DEVOLUTION OF THE FISHER EQUATION: Rational Appreciation to Money Illusion*

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Abstract

In *Appreciation and Interest* Irving Fisher (1896) derived an equation connecting interest rates in any two standards of value. The original Fisher equation (OFE) was expressed in terms of the *expected* appreciation of money [percent change in E(1/P)] whereas the ubiquitous conventional Fisher equation (CFE) uses expected goods inflation [percent change in E(P)]. Since Jensen's inequality implies the non-equivalence of the two equations, the OFE is not subject to standard criticisms of non-rationality leveled against the CFE. The puzzling substitution of inflation for expected money appreciation in Fisher (1930) is resolved by taking into account Fisher's theory of "money illusion." [JEL: E40, B00, B31]

Key Words: Fisher equation, Fisher hypothesis, Fisher effect, money illusion, nominal interest rate, purchasing power of money, value of money.

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> *At the outset the question arises, how can a merchant be said to foresee the appreciation of money? Appreciation is a subtle conception. Few business men have any clear ideas of it. Economists disagree as to its definition, and statisticians as to its measurement.*
>
> Irving Fisher (1896, p. 35)

I. Introduction

Long after the publication of *Appreciation and Interest* (1896), “appreciation of money” remains a subtle conception. The subtlety extends to Irving Fisher’s theory of the nominal interest rate which continues to be misrepresented and misunderstood in the economic literature. According to the Conventional Fisher Equation (CFE), the nominal interest rate (i) is a linear function of the *ex ante* real interest rate (r) and expected inflation (π):

$$i = r + \pi + r\pi \qquad (1)$$

Expected inflation (expected appreciation of goods) is defined as the percent change in the price of goods (P): $\pi \equiv (EP_{t+1} - P_t)/ P_t$. For “small” values of expected inflation, the CFE embodies the “Fisher effect” of a (near) one-to-one relationship between expected inflation (π) and the nominal interest rate (i).

A careful reading of Fisher’s extensive works leads to three surprising conclusions: (1) Irving Fisher never published an explicit form of the CFE. (2) The Original Fisher Equation (OFE, 1896) was written in terms of the *expected* appreciation of money and, as such, is a distinct equation and not a simple transposition of the CFE or reformulation of its temporal base. (3) Fisher’s (1930) substitution of goods appreciation (π) for expected money appreciation (a) in his empirical equations resulted from his conviction that market expectations are more accurately characterized by “money illusion” than by rational behavior.

The paper traces the evolution of Fisher's theory from its original formulation in terms of rational appreciation to its reconstruction under money illusion. The objective is to recover the OFE and explain its distinction from and displacement by the CFE. The OFE reflects Fisher's early interest in mathematical economics and rational behavior. It was derived under the assumption of rational foresight and measurement. In his later work, Fisher (1930) introduced an *implicit* form of the CFE that substituted the appreciation of goods (π) for the expected appreciation of money (a). This change of emphasis resulted from Fisher's growing skepticism about the rationality of market expectations and measurements. It assumed that market participants make decisions based on money illusion. The modern form of the CFE is a curious hybrid of Fisher's old and new thinking. It assumes farsighted behavior, but it adopts the definition of "appreciation" that Fisher and modern financial economists associate with non-rational behavior.

What did Fisher mean by "money appreciation"? In general, he meant a rise in the value of a monetary asset relative to another asset or good. Appreciation under a modern fiduciary standard refers to the rate of change in the value of (paper) money expressed in terms of commodities. A modern name for this concept is the real return on money (Eden, 1976). If the value of goods is P, then the value of one unit of money (v) is 1/P. Where Fisher used "expected appreciation of money," modern economists usually substitute "expected deflation." Adding to the confusion is the fact that the concepts are interchangeable in the special case where current and future prices are known with perfect certainty and where rates of change are calculated with reference to a common base. In the case of uncertainty (where expectations must be formed over uncertain future price levels), the mathematical definitions are *not* equivalent. This is a consequence of Jensen's inequality.[1] The non-equivalence of the two specifications in the uncertainty case is true even if calculations of the *expected* rate of change use a common base or are measured at a single point (instantaneous rate of change).

Fisher, as he readily acknowledged, was not the first person to advance the theory that the interest rate adjusts to changes in money value. Humphrey (1983) traces the lengthy development of this idea. Fisher’s contribution was being the first to write an equation for the relationship (Humphrey, 1983; Dimand, 1999a). Fisher was also the first to clearly derive the equation. His original equation, however, is *not* the one which is commonly attributed to him. This is fortuitous in the sense that the CFE is a misspecification of the relationship between nominal and real yields when market participants form *rational* expectations over *uncertain* future prices (Eden, 1975, 1976; Blejer and Eden, 1979; Kochin, 1980; and Benninga and Protopapadakis, 1983). But what if market expectations are based on money illusion rather than rational behavior? What equation is appropriate then? Ironically, it was Fisher’s (1930) rejection of the rationality postulate that led to the conventional specification.

