold and silver (IX), and money and commodities (ch. X). Part II of *Appreciation and Interest* uses bond market and price data from seven countries to examine the extent to which market interest rates adjust to the "appreciation of money in commodities." Fisher's examples reflect the period of investigation: money is the (relatively) appreciating standard and (aggregate) commodity is the (relatively) depreciating standard. Money appreciates when commodity prices (P) go down and depreciates when prices go up.

Fisher's assumption of perfect foresight in his mathematical derivations was merely for convenience. In part II, Fisher (1896, p. 43) actually used the equation derived in part I to obtain a measure of "expected appreciation." He achieved this remarkable feat by exploiting the difference in the yields of commodity (gold coin) bonds and paper (currency) bonds. Knowing the paper yield (i) and the commodity yield (j), Fisher used the OFE to solve for the *expected* appreciation of money (a); that is, "that rate of appreciation which would have made the two interest rates equally profitable" (Fisher, 1930, p. 42-43, n. 4). He compared this expected appreciation with the realized *(ex post)* appreciation of money (a*) and discovered that expected appreciation consistently under predicted actual appreciation. This discovery, reinforced by subsequent research, called into question the rationality assumption implicit in the OFE. Table 1 reproduces Fisher's original table where one may easily verify that Fisher's original theory was in terms of the *expected* value of money (a).

TABLE 1: Fisher's Calculations of Expected and Actual Appreciation, 1870-78

RATE OF INTEREST REALIZED FROM DATES MENTIONED TO JANUARY 1, 1879, (DATE OF RESUMPTION).				
	[Gold] Coin *j*	[Paper] Currency *i*	Appreciation of Currency in Gold	
			Expected a	Actual (α)
January, 1870	7.1	6.3	.8	2.1
July, 1870	6.2	5.7	.5	1.4
January, 1871	6.7	6.3	.4	1.3
July, 1871	6.4	5.7	.7	1.8
January, 1872	5.9	5.7	.2	1.3
July, 1872	6.2	5.7	.5	2.1
January, 1873	6.5	6.2	.3	2.0
July, 1873	6.2	6.0	.2	2.8
January, 1874	5.6	6.1	-.5	2.1
July, 1874	5.7	5.8	-.1	2.4
January, 1875	6.0	5.4	.6	3.1
July, 1875	6.1	4.2	1.8	4.9
January, 1876	5.4	4.1	1.2	4.3
July, 1876	5.2	2.4	2.7	4.9
January, 1877	5.5	4.0	1.4	3.5
July, 1877	5.7	3.1	2.5	3.6
January, 1878	8.2	6.0	2.1	2.8
July, 1878	4.8	2.6	2.1	1.4

Source: Fisher (1896), p. 42. Entries in brackets are not in original.

IV. Money Illusion and the Conventional Specification

Of Irving Fisher's works on interest rate behavior, the one which is most frequently cited is *The Theory of Interest.* Published in 1930, this book is often erroneously credited as the source of the Fisher equation and Fisher hypothesis. Fisher (1930, p. 451) clearly states that "the main object of this book is to show how the rate of interest would behave if the purchasing power of money were stable." Fisher's 1930 work develops a micro-based theory of the determination of the real interest rate. It is only at the end of the book that Fisher reminds us that, in the short run, the real interest rate is also influenced by monetary phenomenon. He presents his recent empirical work on the link between prices and interest rates.

Although one will not find an explicit representation of the CFE in any of Fisher's works, it is easy to see how a reader who consults only *The Theory of Interest* would find support for the conventional interpretation. Setting a pattern for subsequent research, Fisher (1930, ch. XIX) examined the correlation between the nominal interest rate and the price of commodities. As is well known, he found a positive but weak contemporaneous relationship between the rate of change of commodity prices and the nominal interest rate. Applying a distributed lag model, Fisher found that *past* inflation influenced the nominal interest rate with a long and variable lag.

Fisher's *empirical* model is the source of confusion over the Fisher equation. Why did Fisher switch from expected money appreciation to lagged inflation in his post-1896 empirical work? One cannot be absolutely sure, but the change in emphasis is dramatic in light of Fisher's previous insistence on using money value. The most likely answer can be found in Fisher's psychological theory of expectations. Fisher's early empirical work (1896) led him to question the rationality of market expectations. Conversations with businessmen and workers further convinced him that the value of money was too subtle a concept for ordinary people to

comprehend. Even hyperinflation could not unveil the money illusion:

> The most striking case which I encountered of this pervasive money illusion was that in Germany following World War I. Germany's inflation, of course, was not so much check-book money as printing-press money – money printed to pay the Government's debts. As a result, the mark of 1922 would buy only one-fiftieth as much as the mark of 1914. Yet when I visited Germany in that year expressly to find out what the Germans knew about this fall of the mark, I found that 19 out of 20 whom I interrogated did not realize that the rise of prices had anything to do with the mark. They imagined it was all due to such factors as the Allied Blockade making goods scarce. They simply took for granted that the mark of 1922 was the same mark as that of 1914. They measured all values in marks just as they always had. (Fisher, 1946)

The popular view, according to Fisher (1930, p. 399) is that "money itself does not change." If this is the case, then bond market participants do not form expectations over the value of money and a viable empirical model of the interest rate can not be conditioned on the OFE.