II. The Original Fisher Equation

Despite frequent assertions to the contrary, the original source of the “Fisher equation” is not *The Theory of Interest*. To uncover the nature of the theoretical relationship between *ex ante* real and nominal interest rates, we must take Fisher's (1930, p. 39) advice and consult *Appreciation and Interest*. Careful reading and patience are required for an accurate understanding of the theory. It must be remembered that Fisher was writing in response to the bimetallic controversy, the most important economic issue of the period. His purpose was to show the relationship between interest rates expressed in different standards (e.g. gold and silver or gold and wheat). In *Interest and Appreciation* Fisher (1896) put forth a “multiple theory of interest.” What does this mean? It does not mean that he advanced a theory for the simultaneous determination of multiple interest rates. On the contrary, Fisher derived a single equation that showed the relationship between any two interest rates in any *two* standards of value. For any transaction, there are as many interest rates as there are monetary standards.

Fisher (1896, p. 88) was familiar with the concepts of "real" and "nominal" interest rates, but he preferred not to use this Marshallian terminology (see chapter XII). Fisher's theory was more general in the sense that it connected any two interest rates in any two standards. According to Fisher (1896, p. 92), "[t]hese rates are mutually connected and our task has been merely to state the law of that connection. We have not attempted the bolder task of explaining the rates themselves." Fisher's initial attempt at "the bolder task" was *The Rate of Interest*, published in 1907. His definitive treatment had to await his 1930 work.

The original Fisher equation is a no-arbitrage condition that makes the interest rate in one standard of value equivalent to the interest rate in another standard of value. Fisher's point is that a loan contract that specifies payment in terms of money type X can be rewritten in equivalent form in terms of asset (or good) type Y. If money X is expected to appreciate relative to asset Y, then the interest rate (j) in the relatively depreciating standard (Y) should be greater than the interest rate (i) in the appreciating standard (X).

Fisher-like equations can be derived from a variety of models incorporating interest rate(s), the price level, and production, but only models which impose the OFE (or CFE) as a no-arbitrage constraint are consistent with Fisher's original analysis. In Fisherian analysis, additional equations are required to explain variations in the real interest rate (equations for time preference and investment opportunities) and money value (the quantity theory). The uncovered interest parity (UIP) condition is an application of Fisher's analysis and, as such, is subject to the same measurement issues.

Although a skilled mathematician, Fisher took great pains to make his works accessible and relevant to sophisticated laymen. *Appreciation and Interest* is no exception, but the nature of the subject matter imposes considerable demands on the reader. Modern readers, apparently, do not have the required patience. Fisher explicates the relationship between appreciation and interest by guiding the reader through a series of progressively more complicated calculations supported by numerous illustrations drawn from everyday business experience under a bimetallic standard. As Fisher (1896, chapter X) showed, the analysis can also be applied to a fiduciary standard in which paper money exchanges for commodity money.

The essence of Fisher's approach can be captured by a simple two period present value model.[2] A modern touch is the introduction of the expectation operator (E) which corresponds to Fisher's concept of "mathematical value" (1906, pp. 403-06). Following Fisher (1896), taxes are ignored and risk neutrality is assumed. Fisher (1896, 1906, and 1907) was aware that incorporating such considerations would lead to a modification in his analysis.[3]

Consider a contract in which future payment is to be made in paper money (dollars). The present value ($P_{B,t}$) of a future (dollar) benefit (D) sold at discount in period t and at a nominal (paper money) interest rate, i, is:

$$P_{B,t} = \frac{D_{t+1}}{1+i} \tag{2}$$

An alternative contract is one in which the future payment would be made in bushels (B) of commodity money (wheat in many of Fisher's examples). For market participants to be indifferent between the two contracts, the number of bushels (B) to be paid in the *initial* period must be equivalent in value to the number of dollars (D) required in the money contract. Future payments must take into account changes in the expected terms of trade between money and commodities. Imposing the no-arbitrage condition, the commodity (real) value of future money payments, $D_{t+i}E(v_{t+i})$ is equal to the equivalent amount of future commodities, B_{t+1}.

Define the commodity rate of interest as j and the terms of trade between money and commodities as v. Using these definitions, the real present value of the future (paper) money payment or receipt can be expressed:

$$P_{B,t}v_t = \frac{D_{t+1}Ev_{t+1}}{1+j} \tag{3}$$

Equation (3) gives the present value in a commodity standard of a future money payment. Equating the price of the asset from equations (2) and (3), canceling, transposing, and rearranging terms yields:

$$\frac{1+i}{1+j} = \frac{1}{1+\mathrm{a}} \tag{4}$$

where a ≡ Ev_{t+1} - v_t)/ v_t. Cross multiplying and collecting terms results in the OFE:

$$j = i + a + ia \tag{5}$$

Note that the original "Fisher effect" expresses a relationship between the nominal interest rate and expected appreciation of money (a). For "small" values of the interactive term, there is a (near) one-to-one relationship.

The OFE is an equation for the *ex ante* real return (j). A definition of the *ex post* real return (j*) is provided by the Fisher identity. The *Fisher identity* can be obtained by using *ex post* values of the price level (P) and money appreciation (a*) in the derivation of the Fisher equation. The Fisher identity, written in terms of the actual appreciation of money (a*), is:

$$j^* \equiv i + a^* + ia^* \tag{6}$$

where a* ≡ v_{t+1} - v_t)/ v_t. In an uncertain world, the *ex pos*t (realized) real return (j*) will, in general, turn out to be different from the *ex ante* real return (j). For simplicity, Fisher's formal analysis was based on the exceptional case in which *ex ante* appreciation happens to equal *ex post* appreciation (a = a*). In his verbal discussion and his empirical work, however, Fisher used the general equation where foresight is imperfect.

III. Lost in Translation: OFE versus CFE

The OFE was derived by taking expectations over the value of money [E(1/p)]. The

CFE [equation (1)] can easily be derived from the same framework and procedures if expectations are taken over the value of goods [E(P)] and expected inflation is defined with reference to the current price level: $\pi \equiv (EP_{t+1} - P_t)/ P_t$. The CFE is a prominent feature of standard macroeconomic and monetary models. Examples of popular textbooks that use the CFE include Mankiw (2007), Romer (2006), and Woodford (2003).

The original Fisher equation (OFE), when applied to a world of paper money ("money") and commodity money ("commodities"), is expressed in terms of the *expected* appreciation of money (a) and written with the commodity (real) interest rate (j), Fisher's "virtual interest in commodities," as the left-hand variable. In this form, the original equation can easily be misinterpreted as the CFE. Hirshleifer (1970, pp. 135-36), in an influential work, reinforced the conventional view by representing anticipated inflation with the letter "a."

A more fundamental misunderstanding concerns the relationship between expected inflation (π) and expected appreciation of money (a). The CFE is not the same equation as the OFE for two reasons: (1) the interactive terms are different ($r\pi \neq - ia$) and (2) the measures of *expected* appreciation are different ($\pi \neq - a$). These inequalities occur even in the perfect certainty case due to differences in the base period used for calculating discrete changes.[4] The OFE calculation uses the current value of money (v_t) as the point of reference in the definition of the appreciation of money. The CFE calculation uses the future value of money (v_{t+1}) as the implicit reference point (the denominator in calculations of discrete percentage changes). Under perfect foresight the OFE and CFE can be made compatible by using a common definition of the *ex post* (i.e. realized) real return ($j^* = r^*$). In this special case, the CFE provides an equivalent definition of the *ex post* real return if expected inflation (π) is identically equal to *ex post* inflation (π^*) and inflation is calculated with respect to the correctly anticipated *future* price level [i.e., $\pi^* \equiv (P_{t+1} - P_t)/ P_{t+1} = -a^*$].

The Fisher equations are not to be confused with the unique definition of the *ex post* real rate of interest provided by the Fisher identity. The CFE and OFE are equivalent equations only when two conditions are satisfied: (1) the current and future price levels are known with perfect certainty and (2) a common base period is used. The fundamental difference in the OFE and CFE stems from different definitions of *expected* appreciation under uncertainty and *not* from different base periods. Under uncertainty, *expected* appreciation of money (a) will not equal *expected* deflation ($-\pi$) due to Jensen's inequality. For a given nominal interest rate, the CFE and OFE imply different measures of the *ex ante* real return ($j \geq r$).

Historians of thought, while employing Fisher's (1896) terminology, have inadvertently contributed to the misunderstanding of Fisher's theory. Tobin (1997, p. 374), Howitt (1992, 2, p. 123), and Dimand and Geanakoplos (2005), for example, describe the Fisher equation using the conventional specification (1). Dimand (1999a) accurately reproduces the original Fisher equation (5) and points out that Fisher's money and commodities model used the "(expected) purchasing power of money." In defining "expected inflation as the difference between real and nominal interest rates," however, Dimand (1997, p. 442; 1999a, p. 748; 1999b, p. 36) assumes that there is no difference between the OFE and the CFE. Humphrey (1983), in an otherwise illuminating discussion, uses the conventional, rather than the original, Fisher equation in describing Fisher's contribution to the history of thought. The mistake is in viewing the two equations as equivalent. The CFE is a rival equation not a transposition of the OFE.