In 1896, Fisher had yet to introduce the concept of "money illusion." Fisher (1896, p. 11) made clear that he was "regarding money as a standard of value and not as a medium of exchange." Money is a measure of value just as a yard is a measure of length. Contracts, whether expressed in money or yards, should be adjusted to take proper account of changes in the units of measurement:

> It is clear that if the unit of length were changed and its change were foreknown, contracts would be modified accordingly. Suppose a yard were defined (as once it probably was) to be the length of the king's girdle, and suppose the king to be a child. Everybody would then know that the "yard" would increase with age and a merchant who should agree to deliver 1,000 "yards" ten years hence, would make his terms correspond to his expectations. (1896, p. 1)

Fisher derived the OFE under the assumption of rational measurement, but his early empirical work suggested that interest rates fell significantly short of anticipating subsequent appreciation.

As early as 1896, Fisher was beginning to have second thoughts about rational behavior: "If you ask a merchant whether he takes account of appreciation, he will say he never thinks of it, that he always regards a dollar as a dollar. Other things may change in terms of

money, but money itself he is accustomed to think of as the one fixed thing." In spite growing doubts, Fisher (1896) left open the possibility that inadequate interest rate adjustment might be due to "imperfection of foresight." He recognized that the mere possibility of a monetary regime change would provide rational grounds for such imperfection (Fisher, 1896, chapter VIII). At the end of his career, Fisher (1946) admitted his reluctance to shed the rationality assumption:

> It took me a long time to realize how pervasive is this money illusion. In fact, it dawned on me only after I had published *Stabilizing the Dollar*, which contained my first suggestion as to how to stabilize. I found people saying of this book: "But does the dollar need any stabilizing? If so, that's news to me."

Fisher's own illusion about educating the masses on the value of the dollar was gradually undermined by his empirical studies and his business dealings.

Short-run fluctuations in real interest rates and output played a key role in Fisher's subsequent work on the monetary theory of the business cycle (Dimand, 1999b). From 1920 onwards, Fisher's business cycle works (including those on interest rates) embodied the money illusion hypothesis. Fisher's adoption of the money illusion hypothesis reflects a natural evolution in his thinking concerning the causes and consequences of the "dance of the dollar."

Money illusion is traditionally defined as a situation where market participants make economic decisions based on money prices rather than theoretically correct relative prices and real wealth (Patinkin, 1965, pp. 22-23). Money illusion, in this sense, is a violation of the "homogeneity postulate" (Leontief, 1936). Workers suffering from money illusion bargain in terms of money wages rather than real wages. Business managers, to the extent they suffer from the disease, fail to adequately take account of the general price level in making pricing and output decisions. Fisher used such notions throughout his collected works, especially in *The Money Illusion* and other business cycle writings where some type of fooling assumption is required to explain output and employment effects of monetary disturbances.

Money illusion, according to Fisher (1928, p. 4) is "the failure to perceive that the dollar, or any other unit of money, expands or shrinks in value." Money illusion is tantamount to a failure to properly measure the appreciation of money (the growth of the king's girdle). The Patinkin form of money illusion is an extreme case when money (the yardstick) is perceived not to change in value at all; when a yard is a yard and "a dollar is a dollar" (Fisher, 1896, p. 35; 1930, p. 399).

The presence of money illusion rules out the direct impact of expected appreciation on interest rates as presumed in the OFE. The possibility remains of a more round-about influence. In various writings, Fisher conjectured that changes in commodity inflation would have an *indirect* and *lagged* impact on the nominal interest rate and other variables. In *The Theory of Interest*, for example, Fisher (1930, pp. 399-400) described the adjustment process of interest rates under money illusion:

> Most people are subject to what may be called "the money illusion," and think instinctively of money as constant and incapable of appreciation or depreciation. Yet it may be true that they do take account, to some extent at least, even if unconsciously, of a change in the buying power of money, under guise of a change in the level of prices in general. If the price level falls in such a way that they may expect for themselves a shrinking margin of profit, they will be cautious about borrowing unless interest falls, and this very unwillingness to borrow, lessening the demand in the money market, will tend to bring interest down. On the other hand, if inflation is going on, they will scent rising prices ahead and so rising money profits, and will be stimulated to borrow unless the rate of interest rises enough to discourage them, and their willingness to borrow will itself tend to raise interest.