The continuing popularity of the CFE in the conventional economics literature is puzzling. It is a well-established proposition in the finance literature that the CFE provides a biased estimate of the relationship between nominal and real bond yields when expectations are formed rationally over uncertain future prices (Benniga and Protopapadakis, 1983; Blejer and

Eden, 1979; and Kochin, 1980). What is the extent of the bias? If the size of the bias is small, then the CFE is a reasonable approximation to the OFE. Theoretical models suggest that this is a risky assumption, particularly in cases where price level volatility is large (Eden, 1975, 1976; Sarte, 1998), expectation horizons are long (McCulloch and Kochin, 2000), or individual expectations are diffuse (Kochin, 1980). Fama (1975, 1976), in an apparent attempt to avoid inflation-uncertainty bias, wrote the Fisher relationship in terms of the expected value of money [E(1/P)].[5] In doing so, he inadvertently rediscovered the OFE.

IV. Money Appreciation and the Original Equation

Why did Fisher (1896) insist on formulating the problem in terms of the expected appreciation of money (a) rather than expected deflation of commodities (π)? One cannot be sure, but one should not overlook a simple explanation. As a neoclassical economist, Fisher defined the value of money as the inverse of the price of goods.[6] It verges on the obvious that if you use the wrong definition for the price of money, you will end up with a biased measure of the expected appreciation of money. Fisher's preference for money appreciation (rather than goods depreciation) has, however, an economic justification. Although not established by Fisher, only the OFE yields an unbiased prediction under risk neutrality of the expected *ex post* real rate of interest (the average real interest rate that would result from repeated sampling from the population of realized real interest rates). [See *Supplementary Notes*.]

Jensen did not publish his formal proof concerning convex functions until 1906. Did Fisher (1896) have an understanding, intuitive or otherwise, of Jensen's inequality? One cannot say with absolute certainty, but a careful reading of his subsequent work on index numbers suggests he did. Schumpeter (1954, pp. 1091) points out that the work of Fisher and others on index numbers was the "statistical complement" to the "theoretical discussion on the purchasing power of money." Fisher knew that the choice of definition for money appreciation has important consequences. As "the greatest expert of all time on index numbers" (Tobin, 1987, p. 369), Fisher understood that care must be exercised in the calculation of mean values.

A special case of Jensen's inequality is the difference between arithmetic and harmonic means. Fisher knew and often used the fact that the arithmetic mean [A(P)] of a variable P is greater than (or equal to) its harmonic mean [H(P)].[7] The harmonic mean, H(P), is equal to one over the arithmetic mean of the variable's inverse: H(P) = 1/[A(1/P). That is, A(P) ≥ H(P) = 1/[A(1/P)]. This implies: A(1/P) ≥ 1/A(P). In this special case, all prices are known with certainty and given equal weights (1/n). The generalization to the uncertainty case is straight-forward. If P is a discrete random variable, then the mathematical expectation of P, E(P), is a weighted average of individual prices with the weights of individual prices being probabilities that sum to one. Replacing the arithmetic operator (A) with the expectation operator (E), results in the inequality of current interest: E(1/P) ≥ 1/E(P).

Fisher's choice of terminology is further evidence of his sophisticated understanding. When explaining the *theoretical* connection between nominal and real interest rates, Fisher consistently used terms such as "expected change in the value of money," "expected appreciation of money," or "expected change in the purchasing power of money." Fisher (1896, 1905, and 1930) used the same terminology in all of his major works on the theory of interest. Fisher bemoaned, over and over again, the apparent inability of people to grasp the concept of money value. He did not take it to be a matter of inconsequence that people found it easier to calculate in terms of prices than money values. Indeed, Fisher insisted that his weekly Index Number of Wholesale Prices be published as the *inverse* of the original price series. According to his son, Irving N. Fisher (1956, p. 35):

> The chief purpose of this newspaper publication was to invert the ordinary index number representing the price level, thereby obtaining an index number representing the purchasing power of the dollar, the idea being to accustom the public to the thought that the dollar is not a constant but a variable.

Taking a tip from Mehrling (2001), one may see in Fisher's dogged persistence a conviction that a proper measure of money value would enhance social welfare.

The strongest argument for Fisher's precocity lies in his empirical studies. In his extensive empirical investigations, reported in detail in 1896, appreciation was consistently calculated as the *expected* change in the *reciprocal* of the price level. It was the OFE, not the CFE, that Fisher chose when he processed the data from the uncertain world.

The empirical question Fisher (1896) attempted to address was the extent to which *ex post* appreciation (a*) was captured by *expected* appreciation (a). In testing his theory, Fisher (1896) used a variety of alternative definitions of money X and asset/good Y: gold and wheat (ch. II), gold and paper (ch. VIII), gold and silver (ch. IX), and money and commodities (ch. X). It is not until part II that he introduces the modern convention of using fiduciary money and (aggregate) commodities as the two standards. It is in part II that Fisher drops the simplifying assumption of perfect foresight and makes clear that the OFE is in terms of *expected* appreciation. Fisher uses bond market and price data from seven countries to examine the extent to which market interest rates adjust to the "appreciation of money in commodities." Fisher's examples reflect the period of investigation: money is the (relatively) appreciating standard and (aggregate) commodity is the (relatively) depreciating standard. Money appreciates when commodity prices (P) go down and depreciates when prices go up.

In part II, Fisher (1896, p. 43) uses the equation derived in part I to obtain a measure of "expected appreciation" of money. He achieved this remarkable feat by exploiting the difference in the yields of commodity (gold coin) bonds and paper (currency) bonds. The "realized" (i.e. market determined) yields on the commodity and on paper were calculated independently as "the rate of interest which will render the 'present value', at the date of purchase, of all the future benefits to January, 1879, equal to the purchase price" (p. 42, n. 4).

Using the resulting paper yield (i) and the commodity yield (j), Fisher employed the OFE to solve for the *expected* appreciation of money (a); that is, "that rate of appreciation which would have made the two interest rates equally profitable" (p. 43, n. 4). He compared this *forward-looking* measure of expected appreciation with the realized *(ex post*) appreciation of money and discovered that expected (*ex ante*) appreciation (a) consistently *under*-predicted actual appreciation (a*). Table 1 reproduces Fisher's original table and results.

TABLE 1: Fisher's Calculations of Expected and Actual Appreciation, 1870-78

RATE OF INTEREST REALIZED FROM DATES MENTIONED TO JANUARY 1, 1879, (DATE OF RESUMPTION).				
	[Gold] Coin *J*	[Paper] Currency *I*	Appreciation of Currency in Gold	
			Expected a	Actual [a*]
January, 1870	7.1	6.3	.8	2.1
July, 1870	6.2	5.7	.5	1.4
January, 1871	6.7	6.3	.4	1.3
July, 1871	6.4	5.7	.7	1.8
January, 1872	5.9	5.7	.2	1.3
July, 1872	6.2	5.7	.5	2.1
January, 1873	6.5	6.2	.3	2.0
July, 1873	6.2	6.0	.2	2.8
January, 1874	5.6	6.1	-.5	2.1
July, 1874	5.7	5.8	-.1	2.4
January, 1875	6.0	5.4	.6	3.1
July, 1875	6.1	4.2	1.8	4.9
January, 1876	5.4	4.1	1.2	4.3
July, 1876	5.2	2.4	2.7	4.9
January, 1877	5.5	4.0	1.4	3.5
July, 1877	5.7	3.1	2.5	3.6
January, 1878	8.2	6.0	2.1	2.8
July, 1878	4.8	2.6	2.1	1.4

Source: Fisher (1896), p. 42. Entries in brackets are not in the original.

V. Money Illusion and the Conventional Specification

Of Irving Fisher's works on interest rate behavior, the one which is most frequently cited is *The Theory of Interest*. Published in 1930, this book is often erroneously credited as the source of the Fisher equation and Fisher hypothesis. Fisher (1930, p. 451) clearly states that "the main object of this book is to show how the rate of interest would behave if the purchasing power of money were stable." Fisher's 1930 work develops a micro-based theory of the determination of the real interest rate. It is only at the end of the book that Fisher reminds us that, in the short run, the real interest rate is also influenced by monetary phenomenon. He presents his recent empirical work on the link between prices and interest rates.

Although one will not find an explicit representation of the CFE in any of Fisher's works, it is easy to see how a reader who consults only *The Theory of Interest* would find support for the conventional interpretation. Setting a pattern for subsequent research, Fisher (1930, ch. XIX) examined the correlation between the nominal interest rate and the rate of change in the price of commodities. As is well known, he found a weak contemporaneous correlation between the rate of change of commodity prices and the nominal interest rate. Applying a distributed lag model of his own invention (1925, 1938), Fisher found that *past* inflation influenced both long-term and short-term interest rates with a long and variable lag.