According to Fisher, price changes may have an impact on interest rates even in the presence of imperfect foresight and money illusion. Sluggish price changes and the resulting trade fluctuations put indirect pressure on the market for loans and the rate of interest. To capture the lagged effect of prices on interest rates, Fisher (1930) developed the distributed lag model.

Fisher's concept of money illusion did not rule out the possibility of (imperfect) foresight with respect to the price of goods. "The businessman," Fisher (1930, p. 400)

observed, "makes a definite effort to look ahead not only as to his own particular business but as to general business conditions, including the trend of prices." Furthermore, "(e)vidence that an expected change in the price level does have an effect on the money rate of interest may be obtained from several sources" (Fisher, 1930, p. 400). Based on empirical observation, Fisher came to believe that market participants exhibit complex psychological behavior: both foresight and illusion influence market outcomes. Fisher would not be surprised by modern psychological studies that find inconsistencies and inaccuracies in people's calculation of money values (cf. Safir, Diamond, and Tversky, 1997; Fehr and Tyran, 2001). The interaction between money illusion and (imperfect) foresight provided Fisher a rationale for replacing expected appreciation with lagged inflation.

Friedman and Schwartz (1982, p. 547) note that Fisher's 1930 empirical work has "less economics" than his earlier works (1896, 1907). It is stretching matters, however, to attribute the loss of economics to his adoption of the adaptive expectations hypothesis.[14] The concept of adaptive expectations is an interpretation superimposed on Fisher's (1930) lagged adjustment model by subsequent researchers. What constitutes the "loss" is the switch in emphasis from market rationality to market psychology. Fisher's empirical model could not assume full rationality if market psychology was dominated by widespread money illusion. If Fisher's theory of inflation psychology is correct, then empirical studies using a backward-looking specification should more accurately predict the behavior of the nominal interest rate than those based on a forward-looking specification.

We are now in a position to understand why Fisher called appreciation a "subtle

[14] Much of the modern research on the Fisher equation is critical of Fisher's empirical methodology and findings. In a meticulous study using over a century of data, Friedman and Schwartz (1982, ch. 10) reach conclusions that are broadly consistent with those of Fisher (1930). Money illusion has also received empirical support from a number of recent studies (Fehr and Tyran, 2001; Shafir et. al., 1997).

conception." In addition to the proper definition of money value, the debate over the specification of the Fisher equation involves two subtle issues of measurement. To return to Fisher's colorful analogy, measurement problems may surface if the yardstick (money) depends on the size of the king's girdle (money value). One measurement problem results if people suffer from money illusion; that is, they fail to perceive that the size of the yardstick is changing. But realization that money value changes is not enough to eliminate measurement issues. How do we measure non-girdle things when the yardstick is changing and, especially, when there is uncertainty about its true length? One must not only recognize that the value of money changes, but must use a proper measure of the magnitude of the expected change. Hence, a second measurement problem occurs if one uses an improperly calibrated yardstick to measure the extent of the change in the king's girdle (e.g. the Jensen inequality problem).

V. Conclusion

Irving Fisher fathered two equations describing the relationship between nominal and real interest rates. The OFE gives the "exact theoretical relationship between the rates of interest measured in any two diverging standards of value and the rate of foreseen appreciation or depreciation of one of these two standards relatively to the other..." (Fisher, 1930, p. 39). The OFE is the product of Fisher's work as a theoretical economist. It reflects his belief, confirmed by modern financial economists, that rational behavior requires nominal interest rates to respond to changes in the expected value of money *not* the value of goods. The second equation, which is the conventional specification, is written in terms of expected inflation (π) and has its origins in *The Theory of Interest.* The displacement of money appreciation by goods inflation reflects Fisher's (1930) views as an amateur psychologist and an applied statistician. The CFE, which is an approximation to Fisher's preferred specification, uses an incorrect measure of the expected value of money under rational expectations.

Economists, the "guardians of rationality," often perceive unfulfilled genius in Fisher's works (Schumpeter, 1948; Allen, 1993). If Fisher had followed his scientific bent and forsaken his quixotic campaigns, the argument runs, economic science would have leapt forward by decades. Perhaps, but Fisher's faith in economic rationality was badly shaken by his contact with lesser mortals. Fisher (1946, p. 33) believed that the "study of mere market value" would lead one astray if markets are characterized by such non-rational phenomenon as money illusion, systematic mismeasurement, and irresponsible social behavior. In Fisher's world, social progress requires enlightened leadership as well as science.

APPENDIX A

Derivation of the OFE

The original Fisher (1896) analysis was spread over four chapters. The condensed analysis that follows uses Fisher's seminal idea (chapter V, p. 27) to link the present values of two equivalent bonds in different monetary standards and solve for the underlying interest rate relationship. As a concession to modernity, a standard "nominal" bond is compared with an equivalent valued "real" bond (i.e. one indexed for changes in the expected value of money). Following Fisher (1896), taxes and risk considerations are ignored.

Consider a bond in a representative bond market. In discrete time, the formula for the nominal price of the bond is:

$$P_{B,0} = \sum_{t=1}^{n} \frac{C}{(1+i)^t} + \frac{D_n}{(1+i)^n} \tag{A1}$$

where $P_{B,0} \equiv$ bond price in the reference period, $C \equiv$ coupon value, $D_n \equiv$ future value at end of the holding period, and $i \equiv$ pre-tax nominal yield (for holding period).

The real price of the bond can be expressed:

$$P_{B,0}v_0 = \sum_{t=1}^{n} \frac{Cv_t^*}{(1+j)^t} + \frac{D_n v_n^*}{(1+j)^n} \tag{A2}$$

where $v^*_t \equiv$ (expected) value of money in the commodity standard in period t and $j \equiv$ pre-tax real (holding period) yield. The value of money in the current period (v_0) is simply the inverse of the price level ($1/P_0$).

Define period 0 as the base period such that $v_0 = 1$. Equate the right sides of equations A1 and A2. Divide both sides of the combined equation by C. Invert and expand the combined equation to obtain equation A3:

$$\frac{1}{v_1^*}(1+j)+\frac{1}{v_2^*}(1+j)^2+\ldots+\frac{1}{v_n^*}(1+j)^n+\frac{1}{v_n^*}(1+j)^n\left(\frac{C}{D_n}\right)$$
$$=(1+i)+(1+i)^2+\ldots(1+i)^n+(1+i)^n\left(\frac{C}{D_n}\right) \tag{A3}$$

Assuming that expected value of money (v*) appreciates at a constant rate (a), we can write:

$$v_{t+1}^* = (1+\mathrm{a})v_t^* \tag{A4}$$

for all t (t = 0,...,n). Define: I ≡ (1+i), A ≡ 1/(1+a), and J ≡ (1+j). Using these definitions and equation A5, one can substitute progressively into equation A3 and, after simplification, obtain:

$$AJ+(AJ)^2+(AJ)^n+(AJ)^n\left(\frac{C}{D_n}\right)$$
$$=I+I^2+\ldots+I^n+I^n\left(\frac{C}{D_n}\right) \tag{A5}$$

This implies

$$AJ = I \tag{A6}$$

or

$$j = i+\mathrm{a}+i\mathrm{a} \tag{A7}$$

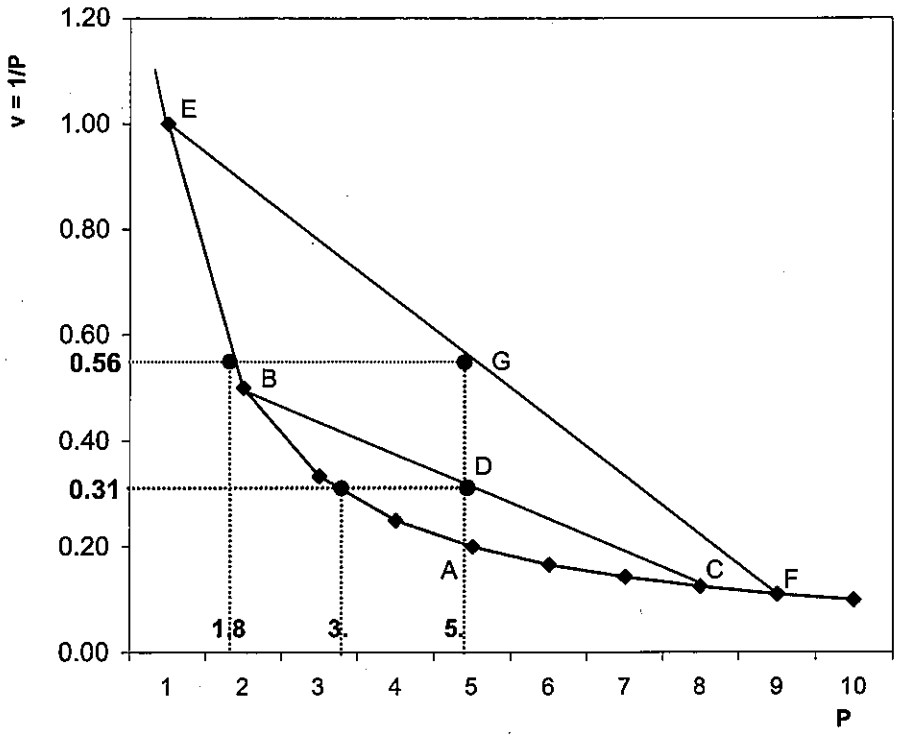
This is the OFE expressed in terms of the expected appreciation of money (a).[15]