Fisher's *empirical* model is the source of confusion over the Fisher equation. Why did Fisher switch from expected money appreciation to lagged inflation in his post-1896 empirical work? One cannot be absolutely sure, but the change in emphasis is dramatic in light of Fisher's previous insistence on using money value. The most likely answer can be found in Fisher's psychological theory of expectations. Fisher's early empirical work (1896) led him to question the rationality of market expectations. Conversations with businessmen and workers further convinced him that the value of money was too subtle a concept for ordinary people to comprehend. Even hyperinflation could not unveil the money illusion:

> The most striking case which I encountered of this pervasive money illusion was that in Germany following World War I. Germany's inflation, of course, was not so much check-book money as printing-press money – money printed to pay the Government's debts. As a result, the mark of 1922 would buy only one-fiftieth as much as the mark of 1914. Yet when I visited Germany in that year expressly to find out what the Germans knew about this fall of the mark, I found that 19 out of 20 whom I interrogated did not realize that the rise of prices had anything to do with the mark. They imagined it was all due to such factors

> as the Allied Blockade making goods scarce. They simply took for granted that the mark of 1922 was the same mark as that of 1914. They measured all values in marks just as they always had. (Fisher, 1946)

The popular view, according to Fisher (1930, p. 399) is that "money itself does not change." If this is the case, then bond market participants do not form expectations over the value of money and a viable empirical model of the interest rate cannot be conditioned on the OFE.

In 1896, Fisher had yet to introduce the concept of "money illusion." Fisher (1896, p. 11) made clear that he was "regarding money as a standard of value and not as a medium of exchange." Money is a measure of value just as a yard is a measure of length. Contracts, whether expressed in money or yards, should be adjusted to take proper account of changes in the units of measurement:

> It is clear that if the unit of length were changed and its change were foreknown, contracts would be modified accordingly. Suppose a yard were defined (as once it probably was) to be the length of the king's girdle, and suppose the king to be a child. Everybody would then know that the "yard" would increase with age and a merchant who should agree to deliver 1,000 "yards" ten years hence, would make his terms correspond to his expectations. (1896, p. 1)

Fisher derived the OFE under the assumption of rational measurement and rational expectations, but his early empirical work suggested that interest rates fell significantly short of anticipating subsequent money appreciation.

As early as 1896, Fisher was beginning to have second thoughts about rational behavior: "If you ask a merchant whether he takes account of appreciation, he will say he never thinks of it, that he always regards a dollar as a dollar. Other things may change in terms of money, but money itself he is accustomed to think of as the one fixed thing." In spite growing doubts, Fisher (1896) left open the possibility that inadequate interest rate adjustment might be due to "imperfection of foresight." Anticipating the regime switching literature (Barsky, 1987), Fisher recognized that the mere possibility of a monetary regime change would provide rational grounds for such imperfection (Fisher, 1896, chapter VIII). At the end of his career, Fisher (1946) admitted his reluctance to shed the rationality assumption:

> It took me a long time to realize how pervasive is this money illusion. In fact, it dawned on me only after I had published *Stabilizing the Dollar*, which contained my first suggestion as to how to stabilize. I found people saying of this book: "But does the dollar need any stabilizing? If so, that's news to me."

Fisher's own illusion about educating the masses on the value of the dollar was gradually undermined by his empirical studies and his business dealings.

Short-run fluctuations in real interest rates and output played a key role in Fisher's subsequent work on the monetary theory of the business cycle (Dimand, 1999b). From 1920 onwards, Fisher's business cycle works (including those on interest rates) embodied the money illusion hypothesis. Fisher's adoption of the money illusion hypothesis reflects a natural evolution in his thinking concerning the causes and consequences of the "dance of the dollar."

Money illusion is traditionally defined as a situation where market participants make economic decisions based on money prices rather than theoretically correct relative prices and real wealth (Patinkin, 1965, pp. 22-23). Money illusion, in this sense, is a violation of the "homogeneity postulate" (Leontief, 1936). Workers suffering from money illusion bargain in terms of money wages rather than real wages. Business managers, to the extent they suffer from the disease, fail to adequately take account of the general price level in making pricing and output decisions. Fisher used such notions throughout his collected works, especially in *The Money Illusion* and other business cycle writings where some type of fooling assumption is required to explain output and employment effects of monetary disturbances.

Money illusion, according to Fisher (1928, p. 4) is "the failure to perceive that the dollar, or any other unit of money, expands or shrinks in value." Money illusion results in an incorrect measure of the *change* in the appreciation of money (the growth of the king's girdle). The Patinkin form of money illusion is an extreme case when money value (the yardstick measurement) is perceived not to change in value at all; when a yard is a yard and "a dollar is a dollar" (Fisher, 1896, p. 35; 1930, p. 399).

The presence of money illusion limits the *direct* impact of expected appreciation on interest rates. The possibility remains of a round-about influence. In various writings, Fisher conjectured that changes in commodity inflation would have an *indirect* and *lagged* impact on the nominal interest rate and other variables. In *The Theory of Interest*, for example, Fisher (1930, pp. 399-400) described the adjustment process of interest rates under money illusion:

> Most people are subject to what may be called "the money illusion," and think instinctively of money as constant and incapable of appreciation or depreciation. Yet it may be true that they do take account, to some extent at least, even if unconsciously, of a change in the buying power of money, under guise of a change in the level of prices in general. If the price level falls in such a way that they may expect for themselves a shrinking margin of profit, they will be cautious about borrowing unless interest falls, and this very unwillingness to borrow, lessening the demand in the money market, will tend to bring interest down. On the other hand, if inflation is going on, they will scent rising prices ahead and so rising money profits, and will be stimulated to borrow unless the rate of interest rises enough to discourage them, and their willingness to borrow will itself tend to raise interest.

According to Fisher, price changes may have an impact on interest rates even in the presence of imperfect foresight and money illusion. Sluggish price changes and the resulting trade fluctuations put indirect pressure on loan markets and interest rates. To capture the lagged effect of prices on interest rates, Fisher (1925, 1930) developed the distributed lag model.

Fisher's concept of money illusion did not rule out the possibility of imperfect foresight with respect to the price of goods. "The businessman," Fisher (1930, p. 400) observed, "makes a definite effort to look ahead not only as to his own particular business but as to general business conditions, including the trend of prices." Furthermore, "(e)vidence that an expected change in the price level does have an effect on the money rate of interest may be obtained from several sources" (Fisher, 1930, p. 400). Based on empirical observation, Fisher came to believe that market participants exhibit complex psychological behavior: both foresight and illusion influence market outcomes. Fisher would not be surprised by modern psychological studies that find inconsistencies and inaccuracies in people's calculation of money values (cf. Safir, Diamond, and Tversky, 1997; Fehr and Tyran, 2001). The interaction between money illusion and imperfect foresight provided Fisher a rationale for replacing expected appreciation with lagged inflation.

Friedman and Schwartz (1982, p. 547) note that Fisher's 1930 empirical work has "less economics" than his earlier works (1896, 1907). It is stretching matters, however, to attribute the loss of economics to his adoption of the adaptive expectations hypothesis.[8] The concept of adaptive expectations is an interpretation superimposed on Fisher's (1930) lagged adjustment model by subsequent researchers. What constitutes the "loss" is the switch in emphasis from market rationality to market psychology. Fisher's empirical model could not assume full rationality if market psychology was dominated by widespread money illusion. If Fisher's theory of inflation psychology is correct, then empirical studies using a backward-looking specification should more accurately predict the behavior of the nominal interest rate than those based on a forward-looking specification.

We are now in a position to understand why Fisher called appreciation a "subtle conception." The debate over the specification of the Fisher equation involves two subtle issues of measurement that Fisher never adequately disentangled. To return to Fisher's colorful analogy, measurement problems arise if the length of the yardstick depends on the size of the king's girdle. One measurement problem results if people suffer from girdle illusion (Patinkin-style money illusion); that is, they fail to adjust the yardstick with the changing size of the king's girdle. Realization that the yardstick changes, however, is not enough to eliminate measurement issues. A second measurement problem occurs if one uses an improper yardstick (the Jensen inequality problem) in calculating the actual and expected rates of change in the king's girth (the expected rate of change in money value). It matters, for example, whether the yardstick is made of wood (linear measurement) or cloth (convex measurement). The point of reference on the king's body (base period problem) is also important for such calculations.

Rational appreciation requires both a correct definition of money value and the absence of money illusion and imperfect foresight. In developing his theoretical model in 1896, Fisher assumed implicitly that both conditions applied. The empirical work reported in 1896 was based on a rational definition of money value, but the findings were inconsistent with rational expectations of changing money value. Market expectations, Fisher surmised, were corrupted by money illusion and imperfect foresight. Under these circumstances, Fisher's earlier emphasis on a correct definition of money value lost its relevance. If the representative market participant believes that money value doesn't change, then, for empirical purposes, there is no point in quibbling over a proper definition of the concept. Fisher's 1930 empirical work modeled the *indirect and lagged* influence of changing price levels on nominal interest rates.[9]

The implicit form of the CFE used by Fisher assumed both money illusion and improper measurement of money value. By the early 1970s Fisher's (1930) distributed lag model of inflation was commonly interpreted as a form of the adaptive expectations hypothesis. The modern specification of the CFE has restored some semblance of rationality by emphasizing forward-looking forecasts of goods prices. Although it is now common to superimpose the rational expectations hypothesis on the CFE, Fisher would not be satisfied. To Fisher, rational appreciation requires both proper measurement of expected money value and unbiased expectations. Full rationality requires a return to the 1896 vision of Fisher. This, in fact, was the course taken by Fama (1975, 1976).