[15] In the above analysis, the assumptions of constant periodic rates of interest and appreciation were made for convenience only. When interest rates and appreciation rates vary, the Fisher identity can be interpreted as a relationship between the yield to maturity (Fisher's rate of return over cost) and the *average* rate of appreciation of money. In this case, equations A1 and A2, which use appropriately weighted average yields, are used in place of the actual expressions for which they are equivalent in present value. Likewise, the rate of appreciation of money (a) may be interpreted as the average rate that generates a set of prices which, when substituted for the actual prices in equation A2, would yield the same present value. This is precisely the argument made by Fisher (1896, pp. 27-28 and 1906, pp. 392-93).

APPENDIX B

A Graphical Illustration of Jensen's Inequality

The graph below plots the relationship between P and 1/P, a rectangular hyperbola. It illustrates the difference between the arithmetic mean and the harmonic mean. The midpoint (D) of the secant line AB shows the arithmetic average of 2 and 8. The arithmetic mean is 5. The harmonic mean of the same numbers is 3.2. Consider what happens under a mean preserving spread. The midpoint (G) of the secant line EF gives the average value of 1 and 9. The arithmetic mean remains at 5, but the harmonic mean drops to 1.8. Increasing the variability of prices widens the gap between the harmonic and arithmetic means. This is a particular example of Jensen's Inequality which would be quite familiar to Fisher (see, for example, the appendix to chapter II of *The Purchasing Power of Money*).



APPENDIX C

Rationality of the OFE

In a rational world, a necessary condition for OFE rationality is the requirement that it provide an unbiased prediction of the *ex post* real rate of interest. An equation for the *ex post* real rate of interest may be obtained by deriving the (*ex post*) OFE using actual (*ex post*) values of money (v = 1/P) and the *ex post* real interest rate (j*) in place of the *ex ante* values. The result is the *Fisher identity*:

$$j / i + a^* + ia^* \tag{C1}$$

where a* / Δv/v and v = 1/P.

The problem is to show that the *ex ante* real return (j) provided by the OFE gives an unbiased forecast of the expected *ex post* real return (Ej*). To begin, assume that there is a finite probability distribution (γ) which associates a probability (γ_k) with each vector of possible future price levels and, hence, with each possible appreciation rate (a_k) and real return (j_k). Using the Fisher identity and the probability distribution, calculate the *ex ante* real return:

$$Ej = \gamma_1 j_1 + \gamma_2 j_2 + \ldots + \gamma_n j_n \tag{C2}$$

The *ex post* real return for price vector k is:

$$j_k = i + a_k^* + i a_k^* \text{ with k = 1, 2, ..., n.} \tag{C3}$$

Substituting equation (C3) into equation (C2) gives the expected real return:

$$Ej = i + \sum_{k=1}^{n} \gamma_k a_k^* + i\left(\sum_{k=1}^{n} \gamma_k a_k^*\right) \tag{C4}$$

Using the probability distribution, we calculate the *expected* depreciation of money:

$$a = \gamma_1 a_1^* + \gamma_2 a_2^* \ldots + \gamma_n a_n^* \tag{C5}$$

Substituting equation (C5) into Fisher's *ex ante* real return equation (C4) and consolidating terms establishes that j = Ej*. The OFE provides an unbiased prediction of the expected real

return for a given nominal rate of interest.

It can now be verified that the CFE gives a biased prediction of the *ex post* real return (j) even if *ex post* money appreciation is defined to be equal to *ex post* inflation: $a^* \neq -\pi^*$. A counter-factual proof is offered. In order for the *ex ante* real return (r) of the CFE to be equal to the *ex ante* real return (j) of the OFE, it would have to be the case that $a = -\pi$. This is not true in the uncertainty case. According to Jensen's inequality, $E(1/P) \exists 1/EP$. This implies that $|a| \# |\pi|$ and, therefore, that $j \exists r$. In general, the CFE *under predicts* the *ex post* real return.

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