VI. Conclusion

Irving Fisher fathered two equations describing the relationship between interest rates in different standards of value (e.g. nominal and real interest rates). The OFE gives the "exact theoretical relationship between the rates of interest measured in any two diverging standards of value and the rate of foreseen appreciation or depreciation of one of these two standards relatively to the other..." (Fisher, 1930, p. 39). The OFE is the product of Fisher's work as a theoretical economist. It reflects his belief, confirmed by modern financial economists, that rational behavior requires nominal interest rates to respond to changes in the expected value of money *not* the value of goods.

The CFE was never explicitly written down by Fisher, but it may be derived from Fisher's analytical framework by assuming that market participants form expectations over the value of goods. The displacement of money appreciation by goods inflation reflects Fisher's (1930) views as an amateur psychologist and an applied statistician. The CFE, which is an approximation to Fisher's theoretical specification, uses an incorrect definition of the expected appreciation of money. Nonetheless, the implicit form of the CFE used by Fisher (1930) is the preferred empirical specification when, due to money illusion and imperfect foresight, interest rates respond indirectly and with a distributed lag to the changing value of goods.

Economists, the "guardians of rationality," often perceive unfulfilled genius in Fisher's works (Schumpeter, 1948; Allen, 1993). If Fisher had followed his scientific bent and forsaken his quixotic campaigns, the argument runs, economic science would have leapt forward by decades. Perhaps, but Fisher's faith in economic rationality was badly shaken by his contact with lesser mortals. Fisher (1946, p. 33) believed that the "study of mere market value" would lead one astray if markets are characterized by such non-rational phenomenon as money illusion, systematic mismeasurement, and irresponsible social behavior. In Fisher's world, social progress requires enlightened leadership as well as science.

[1] For a non-degenerate random variable (P), Jensen's inequality implies: $E(1/P) \geq 1/E(P)$. Consider a simple example. Assume that the price of goods (P) is a discrete random variable that may take on the value of either 10 or 20 with equal probability ($p_1 = p_2 = 0.5$). In this example, E(1/P) = 0.075 > 1/E(P) = 0.067. Suppose P=20 in period t is known with certainty. In this case, the expected rate of change of P between t and t+1 is equal to -0.250 and the expected rate of change of 1/P is 0.050. The difference in the estimates of expected appreciation is 200 basis points! The choice of estimator is not a matter of indifference.
[2] *Supplementary Notes* contains a generalization of Fisher's original n-period model.
[3] In *The Nature of Capital and Income*, Fisher (1906, pp. 398-11) showed how the present value formula can be modified to incorporate taxes and risk. It is a small step to the derivation of a risk-adjusted and/or tax-adjusted Fisher equation (see *Supplementary Notes*).
[4] Fisher (1896, ch. II) was aware of the inequality between money appreciation and goods depreciation and showed that it applies even in the perfect foresight case where *ex ante* appreciation (a) is equal to *ex post* appreciation (a*). Assuming perfect foresight, Fisher (1896, ch. II) showed that the interest rate relationship could be rewritten in terms of the rate of depreciation (d) of asset Y relative to asset/good X: j = i + d + jd. In this special case where relative price changes are known with perfect certainty, Fisher (1896, p. 11) shows that the rate of appreciation (a) exceeds the rate of depreciation (d).
[5] The issue of CFE bias has been engaged in the empirical literature. Since the source of the bias in the CFE is the failure to properly account for price-level variability, regression equations for nominal interest rates sometimes include a measure of inflation variability. The sign, magnitude, and statistical significance of this coefficient have varied across studies. See Chan (1994), Ireland (1998), Sarte (1998), and Shome, Smith, and Pinkerton (1988).
[6] Alternative definitions of money value include the interest rate (the inter-temporal value of money) and the foreign exchange rate. Fisher dismissed such alternatives as inappropriate.
[7] Coggeshall (1886-87), who Fisher (1927, p. 81n) cited, advocated use of the harmonic mean a full decade before the publication of *Appreciation and Interest*. In general, the harmonic mean provides a more accurate measure of average rates of change than the arithmetic mean. This fact has recently been rediscovered (cf. Karatzas et. al., 2003).
[8] Much of the modern research on the Fisher equation is critical of Fisher's empirical methodology and findings. In a meticulous study using over a century of data, Friedman and Schwartz (1982, ch. 10) reach conclusions that are broadly consistent with those of Fisher (1930). Money illusion has also received empirical support from a number of recent studies (Fehr and Tyran, 2001; Shafir et. al., 1997).
[9] Fisher calculated the correlation between interest rates and a weighted average of lagged inflation rates using both British and U.S. data for various time periods. The simple correlation rose as high as +0.98 when British inflation rates were distributed over 28 years. Fisher also considered the possibility that interest rate changes might *lead* changes in inflation, but he found only limited empirical support for such forward-looking behavior.

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